

Effect of random interactions in spin baths on decoherence

S. Camalet and R. Chitra

Laboratoire de Physique Théorique de la Matière Condensée, UMR 7600, Université Pierre et Marie Curie, Jussieu, Paris-75005, France

(Received 24 October 2006; published 29 March 2007)

We study the decoherence of a central spin-1/2 induced by a spin bath with intrabath interactions. Since we are interested in the cumulative effect of interaction and disorder, we study baths comprising Ising spins with random ferromagnetic and antiferromagnetic interactions between the spins. Using the resolvent operator method which goes beyond the standard Born-Markov master equation approach, we show that, in the weak coupling regime, the decoherence of the central spin at all times is entirely determined by the local-field distribution or equivalently, the dynamical structure factor of the Ising bath. We present analytic results for the Ising spin chain bath at arbitrary temperature for different distributions of the intrabath interaction strengths. We find clear evidence of non-Markovian behavior in the low temperature regime. We also consider baths described by Ising models on higher-dimensional lattices. We find that interactions lead to a significant reduction of the decoherence. An important feature of interacting spin baths is the saturation of the asymptotic Markovian decay rate at high temperatures, as opposed to the conventional Ohmic boson bath.

DOI: [10.1103/PhysRevB.75.094434](https://doi.org/10.1103/PhysRevB.75.094434)

PACS number(s): 75.40.Gb, 03.65.Yz, 75.10.Nr

I. INTRODUCTION

Recent developments in nanophysics have made possible the use of the charge and spin dynamics of electrons to develop new technologies like spintronics and qnantronics. This has moreover led to the possibility of using the electron spin, or other more complex entities like the phase in Josephson junctions to fabricate qubits for quantum computing. However, the utility of these nanosystems as qubits is strongly limited by their coupling to the omnipresent dissipative environment. The environment destroys the coherence of the qubit over a certain time scale and a lot of recent theoretical and experimental activity has focused on ways and means to increase this time scale. This clearly emphasizes the importance of understanding the effects engendered by the coupling of a two level system to a dissipative bath.

The fact that the environment plays a crucial role in the physics of small quantum systems has been well known since the pioneering work of Ref. 1, where it was shown that the coupling of a two level system to an Ohmic boson bath could effectively suppress the tunneling of the two level system. In the context of decoherence, the most commonly studied problem is the spin-boson model² which describes the effect of a dissipative bosonic bath on a central spin, where the spin can be an effective description of a system whose discrete lowest energy levels dominate the physics at low enough temperatures and the bosons are often the phonons present in the system. A physical manifestation of the spin boson problem is a nanomagnet (described by a giant spin) coupled to phonons.³ However, for many practical realizations of a central spin or a qubit (spin-1/2), a spin bath comprising other spins might be the principal source of decoherence. This is indeed the case in semiconducting quantum dots, where the nuclei with nonzero spins constitute the spin bath and interact with the central electronic spin in the dot via the hyperfine interaction.^{4,5} Another manifestation of a spin bath occurs in Si:P.⁶ The abundance of spin baths in real systems, necessitates an understanding of their effect on

decoherence. Unlike the case of bosonic baths often modeled as a collection of harmonic oscillators, spin baths can exhibit a wide range of phenomena depending on the interactions between the spins, residual anisotropies, etc. Clearly one expects any resulting decoherence of the central spin to depend rather crucially on the underlying nature of the spin bath and its coupling to the central spin.

Earlier studies which considered independent spins in the bath seemed to indicate that spin baths were not qualitatively different from bosonic baths.^{7,8} More interestingly, recent studies of decoherence induced by spin baths described by mean field Hamiltonians have demonstrated that interactions between the bath spins can be used as a lever to augment the time scales over which the system decoheres.⁹⁻¹¹ These results were however obtained either numerically for a bath with a small number of spins or for the special case where the bath Hamiltonian commutes with the bath-central spin coupling Hamiltonian leading to an effective classical decoherence. A more robust treatment of intrabath interactions was presented in Ref. 12 where the authors studied numerically the zero temperature *quantum* decoherence of two coupled spins engendered by a bath described by the random transverse Ising model. The authors used this model to argue that the central spin decoheres differently, depending on whether the spin bath has a regular or chaotic spectra. Despite their various drawbacks, these works collectively highlight the importance of interactions and disorder in the bath. Moreover, disordered spin baths warrant further attention because both interactions (often dipolar) and disorder are present in real spin bath systems like quantum dots in semiconducting heterojunctions and in Si:P.

In this paper, we reexamine the decoherence induced by disordered interacting spin baths at finite temperatures. More precisely, we study the effect of an Ising bath with random spin-spin interactions on the coherence of a single spin-1/2. The random interactions are characterized by their variance Δ^2 where Δ is analogous to the cutoff frequency for a bosonic bath as well as a mean value J_0 which has no bosonic counterpart. Our choice of an Ising bath is primarily

to facilitate an analytical study of the problem at finite temperatures in the thermodynamic limit. To ensure a quantum decoherence of the central spin in our model, the central spin is coupled to the transverse spin components of the bath. To better comprehend the effect of the bath, we consider a model in which the time evolution of the central spin is exclusively governed by its coupling to the bath. For such a model, the problem is exactly soluble for a bath comprising independent spins and/or bosons. However, when intrabath interactions are present, no exact solution can be obtained and one must take recourse to approximate methods. In this paper, we only study the limit of a weak coupling of the central spin to the bath, where robust analytical methods are available to study the problem in an unbiased manner. We use the resolvent operator method, which takes us beyond the often used Markovian master equation approaches to study the decoherence of the central spin induced by a perturbative coupling to the Ising spin bath. An interesting aspect of our work is that in the absence of any dynamics intrinsic to the central spin, for weak coupling to the bath, the decoherence is primarily dictated by the local-field distribution or equivalently, the dynamic structure factor of the bath spins. For a bath described by an Ising spin chain, we obtain the Markovian decoherence time scale and the non-Markovian corrections as a function of the temperature and the parameters of the bath. We also discuss the case of Ising spins on various lattices in the high temperature regime and the case of the infinite-ranged Sherrington-Kirkpatrick spin-glass model.

The paper is organized as follows: we present the model and derive a general expression for the decoherence of the central spin in Sec. II followed by a discussion of the weak coupling regime in Sec. III. We then present our results for different disordered Ising spin baths in Sec. IV.

II. MODEL

We present the model used to study the decoherence of the central spin σ_c , weakly coupled to a bond disordered bath of N Ising spins σ_i in the thermodynamic limit $N \rightarrow \infty$. The total Hamiltonian describing the combined system of the central spin and the spin bath is given by

$$H = H_B + \sigma_c^x V \quad (1)$$

$$\equiv - \sum_{(ij)} J_{ij} \sigma_i^z \sigma_j^z - \sigma_c^x \sum_i \lambda_i \sigma_i^x, \quad (2)$$

where σ_c^x is the x -component Pauli operator of the central spin and σ_i^x and σ_i^z denote the Pauli operators of the bath spins and J_{ij} are the interaction strengths between the bath spins. Depending on the details of the model studied, (ij) could represent interactions between nearest neighbor spins or interactions of infinite range. In contrast to the models studied in Ref. 9, where the bath Hamiltonian H_B and the bath operator V that couples to the central spin commute, here our choice of V is such that $[H_B, V] \neq 0$. The central spin and spin bath coupling is characterized by the parameters λ_i . Since, we are interested in the influence of disorder as well

as the tendency of the system to order, we consider random interaction energies J_{ij} which are quenched random variables drawn from a distribution $p(J)$ with mean J_0 and variance Δ^2 . Though the central spin does not have any intrinsic dynamics, its coupling to the bath generates a nontrivial dynamical behavior. We note that H_B is the usual Ising spin-glass Hamiltonian which has been well studied in the past.¹³ Depending on the distribution of the spin-spin interactions and the dimensionality, this model can exhibit ferromagnetic, antiferromagnetic or even spin-glass order in some temperature range. Since these phenomena have ramifications for the collective behavior of the bath, it is reasonable to expect the resulting decoherence to depend crucially on the underlying order in the bath.

It is important to note that the formalism developed in this section and Sec. III is *a priori* applicable to any bath Hamiltonian H_B (bosonic baths, Heisenberg spin baths, baths with both spins and bosons, etc). For a Hamiltonian of the form (2), since σ_c^x is a constant of motion, it is convenient to directly study the time evolution of the reduced density matrix of the central spin,

$$\rho(t) = \text{Tr}_B(e^{-iHt} \Omega e^{iHt}), \quad (3)$$

where Ω is the initial density matrix of the composite system consisting of the central spin and the bath and Tr_B denotes the partial trace over the bath degrees of freedom. We use the units $\hbar = k_B = 1$ in this paper. Denoting the eigenstates of σ_c^x by $|\leftarrow\rangle$ and $|\rightarrow\rangle$, we see that due to the property of the density matrix $\text{Tr}[\rho(t)] = 1$ and the stationarity of $\text{Tr}[\rho(t)\sigma_c^x]$, the diagonal elements of the density matrix given by the populations $\langle \leftarrow | \rho(t) | \leftarrow \rangle$ and $\langle \rightarrow | \rho(t) | \rightarrow \rangle$ remain constant. Only the coherences $\langle \rightarrow | \rho(t) | \leftarrow \rangle = \langle \leftarrow | \rho(t) | \rightarrow \rangle^*$ representing the off-diagonal elements of the density matrix $\rho(t)$ change with time. Often these coherences vanish at long times resulting in a decoherence of the central spin, i.e., the asymptotic state of the central spin is a mixture of the states $|\leftarrow\rangle$ and $|\rightarrow\rangle$ irrespective of the nature of the initial state Ω .

To simplify the calculation, we assume that at time $t=0$, the central spin and the bath are disentangled resulting in a factorizable initial density matrix, $\Omega = \rho(0) \otimes \rho_B$. We further suppose that initially the central spin is in a pure state $|\psi\rangle = \alpha |\leftarrow\rangle + \beta |\rightarrow\rangle$ ($\rho(0) = |\psi\rangle\langle\psi|$) and that the bath is at thermal equilibrium with temperature $T \equiv 1/\beta$,

$$\rho_B = \frac{e^{-\beta H_B}}{Z}, \quad (4)$$

where $Z = \text{Tr} \exp(-\beta H_B)$ is the bath partition function. With these initial conditions we obtain the time evolved reduced density matrix

$$\rho(t) = |\alpha|^2 |\leftarrow\rangle\langle\leftarrow| + |\beta|^2 |\rightarrow\rangle\langle\rightarrow| + M(t) \alpha^* \beta |\rightarrow\rangle\langle\leftarrow| + M(t)^* \alpha \beta^* |\leftarrow\rangle\langle\rightarrow|, \quad (5)$$

where the factor

$$M(t) = \text{Tr}(e^{-i(H_B+V)t} \rho_B e^{i(H_B-V)t}) \quad (6)$$

is a measure of the decoherence induced by the bath at time t . Here Tr denotes the usual trace as H_B and V are operators

in the bath Hilbert space. Under a rotation of π around the z axis, while H_B and ρ_B remain unchanged, $V \rightarrow -V$. Consequently the coherence $M(t)$ is a real number. As mentioned in the introduction, though the coherence M is easy to evaluate for baths consisting of noninteracting spins and/or bosons (see Appendix A), it is rather difficult to estimate for an interacting spin bath for arbitrary values of the coupling to the central spin. We remark that $M(t)$ is related to Loschmidt echoes of the bath which characterize its sensitivity to perturbations in its equations of motion.^{14–16}

III. WEAK COUPLING REGIME

In this section, we present a perturbative formalism to calculate $M(t)$ valid for weak coupling to the bath, i.e., the energy scale of the operator V is much smaller than all the energy scales of the bath. We use the resolvent operator method which goes beyond the Born-Markov approach and, though perturbative, is intrinsically capable of handling non-Markovian time evolutions. We obtain a tractable expression for the decoherence in the weak coupling regime.

A. Resolvent operator method

To determine the complete time evolution of the coherence M in the weak coupling limit $V \rightarrow 0$, we first express it in terms of a *self-energy* Σ .^{17,18} To obtain the self-energy, it is convenient to work with the Laplace transform of $M(t)$,

$$\tilde{M}(z) = -i \int_0^\infty dt e^{izt} M(t), \quad (7)$$

where z is a complex variable with $\text{Im } z > 0$. As shown in Appendix B, this Laplace transform can be written as

$$\tilde{M}(z) = [z - \Sigma(z)]^{-1}, \quad (8)$$

where the self-energy Σ is given by

$$\Sigma(z) = \text{Tr}[\mathcal{L}_V \rho_B + \mathcal{L}_V \mathcal{Q}(z - \mathcal{Q} \mathcal{L} \mathcal{Q})^{-1} \mathcal{Q} \mathcal{L}_V \rho_B]. \quad (9)$$

In the above expression, the superoperators \mathcal{Q} , \mathcal{L}_B , \mathcal{L}_V , and \mathcal{L} are defined by their actions on any operator A acting on the bath Hilbert space: $\mathcal{Q}A = A - \text{Tr}(A)\rho_B$, $\mathcal{L}_B A = [H_B, A]$, $\mathcal{L}_V A = VA + AV$, and $\mathcal{L} = \mathcal{L}_B + \mathcal{L}_V$. As ρ_B is a density matrix, \mathcal{Q} is a projection operator, i.e., $\mathcal{Q}^2 = \mathcal{Q}$. Note that \mathcal{L}_V is not a Liouville operator whereas \mathcal{L}_B is the Liouville operator corresponding to the bath Hamiltonian H_B . We reiterate that the above derivation for (8) and (9) is independent of the specific Hamiltonian H_B and coupling operator V considered in this paper. The self-energy Σ can now be expanded perturbatively in the interaction operator V . The second order result is given by the expression (9) with \mathcal{L} replaced by the bath Liouvillian \mathcal{L}_B . The first-order term $\text{Tr}(\mathcal{L}_V \rho_B) = 2 \text{Tr}(V \rho_B)$ vanishes for the Ising spin bath Hamiltonian H_B and the interaction operator V defined by (2). Therefore, the first non-zero contribution to the self-energy is given by the second-order term Σ_2 which can be rewritten in terms of the time-dependent symmetrized correlation function of V (see Appendix C),

$$\Sigma_2(z) = -2i \int_0^\infty dt e^{izt} [\langle V(t)V \rangle + \langle VV(t) \rangle], \quad (10)$$

where $\langle \dots \rangle$ refers to the thermal average over bath spin configurations. Neglecting higher order contributions to Σ in (8) is equivalent to the Born approximation.² We will see in the following that this approximation can describe the decoherence at all time scales whereas a direct expansion of the coherence M gives only the short-time evolution. We remark that in the case of Heisenberg spins, an underlying magnetic order could result in a first order contribution to the self-energy which would then lead to an oscillatory behavior of $M(t)$.

The advantage of the resolvent operator formalism is that we can use the analyticity properties of the self-energy Σ to obtain a tractable expression for the coherence M . As shown in Appendix B, since H_B and V are Hermitian operators, the spectrum of the operator \mathcal{L} is real and hence the self-energy Σ is analytic in the upper (lower) half-plane. Furthermore, since the spectrum of \mathcal{L} in the thermodynamic limit is expected to be a continuum for the models considered in this paper, the self-energy Σ manifests a branch cut on the real axis. The coherence M can thus be written in terms of the real functions Λ and Γ defined by

$$\Lambda(E) - i\Gamma(E) = \lim_{\eta \rightarrow 0^+} \Sigma(E + i\eta), \quad (11)$$

where E is real. Since $M(t)$ is real, the functions Λ and Γ satisfy $\Lambda(-E) = -\Lambda(E)$ and $\Gamma(-E) = \Gamma(E)$. Performing the inverse Laplace transform of (8) and taking the limit $\eta \rightarrow 0^+$, we obtain

$$\Theta(t)M(t) = \frac{i}{2\pi} \int dE \frac{e^{-itE}}{E - \Lambda(E) + i\Gamma(E)}, \quad (12)$$

where $\Theta(t)$ is the Heaviside step function. Moreover, integrating $\Sigma(z)/(z-E)$ along an appropriate contour in the upper half-plane, one obtains the Kramers-Kronig-like dispersion relation

$$\Lambda(E) - i\Gamma(E) = -\frac{i}{\pi} \mathcal{P} \int \frac{dE'}{E' - E} [\Lambda(E') - i\Gamma(E')], \quad (13)$$

where \mathcal{P} denotes the Cauchy principal value. This shows that $\Theta(t)M(t)$ is exclusively determined by Γ (or Λ).

B. Non-Markovian evolution

In this section, we analyze (12) to determine $M(t)$ at any time t in the weak coupling regime. We first note that differentiating (12) with respect to the time t yields the conditions $M(0) = 1$ and $\partial_t M(0) = 0$. Differentiating (12) further and taking the limit $V \rightarrow 0$, we find, to second order in V ,

$$\begin{aligned} \Theta(t) \partial_t^2 M &\simeq -\frac{1}{2\pi} \int dE e^{-iEt} [\Gamma_2(E) + i\Lambda_2(E)] \\ &\simeq -\frac{1}{\pi} \Theta(t) \int dE e^{-iEt} \Gamma_2(E), \end{aligned} \quad (14)$$

where Γ_2 and Λ_2 denote the second-order terms of the functions Γ and Λ . Note that to obtain the second equality we have used the relation (13) and the Fourier transform of $2\Theta(t)-1$. A solution to the above differential equation, for $t>0$, is

$$M(t) \approx 1 - \frac{2}{\pi} \int dE \frac{\sin(tE/2)^2}{E^2} \Gamma_2(E), \quad (15)$$

where we have taken into account the symmetry of the function Γ_2 . Though this Fermi golden rule like approximation yields the correct short time behavior, it leads to the false result $M(t) \approx -\Gamma_2(0)t$ for $t \rightarrow \infty$ and hence is invalid for an arbitrary time t .

To obtain the long-time decoherence, we evaluate the integral (12) using the analytic continuation of Σ from the upper half-plane to the lower half-plane (second Riemann sheet). If the functions Γ and Λ are analytic in the vicinity of $E=0$, the analytic continuation of Σ in the second Riemann sheet is given by

$$\tilde{\Sigma}(z) = -i\Gamma(0) + \Lambda'(0)z + O(|z|^2) \quad (16)$$

for small z where Λ' denotes the derivative of the function Λ with respect to E . Note that the symmetry properties of the functions Λ and Γ ensure $\Lambda(0)=0$ and $\Gamma'(0)=0$. The coherence (12) is principally determined by the singularities of $[z - \tilde{\Sigma}(z)]^{-1}$, one of which is a pole at $z_0 = -i\Gamma_2(0) + O(V^4)$ close to the real axis. Every other singularity lies beyond a finite distance ϵ^{-1} from the real axis determined by the temperature and the bath parameters. Consequently, for times $t \gg \epsilon$, the residue $\exp(-iz_0 t)[1 - \partial_z \tilde{\Sigma}(z_0)]^{-1}$ dominates and thus in the weak coupling limit $V \rightarrow 0$,

$$\ln M(t) \approx -\Gamma_2(0)t + \Lambda'_2(0). \quad (17)$$

Combining the approximations for short times and long times given by (15) and (17), respectively, we see that in the weak coupling regime the decoherence at any time t is described by

$$\ln M(t) \approx -\frac{2}{\pi} \int dE \frac{\sin(tE/2)^2}{E^2} \Gamma_2(E). \quad (18)$$

This equation shows that $M(t)$ in the weak coupling regime is determined by the entire function Γ_2 . For asymptotic times, we see from (18) that the coherence of the central spin is essentially given by $M(t) \approx \exp[-\Gamma_2(0)t]$ which is simply the solution of the Markovian master equation obtained within the Born-Markov approximation.² However, for the short and intermediate time evolution of M , the full energy dependence of Γ_2 comes into play which can then lead to a *nonexponential decay*, i.e., non-Markovian behavior of the coherence. Depending on the temperature and bath parameters, the asymptotic Markovian regime could even disappear, provided that $\Lambda'_2(0)$ goes to infinity. We remark that Eq. (18) is valid to all orders in V for a bath of independent bosons.

IV. DISORDERED ISING SPIN BATHS

In this section, we use the formalism of the preceding section to determine the decoherence induced by various disordered Ising spin baths. Though the spin bath models of real systems are expected to be more complicated than the Ising Hamiltonians considered here, we nonetheless study these systems in various dimensions to understand the effect of these simpler systems on the decoherence. More precisely, we study the effect of the disordered Ising spin chain on the coherence M of the central spin as a function of the temperature and the bath parameters J_0 and Δ . We also make predictions for Ising models in higher dimensions in the high temperature regime and for the infinite-ranged Ising model also known as the Sherrington-Kirkpatrick model. Before we embark on in-depth calculations of M , we show that for any Ising bath, the decoherence of the central spin in the weak coupling regime is intimately linked to the local-field distribution of the bath.

A. Local-field distribution

As shown in Sec. III, the decoherence in the weak coupling regime is determined by the time-dependent correlation function $\text{Re}\langle V(t)V \rangle$, through (18) and (10). For the case of the Ising bath Hamiltonian defined in (2), this correlation can be obtained from the local-field distribution of the bath. To see this, it is useful to work in the eigenbasis $|\{\sigma_i\}\rangle$ of H_B : $H_B|\{\sigma_i\}\rangle = -\sum_{(ij)} J_{ij} \sigma_i \sigma_j |\{\sigma_i\}\rangle$. As the matrix element $\langle \{\sigma_i\} | \sigma_k^x | \{\sigma'_i\} \rangle$ is nonvanishing only for spin configurations $\{\sigma_i\}$ and $\{\sigma'_i\}$ where $\sigma'_i = \sigma_i(1 - 2\delta_{ik})$, the time-dependent correlation of V is a sum of local correlations, $\langle V(t)V \rangle = \sum_k \lambda_k^2 \langle \sigma_k^x(t) \sigma_k^x \rangle$. We find

$$\text{Re}\langle V(t)V \rangle = \frac{1}{Z} \sum_{k, \{\sigma_i\}} \lambda_k^2 e^{\beta \sum_{(ij)} J_{ij} \sigma_i \sigma_j} \cos\left(2t \sum_i^{(k)} J_{ki} \sigma_i\right), \quad (19)$$

where $\sum_i^{(k)}$ denotes a sum over the spins σ_i interacting with the spin σ_k . Note that the term $\sum_i^{(k)} J_{ki} \sigma_i$ in (19) is the effective local field acting on the spin at site k generated by the configuration $\{\sigma_i\}$. Since $\text{Re}\langle \sigma^x(t) \sigma^x \rangle = \cos(2th)$ for an isolated spin σ in a field h parallel to the z axis, we rewrite (19) as

$$\text{Re}\langle V(t)V \rangle = \sum_k \lambda_k^2 \int dh P_k(h) \cos(2th), \quad (20)$$

where $P_k(h) = \langle \delta(h - \sum_i^{(k)} J_{ki} \sigma_i^z) \rangle$ can be interpreted as the distribution of the local field h at site k at temperature T . This interpretation is also valid for the bath thermodynamic quantities such as magnetization or specific heat.¹⁹

It is now rather straightforward to obtain the function Γ_2 which is the crucial ingredient to determine the coherence M in the weak coupling regime. Since the disorder average of the correlation $\langle \sigma_k^x(t) \sigma_k^x \rangle$ is site independent, the central spin and spin bath coupling only contributes a multiplicative factor $\sum_k \lambda_k^2$ to the disorder average of Γ_2 . Consequently, in the rest of the paper, we use the conventional coupling $\lambda_k = \lambda N^{-1/2}$.^{9,20} With this choice, we obtain

$$\Gamma_2(E) = 2\pi\lambda^2 P(E/2), \quad (21)$$

where the local-field distribution P is the spatial average

$$P(h) = \frac{1}{N} \sum_k P_k(h) = \frac{1}{N} \sum_k \left\langle \delta \left(h - \sum_i^{(k)} J_{ki} \sigma_i^z \right) \right\rangle. \quad (22)$$

The thermodynamic limit of this expression is unambiguously defined and P is self-averaging,²¹ i.e., in the limit $N \rightarrow \infty$, P is given by bond disorder average of any distribution P_k . Though the function Γ_2 might not be strictly self-averaging for more realistic λ_k , we nonetheless expect similar qualitative results if the central spin is coupled to a sufficiently large number of bath spins. For an Ising spin system, the local-field distribution P is temperature dependent and determines the thermodynamic quantities and the dynamic linear response of the system.^{19,22} Equation (21) shows that the local-field distribution is also an important characteristic of an Ising system considered as a bath.

We remark that for a bath of independent spins,⁷ i.e., $H_B = -\sum_i h_i \sigma_i^z$ in (2), the function Γ_2 is given by (21) with the field distribution $P(h) = \sum_k (\lambda_k/\lambda)^2 \delta(h - h_k)$. In this case, since the spins are noninteracting, the distribution P is arbitrary and temperature independent and the ensuing decoherence of the central spin is temperature independent. This feature of a temperature independent decoherence induced by a bath of independent spins seen in the weak coupling limit is also seen in the exact nonperturbative result for the coherence $M(t)$ obtained in Appendix A.

B. Ising spin chain

In this section we consider a one-dimensional (1D) Ising bath described by the Hamiltonian

$$H_B = - \sum_{i=1}^{N-1} J_i \sigma_i^z \sigma_{i+1}^z. \quad (23)$$

The spin at site i interacts with its nearest neighbors with interaction strengths J_i and J_{i-1} . The J_i are a quenched set of random bonds drawn independently from a distribution p with mean J_0 and variance Δ^2 .

1. Born self-energy

To obtain the coherence M we need the time-dependent correlation of the coupling operator $V = -\sum_i \lambda_i \sigma_i^x$. As shown earlier, this correlation is given by $\text{Re}\langle V(t)V \rangle = \sum_k \lambda_k^2 \text{Re}\langle \sigma_k^x(t) \sigma_k^x \rangle$. Here the time-dependent spin-spin correlations can be written in terms of the static correlation function as

$$\begin{aligned} \text{Re}\langle \sigma_k^x(t) \sigma_k^x \rangle &= \cos(2tJ_{k-1}) \cos(2tJ_k) \\ &\quad - \langle \sigma_{k+1}^z \sigma_{k-1}^z \rangle \sin(2tJ_{k-1}) \sin(2tJ_k), \end{aligned} \quad (24)$$

where $\langle \sigma_{k+1}^z \sigma_{k-1}^z \rangle = \tanh(\beta J_{k-1}) \tanh(\beta J_k)$ is related to the derivative of the partition function

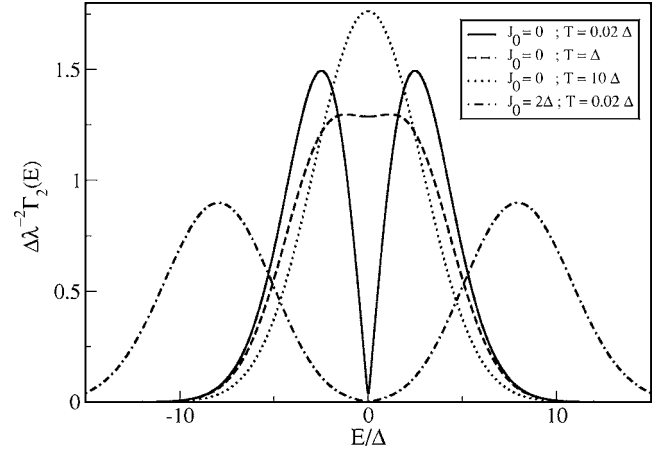


FIG. 1. The real part of the dimensionless second-order self-energy $\Gamma_2 \Delta / \lambda^2$ as a function of E/Δ for a Gaussian bond distribution with mean values $J_0 = 0, 2\Delta$ and temperatures $T = 0.02\Delta, \Delta, 10\Delta$.

$$Z = 2^N \prod_{i=1}^{N-1} \cosh(\beta J_i) \quad (25)$$

with respect to the interaction strengths J_k and J_{k-1} . The choice $\lambda_k = \lambda N^{-1/2}$ yields a variance of $\text{Re}\langle V(t)V \rangle$ of order N^{-1} and a mean of order N^0 . Consequently, $\text{Re}\langle V(t)V \rangle$ and hence Σ_2 are self-averaging in the thermodynamic limit. The second-order self-energy is thus given by the average of (10) over bond disorder. We obtain

$$\begin{aligned} \Gamma_2(E) &= 2\pi\lambda^2 \int dJ p_e(J) p_e(J + E/2) \\ &\quad \times [1 - \tanh(\beta J) \tanh(\beta J + \beta E/2)], \end{aligned} \quad (26)$$

where $p_e(J) = [p(J) + p(-J)]/2$ is the symmetrized bond distribution. Note that the integral on the right-hand side is the local-field distribution (22) for the spin chain.^{21,23} Equation (26) is valid for all bond distributions p . Moreover, for distributions p symmetric around their mean values, as Γ_2 depends only on the symmetrized distribution p_e the self-energy Σ_2 is the same for opposite means $\pm J_0$. This shows that, in this case, though the decoherence is influenced by the interactions in the bath, it is insensitive to the underlying ferromagnetic or antiferromagnetic nature of the interactions. This feature can be understood as follows. The spin-chain Hamiltonian (23) and the local field operator $J_{k-1} \sigma_{k-1}^z + J_k \sigma_{k+1}^z$ are invariant under the transformation $[J_i, \sigma_i^z] \rightarrow [-J_i, (-1)^{i-k} \sigma_i^z]$. Consequently, the local-field distribution P and hence the function Γ_2 are invariant under the transformation $J_0 \rightarrow -J_0$. To complete the determination of the second-order self-energy, its real part Λ_2 can be obtained from the function Γ_2 using (13). In Fig. 1, we plot Γ_2 for different values of J_0 and the temperature T .

2. Markovian evolution

As shown in Sec. III, in the weak coupling limit $\lambda \rightarrow 0$, the evolution of the coherence M is essentially Markovian:

$M(t) \simeq \exp(-\gamma t)$. For the disordered Ising chain, the decoherence rate γ is given, to lowest order in λ , by

$$\gamma = \Gamma_2(0) = 2\pi\lambda^2 \int dJ p_e(J)^2 [1 - \tanh(\beta J)^2]. \quad (27)$$

The rate γ increases monotonically from $\gamma=0$ at $T=0$ and saturates to the value

$$\gamma_\infty = 2\pi\lambda^2 \int dJ p_e(J)^2 \quad (28)$$

as $T \rightarrow \infty$. At low temperatures, $\gamma \simeq 4\pi\lambda^2 p(0)^2 T$ for disorder distributions with $p(0) \neq 0$. The vanishing of γ at zero temperature can be understood as follows.²¹ At $T=0$, the local field distribution is completely dictated by the spin structure of the ground state $|\{\sigma_i\}\rangle$ of the bath Hamiltonian (23). Since $J_i \sigma_i \sigma_{i+1} > 0$ for any pair of neighboring spins in the ground state, this inevitably leads to nonvanishing local fields $J_{i-1} \sigma_{i-1} + J_i \sigma_{i+1} = (|J_{i-1}| + |J_i|) \sigma_i$. This implies $P(h=0) = 0$ and hence $\gamma = 2\pi\lambda^2 P(0) = 0$.

We now study the influence of the bond distribution on the rate γ . In the weak disorder regime $\Delta \ll |J_0|$, the distribution $p_e(J)^2$ practically vanishes for $J \neq \pm J_0$ thus the rate γ is given by

$$\gamma \simeq \pi\lambda^2 \int dJ p(J)^2 [1 - \tanh(\beta J_0)^2]. \quad (29)$$

In this regime, the temperature variations of γ are independent of the form of the bond distribution p and are exclusively determined by the mean value J_0 . In Fig. 2, we plot the temperature dependence of γ for various values of the mean interaction strength J_0 in the case of a Gaussian bond distribution,

$$p(J) = \frac{e^{-(J - J_0)^2/2\Delta^2}}{\sqrt{2\pi}\Delta}. \quad (30)$$

The linear regime at low temperature given by $\gamma \simeq 2(\lambda/\Delta)^2 \exp(-J_0^2/\Delta^2) T$ is visible only for $|J_0| < 2\Delta$. For higher interaction strengths J_0 , γ remains practically zero in the low-temperature regime. The maximal rate obtained as $T \rightarrow \infty$ is given by (28): $\gamma_\infty = \sqrt{\pi}\lambda^2 [1 + \exp(-J_0^2/\Delta^2)]/2\Delta$. For $|J_0| > 2\Delta$, the high-temperature rate γ_∞ is essentially independent of J_0 and the temperature dependence of γ is well described by the weak disorder approximation $\gamma \simeq (\sqrt{\pi}\lambda^2/2\Delta) \times [1 - \tanh(\beta J_0)^2]$. Note that the agreement with this approximation improves with increasing J_0 .

We now consider a uniform distribution for the intrabath interaction strength,

$$p(J) = (2\sqrt{3}\Delta)^{-1} \quad \text{for } |J - J_0| < \sqrt{3}\Delta \\ = 0 \quad \text{otherwise.} \quad (31)$$

In this case, the integral (27) can be evaluated exactly for any temperature T ,

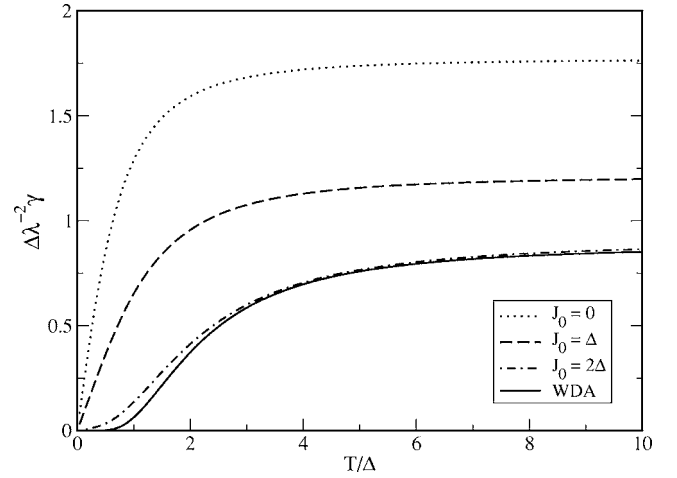


FIG. 2. Dimensionless rate $\gamma\Delta/\lambda^2$ as a function of the temperature T/Δ for a Gaussian bond distribution with mean $J_0=0, \Delta$, and 2Δ . The weak disorder approximation (WDA) given by Eq. (29) is shown for $J_0=2\Delta$.

$$\gamma = \frac{\pi\lambda^2 T}{12\Delta^2} \left[\tanh\left(\frac{|J_0| + \sqrt{3}\Delta}{T}\right) - 3 \tanh\left(\frac{|J_0| - \sqrt{3}\Delta}{T}\right) \right] \\ \text{for } |J_0| < \sqrt{3}\Delta \\ = \frac{\pi\lambda^2 T}{12\Delta^2} \left[\tanh\left(\frac{|J_0| + \sqrt{3}\Delta}{T}\right) - \tanh\left(\frac{|J_0| - \sqrt{3}\Delta}{T}\right) \right] \\ \text{for } |J_0| > \sqrt{3}\Delta. \quad (32)$$

For $|J_0| < \sqrt{3}\Delta$, $\gamma \propto T$ at low temperatures with a slope $\pi\lambda^2/3\Delta^2$ independent of J_0 whereas, for $|J_0| > \sqrt{3}\Delta$ the low temperature behavior is not linear. For $|J_0| > 4\Delta$, the agreement between the exact result and the weak disorder approximation $\gamma \simeq (\pi\lambda^2/2\sqrt{3}\Delta)[1 - \tanh(\beta J_0)^2]$ is excellent. Moreover, in this regime, the Gaussian and uniform bond distributions cannot be distinguished ($\sqrt{\pi/3} \simeq 1.02$). An interesting feature of our results is that a nonzero J_0 favors the coherence of the central spin via a robust short range ordering of the bath spins.

Comparing the above results with those obtained for a bath comprising free spins, we see that the time scale of the decoherence generated by the spin chain bath $\gamma_\infty^{-1} \sim \Delta/\lambda^2$ (for weak coupling $\lambda \ll \Delta$) is much longer than the decoherence time λ^{-1} (for $\lambda \ll \sqrt{N}$) obtained in the free case (see Appendix A). As we will demonstrate later, $\gamma_\infty^{-1} \gg \lambda^{-1}$ for any dimensionality of the spin bath lattice. This clearly illustrates the fact that interactions in the bath significantly slow down the decoherence of the central spin.

Another interesting comparison is to an Ohmic boson bath. Contrary to the Ising bath, the decoherence rate γ_{bos} of an Ohmic boson bath is proportional to T in the whole temperature range and thus does not saturate at high temperatures. This forces the question as to whether interactions between the bosons also lead to a saturation of the rate γ_{bos} . This warrants further work which is beyond the scope of the present paper. If the central spin is coupled both to a spin

bath and a boson bath, the resulting coherence $M(t)$ is given by the product of (6) and a similar factor, with H_B replaced by the boson bath Hamiltonian H_b and the interaction operator V by an analogous boson operator V_b . Then in the Markovian regime at weak coupling, the total decoherence rate is simply the sum $\gamma_{\text{bos}} + \gamma$. Consequently, at high temperatures the Ohmic boson bath dominates but for temperature $T \lesssim \Delta$ the Ising bath must be taken into account. The question of their relative dominance depends on the various bath coupling constants and can vary from system to system.

3. Non-Markovian regime

Here, we discuss the non-Markovian aspects of the decoherence of the central spin essentially seen at low temperatures and at short and intermediate time scales. We first consider the case of a bond distribution with zero mean. The time evolution of the coherence $M(t)$ within the Born approximation, shown in Fig. 3, is obtained by a numerical evaluation of $\Gamma_2(E)$, $\Lambda_2(E)$, and $M(t)$ using the expressions (26), (13), and (12) for a Gaussian bond distribution (30) with $J_0=0$. Note that the agreement with the weak coupling approximation (18) is remarkably good even for reasonably large values of λ , i.e., of the order of 0.1Δ . Though, in Fig. 3, we illustrated the equivalence of the Born and weak coupling approximations for $M(t)$ in the weak coupling regime, we nonetheless expect, based on the analyticity arguments presented in Sec. III, the weak coupling approximation (18) to be an exact description of the full coherence M for low enough λ .

We now discuss the influence of temperature on the behavior of the coherence $M(t)$. In Fig. 3, we see that at high temperatures, M remains practically constant for times $t \lesssim \Delta^{-1}$ and then decays exponentially as described by (17). For temperatures $T \lesssim \Delta$, the Markovian regime is preceded by an intermediate time regime $\Delta^{-1} \lesssim t \lesssim \beta$. The difference between the high-temperature and the low-temperature decoherence can be traced back to the temperature dependence of the function Γ_2 . At high enough temperatures, since $\Gamma_2(E)$ is essentially a peak of width Δ , one crosses over from the short time regime to the Markovian regime for $t \approx \Delta^{-1}$. As the temperature is lowered, $\Gamma_2(0)$ steadily decreases but the curvature $\Gamma_2''(0)$ remains negative. However, below a certain temperature $\Gamma_2''(0)$ becomes positive, see Fig. 1 and the function Γ_2 can be effectively characterized by two energy scales: T and Δ . At low temperatures, Γ_2 remains practically constant for $|E| \lesssim T$, increases linearly for larger energies with a slope $(\lambda/\Delta)^2$ and finally vanishes for $|E| \gtrsim \Delta$. These three energy regimes result in three different time regimes for the coherence (18). The short-time behavior ($t \lesssim \Delta^{-1}$) is determined by the long-energy tails of Γ_2 . For intermediate times $\Delta^{-1} \lesssim t \lesssim \beta$, the linear regime of Γ_2 yields a power law decay of the coherence $M(t) \propto t^{-\epsilon}$ where the exponent is given by

$$\epsilon = \frac{2}{\pi} \left(\frac{\lambda}{\Delta} \right)^2. \quad (33)$$

For long times ($t \gtrsim \beta$) the integral (18) is dominated by the energies $|E| \lesssim T$ for which $\Gamma_2(E) \approx \gamma$ and hence the decoherence is essentially exponential with the rate γ . However,

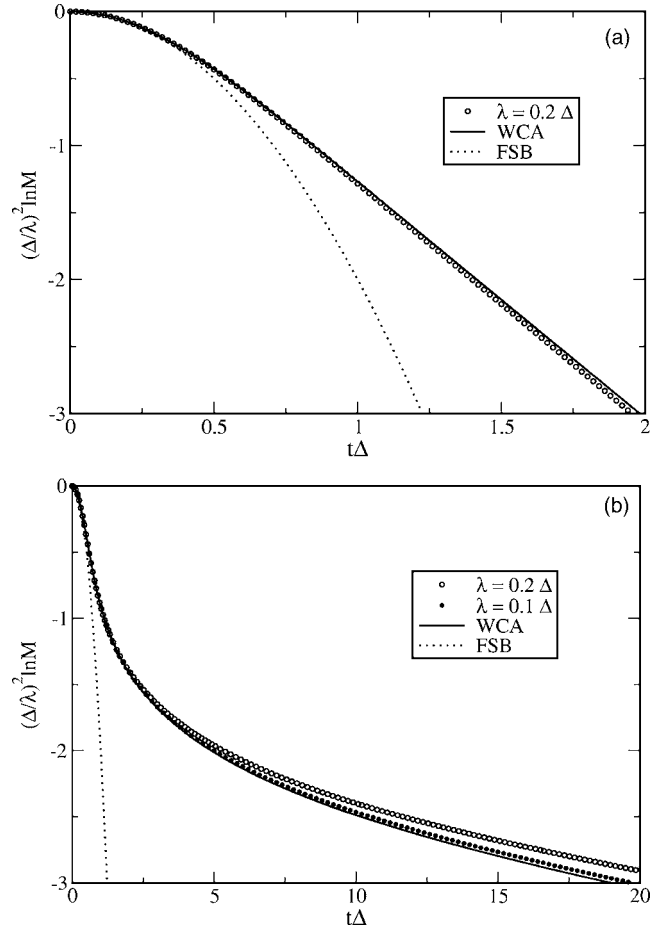


FIG. 3. $(\Delta/\lambda)^2 \ln M$ as a function of time $t\Delta$ within the Born approximation for a Gaussian bond distribution with zero mean for coupling strengths $\lambda=0.1\Delta$, 0.2Δ and temperatures (a) $T=10\Delta$ and (b) $T=0.02\Delta$. The full line is the weak coupling approximation (WCA) and the dotted line corresponds to a free spin bath (FSB).

one should be cautious about extending the above results to ultralow temperatures because the contribution of the energies $|E| \gtrsim T$, given by $\Lambda_2'(0)$, diverges in the limit $T \rightarrow 0$. This divergence is logarithmic with a prefactor ϵ . At zero temperature, the Markovian regime disappears and the coherence vanishes in the limit $t \rightarrow \infty$ according to the power law $t^{-\epsilon}$. The low temperature behavior of $M(t)$ obtained here is very similar to the decoherence induced by a boson bath in the strict Ohmic case,² i.e., for an Ohmic spectral function with a cutoff frequency ω_c far larger than T and $1/t$. In this case, the coherence of the spin coupled to the boson bath is given by $\ln M(t) = -K \ln[(\omega_c/T\pi) \sinh(\pi T t)]$ where K is the spin-bath coupling strength. We recover a Markovian behavior, $\ln M(t) \approx K \ln(2\pi T/\omega_c) - K\pi T t$, for times $t \gtrsim \beta$ and a power law, $M(t) = (\omega_c t)^{-K}$, at zero temperature.

We now present a more detailed comparison of our numerical results with the weak coupling approximation (18), shown in Fig. 3. For given λ and T , the Born approximation deviates from the expression (18) as time increases. This difference, which is more pronounced at low temperatures, can be easily quantified in terms of higher order corrections stemming from the expansion of the analytically continued

self-energy discussed in Sec. III B. At long times, the first correction to $\ln M(t)$ arises at $O(\lambda^4)$ and takes the form

$$\ln M(t) - [\Lambda_2'(0) - \Gamma_2(0)t] \simeq -\Gamma_2(0)\Gamma_2''(0) + \frac{1}{2}[\Lambda_2'(0)]^2 - \Gamma_2(0)\Lambda_2'(0)t. \quad (34)$$

In the high temperature regime, since $\Lambda_2'(0)$ is positive, the weak coupling approximation given by the second-order terms in (34) slightly overestimates $\ln M$. As the temperature is lowered, the evolution of the function Γ_2 discussed earlier (see Fig. 1) results in a sign change of $\Lambda_2'(0)$. Thus for low enough temperatures, $\Lambda_2'(0)$ is negative and the weak coupling approximation underestimates $\ln M$. This analysis is indeed very consistent with our results shown in Fig. 3. This discussion clearly highlights the efficiency of our approach based on the analyticity properties of the self-energy, obtained within any approximation, to describe the long-time decoherence. We remark that other fourth order corrections to $\ln M(t)$ exist beyond the Born approximation.

We now consider the case $J_0 \neq 0$. For weak disorder $|J_0| \gg \Delta$, the function Γ_2 at low temperature consists of two Gaussian peaks of width $2\sqrt{2}\Delta$ centered around $\pm 4J_0$. The low-temperature self-energy in this regime is thus very different from that for $J_0=0$, see Fig. 1. Nonetheless, the qualitative behavior of the coherence M at intermediate and long times, determined by the low-energy Γ_2 , is very similar to the one discussed above for $J_0=0$. For short times $t \lesssim \Delta^{-1}$, contrary to the $\ln M(t) \propto -t^2$ behavior seen for $J_0=0$, here the coherence $M(t)$ oscillates at the frequency $2J_0/\pi$. At zero temperature, the long-time decoherence decays as a power law, $M(t) \propto t^{-\eta}$ where the exponent $\eta = \epsilon \exp(-J_0^2/\Delta^2)$. As seen earlier for the Markovian decay rate γ , the low-temperature behavior seen here also signals a slowing down of the decoherence by a nonzero J_0 .

C. Higher-dimensional spin lattices

Here we consider other geometries for an Ising spin bath described by the Hamiltonian $H_B = -\sum_{\langle ij \rangle} J_{ij} \sigma_i^z \sigma_j^z$ where the spins occupy the sites i of a regular lattice of arbitrary dimensionality and are coupled by nearest-neighbor interactions. The bonds J_{ij} are drawn independently from the Gaussian distribution p with mean J_0 and variance Δ^2 given by (30). An important difference between the spin-chain model and the generic Ising model on higher-dimensional lattices is the presence of frustration arising from geometric constraints and/or randomness. Though frustration can give rise to novel ground states and related dynamical behavior, it renders any analytic study of these models very difficult. In this section, we focus on the high temperature regime where one can use a controlled analytical method like the high temperature series expansion to study the effect of higher dimensions on the decoherence.

To obtain the coherence M at high temperatures, we expand the second-order self-energy (10) in terms of the inverse temperature β . To do so, we first rewrite the time-dependent correlation (19) as

$$\begin{aligned} \text{Re}\langle V(t)V \rangle &= \frac{1}{Z'} \sum_k \lambda_k^2 \sum_{\{\sigma_j\}} \prod_{\langle ij \rangle} (1 + \sigma_i \sigma_j \kappa_{ij}) \\ &\quad \times \text{Re} \prod_i^{(k)} [\cos(2tJ_{ik}) + i \sigma_i \sigma_k \sin(2tJ_{ik})], \end{aligned} \quad (35)$$

where $\prod_i^{(k)}$ denotes a product over the nearest neighbors of site k , $Z' = \sum_{\{\sigma_j\}} \prod_{\langle ij \rangle} (1 + \sigma_i \sigma_j \kappa_{ij})$ and $\kappa_{ij} = \tanh(\beta J_{ij})$. Since $\kappa_{ij} \rightarrow 0$ as $\beta \rightarrow 0$, we consider the above expression as a power series in κ_{ij} . Multiplying out the two products in Eq. (35) generates a series of products of nearest-neighbor spin pairs: $(\sigma_i \sigma_j \sigma_k \sigma_l \dots)$. Since Eq. (35) involves sums over configurations $\{\sigma_j\}$, each of these spin-pair products contributes only if it simplifies to 1. This implies, for instance, that the n th order terms in the expansion of Z' in κ_{ij} correspond to closed loops comprising n bonds on the lattice. An immediate consequence is that $Z' = 1$ up to third order. However, in the expression (35) there exists another type of relevant spin-pair products which involve repeated bonds. We discuss these terms in the following.

As $T \rightarrow \infty$, for equal couplings to the central spin $\lambda_k = \lambda N^{-1/2}$, the time-dependent correlation (35) becomes $\text{Re}\langle V(t)V \rangle = \lambda^2 N^{-1} \sum_k \prod_i^{(k)} \cos(2tJ_{ik})$. Note that this correlation remains unchanged under $J_{ik} \rightarrow -J_{ik}$. This invariance stems from the equiprobability of every spin configuration at infinite temperature. This infinite-temperature correlation is self-averaging in the thermodynamic limit and hence the corresponding second order self-energy is given by the average of (10) over the bond distribution. This leads to

$$\Gamma_2(E) = 2\lambda^2 \int_{-\infty}^{\infty} dt e^{iEt} e^{-2\Delta^2 t^2} \cos(2J_0 t)^s, \quad (36)$$

where s is the coordination number of the lattice. As remarked earlier for the 1D case, here also Γ_2 is the same for $\pm J_0$ for bond distributions symmetric around their mean values. Equation (36) shows that in the infinite temperature limit the only characteristic of the bath lattice which intervenes in the decoherence is s . In particular, there is no explicit dependence on the dimensionality. However, we shall show later that the detailed geometric characteristics of the lattice manifest themselves in the higher order corrections. Though static properties of the spin system are independent of J_0 and Δ in the infinite temperature limit, these parameters strongly influence the correlation $\text{Re}\langle V(t)V \rangle$ and hence the decoherence. Using (36), for the case $J_0=0$, we obtain from (18)

$$\ln M(t) = -2\sqrt{\pi}\lambda^2 \pi \text{erf}(t/\tau) + 2\lambda^2 \tau^2 (1 - e^{-t^2/\tau^2}), \quad (37)$$

where erf is the error function and the characteristic time τ is defined by

$$\tau = \Delta^{-1} (2s)^{-1/2}. \quad (38)$$

For times $t \gg \tau$, we recover the Markovian regime where $\ln M(t) \simeq -2\sqrt{\pi}\lambda^2 \pi + 2\lambda^2 \tau^2$. For $t \ll \tau$, one finds the usual short time evolution: $\ln M(t) \simeq -2\lambda^2 t^2$. The same time re-

gimes exist for $J_0 \neq 0$. For short times $t \ll \tau$ we find, for an even s ,

$$\ln M(t) = -\frac{\lambda^2}{2^{s-2}J_0^2} \sum_{n=0}^{s/2-1} \frac{s! \sin[J_0 t(2n-s)]^2}{n!(s-n)!(2n-s)^2} - \frac{s! \lambda^2 t^2}{2^{s-1}[(s/2)!]^2}. \quad (39)$$

If s is an odd number, the short-time decoherence is described by (39) without the quadratic term in t and with a sum running from 0 to $(s-1)/2$. Clearly the effect of a non-zero J_0 is to induce oscillations in the coherence $M(t)$. The decoherence rate in the Markovian regime is given by

$$\gamma_\infty = \frac{\lambda^2}{2^{s-1}\Delta} \sqrt{\frac{\pi}{2s}} \sum_{n=0}^s \frac{s!}{n!(s-n)!} e^{-J_0^2(2n-s)^2/2s\Delta^2}. \quad (40)$$

For $s=2$ this expression simplifies to the result (28) obtained for the spin chain. As anticipated, the decoherence time γ_∞^{-1} is of the order of Δ/λ^2 and hence far longer than that for free spins which is of the order of λ^{-1} . The infinite-temperature rate manifests the influence of the lattice geometry: for lattices with an odd s like the honeycomb lattice ($s=3$), $\gamma_\infty \rightarrow 0$ in the weak disorder limit $\Delta \ll |J_0|$ whereas for an even s , triangular and square lattices, for example, $\gamma_\infty \neq 0$ in this limit.

We now evaluate the leading order corrections in β to the decoherence rate γ . To illustrate the significance of the lattice geometry we consider various bidimensional lattices. For a triangular lattice, the lowest order terms in the sum over the configurations $\{\sigma_i\}$ in (35) take the form $-\sin(2tJ_{ik})\kappa_{ij}\sin(2tJ_{jk})$ where the site i is a neighbor of the site k and the site j is a neighbor of both sites k and i . Consequently, the rate up to first order reads

$$\gamma^T = \gamma_\infty^T - \frac{\sqrt{3}\pi\lambda^2\beta J_0}{16\Delta} (2 - 2e^{-4J_0^2/3\Delta^2} + e^{-J_0^2/3\Delta^2} - e^{-3J_0^2/\Delta^2}) + O(\beta^2), \quad (41)$$

where γ_∞^T is given by (40) with $s=6$. The explicit appearance of J_0 in this expansion is linked to the fact that the first nonzero correction for this lattice occurs at first order in β . Moreover, since this correction is positive (negative) for baths with a majority of antiferromagnetic (ferromagnetic) bonds, the ensuing decoherence time is longer for a predominantly ferromagnetic bath. This clearly highlights the importance of frustration in determining decoherence. Thus at high temperatures, the decoherence induced by the triangular lattice is very different than that by the linear chain: the convergence of γ^T to γ_∞^T is slower and depends on the sign of J_0 . We also remark that though the cubic and triangular lattices have the same coordination $s=6$ and hence the same infinite-temperature rate, the corrections are different since there is no three-bond closed loop on the cubic lattice. For the honeycomb lattice, the first corrections arise at second order. These corrections correspond to loops with two repeated bonds: $-\kappa_{ki}\sin(2tJ_{ik})\kappa_{kj}\sin(2tJ_{jk})$. Here the contribution of closed loops becomes relevant at higher orders. The resulting decoherence rate is

$$\gamma^H = \gamma_\infty^H - \sqrt{\frac{\pi}{24}} \frac{\lambda^2\beta^2}{\Delta} [(3\Delta^2 + 4J_0^2)e^{-J_0^2/6\Delta^2} + \Delta^2 e^{-3J_0^2/2\Delta^2}] + O(\beta^4), \quad (42)$$

where γ_∞^H is given by (40) with $s=3$. Finally for the square lattice we obtain the decoherence rate,

$$\gamma^S = \gamma_\infty^S - \frac{1}{4} \sqrt{\frac{\pi}{2}} \frac{\lambda^2\beta^2}{\Delta} \left[\left(\frac{3}{4}\Delta^2 - 2J_0^2 \right) e^{-2J_0^2/\Delta^2} + 3(\Delta^2 + J_0^2)e^{-J_0^2/2\Delta^2} + \frac{9}{4}\Delta^2 + 5J_0^2 \right] + O(\beta^4), \quad (43)$$

where γ_∞^S is given by (40) with $s=4$. We mention that for the square lattice both closed loops and loops with repeated bonds contribute to the lowest order correction. Interestingly, the convergence of the rates γ^H and γ^S to their respective infinite-temperature limits is reminiscent of that of the chain. Moreover, as seen for the spin chain, our results for γ^H and γ^S are independent of the sign of J_0 . This feature can be attributed to the bipartite nature of these lattices. As explained for the chain, the respective local field operators and Hamiltonians are invariant under a sign change of the bonds J_{ij} coupled with a suitable unitary transformation for the spins. Since this argument is valid in the entire paramagnetic phase, the higher order corrections to γ^H and γ^S are expected to be independent of the sign of J_0 in this phase. On the other hand, due to its nonbipartite nature no analogous transformation exists for the triangular lattice. It would be interesting to study the evolution of these rates behavior as one lowers temperature and enters a nonparamagnetic phase.

D. Infinite-ranged Ising bath

We now consider a bath described by the mean field Sherrington-Kirkpatrick model where all the Ising spins interact with each other. We restrict ourselves to the well studied case of spin-spin interaction strengths distributed around a zero mean with a variance Δ^2 . This model exhibits a spin-glass phase transition at a finite temperature $T_{sg} = \Delta$ below which the spins freeze.²⁴ The spin-glass phase is characterized by the large number of metastable states present which then lead to anomalous dynamic behavior. An interesting question is whether the decoherence manifests novel features as it goes from the high temperature paramagnetic phase to the low temperature spin-glass phase.

As shown in Sec. IV, the coherence $M(t)$ of the central spin in the weak coupling regime can be obtained through a knowledge of the local-field distribution $P(h)$. Though the infinite-range model has been extensively studied in the past using the replica approach and numerical simulations, it is not easy to obtain the local-field distribution for all fields and temperatures. Here, we do not delve into the problem of calculating $P(h)$ but use existing results²¹ to make predictions for the coherence of the central spin coupled to this mean field bath. In the high temperature paramagnetic phase, the local-field distribution is

$$P(h) = \frac{1}{2\Delta\sqrt{2\pi}} \left(e^{-(h - \beta\Delta^2)^2/2\Delta^2} + e^{-(h + \beta\Delta^2)^2/2\Delta^2} \right). \quad (44)$$

In the spin-glass phase, calculations based on the replica formalism yield the following result for $P(0)$ provided the temperature $T \approx T_{sg}$:

$$P(0) = \frac{1}{\Delta\sqrt{2\pi e}} \left[1 - \left(1 - \frac{T}{T_{sg}} \right) - \frac{1}{3} \left(1 - \frac{T}{T_{sg}} \right)^4 \right] + \dots \quad (45)$$

These analytic results are sufficient to determine the decoherence in the Markovian regime: the decoherence rate is given by $\gamma = \sqrt{2\pi}\lambda^2 \exp[-\frac{1}{2}(T_{sg}/T)^2]/\Delta$ for $T > T_{sg}$ and $\gamma = \sqrt{2\pi}/e\lambda^2 [1 - (1 - T/T_{sg}) - \frac{1}{3}(1 - T/T_{sg})^4]/\Delta$ for temperatures in the vicinity of the spin-glass transition temperature, i.e., $T \approx T_{sg}$. Clearly, in the Markovian regime, one does not see any sign of the spin-glass transition. Note that γ saturates at infinite temperature to a value comparable to that obtained earlier for finite-ranged lattice models. This implies that even in the case of a highly frustrated bath, the central spin decoheres at time scales longer than those for free spins.

For lower temperatures, only numerical solutions exist for the local-field distribution.^{21,25} These results suggest a continuous variation of $P(h)$ with temperature. The only significant signature of the transition is a flattening of P at $T = T_{sg}$ which has no manifest effect on the decoherence. Moreover, numerical extrapolations of $P(0)$ to low temperatures indicate a rate $\gamma \sim (\lambda/\Delta)^2 T$ which is very similar to the linear T behavior seen in the Ising spin chain system at low enough temperatures. Again, since $\gamma = 0$ at zero temperature, the decoherence is no longer Markovian. The form of $M(t)$ is then dictated by the low-energy behavior of Γ_2 through (18). Based on the numerical inference $P(h) \propto h$ for $T = 0$, the central spin is expected to decohere as a power law at $T = 0$.^{21,25} To conclude, we see that both the thermal transition and the spin-glass order of the Sherrington-Kirkpatrick model do not have any palpable effect on the asymptotic decoherence in the weak coupling regime.

V. CONCLUSIONS

In this paper, we have studied the decoherence induced by a Ising spin bath with random intrabath interactions. The resolvent operator formalism was used to determine the coherence M of the central spin for weak coupling to the bath. We then obtained detailed analytical results for the disordered Ising spin-chain bath for arbitrary temperature. The decoherence was found to be independent of the sign of the mean intrabath interaction strength J_0 for symmetric bond distributions. Three regimes were identified in the time evolution of $M(t)$: a short time Gaussian decay, an intermediate time power law behavior and the usual asymptotic Markovian regime. The relative sizes of these regimes are fixed by temperature. At zero temperature, the Markovian regime was found to vanish and the decoherence is essentially described by a power law decay. We also studied the decoherence induced by an infinite-ranged Ising spin-glass bath and Ising

models on lattices in dimensions greater than one. For all these baths, the Markovian rate was found to saturate to a finite value at infinite temperature, which is much smaller than the corresponding rate for a free spin bath. Our results clearly indicate that intrabath interactions significantly increase the time scales over which the central spin decoheres.

For the infinite-ranged Ising spin glass, our conclusions based on existing results suggest that the thermal spin-glass transition has no visible effect on the decoherence. Plausibly this is an artefact of the infinite-ranged interactions and/or the Ising nature of the spins. This raises the general issue of the influence of thermal and quantum phase transitions and the resulting orders in finite-ranged spin baths on the decoherence of the central spin. In most realistic cases, the spin environment consists of Heisenberg spins. In this case, one expects the dynamics of the bath to be richer and this may have interesting consequences for the decoherence. This however, is beyond the scope of the analytic work presented in this paper. An interesting question is the effect of a strong coupling between the central spin and the bath. For a bath of independent spins and/or bosons the results obtained in the weak coupling regime are qualitatively valid even for strong coupling. However, in the presence of interactions, this is not necessarily the case and one can expect interesting dynamical behaviors. A natural extension of our work would be to include the intrinsic dynamics of the central spin and study the relaxation induced by the spin bath. These and other questions are left for future work.

ACKNOWLEDGMENT

One of the authors (R.C.) acknowledges support from ACI Grant No. JC-2026.

APPENDIX A: INDEPENDENT SPIN BATH

Here we derive an exact expression for the coherence $M(t)$ given by (6) in the case of a bath comprising noninteracting spins described by the Hamiltonian $H_B = -\sum_i h_i \sigma_i^z$. Since the spins are independent, the trace (6) can be factorized as

$$M(t) = \prod_{k=1}^N \frac{\text{Tr}(e^{\beta h_k \sigma_k^z} e^{-it(h_k \sigma_k^z - \lambda_k \sigma_k^x)} e^{it(h_k \sigma_k^z + \lambda_k \sigma_k^x)})}{2 \cosh(\beta h_k)}. \quad (A1)$$

To evaluate the factors in this expression we require the diagonal elements of 2×2 matrices of the form $\exp[ibA(-a)] \exp[ibA(a)]$ where $A(a) = a\sigma^z + \sigma^x$. Diagonalizing $A(a)$, we find that these diagonal elements are equal and given by

$$\langle \sigma | e^{ibA(-a)} e^{ibA(a)} | \sigma \rangle = \frac{1}{1 + a^2} [a^2 + \cos(2b\sqrt{1 + a^2})]. \quad (A2)$$

The resulting coherence $M(t)$ is independent of the temperature and can be written as

$$\ln M(t) = \sum_{k=1}^N \ln \left(1 - \frac{2\lambda_k^2}{\lambda_k^2 + h_k^2} \sin(t\sqrt{\lambda_k^2 + h_k^2}) \right). \quad (\text{A3})$$

For $h_k=0$, the above expression leads to the coherence, $M(t)=\prod_k \cos(2\lambda_k t)$ which then culminates in a Gaussian decay $M(t)=\exp(-2t^2\sum_k\lambda_k^2)$ for weak coupling to the bath. For nonzero local fields h_k the coherence in the weak coupling regime reads

$$\ln M(t) = -2\lambda^2 \int dh P(h) \frac{\sin(ht)^2}{h^2}, \quad (\text{A4})$$

where the coupling strength λ and the field distribution P are defined by $\lambda^2=\sum_k\lambda_k^2$ and $P(h)=\sum_k(\lambda_k/\lambda)^2\delta(h-h_k)$. This result is the same as that obtained earlier in (18) for the specific bath considered here.

APPENDIX B: DERIVATION OF THE SELF-ENERGY

In this Appendix, we derive the expressions (8) and (9) for the Laplace transform $\tilde{M}(z)$ of the coherence (6) and we show that $\tilde{M}(z)$ is analytic in the upper and lower half-planes. We first rewrite the expression (6) as

$$M(t) = \text{Tr}(e^{-i\mathcal{L}t}\rho_B), \quad (\text{B1})$$

where \mathcal{L} is a superoperator in the bath Liouville space defined by $\mathcal{L}A=(H_B+V)A-A(H_B-V)$ where A is any operator in the bath Hilbert space. The Laplace transform (7) can then be written as $\tilde{M}(z)=\text{Tr}[G(z)\rho_B]$ where $G(z)=(z-\mathcal{L})^{-1}$ is the resolvent of the operator \mathcal{L} .

We now introduce the superoperators \mathcal{P} and \mathcal{Q} defined by $\mathcal{Q}=1-\mathcal{P}$ and $\mathcal{P}A=\text{Tr}(A)\rho_B$ where A is any operator in the bath Hilbert space. Since $\text{Tr}(\rho_B)=1$, \mathcal{P} and \mathcal{Q} are projection operators. Using \mathcal{P} , \mathcal{Q} and $(z-\mathcal{L})G(z)=1$, we obtain the following coupled equations:

$$\mathcal{P}(z-\mathcal{L})\mathcal{P}\mathcal{P}G(z)\rho_B + \mathcal{P}(z-\mathcal{L})\mathcal{Q}\mathcal{Q}G(z)\rho_B = \rho_B,$$

$$\mathcal{Q}(z-\mathcal{L})\mathcal{P}\mathcal{P}G(z)\rho_B + \mathcal{Q}(z-\mathcal{L})\mathcal{Q}\mathcal{Q}G(z)\rho_B = 0 \quad (\text{B2})$$

for the operators $\mathcal{P}G(z)\rho_B$ and $\mathcal{Q}G(z)\rho_B$. Solving the latter for $\mathcal{Q}G(z)\rho_B$ in terms of $\mathcal{P}G(z)\rho_B$ and then substituting in the former yields

$$\mathcal{P}[z-\mathcal{L}-\mathcal{L}\mathcal{Q}(z-\mathcal{Q}\mathcal{L}\mathcal{Q})^{-1}\mathcal{Q}\mathcal{L}]\mathcal{P}G(z)\rho_B = \rho_B. \quad (\text{B3})$$

Finally tracing both sides gives $[z-\tilde{\Sigma}(z)]\tilde{M}(z)=1$ where $\tilde{\Sigma}(z)$ is given by (9).

We now discuss the analyticity properties of the Laplace transform $\tilde{M}(z)$. Consider an eigenoperator A of \mathcal{L} with the eigenvalue λ , $\mathcal{L}A=\lambda A$. The scalar product $\text{Tr}(A^\dagger\mathcal{L}A)=\lambda\text{Tr}(A^\dagger A)$ can also be written

$$\begin{aligned} \text{Tr}(A^\dagger\mathcal{L}A) &= \text{Tr}[A^\dagger(H_B+V)A - (H_B-V)A^\dagger A]^* \\ &= \text{Tr}(A^\dagger\mathcal{L}A)^* = \lambda^*\text{Tr}(A^\dagger A), \end{aligned} \quad (\text{B4})$$

where the first equality is obtained using the Hermiticity of H_B and V and the second one using the cyclic property of the trace. Consequently, the eigenvalues of \mathcal{L} are real and thus the resolvent $G(z)$ and the Laplace transform $\tilde{M}(z)$ are analytic in the upper and lower half planes.

APPENDIX C: SECOND-ORDER SELF-ENERGY

Here, we show that the second-order self-energy Σ_2 can be rewritten in terms of the time-dependent correlation of the interaction operator V as given by (10). Let $|\alpha\rangle$ denote the eigenstates of the Hamiltonian H_B and $|\alpha,\beta\rangle=|\alpha\rangle\langle\beta|$ denote the eigenstates of the corresponding Liouville operator \mathcal{L}_B . Any superoperator \mathcal{F} in the Liouville space can be expanded in this eigenbasis as

$$\mathcal{F} = \sum_{\alpha,\beta,\gamma,\delta} (\alpha,\beta|\mathcal{F}|\gamma,\delta)|\alpha,\beta\rangle(\gamma,\delta|, \quad (\text{C1})$$

where the scalar product in the Liouville space is defined by $(A|B)=\text{Tr}(A^\dagger B)$. The following decompositions are useful for our purpose:

$$\mathcal{P} = \rho_B \sum_{\beta} (\beta,\beta| = \sum_{\alpha,\beta} \langle\alpha|\rho_B|\alpha\rangle|\alpha,\alpha\rangle(\beta,\beta|$$

$$(z-\mathcal{Q}\mathcal{L}_B\mathcal{Q})^{-1} = \sum_{\alpha,\beta} \frac{1}{z-E_\alpha+E_\beta} |\alpha,\beta\rangle(\alpha,\beta|$$

$$\mathcal{L}_V = \sum_{\alpha,\beta,\gamma} [\langle\alpha|V|\gamma\rangle|\alpha,\beta\rangle(\gamma,\beta| + \langle\gamma|V|\beta\rangle|\alpha,\beta\rangle(\alpha,\gamma|], \quad (\text{C2})$$

where $\mathcal{P}=1-\mathcal{Q}$ and E_α is the eigenenergy corresponding to the eigenstate $|\alpha\rangle$. Using these results we find

$$\begin{aligned} \Sigma_2(z) &= 2 \sum_{\alpha,\beta} \langle\alpha|\rho_B|\alpha\rangle|\langle\alpha|V|\beta\rangle|^2 \\ &\quad \times \left(\frac{1}{z-E_\alpha+E_\beta} + \frac{1}{z-E_\beta+E_\alpha} \right). \end{aligned} \quad (\text{C3})$$

Comparing this expression with the time-dependent correlation

$$\langle V(t)V \rangle = \text{Tr}(\rho_B e^{itH} V e^{-itH} V) = \sum_{\alpha,\beta} \langle\alpha|\rho_B|\alpha\rangle|\langle\alpha|V|\beta\rangle|^2 e^{it(E_\alpha-E_\beta)} \quad (\text{C4})$$

we infer that the Laplace transform of $4\text{Re}\langle V(t)V \rangle$ is $\Sigma_2(z)$ as given by (10).

- ¹A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. **46**, 211 (1981); Ann. Phys. **149**, 374 (1983).
- ²U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 1993).
- ³N. V. Prokof'ev and P. C. E. Stamp, Rep. Prog. Phys. **63**, 669 (2000).
- ⁴W. A. Coish and D. Loss, Phys. Rev. B **70**, 195340 (2004).
- ⁵J. M. Taylor, H.-A. Engel, W. Dur, A. Yacoby, C. M. Marcus, P. Zoller, and M. D. Lukin, Nat. Phys. **1**, 177 (2005); J. M. Taylor, J. R. Petta, A. C. Johnson, A. Yacoby, C. M. Marcus, and M. D. Lukin, cond-mat/0602470 (unpublished).
- ⁶X. Hu, cond-mat/0411012 (unpublished).
- ⁷J. Shao and P. Hänggi, Phys. Rev. Lett. **81**, 5710 (1998).
- ⁸K. M. Forsythe and N. Makri, Phys. Rev. B **60**, 972 (1999).
- ⁹S. Paganelli, F. de Pasquale, and S. M. Giampaolo, Phys. Rev. A **66**, 052317 (2002).
- ¹⁰L. Tessieri and J. Wilkie, J. Phys. A **36**, 12305 (2003).
- ¹¹X.-Z. Yuan and K.-D. Zhu, Europhys. Lett. **69**, 868 (2005).
- ¹²J. Lages, V. V. Dobrovitski, M. I. Katsnelson, H. A. De Raedt, and B. N. Harmon, Phys. Rev. E **72**, 026225 (2005).
- ¹³K. H. Fischer and J. A. Hertz, *Spin Glasses* (Cambridge University Press, Cambridge, 1991).
- ¹⁴R. A. Jalabert and H. M. Pastawski, Phys. Rev. Lett. **86**, 2490 (2001).
- ¹⁵Z. P. Karkuszewski, C. Jarzynski, and W. H. Zurek, Phys. Rev. Lett. **89**, 170405 (2002).
- ¹⁶D. Rossini, T. Calarco, V. Giovannetti, S. Montangero, and R. Fazio, cond-mat/065051 (unpublished).
- ¹⁷S. Dattagupta, H. Grabert, and R. Jung, J. Phys.: Condens. Matter **1**, 1405 (1989).
- ¹⁸C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Processus d'interaction entre photons et atomes* (CNRS Editions, Paris, 1988).
- ¹⁹M. Thomsen, M. F. Thorpe, T. C. Choy, and D. Sherrington, Phys. Rev. B **30**, 250 (1984).
- ²⁰H.-P. Breuer, D. Burgarth, and F. Petruccione, Phys. Rev. B **70**, 045323 (2004).
- ²¹M. Thomsen, M. F. Thorpe, T. C. Choy, D. Sherrington, and H. J. Sommers, Phys. Rev. B **33**, 1931 (1986).
- ²²T. Plefka, Phys. Rev. B **65**, 224206 (2002).
- ²³M. Barma, Solid State Commun. **30**, 11 (1979).
- ²⁴S. Kirkpatrick and D. Sherrington, Phys. Rev. B **17**, 4384 (1978).
- ²⁵K. Nemoto, J. Phys. C **20**, 1325 (1987).