## **Electron-phonon interaction in a quantum wire in the Bloch-Gruneisen regime**

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We investigate the one-dimensional electron gas (1DEG) interaction with three-dimensional (3D) acoustic phonons in a quantum wire (QWR) by calculating the temperature  $T$  and electron concentration  $n_l$  dependence of phonon drag thermopower *S<sup>g</sup>* and energy loss rate *P* in the Bloch-Gruneisen regime taking account of static screening. Electron scattering by acoustic phonons through piezoelectric field and deformation potential is considered. At very low temperature, *S<sup>g</sup>* and *P* are dominated by the contribution due to piezoelectric scattering. The contribution due to deformation potential becomes significant at relatively higher temperature. The power laws for *T* and  $n_l$  dependences of  $S<sup>g</sup>$  and *P* are obtained. The screening affects weakly the power of *T* and significantly the power of  $n_l$ . The interesting feature is that  $S^g$  is reduced by a factor of  $(\nu_s/\nu_F)^2$ , where  $\nu_s$ is the sound velocity and  $\nu_F$  is the Fermi velocity, compared to that in two-dimensional electron gas (2DEG). A qualitative comparison is made between the calculated and experimentally observed energy loss rate in etched InGaAs QWRs. Herring's law  $S^g \mu_p \sim T^{-1} (\mu_p)$  is the phonon limited mobility) is validated in QWR. Our results are compared with those in 2DEG.

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### **I. INTRODUCTION**

In this work we investigate the one-dimensional electron coupling with three-dimensional acoustic phonons in a quantum wire (QWR) in the Bloch-Gruneisen (BG) regime by studying the phonon drag thermopower *S<sup>g</sup>* and hot electron energy loss rate *P*. Both *S<sup>g</sup>* and *P* are sensitive probes of electron-acoustic phonon interactions. The system enters into the BG regime at a very low temperature *T* when acoustic phonon wave vector  $q \ll 2k_F$  (where  $k_F$  is the Fermi wave vector).<sup>[1](#page-5-1)[–3](#page-5-2)</sup> In this regime, the acoustic phonon energy  $\hbar \omega_q$  $\approx k_B T$ . For these temperatures acoustic phonons of  $q \approx 2k_F$ are not excited appreciably and no longer contribute to the electron scattering.<sup>1,[3](#page-5-2)</sup> The characteristic temperature  $T_{BG}$  of the BG regime is roughly determined by the acoustic phonon energy  $2\hbar v_s k_F \approx k_B T_{BG}$  (where  $v_s$  is the velocity of the acoustic wave). When  $T \ll T_{BG}$  only scattering due to phonons with  $q \ll k_F$  will prevail.

In general there has been extensive experimental and theoretical work on both *S<sup>g</sup>* and *P* of two-dimensional electron gas (2DEG) in metal-oxide-semiconductor field-effect transistors (MOSFETs) and GaAs/GaAlAs heterostructures and they are largely well understood. $4-15$  In the BG regime of 2DEG the temperature dependence of  $S<sup>g</sup>$  and  $P$  are found to obey a simple power law  $S^g \sim T^6$  and  $T^4$  (Refs. [5–](#page-5-5)[10](#page-5-6)) and  $P \sim T^7$  $P \sim T^7$  and  $T^5$ ,<sup>7[,11](#page-5-8)[,13](#page-5-9)[–15](#page-5-4)</sup> respectively, due to the screened acoustic phonon deformation potential and piezoelectric scattering in the clean limit. In both the cases screening gives an extra factor of  $T^2$  dependence. Besides, the 2D electron density  $n<sub>s</sub>$  dependence of  $S<sup>g</sup>$  and P are given by the power law *S<sup>g</sup>*,  $P \sim n_s^{-3/2}$ . [7,](#page-5-7)[9](#page-5-10)[,15](#page-5-4) In one-dimensional electron gas (1DEG) there are a few calculations of  $S<sup>g</sup>$  (Refs. [16](#page-5-11) and [17](#page-5-12)) and *P*.<sup>[18](#page-5-13)[,19](#page-5-14)</sup> Kubakaddi and Butcher<sup>16</sup> have carried out the calculations of *S<sup>g</sup>* in 1DEG for screened acoustic deformation and piezoelectric scattering for phonon wave vector component parallel to the length of wire  $q_{\parallel} = 2k_F$ . However, the behavior of  $S^g$  in the BG regime is not examined in the form of a simple power law dependence on the temperature and 1D electron concentration  $n_l$ . Das Sarma and Campos<sup>18</sup> have studied the power loss of 1DEG due to emission of acoustic phonon via screened deformation potential coupling. The *T* dependence of *P* is shown to be *P*  $\sim \exp(-2\hbar v_s k_F/k_B T)$ , in contrast to the simple power law in 2DEG. The authors attribute this behavior to the dominance of  $q_{\parallel} = 2k_F$  scattering at low temperatures because of the peculiar nature of the one-dimensional Fermi surface. However, as we have already mentioned, in the low-temperature regime we consider in this work, phonons with  $q_{\parallel} = 2k_F$  do not contribute to the electron-phonon coupling. Shik and Challis<sup>19</sup> have calculated the power loss in the BG regime and obtained the simple power law of the temperature for unscreened electron-phonon interaction. In the present work, we give the calculations of  $S^g$  and  $P$  in the BG regime where  $q \ll 2k_F$  by studying their *T* and  $n_l$  dependence with the screened electron-acoustic phonon interaction via piezoelectric coupling and deformation potential coupling.

### **II. THEORY**

We model 1DEG confined in *x* and *y* directions with cross-section dimension *a* in an infinite potential well. The electron wave vector is  $k = k_z$  in one dimension. The one electron wave function is  $\psi = L^{-1/2} e^{ikz} \varphi(r)$ , where *L* is the normalization length (of the wire),  $\varphi(r)$  is the eigenfunction in the direction of quantization, and  $r(=x, y)$  is the 2D position vector. Only the lowest subband is assumed to be occupied by the electrons. Electrons are assumed to interact with the 3D phonons of wave vector  $q = (q_\perp, q_\parallel)$ , where  $q_\perp$  is the component of *q* in the *x*-*y* plane and  $q_{\parallel}$  is the component along the length of the wire.

As in 2DEG, the calculations of both *S<sup>g</sup>* and *P* involve transition probability for phonon absorption $16,18$  $16,18$ 

<span id="page-1-0"></span>
$$
P_q^a(k, k') = \left(\frac{2\pi}{\hbar}\right) \frac{N_{qs}|M(k, k')|^2 |F|^2}{|\varepsilon(q_{\parallel})|^2} \delta(E_{k'} - E_k - \hbar \omega_q) \delta_{k', k+q_{\parallel}},
$$
\n(1)

where  $N_{as}$  is the *s* mode phonon Bose distribution function,  $M(k, k')$  is the matrix element of the electron-acoustic phonon interaction, and  $\varepsilon(q_{\parallel}) = 1 - V(q_{\parallel}) \chi_0(q_{\parallel})$  is the static screening function.<sup>20[,21](#page-5-16)</sup> In  $\varepsilon(q_{\parallel})$ ,  $V(q_{\parallel})$  is the Coulomb interaction between the electrons and  $\chi_0(q_\parallel)$  is the static noninteracting polarizability function of the 1DEG. Their full forms are given in the Appendix. The form factor  $|F|^2$  $= | \int d^2r \varphi^*(r) e^{+iq} \psi(r) |^2$  accounts for the *e-p* interaction in the direction of confinement. In the BG regime  $q_{\perp}$  of the phonon interacting with the electrons is limited by  $\hbar v_s q_{\perp}$  $\sim k_B T$ . If  $r_0$  is the confining length, then  $q_{\perp} r_0$  $\sim k_B T / [(2)^{1/2} (\hbar v_s / r_0)] \ll 1$ . Then  $F \approx 1$  and the transition probability does not depend on details of confinement in the BG regime.

As discussed earlier, at very low *T*, only transitions corresponding to  $q_{\parallel} \ll 2k_F$  are to be considered. The Dirac  $\delta$ function in Eq. ([1](#page-1-0)) gives the energy conservation law  $E_{k'}$ function in Eq. (1) gives the energy conservation law  $E_{k'}$ <br>  $-E_k - \hbar \omega_q = 0$ , which leads to  $q_{\parallel} = [v_s/(\hbar k/m)]q$  for  $q_{\parallel} \ll k$ . The coefficient of *q*, i.e.,  $\nu_s / (\hbar k / m) \ll 1$  for degenerate 1DEG. Then the energy conservation law can be fulfilled if  $q_{\parallel} \ll q_{\perp}$  and hence

$$
q_{\parallel} = \frac{\nu_s}{(\hbar k/m)} q_{\perp}.
$$
 (2)

It means that energy conservation leads to a highly anisotropic emission or absorption of phonons in the direction perpendicular to the wire making an angle  $\theta = mv_s/\hbar k_F$  with  $q_{\perp}$  with the width  $\theta$   $(k_B T / E_F)^{1/2}$ .<sup>[3](#page-5-2)[,19](#page-5-14)</sup>

In the BG regime the following simplifications can be made. Both the equations for  $S<sup>g</sup>$  (Refs. [16](#page-5-11) and [17](#page-5-12)) and P (Ref. [18](#page-5-13)) contain a product of electron occupation factors  $f^{\circ}(E_k)[1 - f^{\circ}(E_k + \hbar \omega_q)]$  and it can be approximated by  $\hbar \omega_q (N_q + 1) \delta(E_k - E_F)$  for  $\hbar \omega_q \ll E_F$ . In the long wavelength limit  $V(q_{\parallel}) \cong 2e^2 |\ln(q_{\parallel}a)|/\varepsilon_0$  (Ref. [20](#page-5-15)) and  $\chi_0(q_{\parallel})$  $=-(2m/\pi\hbar^2 k_F)^{21}$  $=-(2m/\pi\hbar^2 k_F)^{21}$  $=-(2m/\pi\hbar^2 k_F)^{21}$  where  $\varepsilon_0$  is the dielectric constant (see Appendix for details). Then the static screening function reduces to the form<sup>18</sup>

$$
\varepsilon(q_{\parallel}) \cong 1 + \frac{4e^2m}{\varepsilon_0 \pi \hbar^2 k_F} |\ln(q_{\parallel} a)|. \tag{3}
$$

The matrix element for the electron-phonon interaction is described by  $16,19$  $16,19$ 

$$
|M(k,k')|^2 = Cq^{\gamma},\tag{4}
$$

where  $C = (\hbar E_l^2 / 2 \rho V \nu_l)$  for the deformation potential coupling with  $\gamma = +1$  and  $C = \hbar (eh_{14})^2 A_s / (2\rho V v_s)$  for piezoelectric scattering with  $\gamma = -1$ ,  $A_l = 9q_\perp^4 q_\parallel^2 / 2q^6$  and  $A_t = (8q_\perp^2 q_\parallel^4)$  $+q_{\perp}^{6}$ //4 $q^{6}$ , respectively, for the longitudinal *(s=l)* and transverse  $(s=t)$  modes. Here  $E_l$  is the deformation potential constant,  $h_{14}$  is the piezoelectric constant, and  $\rho$  is the density of the material.

In what follows we obtain  $S^g$  and  $P$  due to piezoelectric and deformation potential scattering using the above ap<span id="page-1-1"></span>proximations. Following Kubakaddi and Butcher,<sup>16</sup> the phonon drag thermopower in the BG regime is

$$
S_p^g = -\frac{k_B m^2 \Lambda (e h_{14})^2 (k_B T)^2}{2 n_l \rho \hbar^5 k_F^2 \pi^2} \sum_s \frac{A_s}{\nu_s^3} \left(\frac{\nu_s}{\nu_F}\right)^2
$$

$$
\times \int_0^\infty \frac{x^3 dx}{4 \sinh^2(x/2) |\varepsilon(x, T)|^2}
$$
(5)

due to piezoelectric scattering and

<span id="page-1-3"></span>
$$
S_d^g = -\frac{k_B}{|e|} \frac{m^2 \Lambda E_1^2 (k_B T)^4}{2n_B \hbar^7 k_F^2 \pi^2} \sum_s \frac{1}{\nu_s^5} \left(\frac{\nu_s}{\nu_F}\right)^2 \int_0^\infty \frac{x^5 dx}{4 \sinh^2(x/2) |\varepsilon(x, T)|^2}
$$
(6)

due to deformation potential scattering. Here  $\Lambda$  is the phonon mean free path,  $\hbar k_F = m \nu_F$ ,  $A_t \approx 1/2$ ,  $A_l \approx (9/2)(\nu_s/\nu_F)^2$ , and  $\epsilon(x,T) \approx 1 + (4e^2m/\epsilon_0 \pi \hbar^2 k_F) |\ln(ak_B T x/\hbar \nu_F)|$ . We have used  $x = (\hbar v_s / k_B T) q_{\perp}$ . We see that the screening function adds only logarithmic correction to the temperature dependence.

In the BG regime, the power loss of electrons at electron temperature *T*, following Ma *et al.*,<sup>[13](#page-5-9)</sup> is given by

$$
P = F(T) - F(T_l),\tag{7}
$$

<span id="page-1-4"></span>where  $T_l$  is the lattice temperature,

$$
F_p(T) = \frac{m^2 (e h_{14})^2 (k_B T)^3}{n_l \rho \pi^2 \hbar^5 k_F^2} \sum_s \frac{A_s}{\nu_s^2} \int_0^\infty \frac{x^2 dx}{(e^x - 1)|\varepsilon(x, T)|^2} \quad (8)
$$

<span id="page-1-2"></span>for piezoelectric scattering, and

$$
F_d(T) = \frac{m^2 E_1^2 (k_B T)^5}{n_l \rho \pi^2 \hbar^7 k_F^2} \sum_s \frac{1}{\nu_s^4} \int_0^\infty \frac{x^4 dx}{(e^x - 1)|\varepsilon(x, T)|^2} \qquad (9)
$$

for deformation potential scattering.

### **III. RESULTS AND DISCUSSION**

From Eqs.  $(5)-(9)$  $(5)-(9)$  $(5)-(9)$  $(5)-(9)$  $(5)-(9)$  we see that, because of the logarithmic  $q_{\perp}$  dependence of the screening function,  $S^g$  and P are not expressed by a simple power law of *T*. If the weak dependence of  $\varepsilon$  on  $q_{\perp}$  and hence on *T* (with  $q_{\perp} \approx k_B T / \hbar v_s$ ) is ignored, then the integrals in Eqs.  $(5)$  $(5)$  $(5)$ ,  $(6)$  $(6)$  $(6)$ ,  $(8)$  $(8)$  $(8)$ , and  $(9)$  $(9)$  $(9)$  give constant values. This leads to the simple power laws,  $S_p^g$  $\sim T^2$  and  $P_p \sim T^3$  for piezoelectric scattering and  $S_d^g \sim T^4$  and  $P_d \sim T^5$  for deformation potential scattering. These are in contrast with the power laws in 2DEG:  $S^g \sim T^4$  and  $P \sim T^5$ for piezoelectric scattering and  $S^g \sim T^6$  and  $P \sim T^7$  for deformation potential scattering. These differences have the origin in the respective screening functions. In 2DEG,  $\varepsilon \sim q_{\perp}^{-1}$ , in the long wavelength limit, gives extra power of 2 to *T* in *S<sup>g</sup>* and *P*.

The numerical calculations of *S<sup>g</sup>* and *P* are performed for a GaAs/Ga*x*Al1−*x*As QWR with the material parameters *m*  $= 0.067 m_0$ ,  $\varepsilon_0 = 11.97$ ,  $h_{14} = 1.2 \times 10^7$  V/cm,  $E_1 = 11.5$  eV,  $\nu_l$  $= 5.12 \times 10^5$  cm/s,  $v_t = 3.01 \times 10^5$  cm/s, and  $a = 100$  A. In Fig. [1,](#page-2-0) we have shown *S<sup>g</sup>* as a function of temperature *T* in the range  $0.1-1.0$  K for  $n_l$ = $2.0\times10^6$  cm<sup>-1</sup>. The characteristic temperature  $T_{BG}$ = 14.44 K and 24.67 K, respectively, for

<span id="page-2-0"></span>

FIG. 1. The phonon-drag thermopower *S<sup>g</sup>* as a function of temperature *T* in a GaAs/GaAlAs QWR with  $n_l = 2.0 \times 10^6$  cm<sup>-1</sup>. Curve *a* is due to screened deformation potential scattering, curve *b* is due to screened piezoelectric scattering, curve *c* is the total contribution due to screened piezoelectric and deformation potential scattering, and curve *d* is the total contribution due to the unscreened piezoelectric and deformation potential scattering.

transverse and longitudinal acoustic phonons. We find that  $S_p^g$ due to screened piezoelectric scattering (curve b) is dominant over  $S_d^g$  due to screened deformation potential scattering (curve *a*) at very low *T*. However,  $S_d^g$  becomes equally important at  $T \approx 1$  K and dominates over  $S_p^g$  in a relatively high-*T* region. The dominance of  $S_p^g$  in a very low-*T* region is consistent with the results in  $2DEG$ .<sup>6[,8](#page-5-18)[–10](#page-5-6)</sup> The contribution due to piezoelectric scattering continues to dominate up to  $T \sim 3$  K in 2DEG.<sup>6[,9](#page-5-10)[,22](#page-5-19)</sup> We discuss the possible reason for this difference later. It is found that  $S_p^g \sim T^{2.24}$  and  $S_d^g \sim T^{4.25}$ . The weak temperature dependence of screening has increased the power of *T* by about 0.25. We have also shown the total  $S^g$  due to screened interaction (curve  $c$ ) and unscreened interaction (curve *d*). Screening is found to reduce *S<sup>g</sup>* approximately by an order of magnitude. The interesting feature of our calculations is that *S<sup>g</sup>* is being reduced by a factor  $(\nu_s/\nu_F)^2$  compared to that in 2DEG. This reduction is consistent with the observation made with respect to the momentum relaxation rate in  $1DEG<sup>3</sup>$  It may be noted that the phonon drag thermopower depends on the momentum relax-ation due to electron-phonon interaction.<sup>7,[9](#page-5-10)</sup>

It would be interesting to compare phonon drag thermopower  $S^g$  with the diffusion thermopower  $S^d$ . It can be shown that, for 1DEG,  $S^d$  is given by the Mott formula  $S^d = -(\pi^2 k_B^2 T/3|e|E_F)(p+1/2)$ , where *p* is the scattering pa-rameter of the order of 1.<sup>4[,7,](#page-5-7)[16](#page-5-11)</sup> For  $n_l$ = 2.0 × 10<sup>6</sup> cm<sup>-1</sup>, at *T* = 1 K, we find  $S^d$  ≈ -0.2  $\mu$ V/K with *p*=0. This value is very much large compared to  $S^g \approx -1.0 \times 10^{-5} \mu V/K$ . The suppression of  $S^g$  may help for the experimental study of  $S^d$ which is sensitive to the scattering mechanisms through the scattering parameter *p*.

Electron power loss  $P$  as a function of  $T$  (0.1–1 K) is shown in Fig. [2,](#page-2-1) for  $n_l = 2.0 \times 10^6$  cm<sup>-1</sup>. Again, in the temperature region of interest considered here,  $P_p$  due to piezoelectric scattering (curve *b*) is dominant in the very low-*T* 

<span id="page-2-1"></span>

FIG. 2. The electron power loss *P* as a function of temperature *T* in a GaAs/GaAlAs QWR with  $n_l$ = 2.0  $\times$  10<sup>6</sup> cm<sup>-1</sup>. Symbols on the curves represent the same as in Fig. [1.](#page-2-0)

region and  $P_d$  due to deformation potential scattering (curve *a*) is becoming significant in the high-*T* region. However, in 2DEG, *P* due to piezoelectric scattering still remains dominant in this high-*T* regime.<sup>8[,15](#page-5-4)</sup> The *T* dependence of *P* is found to be  $P_p \sim T^{3.23}$  (curve *b*) and  $P_d \sim T^{5.25}$  (curve *a*). The total power loss  $P$  due to these two mechanisms (curve  $c$ ) is smaller approximately by an order of magnitude compared to the total  $P$  due to the unscreened couplings (curve  $d$ ). It is to be noted that *P* in 1DEG is comparable to that in 2DEG[.7,](#page-5-7)[13](#page-5-9)[,15](#page-5-4)

It is worth noting that there exist some experimental results of temperature dependence of energy relaxation rate  $\tau_e^{-1}$ (Ref. [23](#page-5-20)) and electron energy loss rate<sup>24[,25](#page-6-0)</sup> in etched InGaAs QWRs. The energy relaxation time  $\tau_e^{-1}$  is proportional to  $(dP/dT)(1/C)$ , where *C* is the electronic specific heat. Sugaya *et al.*[23](#page-5-20) have determined the energy relaxation time of wet-etched InGaAs QWRs from magneto-resistance measurements and found  $\tau_e^{-1} \sim T^3$  behavior over the temperature between 2 to 5.2 K, which corresponds to  $P \sim T^{5}$ .<sup>[11](#page-5-8)[,14](#page-5-22)</sup> This behavior is agreeing with our calculation for deformation potential scattering which is dominant at higher temperatures. A deviation from this behavior is observed from the low temperature data which is attributed, from the simple estimates, to the possibility of the increased importance of phonon confinement at low temperatures.<sup>23</sup>

Energy loss rate measurements on etched InGaAs QWRs of widths 600 nm and 25 nm show  $P \sim T^5$  and  $T^{1.4}$ , respectively.<sup>24</sup> The  $T^5$  behavior is again agreeing with our calculations of deformation potential scattering. The  $T^{1.4}$  behavior is argued to result from the saturation of electron back scattering processes in the highest occupied 1D subband. $24$ However, there are no indications of phonon confinement in these observations. In recent measurements,  $25$  in InGaAs QWRs, the rapid decay of the energy loss rate is observed for  $T$  $7 \times 7$  K indicating the possible exponential suppression.<sup>18</sup> For  $T>7$  K,  $P \sim T^n$  dependence is seen with  $n \approx 4$  for their WA and WB samples (wider wires) and  $n \approx 3$  for the WC sample (narrower wire). The smaller observed value of *n* is suggested to be due to possible phonon confinement in the

<span id="page-3-0"></span>TABLE I. The temperature *(T)* dependence of momentum relaxation rate  $(\tau_m^{-1})$ , phonon drag thermopower (*S*<sup>*g*</sup>), and energy loss rate *(P)* expressed in the form of power law  $\tau_m^{-1}$ , *S*<sup>*g*</sup>, and *P* $\sim$ *T*<sup>*n*</sup> are given below for different cases of 1D and 2D electron interactions with the 3D acoustic phonons. The power law for energy relaxation rate  $\tau_e^{-1}$  can be obtained by reducing the powers of *T* in *P* by 2.

Electron-phonon scattering mechanisms	$\tau_m^{-1}$		$S^g$		P	
	1DEG	$2$ DEG	1DEG	$2$ DEG	1DEG	$2$ DEG
Screened piezoelectric coupling	$T^{3.23a}$	$T^{5b}$	$T^{2.24c}$	$T^{4d}$	$T^{3.23c}$	$T^5$ e
Screened deformation potential coupling	$T^{5.25a}$	$T^{7b}$	$T^{4.25c}$	$T^{6d}$	$T^{5.25c}$	T <sup>7e</sup>
Unscreened piezoelectric coupling	$T^{3f}$	$T^3g$	$T^{2c}$	$T^{2d}$	$T^3c$	$T^3$ e
Unscreened deformation potential coupling	$T^{5f}$	$T^5$ g	$T^{4c}$	$T^{4d}$	$T^5c$	$T^{5e}$

a Obtained from the results of Ref. [3](#page-5-2) assuming that screening enhances the exponent of *T* by the same magnitude as in *S<sup>g</sup>* and *P*.

b Reference [2.](#page-5-23) <sup>c</sup>T dependences obtained in the present work. d References [5–](#page-5-5)[10.](#page-5-6) e References [7,](#page-5-7) [11,](#page-5-8) and [13–](#page-5-9)[15.](#page-5-4) f Reference [3.](#page-5-2) gReference [26.](#page-6-2)

narrower wire. $25$  However, this behavior is observed for piezoelectric scattering by 3D phonons in our calculations. The exponents seen in the experimental work vary over a wide range of values. Sugaya *et al.*[23](#page-5-20) and Prasad *et al.*[25](#page-6-0) suggest for more extended measurements to confirm the possibility of phonon confinement at very low temperatures and to verify exponential suppression of power loss which is not observed in other such experiments.<sup>23[,24](#page-5-21)</sup>

In Table [I,](#page-3-0) we have listed the *T* dependences of momentum relaxation rate  $\tau_m^{-1}$ ,  $S^g$ , and *P* in 1DEG and 2DEG for screened and unscreened piezoelectric and deformation potential scattering by 3D acoustic phonons. We note that power laws for momentum relaxation time and energy loss rate are same. The *T* dependences of  $\tau_m^{-1}$  for screened electron-phonon interactions in 1DEG are obtained from the results of Ref. [3](#page-5-2) assuming that screening enhances the exponent of  $T$  by the same magnitudes as in  $S^g$  and  $P$ . It is to be noted that unscreened coupling in 2D systems and screened and unscreened coupling in the 1D system in our theory give nearly the same *T* dependence.

In 3DEG, Herring's formula relates  $S<sup>g</sup>$  with phonon limited mobility  $\mu_p$  and this relation is  $S^g \mu_p \sim T^{-1}$ .<sup>[27](#page-6-1)</sup> It is true in 2DEG as well.<sup>7[,9](#page-5-10)</sup> In 1DEG,  $\mu_p \sim T^{-3}$  and  $T^{-5}$  respectively, for unscreened piezoelectric and deformation potential scattering.<sup>3</sup> For these unscreened cases, using Eqs.  $(5)$  $(5)$  $(5)$  and ([6](#page-1-3)), we can see that  $S_p^g \sim T^2$  and  $S_d^g \sim T^4$ . Then,  $S^g \mu_p \sim T^{-1}$  (for both the mechanisms) validating the Herring's law in 1DEG.

In 2DEG  $S^g$  and *P* are shown to be related by *P* =−*S<sup>g</sup> eT*/, [7](#page-5-7) where *v* is a suitable average velocity of the sound and  $\xi$  is a numerical constant of the order of unity. In 1DEG we also find a similar relation between *P* and *S<sup>g</sup>* : *P*  $=-\xi S^g \nu |e| (\nu_F/\nu)^2 T/\Lambda$  differing by a factor  $(\nu_F/\nu)^2$  with  $\xi$  $\approx 0.6$ .

It is interesting to discuss the  $\nu_s$  dependence of  $S^g$  and P in QWR. In piezoelectric coupling both longitudinal and transverse acoustic phonons are involved, and in deformation potential coupling only longitudinal acoustic phonons are involved. In 1DEG, we find the dependences of *S<sup>g</sup>* and *P* on acoustic phonon velocity  $\nu_s$  to be  $S_p^g \sim \nu_s^{-1}$ ,  $S_d^g \sim \nu_s^{-3}$ ,  $P_p$  $\sim \nu_s^{-2}$ ,  $P_d \sim \nu_s^{-4}$ . We observe that the contribution to *S<sup>g</sup>* and *P* due to piezoelectric scattering by transverse acoustic phonons is dominant over the contribution due to piezoelectric scattering by longitudinal acoustic phonons because of the smaller velocity of the transverse acoustic phonons. In 2DEG, the respective dependences of *S<sup>g</sup>* and *P* are  $S_p^g \sim \nu_s^{-5}$ ,  $S_d^g \sim \nu_s^{-7}$ ,  $P_p \sim \nu_s^{-4}$ ,  $P_d \sim \nu_s^{-6}$  $P_d \sim \nu_s^{-6}$  $P_d \sim \nu_s^{-6}$ . *Po*-10[,15](#page-5-4) In 1DEG, the reduced power of  $\nu_s$ , appearing in the denominator of  $S^g$  and *P*, may make the contribution due to deformation potential scattering to become significant at  $\sim$ 1 K and above, compared to  $\sim$ 3 K and above in 2DEG.

The behavior of  $S^g$  and *P* as a function of  $n_l(=1-5)$  $\times$  10<sup>6</sup> cm<sup>-1</sup>) are shown in Fig. [3](#page-4-0) and Fig. [4,](#page-4-1) respectively, at *T*= 1.0 K. The *n<sub>l</sub>* dependences are found to be  $S_p^g \sim n_{l}^{-4.01}$  and  $P_p \sim n_l^{-1.97}$  for piezoelectric scattering, and  $S_d^{\xi} \sim n_l^{-4.08}$  and  $P_d^r \sim n_l^{-2.06}$  for deformation potential scattering. The powers of  $n_l$  due to both the mechanisms are almost the same. This observation is similar to that in  $2DEG<sup>7,9</sup>$  $2DEG<sup>7,9</sup>$  $2DEG<sup>7,9</sup>$  However, these powers are in contrast with  $S^g$  and  $P \sim n_l^{-1.5}$  in 2DEG.<sup>7[,10](#page-5-6)[,15](#page-5-4)</sup> In 1DEG, the factor  $(v_s/v_F)^2$  in  $S^g$  increases the negative power of  $n_l$  by 2. In 2DEG the power of  $n_s$  is not affected by the screening. In 1DEG, the screening function is  $n_l$  dependent and screening is found to decrease the negative power of  $n_l$  by about 1 in  $S^g$  and *P*. When screening is ignored,  $S^g \sim n_l^{-5}$  and  $P \sim n_l^{-3}$ .

It is believed that for lattice matched embedded structures such as GaAs/Ga<sub>1-*x*</sub>Al<sub>*x*</sub>As quantum wells, QWRs and gate

<span id="page-4-0"></span>

FIG. 3. The phonon-drag thermopower  $S^g$  as a function of electron concentration  $n_l$  in a GaAs/GaAlAs QWR at temperature 1.0 K. Symbols on the curves represent the same as in Fig. [1.](#page-2-0)

confined quantum wires the bulk phonon approximation is adequate. However, in embedded structures with an acoustic mismatch (e.g., GaAs/AlAs QWRs) confined phonons, interface modes and extended modes are expected. $28,29$  $28,29$  Also acoustic phonon confinement is expected in the sample with the dimension smaller than the phonon thermal wevelength  $\sim (k_B T/\hbar v_s)^{-1}$  and the phonon coherence length. In a QWR, if phonons are confined in two transverse directions, then conventionally the exponent is expected to reduce by 2. For example, for *P*, this effect will give *T* and  $T^3$  behavior, respectively, for unscreened piezoelectric and deformation potential coupling in our calculations. But, it is also to be noted that acoustic phonon confinement leads to modifications in the dispersion and the density of modes. $28,29$  $28,29$  These modifications may be taken into account in the calculations and to analyze the observed behavior due to the possible confinement. From the calculations of the scattering rates in an embedded QWR with acoustic mismatch it is shown that the deformation coupling between electrons and confined acoustic phonons is extremely small and the magnitude and electron energy dependence of the scattering rates due to extended modes are almost same as those due to the usual three-dimensional bulk phonons.<sup>29</sup>

Early studies in Si MOSFET show anomalously strong attenuation for ballistic phonon propagation<sup>30</sup> and it is accounted for by the specular reflection of the phonons at the  $Si-SiO<sub>2</sub>$  interface.<sup>31</sup> Besides, experimentally, a study of the reflection of phonons at the interface in a Si MOSFET and GaAs/GaAlAs heterojunction has been made.<sup>32</sup> Theoretically, in a study of acoustic phonon emission in Si  $MOSFET^{33}$  and GaAs/GaAlAs heterostructures<sup>34</sup> and absorption of electromagnetic wave in Si MOSFET, $35$  the effect of phonon reflection at the interface is shown to play an important role. The energy relaxation measurements in 2D systems of  $Ge/Si_{0.4}Ge_{0.6}(Si/Si_{0.7}Ge_{0.3})$  show broadly 2D (3D) behavior of phonons indicating the total internal reflection in the Ge sample. $36$ 

However, in later studies, the interpretation of most of the experimental data on the 2DEG transport properties, involv-

<span id="page-4-1"></span>

FIG. 4. The electron power loss *P* as a function of electron concentration  $n_l$  in a GaAs/GaAlAs OWR at temperature 1.0 K. Symbols on the curves represent the same as in Fig. [1.](#page-2-0)

ing electron-phonon interaction, was successful without taking account of the acoustic phonon reflection and interface modes and the bulk phonon approximation is found to be adequate.<sup>37</sup> For example, a detailed experimental and theoretical study of  $S^g$  and  $P$  (Ref. [7](#page-5-7)) and phonon emission study<sup>38</sup> in Si MOSFET have not indicated the phonon reflection at the interface. Similarly, phonon reflection at the interface of GaAs/GaAlAs 2D systems is not found in the experimental and theoretical study of  $S^g$  and *P* (Refs. [4–](#page-5-3)[6](#page-5-17) and  $8-15$  $8-15$ ) and phonon emission.<sup>39,[40](#page-6-15)</sup> The consideration of the above effects on the  $S^g$  and  $P$  of 1DEG in the BG regime could be examined in a future work.

### **IV. CONCLUSIONS**

In conclusion, phonon-drag thermopower  $S^g$  and electron power loss *P* are studied in a quantum wire to probe electron-phonon interaction in the Bloch-Gruneisen regime. The *S<sup>g</sup>* and *P* due to piezoelectric scattering are dominant over those due to the deformation potential scattering in a very low temperature region. Screening reduces the magnitude of *S<sup>g</sup>* and *P* by about an order of magnitude. Unlike the case of 2DEG, the screening increases the power of *T* by about 0.25. However, screening significantly affects the power law for electron concentration. The powers of *T* and  $n_l$ are different from those in 2DEG. More importantly, the *S<sup>g</sup>* is reduced by a factor of  $(\nu_s/\nu_F)^2$  compared to that in 2DEG. A qualitative comparison of our *P* calculations with the experimental results is made in etched InGaAs QWRs. These studies of phonon drag thermopower and electron power loss in the BG regime with the power law dependences on temperature and electron concentration may provide a better understanding of electron-acoustic phonon interactions than the mobility studies in QWRs.

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# **APPENDIX: SOME DETAILS OF DIELECTRIC FUNCTION OF ONE-DIMENSIONAL ELECTRON GAS**

<span id="page-5-25"></span>The static dielectric function for a QWR is given by $20,21$  $20,21$ 

$$
\varepsilon(q_{\parallel}) = 1 - V(q_{\parallel}) \chi_0(q_{\parallel}), \qquad (A1)
$$

where  $V(q_{\parallel})$  is

$$
V(q_{\parallel}) = (2e^{2}/\varepsilon_{o}) \int d^{2}r \int d^{2}r' \varphi^{2}(r) \varphi^{2}(r') K_{0}(q_{\parallel}|r-r'|),
$$
\n(A2)

with  $\varphi(r)$  being the electron envelope function and  $K_n$  the *n*th-order modified Bessel function of the second kind. The static polarizability function  $\chi_0(q_\parallel)$  is given by

$$
\chi_0(q_{\parallel}) = 2 \sum_{k} \frac{f^0(k+q_{\parallel}) - f^0(k)}{E_{k+q_{\parallel}} - E_k}.
$$
 (A3)

<span id="page-5-24"></span>For the case of degenerate 1DEG  $\chi_0(q_\parallel)$  takes the form<sup>21</sup>

$$
\chi_0(q_{\parallel}) = -\frac{2m}{\pi \hbar^2 q_{\parallel}} \ln \left| \frac{2k_F + q_{\parallel}}{2k_F - q_{\parallel}} \right|.
$$
 (A4)

If the square wire is approximated by a cylindrical wire of radius  $R$ , then the wave function is approximately constant inside and zero outside for such a cylindrical symmetry when the electron density is sufficiently high, $^{41}$  i.e.,

$$
|\varphi(r)|^2 = (\pi R^2)^{-1} Y(R - |r|), \tag{A5}
$$

where  $Y(x)$  is the step function. With this approximation, <sup>21[,41](#page-6-16)</sup>

$$
V(q_{\parallel}) = \frac{2e^2}{\varepsilon_0} \frac{2}{(q_{\parallel}R)^2} [1 - 2K_1(q_{\parallel}R)I_1(q_{\parallel}R)].
$$
 (A6)

Here,  $I_n$  is the *n*th-order modified Bessel function of the first kind.

For  $x \le 1$ ,  $K_1(x) \approx (1/x) + (x/2) \ln(x/2)$  and  $I_1(x) \approx x/2$ .<sup>42</sup> Hence, for a QWR of square cross section, in the long wavelength limit  $(q_{\parallel}a \ll 1),^{20}$  $(q_{\parallel}a \ll 1),^{20}$  $(q_{\parallel}a \ll 1),^{20}$ 

$$
V(q_{\parallel}) = (2e^2/\varepsilon_0)|\ln(q_{\parallel}a)|. \tag{A7}
$$

<span id="page-5-27"></span><span id="page-5-26"></span>Moreover, for  $q_{\parallel} \ll 2k_F$  the Taylor expansion of Eq. ([A4](#page-5-24)) gives $^{21}$ 

$$
\chi_0(q_{\parallel}) = -\frac{2m}{\pi \hbar^2 k_F}.\tag{A8}
$$

Finally, by using expressions  $(A1)$  $(A1)$  $(A1)$ ,  $(A7)$  $(A7)$  $(A7)$ , and  $(A8)$  $(A8)$  $(A8)$ , the dielectric function in the temperature range of our interest is written  $as^{18}$ 

$$
\varepsilon(q_{\parallel}) = 1 + \frac{4me^2}{\varepsilon_0 \pi \hbar^2 k_F} |\ln(q_{\parallel} a)|. \tag{A9}
$$

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