Competition between superconductivity and charge density waves

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We derive an effective field theory for the competition between superconductivity (SC) and charge density waves (CDWs) by employing the SO(3) pseudospin representation of the SC and CDW order parameters. One important feature in the effective nonlinear σ model is the emergence of a Berry phase even at half filling, originating from the competition between SC and CDWs, i.e., the pseudospin symmetry. A-well known conflict between the previous studies of Oshikawa [Phys. Rev. Lett. **84**, 1535 (2000)] and Lee and Shankar [Phys. Rev. Lett. **65**, 1490 (1990)] is resolved by the appearance of the Berry phase. The Berry phase contribution allows a deconfined quantum critical point of fractionalized charge excitations with e instead of $2e$ in the SC-CDW quantum transition at half filling. Furthermore, we investigate the stability of the deconfined quantum criticality against quenched randomness by performing a renormalization group analysis of an effective vortex action. We argue that, although randomness results in a weak disorder fixed point differing from the original deconfined quantum critical point, deconfinement of the fractionalized charge excitations still survives at the disorder fixed point owing to a nonzero fixed point value of the vortex charge.

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I. INTRODUCTION

Recently, it was proposed that, when there exist two competing orders characterized by different patterns of symmetry breaking, the two order parameters can acquire some topological Berry phases to allow a continuous quantum phase transition between the two states, although forbidden in the Landau-Ginzburg-Wilson (LGW) theoretical framework without fine tuning of couplings admitting multicritical points.[1,2] In particular, the quantum critical point in this quantum phase transition is quite exotic in the respect that elementary excitations are fractionalized; it is thus called a deconfined quantum critical point.[3,4]

One deconfined quantum critical point was demonstrated in the competition between antiferromagnetic (AF) and valance bond solid (VBS) orders.[3,4] Tanaka and Hu considered an SO(5) superspin representation including both the AF and VBS order parameters, and derived an effective nonlinear σ model for the SO(5) superspin variable from the spinon representation of the Heisenberg Hamiltonian.[1] One crucial feature in their effective field theory is the presence of a Berry phase for the superspin field. They demonstrated that the competition between AF and VBS orders is well described by the SO(5) nonlinear σ model with a topological Berry phase term.

In the present paper we consider another concrete example, the competition between superconductivity (SC) and charge density waves (CDWs), as a simplified version of the competition between AF and VBS. Introducing an SO(3) pseudospin representation to include both the SC and CDW order parameters, we derive an effective nonlinear σ model in terms of the O(3) pseudospin variable from the attractive Hubbard model. Interestingly, a Berry phase term naturally appears in this σ model, allowing a deconfined quantum critical point of fractionalized charge excitations with e instead of $2e$ as a result of the competition between SC and CDWs. Furthermore, we examine the stability of the deconfined quantum criticality against quenched randomness generating two kinds of random potentials, a random mass term and a random fugacity one in the effective vortex action [Eq. (16)]. Performing a renormalization group (RG) analysis of the vortex action [Eq. (16)] in the London approximation [Eq. (17)], we argue that deconfinement of the fractionalized excitations still survives although the presence of disorder leads to a new quantum critical point with finite disorder strength. We find that the stability of the deconfined quantum criticality originates from the existence of the charged critical point.

Before going further, it is valuable to address several important differences between the present work and previous studies. Earlier studies[5] revealed that the half-filled negative-U Hubbard model on a two-dimensional (2D) square lattice is mathematically equivalent to the positive-U Hubbard model, using the particle-hole transformation. This equivalence maps the XY ordered antiferromagnetic phase of the spin system that results for positive U to the superfluid phase of the negative-U problem. Likewise, the Ising antiferromagnet (for positive U) maps to a CDW phase (for negative U). However, in these earlier studies[5] the role of the Berry phase was not investigated clearly; thus the LGW-forbidden continuous transition and deconfined quantum critical points were not found in the context of SC-CDW transitions.

It is interesting to understand the origin of the Berry phase in the negative-U Hubbard model and the positive-U one. The positive-U Hubbard model reduces to the antiferromagnetic Heisenberg model in the large-U limit. In the negative-U Hubbard model the low-energy effective action can be mapped onto an effective model of hard-core lattice bosons with a hopping amplitude of order t^2/U and a repulsive nearest neighbor interaction of the same order in the strong coupling limit $U \rightarrow -\infty$.[6] One can show that this hard-core boson model is equivalent to the antiferromagnetic Heisenberg model, associated with charge degrees of freedom to form a pseudospin.[5,7] The Berry phase in the negative-U Hubbard model originates from the pseudospin (charge) SU(2) symmetry[5] while in the positive-U Hubbard model it comes from the spin SU(2) symmetry. It should be noted that this topological phase appears even at half filling. On the other hand, it was not allowed at half filling in recent studies.[8,9] The Berry phase resulting from the chemical potential in the boson Hubbard-type model[8,9] is different from the present one because the presence of the chemical potential reduces the SU(2) pseudospin symmetry to the U(1) one. This is the reason why there exists only the Berry phase coming from the chemical potential in the boson Hubbard-type model while our effective action has both Berry phases resulting from the SU(2) pseudospin symmetry and the chemical potential. In other words, the competition between SC and CDWs results in a nontrivial Berry phase term even at half filling. Thus, the chemical potential plays the role of an additional Berry phase in the present effective theory. Furthermore, the appearance of the Berry phase at half filling allows other possible disordered phases corresponding to valence bond orders in the pseudospin language. This resolves the well-known conflict between the two previous studies[10,11] that Ref. 11 does not admit a dimerized order while Ref. 10 claims this phase is certainly possible. The emergence of a Berry phase at half filling clearly reveals how the dimerized order appears.

We would like to mention that the present quantum transition occurs between the *XY* ordered phase and the Ising antiferromagnetic one if one maps our negative-*U* problem to the positive-*U* one. This *XY*-Ising antiferromagnetic transition allows the SO(3) pseudospin description for the competition of SC and CDW fluctuations in the context of the negative-*U* Hubbard model. On the other hand, the AF-VBS quantum transition requires the SO(5) superspin description for the competition of AF and VBS fluctuations.[1]

II. EFFECTIVE FIELD THEORY

A. Derivation of the O(3) nonlinear σ model from the attractive Hubbard model

We consider the attractive Hubbard Hamiltonian

$$H = -t\sum_{ij\sigma} c_{i\sigma}^{\dagger} e^{iA_{ij}} c_{j\sigma} - \frac{3u}{2}\sum_{i} c_{i\uparrow}^{\dagger} c_{i\uparrow} c_{i\downarrow}^{\dagger} c_{i\downarrow} - \sum_{i\sigma} v_i c_{i\sigma}^{\dagger} c_{i\sigma}. \tag{1}$$

Here t is the hopping integral of electrons, and u the strength of on-site Coulomb repulsions. A_{ij} is an external (static) electromagnetic field, and v_i a quenched random potential.

The local interaction term can be decomposed into pairing and density channels in the following way:

$$-\frac{3u}{2}\sum_{i} c_{i\uparrow}^{\dagger} c_{i\uparrow} c_{i\downarrow}^{\dagger} c_{i\downarrow} = -\frac{u}{2}\sum_{i} c_{i\uparrow}^{\dagger} c_{i\downarrow}^{\dagger} c_{i\downarrow} c_{i\uparrow} - \frac{u}{2}\sum_{i}\left(\sum_{\sigma} c_{i\sigma}^{\dagger} c_{i\sigma} - 1\right)^2 - \frac{u}{2}\left(\sum_{\sigma} c_{i\sigma}^{\dagger} c_{i\sigma} - 1\right).$$

Performing the Hubbard-Stratonovich transformation for the pairing and density interaction channels, we find an effective Lagrangian in the Nambu-spinor representation

$$Z = \int D[\psi_i, \psi_i^{\dagger}, \Phi_i^R, \Phi_i^I, \varphi_i] \exp\left(-\int d\tau L\right),$$

$$\begin{aligned} L = {} & \sum_i \psi_i^{\dagger}(\partial_\tau \mathbf{I} - \mu\tau_3)\psi_i - t\sum_{\langle ij\rangle}(\psi_i^{\dagger}\tau_3 e^{iA_{ij}\tau_3}\psi_j + \text{H.c.}) \\ & - \sum_i (\Phi_i^R \psi_i^{\dagger}\tau_1\psi_i + \Phi_i^I \psi_i^{\dagger}\tau_2\psi_i + \varphi_i\psi_i^{\dagger}\tau_3\psi_i) \\ & + \frac{1}{2u}\sum_i (\Phi_i^{R2} + \Phi_i^{I2} + \varphi_i^2) - \sum_i v_i(\psi_i^{\dagger}\tau_3\psi_i + 1). \end{aligned} \tag{2}$$

Here ψ_i is the Nambu spinor, given by $\psi_i = \binom{c_{i\uparrow}}{c_{i\downarrow}^{\dagger}}$. Φ_i^R and Φ_i^I are the real and imaginary parts of the superconducting order parameter, respectively, and φ_i the effective density potential. μ is the electron chemical potential which differs from its bare value μ_b as $\mu = \mu_b + u/2$.

Introducing a pseudospin vector $\vec{\Omega}_i \equiv (\Phi_i^R, \Phi_i^I, \varphi_i)$, one can express Eq. (2) in a compact form as

$$Z = \int D[\psi_i, \psi_i^{\dagger}, \vec{\Omega}_i] \exp\left(-\int d\tau L\right),$$

$$\begin{aligned} L = {} & \sum_i \psi_i^{\dagger}\partial_\tau\psi_i - t\sum_{\langle ij\rangle}(\psi_i^{\dagger}\tau_3 e^{iA_{ij}\tau_3}\psi_j + \text{H.c.}) - \sum_i \psi_i^{\dagger}(\vec{\Omega}_i \cdot \vec{\tau})\psi_i \\ & + \frac{1}{4u}\sum_i \text{tr}[\vec{\Omega}_i \cdot \vec{\tau} - (\mu + v_i)\tau_3]^2 - \sum_i v_i, \end{aligned} \tag{3}$$

where we used the shift of $\varphi_i \to \varphi_i - \mu - v_i$. Integrating over the pseudospin field $\vec{\Omega}_i$, Eq. (3) recovers the Hubbard model Eq. (1).

In this paper we consider only phase fluctuations in $\vec{\Omega}_i$, assuming amplitude fluctuations frozen, thus setting it as $\vec{\Omega}_i = m\vec{n}_i$ with an amplitude m. Since our starting point is a nonzero amplitude of the pseudospin field, we utilize a strong coupling approach decomposing the directional fluctuating field $\vec{n}_i$ into two complex boson fields, the so-called CP^1 representation,[12]

$$\vec{n}_i \cdot \vec{\tau} = U_i \tau^3 U_i^{\dagger},$$

$$U_i = \begin{pmatrix} z_{\uparrow} & -z_{\downarrow}^{\dagger} \\ z_{\downarrow} & z_{\uparrow}^{\dagger} \end{pmatrix}, \tag{4}$$

where U_i is an SU(2) matrix field in terms of a complex boson field $z_{i\sigma}$ with pseudospin σ. Using the CP^1 representation in Eq. (3) and performing the gauge transformation

$$\Psi_i = U_i^{\dagger}\psi_i, \tag{5}$$

Eq. (3) reads

$$Z = \int D[\Psi_i, \Psi_i^{\dagger}, U_i] \exp\left(-\int d\tau L\right),$$

$$L = \sum_i \Psi_i^\dagger(\partial_\tau \mathbf{I} - m\tau_3 + U_i^\dagger \partial_\tau U_i)\Psi_i - t\sum_{\langle ij\rangle} (\Psi_i^\dagger U_i^\dagger \tau_3 e^{iA_{ij}\tau_3} U_j i_j + \text{H.c.}) + \frac{1}{4u}\sum_i \text{tr}[m\tau_3 - (\mu + v_i)U_i^\dagger \tau_3 U_i]^2 - \sum_i v_i. \quad (6)$$

Since Eq. (6) is quadratic for the spinor field Ψ_i, one can formally integrate out the spinor field to obtain

$$S_{eff} = -\text{tr}\ln[\partial_\tau \mathbf{I} - m\tau_3 + U_i^\dagger \partial_\tau U_i - t_{ij}U_i^\dagger \tau_3 e^{iA_{ij}\tau_3} U_j] + \int d\tau\left[-\frac{m}{2u}\sum_i (\mu + v_i)\text{tr}[U_i^\dagger \tau_3 U_i \tau_3] + \sum_i \left(\frac{v_i^2 + \mu^2 + m^2 + \mu v_i}{2u} - v_i\right)\right]. \quad (7)$$

Expanding the logarithmic term for $U_i^\dagger \partial_\tau U_i$ and $U_i^\dagger \tau_3 e^{iA_{ij}\tau_3} U_j$, we obtain

$$S_{eff} \approx \sum_i \text{tr}[G_0(U_i^\dagger \partial_\tau U_i)] + \frac{1}{2}\sum_i \text{tr}_j[G_0 t_{ij} U_i^\dagger \tau_3 e^{iA_{ij}\tau_3} U_j G_0 t_{ji} U_j^\dagger \tau_3 e^{-iA_{ij}\tau_3} U_i] + \int d\tau\left[-\frac{m}{2u}\sum_i (\mu + v_i)\text{tr}[U_i^\dagger \tau_3 U_i \tau_3] + \sum_i \left(\frac{v_i^2 + \mu^2 + m^2 + \mu v_i}{2u} - v_i\right)\right], \quad (8)$$

where $G_0 = -(\partial_\tau \mathbf{I} - m\tau_3)^{-1}$ is the single-particle propagator. The first term leads to a Berry phase while the second results in an exchange interaction term. The resulting effective action is obtained to be without the electromagnetic field A_{ij},

$$S_{eff} = iS\sum_i \omega(\{\mathbf{S}_i(\tau)\}) + \int_0^\beta d\tau H_{eff},$$

$$H_{eff} = -J\sum_{ij} (S_i^x S_j^x + S_i^y S_j^y) + V\sum_{ij} S_i^z S_j^z - \sum_i (\mu + v_i)S_i^z, \quad (9)$$

where the effective exchange coupling strength is given by $J = V = 2t^2/m$.[13,14] It is interesting that the effective Hamiltonian for the competition between SC and CDW is obtained to be the Heisenberg model in terms of the O(3) pseudospin variable. One important message in this effective action is that the Berry phase term $iS\Sigma_i \omega(\{\mathbf{S}_i(\tau)\})$ should be taken into account for the SC-CDW transition even at half filling. Furthermore, the chemical potential plays the same role as an external magnetic field, and the disorder potential as a random magnetic field.

If we consider half filling without disorder, i.e., $\mu = v_i = 0$, the XY order of $\langle S_i^\pm \rangle \neq 0$ and $\langle S_i^z \rangle = 0$ is expected in the case of $J \gg V$, identified with SC. On the other hand, the Ising order of $\langle S_i^z \rangle \neq 0$ and $\langle S_i^\pm \rangle = 0$ arises in the case of $V \gg J$, corresponding to a CDW because of the Berry phase, as will be discussed below. One important question in this paper is how the SC-CDW transition appears in the presence of disorder.

It is easy to show that the Heisenberg model with ferromagnetic XY couplings is the same as that with antiferromagnetic ones. Performing the Haldane mapping of the antiferromagnetic Heisenberg model[14] with a magnetic field in the z direction, we obtain the O(3) nonlinear σ model

$$S_\sigma = iS\sum_i (-1)^i \omega(\{\mathbf{n}_i(\tau)\}) + \frac{1}{g}\int_0^{c\beta} dx_0 \int d^d\mathbf{x}\{(\partial_0 n_z)^2 + (\partial_0 n_x - i[\mu + v]n_y)^2 + (\partial_0 n_y + i[\mu + v]n_x)^2 + (\nabla_{\mathbf{x}}\mathbf{n})^2\}, \quad (10)$$

where c is the velocity of spin waves, and g is the coupling strength between spin wave excitations. As Tanaka and Hu derived an effective SO(5) nonlinear σ action of the superspin field for the AF-VBS transition, we derived an effective SO(3) nonlinear σ action of the pseudospin field for the SC-CDW transition. Furthermore, this effective σ action includes not only doping contributions but also disorder effects. On the other hand, in the SO(5) superspin σ model it is not clear how the doping effect modifies the effective action because a chemical potential term breaks the relativistic invariance. In this case it is not clear even how to obtain the topological term. In the following we discuss how this σ action describes the competition between SC and CDWs in the presence of quenched disorder by focusing on the role of the Berry phase.

Without loss of generality we use the parametrization

$$\vec{n}_i = (\sin(u\vartheta_i)\cos\varphi_i, \sin(u\vartheta_i)\sin\varphi_i, \cos(u\vartheta_i)), \quad (11)$$

where u is an additional timelike parameter for the Berry phase term.[14] We note that $n_i^+ = \sin\vartheta_i e^{i\varphi_i}$ corresponds to the pairing potential $\Phi_i = \Phi_i^R + i\Phi_i^I$. Inserting Eq. (11) into Eq. (10) and performing the integration over u in the Berry phase term, we obtain the following expression for the nonlinear σ model:

$$S_{eff} = iS\sum_i (-1)^i \int_0^{c\beta} dx_0 (1 - \cos\vartheta_i)\dot{\varphi}_i + \int_0^{c\beta} dx_0 \int d^d\mathbf{x}\frac{1}{g}[\sin^2\vartheta(\partial_\mu\varphi)^2 + (\partial_\mu\vartheta)^2] + \int_0^{c\beta} dx_0 \int d^d\mathbf{x}\frac{1}{g}[-(\mu + v)^2 \sin^2\vartheta + 4i(\mu + v)\dot{\varphi}\sin^2\vartheta] + S_I,$$

$$S_I = I\int_0^{c\beta} dx_0 \int d^d\mathbf{x}\cos^2\vartheta, \quad (12)$$

where we introduced the action S_I favoring the XY order. This procedure is quite parallel to that in the SO(5) σ model.[1] The chemical potential favors the XY order without the "easy plane" anisotropy term. The easy plane anisotropy allows us to set $\vartheta_i = \pi/2$. In this case Eq. (12) reads

$$S_{XY} = i\pi\sum_i \left((-1)^i + \frac{8}{g}(\mu + v_i)\right)q_i + \int_0^{c\beta} dx_0 \int d^d\mathbf{x}\left(\frac{1}{2u_\varphi}\dot{\varphi}^2 + \frac{\rho_\varphi}{2}(\nabla_\mathbf{x}\varphi)^2\right). \quad (13)$$

Here $q_i = (1/2\pi)\int_0^{c\beta} dx_0 \dot{\varphi}_i$ is an integer representing an instanton number, here a vortex charge, and the pseudospin value $S=1/2$ is used. Anisotropy in time and spatial fluctuations of the φ fields are introduced by u_φ and ρ_φ. The effective field theory for the SC-CDW transition is given by the quantum XY model with Berry phase in the easy plane limit of Eq. (10). It is clear that the topological phase appears even at half filling as a result of the competition between SC and CDWs. The chemical potential plays the role of an additional Berry phase in the phase field φ.

B. Effective vortex action with both external and random dual magnetic flux

To take into account the Berry phase contribution, we resort to a duality transformation, and obtain the dual vortex action

$$S_v = -t_v \sum_{nm} \Phi_n^\dagger e^{i\bar{c}_{nm} + ic_{nm}}\Phi_m + V(|\Phi_n|) + \frac{1}{2e_v^2}\sum_\mu (\partial \times c)_\mu^2 - \frac{4}{ge_v^2}\sum_\mu v_i(\nabla \times c)_i. \quad (14)$$

Here Φ_n is the vortex field residing in the (2+1)D dual lattice n of the original lattice $\mu=(\tau,i)$, and c_{nm} the vortex gauge field. $V(|\Phi_n|)$ is the effective vortex potential. e_v is a coupling constant of the vortex field to the vortex gauge field. $\bar{c}_{nm}$ is the background gauge potential for the vortex field, resulting from the Berry phase contribution and satisfying at half filling

$$(\nabla \times \bar{c})_i = (-1)^i \pi.$$

The randomness v_i plays the role of a dual random magnetic field in vortices.

In the mean-field approximation ignoring the vortex gauge fluctuations c_{nm}, one finds that the vortex problem coincides with the well-known Hofstadter one. If one considers a dual magnetic flux $f=p/q$ with relatively prime integers p,q (here, $p=1$ and $q=2$), the dual vortex action has q-fold degenerate minima in the magnetic Brillouin zone. Low-energy fluctuations near the q-fold degenerate vacua are assigned to be ψ_l with $l=0,\ldots,q-1$. Balents *et al.* constructed an effective LGW free-energy functional in terms of low-energy vortex fields Ψ_l, given by linear combinations of ψ_l.[8] Constraints for the effective potential of Ψ_l are the symmetry properties associated with lattice translations and rotations in the presence of the dual magnetic field. In the present $q=2$ case (corresponding to a π flux phase) there are two degenerate vortex ground states at momentum (0,0) and (π,π). Introducing the linear-combined vortex fields of $\Psi_0 = \psi_0 + i\psi_1$ and $\Psi_1 = \psi_0 - i\psi_1$ where ψ_0 and ψ_1 are the low-energy vortex fluctuations around the two degenerate ground states, respectively, and considering the symmetry properties mentioned above, one can find an effective low-energy action. However, one important difference from the previous study[8] due to the contribution of the random Berry phase should be taken into account carefully. A cautious person may doubt if it is meaningful to consider the magnetic Brillouin zone in the presence of randomness. Actually, this is a correct question. In this paper we assume the existence of the magnetic Brillouin zone since we are interested in the limit of weak randomness.

Based on symmetry properties of the square lattice under π flux, we write down the effective action for low-energy vortices with randomness

$$S_{eff} = \int d\tau d^2r\Big(|(\partial_\mu - ic_\mu)\Psi_0|^2 + |(\partial_\mu - ic_\mu)\Psi_1|^2 + m^2(|\Psi_0|^2 + |\Psi_1|^2) + u_4(|\Psi_0|^2 + |\Psi_1|^2)^2 + v_4|\Psi_0|^2|\Psi_1|^2 - v_2(\Psi_0^*\Psi_1 + \text{H.c.}) + \frac{1}{2e_v^2}(\partial \times c)^2\Big) - \int d\tau d^2r v(\partial \times c)_\tau. \quad (15)$$

In the effective vortex potential m^2 is the vortex mass, u_4 the local interaction, v_4 the cubic anisotropy, and v_2 breaks the U(1) phase transformation $\Psi_{0(1)} \to e^{i\varphi_{0(1)}}\Psi_{0(1)}$ in the presence of a random Berry phase for vortices. There are two important differences between the cases with and without disorder. In the absence of disorder the v_2 term is given by $-v_8[(\Psi_0^*\Psi_1)^4 + \text{H.c.}]$ owing to the fourfold symmetry.[3,8] However, the presence of weak disorder implies that lattice translations and rotations are no longer symmetries. This reduces the fourth power to the first one. Furthermore, we estimate that v_2 is a random variable depending on disorder. One can regard v_2 as an instanton fugacity.[3,4] Thus, the estimation of the random variable v_2 means that disorder makes the instanton fugacity random. As another contribution of disorder v is a dual random magnetic field in the last term. This term generates different kinds of random potentials, as will be seen later.

Based on the effective vortex potential Eq. (15), one can perform a mean-field analysis in the absence of disorder $(v=0)$.[15] Condensation of vortices occurs in the case of $m^2<0$ and $u_4>0$. The signs of v_4 and v_8 determine the ground state. For $v_4<0$, both vortices have a nonzero vacuum expectation value $|\langle\Psi_0\rangle| = |\langle\Psi_1\rangle| \neq 0$, and their relative phase is determined by the sign of v_8. In the case of $v_8>0$ the resulting vortex state corresponds to a columnar dimer order, breaking both the rotational and translational symmetries. In the case of $v_8<0$ the resulting phase exhibits a plaquette pattern, breaking the rotational symmetries. On the other hand, if $v_4>0$, the ground states are given by either $|\langle\Psi_0\rangle| \neq 0$, $|\langle\Psi_1\rangle| = 0$ or $|\langle\Psi_0\rangle| = 0$, $|\langle\Psi_1\rangle| \neq 0$, and the sign of v_8 is irrelevant. In this case an ordinary charge density wave order at wave vector (π,π) is obtained, breaking the translational symmetries. This mean-field analysis coincides with that in Ref. 3.

At the critical point $m^2=0$ the eighth-order term is certainly irrelevant owing to its high order. Furthermore, the cubic anisotropy term (v_4) is well known to be irrelevant in the case of $q<q_c=4$, ignoring vortex gauge fluctuations.[16] As a result, the Heisenberg fixed point ($v_4^*=0$ and $u_4^*\neq 0$) appears in the limit of zero vortex charge ($e_v\to 0$). Allowing the vortex gauge fields at the Heisenberg fixed point, the Heisenberg fixed point becomes unstable, and a new fixed point with a nonzero vortex charge appears as long as the cubic anisotropy v_4 is assumed to be irrelevant.[17,18] This charged fixed point seems to be qualitatively the same as that obtained in the absence of the dual magnetic field, i.e., the $q=1$ case. However, one important difference is that the dual flux quantum (corresponding to the electromagnetic charge of the original boson) experienced by the vortex field $\Psi_{0(1)}$ is halved due to the two flavors of vortices.[8] This implies that the boson excitations dual to the vortices carry an electromagnetic charge e instead of $2e$. These fractionalized excitations are confined to appear as the usual Cooper pair excitations with charge $2e$ away from the quantum critical point, resulting from the eighth-order term to break the U(1) gauge symmetry.[4] However, as mentioned above, this v_8 term becomes irrelevant at the critical point, indicating that the charge-fractionalized excitations are deconfined to appear as elementary excitations. Thus, the SC-CDW transition at half filling occurs via a deconfined quantum critical point like the AF-VBS transition.[3] This conclusion does not depend on whether the cubic anisotropy is relevant or not at the charged critical point. Even if v_4 is relevant at the isotropic charged fixed point and causes a new anisotropic charged fixed point, the eighth-order term associated with charge fractionalization will be irrelevant.

III. ROLE OF DISORDER IN THE DECONFINED QUANTUM CRITICAL POINT

Now we investigate the role of disorder in the deconfined quantum critical point. In order to take into account the random potentials by disorder, we use the replica trick to average over disorder. The random magnetic field v and the random fugacity v_2 in the vortex action Eq. (15) would cause

$$-\sum_{k,k'=1}^{N}\int d\tau d\tau_1\int d^2r\frac{\Im}{2}(\partial\times c_k)_\tau(\partial\times c_{k'})_{\tau_1},$$

$$-\sum_{k,k'=1}^{N}\int d\tau d\tau_1\int d^2r\frac{\Re}{2}(\Psi_{0k}^*\Psi_{1k}+\text{H.c.})_\tau \times(\Psi_{0k'}^*\Psi_{1k'}+\text{H.c.})_{\tau_1}$$

for Gaussian random potentials satisfying

$$\langle v(r)\rangle=0,\quad \langle v(r)v(r_1)\rangle=\Im\delta(r-r_1),$$

$$\langle v_2(r)\rangle=0,\quad \langle v_2(r)v_2(r_1)\rangle=\Re\delta(r-r_1)$$

with the strength $\Im$ and $\Re$ of the random potentials, respectively. Here $k,k'=1,\ldots,N$ denote replica indices, and the limit $N\to 0$ is taken at the final stage of calculations. However, inclusion of only this correlation term is argued to be not enough for disorder effects. Because the gauge-field propagator has off-diagonal components in replica indices, the vortex gauge interaction of the order $\Im^2e_v^4$ generates a quartic term including the couplings of different replicas of vortices even if this term is absent initially.[17] The resulting disordered vortex action is obtained as

$$Z_R=\int D\Psi_{0k}D\Psi_{1k}Dc_{k\mu}e^{-S_R},$$

$$S_R=S_v+S_d+S_f,$$

$$S_v=\sum_{k=1}^{N}\int d\tau d^2r\Big(|(\partial_\mu-ic_{k\mu})\Psi_{0k}|^2+|(\partial_\mu-ic_{k\mu})\Psi_{1k}|^2 + m^2(|\Psi_{0k}|^2+|\Psi_{1k}|^2)+u_4(|\Psi_{0k}|^2+|\Psi_{1k}|^2)^2 + v_4|\Psi_{0k}|^2|\Psi_{1k}|^2+\frac{1}{2e_v^2}(\partial\times c_k)^2\Big),$$

$$S_d=-\sum_{k,k'=1}^{N}\int d\tau d\tau_1\int d^2r\frac{\Re}{2}(\Psi_{0k}^*\Psi_{1k}+\text{H.c.})_\tau \times(\Psi_{0k'}^*\Psi_{1k'}+\text{H.c.})_{\tau_1} -\sum_{k,k'=1}^{N}\sum_{q,q'=0}^{1}\int d\tau d\tau_1\int d^2r\frac{W}{2}|\Psi_{qk\tau}|^2|\Psi_{q'k'\tau_1}|^2,$$

$$S_f=-\sum_{k,k'=1}^{N}\int d\tau d\tau_1\int d^2r\frac{\Im}{2}(\partial\times c_k)_\tau(\partial\times c_{k'})_{\tau_1}\qquad(16)$$

with $W>0$. The last term induced by disorder in S_d has the same form as the term resulting from a random mass term. The correlation term S_f between random magnetic fluxes is ignored in this paper. In the small-$\Im$ limit this term was shown to be exactly marginal at one-loop level.[17]

The question is what happens at the deconfined charged critical point when randomness is turned on. It is not an easy task to take into account all of the terms on an equal footing in the RG analysis. To investigate the role of the two disorder-induced terms of S_d at the deconfined charged critical point, one can consider two approximate ways. One is first to examine the random mass term, denoted by the coupling strength W, at the deconfined charged critical point, and then to see what happens if the random fugacity ($\Re$) is turned on at a weak disorder fixed point. The other is first to investigate the effect of the random fugacity term on the deconfined charged critical point, and then to examine the random mass term. In this paper we follow the second approach because our main interest is to see the fate of the deconfined quantum criticality against randomness. It should be noted that the existence of the deconfined quantum criticality is determined by the fugacity term.[4]

To examine the role of the random fugacity term in the charged critical point, we consider a phase-only action ignoring amplitude fluctuations of vortices.[19] This so-called London approximation was also utilized in Refs. 3, 4, and 8. The effective vortex action is obtained to be

$$S_R = \sum_{k=1}^{N} \int d\tau d^2r \left(\sum_{q=0}^{1} \frac{\rho}{2}(\partial_\mu \theta_{qk} - c_{k\mu})^2 + \frac{1}{2e_v^2}(\partial \times c_k)^2 \right) - \sum_{k,k'=1}^{N} \int d\tau d\tau_1 \int d^2r \frac{\mathfrak{R}}{2} \cos(\theta_{0k} - \theta_{1k})_\tau \times \cos(\theta_{0k'} - \theta_{1k'})_{\tau_1}, \quad (17)$$

where ρ is the stiffness parameter proportional to the condensation probability of vortices at the mean-field level. The parameter $\mathfrak{R}$ is also renormalized by the condensation amplitude of the vortices.

To see whether the random cosine term is relevant or not at the charged fixed point, it is necessary to check the existence of the charged critical point without the disorder-induced term. Considering $\mathfrak{R}=0$ in Eq. (17), we obtain the RG equations for the stiffness ρ and the vortex charge e_v^2,

$$\frac{d\rho}{dl} = \rho - \gamma e_v^2 \rho, \quad \frac{de_v^2}{dl} = e_v^2 - 2\lambda e_v^4, \quad (18)$$

where γ and λ are positive numerical constants,[20] and l is the usual scaling parameter. The last term $-\gamma e_v^2\rho$ in the first equation originates from the self-energy correction of the vortex field owing to gauge fluctuations while the term $-\lambda e_v^4$ in the second equation results from that of the gauge field due to screening of the vortex charge. In these RG equations there exist two fixed points; one is the neutral (XY) fixed point of $e_v^{*2}=0$ and $\rho^*=0$ and the other, the charged (IXY) fixed point of $e_v^{*2}=1/(2\lambda)$ and $\rho^*=0$. The neutral fixed point is unstable against a nonzero charge $e_v^2 \neq 0$, and the RG flows in the parameter space of (ρ, e_v^2) converge into the charged fixed point owing to $1-\gamma e_v^{*2} = 1-\gamma/(2\lambda) < 0$.[4]

Next we examine the role of the random fugacity term ignoring vortex gauge fluctuations, i.e., $e_v^2=0$. The random fugacity term can be rewritten in the following way:

$$\frac{\mathfrak{R}}{2}\cos(\theta_{0k} - \theta_{1k})_\tau \cos(\theta_{0k'} - \theta_{1k'})_{\tau_1} = \frac{\mathfrak{R}}{4}\cos[(\theta_{0k} - \theta_{1k})_\tau + (\theta_{0k'} - \theta_{1k'})_{\tau_1}] + \frac{\mathfrak{R}}{4}\cos[(\theta_{0k} - \theta_{1k})_\tau - (\theta_{0k'} - \theta_{1k'})_{\tau_1}]. \quad (19)$$

In this expression we can see that the last term is the most relevant term owing to its sign. Thus, it is reasonable to consider the following action for the RG analysis:

$$S_R \approx \sum_{k=1}^{N} \int d\tau d^2r \left(\frac{\rho}{2}(\partial_\mu \theta_{0k})^2 + \frac{\rho}{2}(\partial_\mu \theta_{1k})^2 \right) - \sum_{k,k'=1}^{N} \int d\tau d\tau_1 \int d^2r \frac{\mathfrak{R}}{4} \cos[(\theta_{0k} - \theta_{1k})_\tau - (\theta_{0k'} - \theta_{1k'})_{\tau_1}].$$

This action was well studied in the context of Anderson localization in one-dimensional systems when the flavor number of bosons is 1.[21] In Ref. 4 we derived RG equations for the two-flavor sine-Gordon action. Similarly, one can easily obtain the following RG equations for the stiffness ρ and the random parameter $\mathfrak{R}$:

$$\frac{d\rho}{dl} = \rho + \beta \mathfrak{R}^2 \frac{2}{\rho},$$

$$\frac{d\mathfrak{R}}{dl} = \left(4 - \alpha\frac{2}{\rho}\right)\mathfrak{R}, \quad (20)$$

with positive numerical constants β and α. In our consideration their precise values are not important. The effect of two flavors appears as the factor 2 in the $1/\rho$ terms. One important difference between the present (2+1)D study and the previous (1+1)D one[21] is that the bare scaling dimensions of ρ and $\mathfrak{R}$ are given by 1 and 4 in (2+1)D while by 0 and 3 in (1+1)D, respectively. This difference results in the fact that there exist no stable fixed points in (2+1)D while in (1+1)D there is a line of fixed points describing the Kosterliz-Thouless transition.[17,21] Both the phase stiffness ρ and the parameter $\mathfrak{R}$ become larger and larger at low energy. This implies that depth of the random cosine potential in Eq. (17) becomes deeper and deeper, making the phase difference $\theta_0-\theta_1$ pinned at one ground position of the cosine potential. This is the signal of confinement between fractionalized excitations θ_0 and θ_1.[4]

Combining Eq. (18) with Eq. (20), we obtain the RG equations for the stiffness ρ, the vortex charge e_v^2, and the random parameter $\mathfrak{R}$:

$$\frac{d\rho}{dl} = \rho - \gamma e_v^2 \rho + \beta \mathfrak{R}^2 \frac{2}{\rho},$$

$$\frac{de_v^2}{dl} = e_v^2 - 2\lambda e_v^4,$$

$$\frac{d\mathfrak{R}}{dl} = \left(4 - \alpha\frac{2}{\rho}\right)\mathfrak{R}. \quad (21)$$

These RG equations tell us that the nonzero fixed point value of the vortex charge ($e_v^{2*}=1/2\lambda$) in the second RG equation makes the stiffness parameter ρ vanish ($\rho^*=0$) in the first RG equation, causing the random parameter to be irrelevant, i.e., $\mathfrak{R}^*=0$ in the third RG equation. This solution is self-consistent with the first RG equation. This result means that, as long as the stable charged fixed point exists, the random fugacity term is irrelevant at the charged critical point. As a result, we find only one stable fixed point of $e_v^{2*}=1/2\lambda$, $\rho^*=0$, and $\mathfrak{R}^*=0$. The deconfined quantum criticality is stable against the random fugacity term.

Now we consider the random mass term at this deconfined charged critical point. At the tree level one can easily check that the random mass term is relevant at the charged critical point, indicating instability of the charged fixed point against disorder. One-loop RG analysis shows that a weak disorder fixed point appears if the cubic anisotropy is irrelevant.[17,18] One important point is that the fixed point value of a vortex charge is nonzero at the weak disorder fixed point, given by the value $e_v^{2*}=1/2\lambda$ of the charged critical point.[17,18] Furthermore, the fixed point value of the phase stiffness would still be zero at the random charged critical point because the vortex condensation should occur at $\rho^*=0$. Based on this discussion, we expect that the random fugacity term would still be zero at the weak disorder fixed point. This implies that, although the dimerized or CDW phases may be unstable owing to disorder, turning into glassy phases, deconfinement of fractionalized charge excitations is expected to survive at the disorder critical point. However, we admit that, because we did not treat the two disorder-induced terms of S_d in Eq. (16) on an equal footing, the present result is not fully justified.

IV. SUMMARY AND DISCUSSION

In summary, we showed that the competition between superconductivity and charge density waves results in a nontrivial Berry phase for the SC and CDW order parameters even at half filling, allowing a deconfined quantum critical point of fractionalized charge excitations with e instead of $2e$. We considered the stability of the deconfined quantum criticality against quenched randomness generating two kinds of random potentials, a random mass term and a random fugacity one in the vortex action. Within the London approximation we showed that the random fugacity term is irrelevant at the charged critical point. Then we discussed the effect of the random mass term on this fixed point, and found that the charged critical point becomes unstable, and a weak disorder fixed point with a nonzero vortex charge appears. We argued that, since the random fugacity term would still be irrelevant at this disorder fixed point owing to the finite fixed point value of the vortex charge, deconfinement of fractionalized excitations survives in the weak disorder limit.

A cautious person may question the relevance of this LGW-forbidden quantum transition because there has been no clear indication in actual physical systems so far. One way to justify this quantum transition is to find its one-dimensional analog. Considering spin fluctuations associated with the AF-VBS transition, its critical field theory is well known to be an effective O(4) nonlinear σ model with a topological θ term as an SU(2) level-1 Wess-Zumino-Witten theory.[2] This effective field theory can be derived from some microscopic models such as the bond-alternating spin chain[22] and the Peierls-Hubbard model[23] via non-Abelian bosonization. We believe that this procedure can be applied to charge fluctuations associated with competition between SC and CDWs. Actually, Carr and Tsvelik investigated the continuous SC-CDW transition in a quasi-one-dimensional system.[24] They considered an effective model of spin-gapped chains weakly coupled by Josephson and Coulomb interactions. They obtained an effective field theory for SC and CDW fluctuations in the framework of the non-Abelian bosonization with weak interchain-interactions. They found its phase diagram to show SC and CDW phases, separated by a line of critical points which exhibits an approximate SU(2) (charge) symmetry. They proposed that the critical line would shrink to a point in two dimensions, identified with the quantum critical point in the SC-CDW quantum transition. Furthermore, they discussed the relevance of their theory by considering the experimental system of $Sr_2Ca_{12}Cu_{24}O_{41}$ built up from alternating layers of weakly coupled CuO_2 chains and Cu_2O_3 two-leg ladders. One important difference is that the effective field theory in Ref. 24 does not include a topological θ term while our field theory does allow the θ term. In this respect the correspondence between the present description and the previous theory[24] is not complete. A further investigation for the one-dimensional system is necessary near future.

Important future work in this direction is to introduce spin degrees of freedom associated with an antiferromagnetic order. Then the resulting effective nonlinear σ model would posses an $SO(4)\cong SU(2)\otimes SU(2)$ symmetry, where the former SU(2) is associated with spin, and the latter SU(2) pseudospin. A topological term would appear in this SO(4) σ model. The competition between antiferromagnetism, superconductivity, and density waves remains to be solved.

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[19] We note that a vortex action is given by a phase-only action in the duality transformation originally. See Refs. 3 and 8.

[20] From the relation of $\rho_R=|\Psi_R|^2=Z_\Psi^{-1}|\Psi_B|^2=Z_\Psi^{-1}\rho_B$ it is necessary to know the wave function renormalization constant Z_Ψ. Here R and B represent *renormalized* and *bare*, respectively. The renormalization factor Z_Ψ can be easily obtained from the one-loop self-energy calculation for the vortex field. The self-energy $\Sigma(p)$ of the vortex field is given by

$$\Sigma(p)=e_v^2\int\frac{d^3k}{(2\pi)^3}\frac{1}{|p-k|^2}(2p-k)_\mu D_{\mu\nu}(k)(2p-k)_\nu,$$

where $D_{\mu\nu}(k)=(1/k^2)(\delta_{\mu\nu}-k_\mu k_\nu/k^2)$ is the propagator of vortex gauge fields in the Landau gauge. We find $Z_\Psi^{-1}=1-\gamma e_v^2$, where γ is a positive numerical constant. In the same way we can obtain the RG equation for the vortex charge e_v^2. From the relation of $e_R^2=Z_c e_B^2$, we find the renormalization factor Z_c of the U(1) gauge field c_μ. It can be derived from the polarization function $\Pi_{\mu\nu}(q)$, given by

$$\Pi_{\mu\nu}(q)=e_v^2\int\frac{d^3k}{(2\pi)^3}\frac{(2q-k)_\mu(2q-k)_\nu}{|q-k|^2|k|^2}.$$

We obtain $Z_c=1-2\lambda e_v^2$, where λ is a positive numerical constant, and the prefactor 2 in the e_v^2 term results from the two flavors.

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