Microcavity enhancement of spontaneous emission for Bloch oscillations

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A theory for the spontaneous emission of a Bloch electron traversing a single energy miniband of a superlattice while accelerating under the influence of a constant external electric field and radiating into a microcavity is presented. In the analysis, the quantum electromagnetic radiation field is described by the dominant microcavity TE_{10} rectangular waveguide mode in the Coulomb gauge, and the instantaneous eigenstates of the Bloch Hamiltonian are utilized as the basis states in describing the Bloch electron dynamics to all orders in the constant external electric field. The results show that the spontaneous emission amplitude, when analyzed over many integral multiple values of the Bloch period, gives rise to selection rules for photon emission in both frequency and wave number with preferred transitions at the Wannier-Stark ladder levels. From these selection rules, the total spontaneous emission probability is derived to first-order perturbation theory in the quantized radiation field. It is shown that the power radiated into the dominant TE_{10} waveguide mode can be enhanced by an *order of magnitude* over the free-space value by tuning the Bloch frequency to align with the waveguide spectral density peak. A general expression for the total spontaneous emission probability is obtained in terms of arbitrary superlattice single band parameters, showing multiharmonic behavior and cavity tuning properties. For GaAs-based superlattices, described in the nearest-neighbor tight-binding approximation, the power radiated into the waveguide from spontaneous emission due to Bloch oscillations in the terahertz frequency range is estimated to be several microwatts.

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I. INTRODUCTION

A theory of spontaneous emission (SE), including the probability for absorption, for a Bloch electron traversing a single energy band in an external constant electric field has been examined recently by the authors and colleagues.[1] The theoretical analysis was fully quantum-mechanical, in that the radiation field was described as the *free-space* quantized electromagnetic field in the Coulomb gauge. Analysis of the probability amplitude, over integral multiples of the Bloch period, resulted in *selection rules* for photon emission in both photon frequency and wave vector, showing preferred transitions to the *Wannier-Stark ladder* levels. Using these selection rules, total SE probability was derived to first-order perturbation theory in the quantized radiation field. Although the output frequency of the radiation could be operationally tuned to span the gigahertz to terahertz spectral range by appropriately fixing the constant electric field, the power output for terahertz emission into free space for a GaAs-based superlattice (SL) was estimated to be about one-tenth of a microwatt.

In this paper, we analyze the SE of radiation emitted by a Bloch electron accelerated through the single miniband of a SL structure, but now with the SL placed in a resonant microcavity. The intent of the resonant microcavity is well known,[2,3] to redistribute the free-space modal spectral density so as to increase its value at some frequencies and to decrease it at others; therefore for an active medium, such as a radiating SL placed in a cavity structure, the SE rate can be enhanced or diminished depending upon the tuning of the emission frequency relative to the cavity mode spectral density peak. We use this tuning property offered by microcavities to increase the power output for enhanced SE from GaAs-based SLs. As examples relevant to the principle idea of this work, many efforts have been focused on increasing of the SE rate in optical microcavities.[4] An increase of the overall emitted THz power of more than one order of magnitude has been reported by placing a surface-field emitter inside a THz cavity.[5]

II. QUANTUM DYNAMICS AND THEORETICAL APPROACH

For purposes of the calculation, we assume that the SL structure is placed in a waveguide with rectangular cross section $L_x \times L_y$ and length L_z, where the coordinate axes are chosen to be along the waveguide edges. The constant or dc electric field $\mathbf{E}$ is applied along the y axis, which is also the SL growth direction. The electromagnetic field inside the waveguide, with assumed perfectly conducting walls, is determined by the guided modes corresponding to standing waves with respect to the X and Y axes [designated by an integer pair (m,n)], and propagating waves along the Z axis characterized by propagation constant q_z. Such modes form a complete and orthogonal basis set for describing the electromagnetic field within the waveguide. In the following, we will consider only transverse electric (TE) modes, where the electric field is perpendicular to the direction of propagation. For practical cases,[6] the most important of all confined modes in this waveguide configuration is the TE_{10} mode ($m=1$, $n=0$), which is the dominant mode of a waveguide with $L_x > L_y$. This mode gives rise to the lowest attenuation, and the corresponding electric field $\mathbf{E}_r$, for the chosen system geometry, is polarized in the direction of the dc field $\mathbf{E}$. Therefore we consider only one excited, TE_{10}, mode, and ignore all the other less effective TE and TM modes. For the remaining TE_{10} mode, the following conditions on $\mathbf{E}_r$ and $\mathbf{H}_r$ are in effect, namely that $E_{r,x}=E_{r,z}=0$ and magnetic field component $H_{r,y}=0$. Therefore it follows that the vector potential $\mathbf{A}_r$ for the waveguide field has only one nonzero (y) component, namely,[7]

$$A_{r,y}=\sum_{q_z}\sqrt{\frac{4\pi\hbar c^2}{\omega_q\varepsilon V}}\sin(q_x x)(\hat{a}_q e^{iq_z z}+\hat{a}_q^{\dagger}e^{-iq_z z}), \tag{1}$$

where $\hat{a}_q^{\dagger}$ and $\hat{a}_q$ are the photon boson creation and annihilation operators, $q_x=\pi/L_x$, c is the velocity of light in vacuum, $V=L_xL_yL_z$, and ε is the dielectric constant of the medium filling the waveguide. The normalization constant in Eq. (1) is chosen in such a way that the Hamiltonian for the quantized radiation field has the form $H_r=\Sigma_q\hbar\omega_q\hat{a}_q^{\dagger}\hat{a}_q$, where $\omega_q=\omega_c[1+(q_z/q_x)^2]^{1/2}$ is the mode dispersion relation, and $\omega_c=q_xc/\sqrt{\varepsilon}$ is the angular cutoff frequency determined by the waveguide geometry. The guided mode wavelength is written as $\lambda=\lambda_c/[(\omega_q/\omega_c)^2-1]^{1/2}$, where $\lambda_c=2L_x$ is the cutoff wavelength.[8]

The Bloch dynamical properties are now considered for the situation in which the electron is confined to a single miniband, "n_0," of a SL with band energy $\varepsilon_{n_0}(\mathbf{K})$, while the *effects of interband coupling*[9] *and electron intraband scattering are ignored*. Therefore the quantum dynamics is described by the time-dependent Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}|\Psi_{n_0}(t)\rangle=H|\Psi_{n_0}(t)\rangle, \tag{2}$$

where the exact Hamiltonian $H=[\mathbf{p}-(e/c)\mathbf{A}]^2/2m_0+V_c(\mathbf{r})+H_r$ can be reduced to a sum of the following separate Hamiltonians with $H=H_0+H_r+H_I$.[1] Here the first two terms represent the Hamiltonian $H_0(t)=[\mathbf{p}+\mathbf{p}_c(t)]^2/2m_0+V_c(\mathbf{r})$, for a single electron in a periodic crystal potential, $V_c(\mathbf{r})$, interacting with a homogeneous electric field, and the Hamiltonian H_r for the cavity mode electromagnetic radiation field. The total vector potential in the exact Hamiltonian consists of $\mathbf{A}=\mathbf{A}_c+\mathbf{A}_r$, where $\mathbf{A}_c=-(c/e)\mathbf{p}_c$ describes the external electric field with $\mathbf{p}_c(t)=e\int_{t_0}^{t}\mathbf{E}(t')dt'$, and $\mathbf{A}_r$, given in Eq. (1), describes the cavity mode quantized radiation field; also, m_0 is the free-electron mass. The Hamiltonian $H_I(t)=-(e/m_0c)\mathbf{A}_r\cdot[\mathbf{p}+\mathbf{p}_c(t)]$, for the first-order interaction between the quantum field and the Bloch electron, couples both subsystems H_0 and H_r, and causes transitions between the accelerated Bloch electron states through photon absorption and emission. Then, starting with the reduced Hamiltonian $H=H_0+H_r+H_I$, use is made of first-order time-dependent perturbation theory to calculate SE transitions probabilities between states of H_0+H_r while regarding $H_I(t)\sim\mathbf{A}_r\cdot[\mathbf{p}+\mathbf{p}_c(t)]$ as a perturbation.[10] The solution to $|\Psi_{n_0}(t)\rangle$ of Eq. (2) can be represented in terms of eigenstates of basis states $|\psi_{n_0\mathbf{k}(t)},\{n_{\mathbf{q},j}\}\rangle=|\psi_{n_0\mathbf{k}(t)}\rangle|\{n_{\mathbf{q},j}\}\rangle$ of the unperturbed Hamiltonian H_0+H_r as

$$|\Psi_{n_0}(t)\rangle=\sum_{\mathbf{k}}\sum_{\{n_{\mathbf{q},j}\}}A_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t)|\psi_{n_0\mathbf{k}(t)},\{n_{\mathbf{q},j}\}\rangle \times\exp\left\{-\frac{i}{\hbar}\int_{t_0}^{t}\left[\varepsilon_{n_0}(\mathbf{k}(t'))+\sum_{\mathbf{q},j}\hbar\omega_q n_{\mathbf{q},j}\right]dt'\right\}, \tag{3}$$

where the summation over $\mathbf{k}$ is carried out over the entire Brillouin zone, and $\{n_{\mathbf{q},j}\}$ is specified over all possible combinations of photon occupation number $n_{\mathbf{q},j}$ with photon wave vectors $\mathbf{q}$ and certain polarization $(j=1,2)$. The instantaneous eigenstates of H_0 are given by[9] $\psi_{n_0\mathbf{k}(t)}(\mathbf{r},t)=\Omega^{-1/2}e^{i\mathbf{K}\cdot\mathbf{r}}u_{n_0\mathbf{k}(t)}(\mathbf{r},t)$, where $u_{n_0\mathbf{k}(t)}(\mathbf{r},t)$ is the periodic part of the Bloch function, $\mathbf{k}(t)=\mathbf{K}+\mathbf{p}_c(t)/\hbar$, and the values of the electron wave vector $\mathbf{K}$ are determined by the periodic boundary conditions of the periodic crystal of volume Ω.

III. PROBABILITY AMPLITUDES, SELECTION RULES, AND TOTAL SPONTANEOUS EMISSION PROBABILITY

A. Probability amplitudes—one-photon emission

For the case of one-photon SE, which assumes that initially no photons are present in the radiation field, the probability amplitude in the wave function of Eq. (3) satisfies the initial condition $A_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t_0)=\{\delta_{n_{\mathbf{q},j},0}\}\delta_{\mathbf{K},\mathbf{K}_0}$ at time $t=t_0$ when the external electric field is turned on. Here, $\mathbf{K}_0$ and $n^0_{\mathbf{q},j}=0$ are the initial values of $\mathbf{K}$ and $n_{\mathbf{q},j}$. The probability amplitude for SE, $A_q(\mathbf{k}_0,t)$, *at any time t*, is now evaluated in first-order perturbation theory[1] as

$$A_q(\mathbf{k}_0,t)=D(q_x/q)^{1/2}\int_{t_0}^{t}dt'v_y(\mathbf{k}_0-\mathbf{q}_s)\times\exp\left\{-\frac{i}{\hbar}\int_{t_0}^{t'}[\varepsilon_{n_0}(\mathbf{k}_0)-\varepsilon_{n_0}(\mathbf{k}_0-\mathbf{q}_s)-\hbar\omega_q]dt_1\right\}, \tag{4}$$

where $D=-i\sqrt{\pi c\alpha/\omega_c\varepsilon V}$, $\alpha=e^2/\hbar c$ is the fine-structure constant, $\mathbf{k}_0(t)=\mathbf{K}_0+\mathbf{p}_c(t)/\hbar$, $\mathbf{q}_s=\{\pm q_x,0,q_z\}$ with the "+" used for $s=1$ and the "−" used for $s=2$, $v_y(\mathbf{k}(t))=(1/\hbar)\nabla_{\mathbf{K_y}}\varepsilon_{n_0}(\mathbf{K})|_{\mathbf{k}(t)}$, the y component of Bloch velocity in the band, and $q=(q_x^2+q_z^2)^{1/2}$. Then the spontaneous emission process results in the *total SE probability at a time t*, $P_e^s(t)$, given by

$$P_e^s(t)=\sum_q\sum_{s=1,2}|A_q(\mathbf{k}_0,t)|^2. \tag{5}$$

In evaluating $A_q(\mathbf{k}_0,t)$, we take into account that the external dc field $\mathbf{E}$ is along the Y axis; then, it follows that $k_{0y}(t)=K_{0y}+eE(t-t_0)/\hbar$. In taking advantage of the *periodic* properties of the terms in Eq. (4), $A_q(\mathbf{k}_0,t)$ is easily evaluated in terms of *clocked* integral multiples of the Bloch period; here, $t=N\tau_B$, where $\tau_B=2\pi/\omega_B$, the time to traverse one period of the Brillouin zone. Then the time integral in Eq. (4) can be replaced by an integral over k_{0y} through the substitution $dt=(\hbar/eE)dk_{0y}$, and the probability amplitude, at integral multiples of the Bloch period, can be expressed in terms of an integral over the single Bloch period, τ_B.[1,9] Thus we obtain

$$|A_q(\mathbf{k}_0,N\tau_B)|^2 = \frac{\sin^2(N\beta_q/2)}{\sin^2(\beta_q/2)}|A_q(\mathbf{k}_0,\tau_B)|^2, \tag{6}$$

where the parameter β_q is given by

$$\beta_q = 2\pi\frac{\omega_q}{\omega_B} + \frac{1}{eE}\int_{K_{0y}}^{K_{0y}+G_y} dk_{0y}[\varepsilon_{n_0}(\mathbf{k}_0) - \varepsilon_{n_0}(\mathbf{k}_0-\mathbf{q}_s)], \tag{7}$$

and $G_y=2\pi/a$, the y component of the SL reciprocal-lattice vector.

B. Selection rules

From Eq. (6), it is seen that the quantity $|A_q(\mathbf{k}_0,N\tau_B)|^2$ will reach its maximum growth value when $\beta_q=2\pi(m+\delta)$, where m is a nonzero integer and $\delta\to 0$; for this limit, the function $\eta(\omega_q)=\sin^2(N\beta_q/2)/\sin^2(\beta_q/2)\to N^2$, i.e., it becomes sharply peaked at the resonances $\beta_q=2\pi m$ with increasing N. It is clear that this condition for maximum growth establishes the *selection rule*,[1,9] *for both the photon emission frequency*, ω_q, *and the key wave-vector component*, q_z. Indeed, from the condition $\beta_q=2\pi m$, it follows from Eq. (7) in the radiative long-wavelength limit ($qa\ll 1$) that one generally[11] has

$$\omega_q = m\omega_B, \quad q_z = \pm q_{zm}, \quad q_{zm} \equiv q_x\left[\left(m\frac{\omega_B}{\omega_c}\right)^2 - 1\right]^{1/2}; \tag{8}$$

this gives two conditions, the "*Wannier-Stark ladder*" resonance frequency condition, and the sustaining wave-vector cavity resonance condition. These quantization conditions are obtained, naturally, through the use of instantaneous eigenstates of the Bloch Hamiltonian without requiring any *ad hoc* assumptions concerning the existence of Wannier-Stark energy states. It is worth emphasizing that Eq. (6) should be considered simultaneously with Eq. (8), i.e., both selection rule conditions for the transition of photon frequency and wave vector should be taken into account explicitly in evaluating the total SE probability spectra given by $|A_q(\mathbf{k}_0,\tau_B)|^2$. In this regard, we note that although Eq. (6) predicts the equal relative probabilities calculated at times $t=\tau_B$ and $t=N\tau_B$ for different harmonics of Bloch frequency through

$$\frac{|A_q(\mathbf{k}_0,N\tau_B)|^2}{|A_q(\mathbf{k}_0,\tau_B)|^2} = \eta(\omega_q=m\omega_B) = N^2, \tag{9}$$

it is also clear from Eqs. (4) and (8) explicitly that the probability amplitude depends on ω_B and q independently. Therefore the *total SE probability* in Eq. (5) must incorporate, and will reflect, the importance of both selection rules.

In summary, it is noted here that the probability amplitude of Eq. (4) is markedly influenced by the time-dependent exponential phase factor. This phase factor measures the electron's field-dependent energy change from state $\varepsilon_{n_0}[\mathbf{k}_0(t)]$ to state $\varepsilon_{n_0}[\mathbf{k}_0(t)-\mathbf{q}_s]$, while simultaneously emitting a single photon $\hbar\omega_q$ to the initially prepared vacuum state of the radiation field. This form of the probability amplitude *does not* follow the usual *golden rule* for *stationary* initial-final state dependence because the instantaneous Bloch eigenstates as well as the Hamiltonian describing the external electric field based on the vector potential gauge are explicitly time dependent. Therefore as an equivalent alternative to the *golden rule*, use is made of the periodic nature of the energy band under consideration, and the maximum growth condition for the probability amplitude is established at integral multiples of the Bloch period, $N\tau_B$ with N large. Thus maximum growth in probability amplitude places a quantization condition on β_q in Eq. (7), namely, that $\beta_q=2\pi m$, which results in the selection rules of Eq. (8) for ω_q and q_z, and allows for the calculation of the total spontaneous emission probability.

C. Total spontaneous emission probability

The total SE probability is evaluated at time $t=N\tau_B$, $P_e^s=P_e^s(N\tau_B)$, by substituting $|A_q(\mathbf{k}_0,N\tau_B)|^2$ from Eq. (6) into Eq. (5), and then summing over q. The sum over q in Eq. (5) has been replaced by an integral over q, taking into account the TE_{10} mode density of states and polarization such that $\Sigma_q(\cdots)\to(L_z/\pi)\int dq(\cdots)/[1-(q_x/q)^2]^{1/2}$. The integral can be evaluated by using the property of the integrand which contains a sharply peaked, symmetric function of q at $q=q_m=(q_x^2+q_{zm}^2)^{1/2}$ [see Eq. (8)]. Thus at every node defined by the resonance conditions, the slowly varying function of q in the integrand can be replaced by its value evaluated at $q=q_m$, and then removed from the integral over q; after that, the remaining integral can be evaluated to obtain

$$P_e^s = N\frac{L_z}{L_x}\frac{\omega_B}{\omega_c}\sum_{l=1}^{l_{max}}\sum_{s=1}^{2}\frac{|A_{q_l}(\mathbf{k}_0,\tau_B)|^2}{[1-(q_x/q_l)^2]^{1/2}}. \tag{10}$$

Here l_{max} follows from $q_{max}=l_{max}(\omega_B/\omega_c)q_x$, and determines the upper limit in the sum over higher Bloch oscillation harmonics.

The calculation of P_e^s in Eq. (10) requires the use of $A_q(\mathbf{k}_0,t)$ in Eq. (4), evaluated when $t=t_0+\tau_B$,[12] and at the selection rules of Eq. (8), that is, when $\hbar\omega_q=m\hbar\omega_B$ and $q=q_m$. In addition, the dependence upon $\mathbf{q}$ in Eq. (4) is made explicit by invoking the assumption of photon long-wavelength limit, which is valid for all periodic potentials of interest, even SLs, where $q\ll\pi/a$. Thus it follows from Eq. (4) that

$$|A_{q_l}(\mathbf{k}_0,\tau_B)|^2 = \frac{q_x}{q_l(\omega_B/2\pi)^2}|DI_l|^2, \tag{11}$$

where

$$I_l = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\vartheta_k v_y(\vartheta_k)\exp(-il\vartheta_k) \tag{12}$$

is the lth Fourier component of the $v_y(\vartheta_k)$, the y component of electron band velocity, and $\vartheta_k=k_{0y}a$; D is defined with Eq.

(4). Since the electron velocity component $v_y(\vartheta_k)$ depends on details of the SL miniband structure, it is then clear that the band-structure dependence appears explicitly in the SE probability amplitude through the integral I_l of Eqs. (11) and (12). From this general expression of Eq. (11), it is immediately apparent that for electron dynamics in a purely harmonic miniband, that is, one in which only one single Fourier component of velocity, $v_y(\vartheta_k)$, is nonvanishing so that $l=1$, then only the probability amplitude corresponding to the fundamental Bloch frequency contributes to the total emission process.

IV. TOTAL SPONTANEOUS EMISSION RADIATION AND BAND STRUCTURE CHARACTERISTICS

The analysis for total SE radiation characteristics is now developed by considering a general form of the electron energy miniband dispersion relation expressed as

$$\varepsilon_{n_0}(\mathbf{K}) = \varepsilon_{n_0}(0) + \sum_{l=1}^{\infty} \Delta_l \sin^2 \frac{laK_y}{2} + \varepsilon_\perp(\mathbf{K}_\perp), \qquad (13)$$

where $\varepsilon_{n_0}(0)$ is the miniband edge, Δ_l is the width of the lth miniband harmonic of the SL, and $\varepsilon_\perp(\mathbf{K}_\perp)$ is the contribution from the perpendicular components of the band. Such a form of the energy-band dispersion in the SL growth direction generally includes long-range coupling over the neighboring QWs with a relative strength measured by the specific value of the ratio $\Delta_{l+1}/\Delta_l < 1$, which is strongly dependent upon the extent of wave-function overlap. In particular, for the well-known case of nearest-neighbor tight-binding (NNTB) approximation, only Δ_1 with $l=1$ with purely harmonic energy dispersion is considered significant, so that next-nearest-neighbor and longer range QW wave-function overlaps are assumed to be negligibly small. The electron group velocity in the general miniband of Eq. (13), for the given K_y in the y direction, is then given by $v_y(K_y)=(1/\hbar)\times[\partial\varepsilon_{n_0}(K_y)/\partial K_y]=\sum_{l=1}^{\infty} v_l \sin(laK_z)$, where $v_l=la\Delta_l/2\hbar$, the maximum velocity associated with the lth miniband of band width, Δ_l. Substituting the expression for $v_y(K_y)$ into Eq. (12), one can find that $I_l=-iv_l/2$. Then the probability amplitude in Eq. (11) reduces to

$$|A_{q_l}(\mathbf{k}_0,\tau_B)|^2 = \pi^2 |D|^2 \frac{\omega_c v_l^2}{\omega_B^3 l}, \qquad (14)$$

where account has been taken for the wave vector $q_l=q_x l\omega_B/\omega_c$. Then, from Eqs. (10) and (14), again using $q_l=q_x l\omega_B/\omega_c$, the total SE probability becomes

$$P_e^s = 2\alpha N \varepsilon^{1/2} \frac{L_x}{L_y} \frac{\omega_c^2}{\omega_B^2} \sum_{l=1}^{l_{max}} \frac{(v_l/c)^2}{l[1-(\omega_c/l\omega_B)^2]^{1/2}}. \qquad (15)$$

This is a general expression for total SE probability which contains contributions from higher harmonics of Bloch frequency. Note that in the NNTB approximation of Eq. (13), obtained by letting $\Delta_l=\Delta_1\delta_{1l}$, so that $v_l=v_1\delta_{1l}$, it follows from Eqs. (15) that

$$P_e^s(l=1) = 2\alpha N \frac{L_x}{L_y} \frac{v_1^2}{c^2} \frac{\varepsilon^{1/2}\omega_c^2}{\omega_B^2(1-\omega_c^2/\omega_B^2)^{1/2}}, \qquad (16)$$

where $v_1=a\Delta_1/2\hbar$. For a general SL miniband, higher-order harmonic terms beyond the first harmonic term of the Bloch frequency appear in the SE probability amplitude [Eq. (14)], and thus appear in the total SE probability [Eq. (15)] due to a nonzero contribution from higher Fourier components of $v_y(\vartheta_k)$ [Eq. (12)] that are in resonance with corresponding harmonics of the miniband energy spectrum. In Eq. (15), $v_l=la\Delta_l/2\hbar$ indexes the strength of the lth harmonic through the parameter Δ_l, and lth harmonic is indexed through $l\omega_B$. In general, high-order contributions are much weaker than the fundamental one because the coefficients Δ_l in Eq. (13) rapidly decrease with increasing l.[13,14] Note that the SE probability of the higher harmonics in Eq. (15) can be enhanced by tuning the emission frequency $\omega_q=l\omega_B$ to align with the spectral peak of the waveguide TE_{10} mode, so that it is close to the waveguide cutoff frequency ω_c; then, the resonance at the fundamental Bloch frequency will be suppressed because it is less than the waveguide cutoff frequency ($\omega_B<\omega_c$). In particular, for $l=2$, the SE probability for the *second harmonic generation* is expressed as

$$P_e^s(l=2) = \alpha N \frac{L_x}{L_y} \frac{v_2^2}{c^2} \frac{\varepsilon^{1/2}\omega_c^2}{\omega_B^2[1-(\omega_c/2\omega_B)^2]^{1/2}}, \qquad (17)$$

where $v_2=a\Delta_2/\hbar$. It is noted therefore that *without scattering* higher-order harmonic generation in the SE spectrum of the accelerated electron from the SL miniband is a signature for band anharmonicity in the SL dispersion.

In noting that the SE probability of Bloch radiation into free space is given by the expression $P_{fs}^s=(2\pi/3)\alpha N(v_1/c)^2$,[1] we can compare both the probabilities for SE at fundamental Bloch frequency into free space and a microcavity analyzing the ratio

$$\eta \equiv \frac{P_e^s}{P_{fs}^s} = \frac{3L_x\omega_c^2\varepsilon^{1/2}}{\pi L_y \omega_B^2[1-(\omega_c^2/\omega_B^2)]^{1/2}}, \qquad (18)$$

where use has been made of Eq. (16) for the $P_e^s=P_e^s(l=1)$. The enhancement factor η given in Eq. (18) is a function of the frequency ratio, ω_B/ω_c, and increases monotonically with decreasing ratio; of course, ω_B/ω_c reflects the detuning of the Bloch frequency relative to the TE_{10} mode cutoff frequency. Taking for an estimation $\varepsilon=12.2$ and $L_x/L_y=2$, one can find that for $\omega_B/\omega_c<1.15$ the enhancement factor η increases precipitously with decreasing ratio; at $\omega_B/\omega_c\simeq 1.15$, $\eta\simeq 10$, indicating that when ω_B is tuned to within $1.15\omega_c$, the SE enhancement will be increased at least by an order of magnitude over unity.

V. SPONTANEOUS EMISSION POWER ESTIMATE

For numerical estimations, we assume a GaAs-based SL structure with the SL lattice parameter $a=100$ Å, vertical dimension 9 μm, and lateral cross section 18×1000 μm^2 embedded into a rectangular waveguide with horizontal and vertical dimensions $L_x/L_y=2$. The electron density in the active region is taken to be 5×10^{16} cm^{-3}. Taking for the SL lowest miniband energy width $\Delta = 20$ meV, the maximum group velocity in the miniband is estimated as $v_{max} = 1.6 \times 10^7$ cm/s. These parameter magnitudes are close to those of GaAs-based SL structures used to study high-frequency microwave generation.[15–17] Spontaneous emission of a photon with the energy 10 meV corresponds to the Bloch frequency $\nu_B = \omega_B / 2\pi = 2.5$ THz. The electric field required to achieve such Bloch frequency is $E = \hbar\omega_B / ea = 10$ kV/cm, and results in the application of 9 V across the vertical dimension of the SL structure.[18] The spontaneous emission probability of radiation into free space can be estimated taking, for example, $N = 100$ as $P^s_{fs} = 4.3 \times 10^{-7}$; and the generation energy per electron $\hbar\omega_B P^s_{fs} = 4.3 \times 10^{-6}$ meV. Since there is a total of $n = 8 \times 10^9$ electrons in the active region of the SL, the generated energy achievable is estimated to be $P_{fs} = n\hbar\omega_B P^s_{fs} = 34.4$ eV, which corresponds to a power output generated into free space $W_{fs} = (\nu_B / N) P_{fs} \simeq 0.14$ μW. Although the power generated into free space is discernibly low for the SE of Bloch oscillation radiation, it follows from Eq. (18) that SE probabilities and rates are substantially modified for the case of radiation into the waveguide mode. We find from Eq. (18) that $\eta = 20$, if we take for the detuning parameter $\omega_B / \omega_c = 1.05$. Then, using the obtained value for the enhancement factor, we estimate the power output generated into the TE_{10} waveguide mode as $W_{wg} \simeq 3$ μW. For purposes of this estimate, the cavity was assumed to be lossless; further studies will consider cavities with finite quality factor.

It is noted that a Bloch oscillation SL does not require controlled inversion population between Wannier-Stark ladder levels to get the desired SE photon frequency; the desired frequency is controlled by the applied field. Whereas in other SL light generating devices, such as quantum cascade lasers, a large inversion population is required to provide stimulated emission with resulting high threshold current densities and high heat dissipation. In this regard, the Bloch oscillator in SE offers a novel option for operating at THz frequencies, provided the power output can be enhanced in the coherent Bloch regime.

VI. SUMMARY

A quantum theory has been presented to describe spontaneous emission of radiation for a Bloch electron accelerating across a single energy band of a SL in a constant electric field while the radiating SL interacts with a resonant microcavity. In the theoretical analysis, use was made of the instantaneous eigenstates of the Bloch Hamiltonian in the external electric field described in terms of the vector potential gauge, and the spontaneous emission rates were calculated using first-order time-dependent perturbation theory in the TE_{10} waveguide mode quantized radiation field. The analysis of spontaneous emission amplitude resulted in transition selection rules for both photon emission frequency and wave number at the Wannier-Stark ladder levels; this result is found naturally through the use of the instantaneous eigenstates of the Bloch Hamiltonian with no *ad hoc* assumptions employed concerning the existence of Wannier-Stark quantized energy levels within the band.

Using the selection rules, the total spontaneous emission probability was calculated for a general SL miniband dispersion, and was shown to depend upon the fundamental and higher-order Bloch frequency harmonics as well as the bandstructure parameters; in the limit of nearest-neighbor tight binding, the results for total spontaneous emission reduced to the expected sole dependence upon the fundamental Bloch harmonic.

Finally, it was shown, within the nearest-neighbor tight-binding model, that the total spontaneous emission probability in the microcavity is substantially enhanced over the comparable free-space value when the Bloch frequency is tuned by means of the external electric field so as to align with the spectral peak of the waveguide mode density of states. In this regard, a theoretical estimate showed that about a one-order-of-magnitude enhancement factor for GaAs-based SLs resulted in a power output of about 3 μW. Here, it is noted that the prime source of the total spontaneous emission enhancement is due to the alignment of the Bloch frequency with the peak of the spectral density of states; in the spirit of the Purcell effect,[2] this will be a general feature of any cavity considered for this analysis.

Future directions of this work include studying the limiting effects of dephasing inhomogeneities such as SL interface scattering and other competitive processes[20–22] so as to more realistically evaluate the optimal magnitudes of power output from the spontaneous emission of SL Bloch oscillation radiation.

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[1] V. N. Sokolov, L. Zhou, G. J. Iafrate, and J. B. Krieger, Phys. Rev. B **73**, 205304 (2006).

[2] E. M. Purcell, Phys. Rev. **69**, 681 (1946).

[3] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).

[4] In *Spontaneous Emission and Laser Oscillation in Microcavities*, edited by H. Yokoyama and K. Ujihara (CRC, Boca Raton, 1995).

[5] C. Janke, P. Haring Bolivar, A. Bartels, H. Kurz, and H. Künzel, Phys. Rev. B **67**, 155206 (2003); M. Zedler, C. Janke, P. H. Bolivar, H. Kurz, and H. Künzel, Appl. Phys. Lett. **83**, 4196 (2003).

[6] G. Gallot, S. P. Jamison, R. W. McGowan, and D. Grischkowsky, J. Opt. Soc. Am. B **17**, 851 (2000).

[7] D. Marcuse, *Principles of Quantum Electronics* (Academic, New York, 1980), p. 426.

[8] N. Marcuvitz, *Waveguide Handbook* (Peregrinus, London, 1993).

[9] J. B. Krieger and G. J. Iafrate, Phys. Rev. B **33**, 5494 (1986); G. J. Iafrate and J. B. Krieger, *ibid.* **40**, 6144 (1989).

[10] J. Singh, *Quantum Mechanics: Fundamentals and Applications to Technology* (Wiley-Interscience, New York, 1997), p. 337.

[11] Note that $\mathbf{q}$ is not collinear with the direction of the applied field $\mathbf{E}$, then the integral in Eq. (7) does not vanish because of the periodicity of $\varepsilon_{n_0}(\mathbf{k}_0)$, and contributes to the selection rule. For $\omega_q = \omega_c[1+(q_z/q_x)^2]^{1/2}$, this corresponds to neglecting a small term $\sim v_\perp/c$ as compared to unity, where $v_\perp$ is the electron band velocity perpendicular to the y axis.

[12] In the photon long-wavelength limit, the probability amplitude in Eq. (4) allows one to factor out the arbitrary initial value of the electron wave vector $\mathbf{K}_0$ resulting from t_0 as a phase factor, so that $A_{q_l}(\mathbf{k}_0, \tau_B) = -D(q_x/q_l)^{1/2}(2\pi/\omega_B)\exp(ilaK_{0y})I_l$, with I_l defined in Eq. (12). Therefore the absolute value of A_{q_l} which gives $|A_{q_l}(\mathbf{k}_0, \tau_B)|^2$ in Eq. (11) is independent of t_0 or $\mathbf{K}_0$, a consequence of periodicity property of the band.

[13] As noted in Refs. 14, higher Bloch frequency harmonic terms that are much weaker than the fundamental term may appear in a pure harmonic energy miniband due to the interaction of Bloch electrons with plasmons or with cyclotron oscilations in an external magnetic field; these terms arise as higher harmonic terms in the semiclassical description of oscillating current. Such effects as well as other effects imposed on dephasing Bloch oscillations due to electron scattering are not treated in this paper.

[14] F. Löser, Yu. A. Kosevich, K. Köhler, and K. Leo, Phys. Rev. B **61**, R13373 (2000); A. B. Hummel, T. Bauer, E. Mohler, and H. G. Roskos, J. Phys.: Condens. Matter **18**, 2487 (2006).

[15] L. Esaki and R. Tsu, IBM J. Res. Dev. **14**, 61 (1970).

[16] E. Schomburg, T. Blomeier, K. Hofbeck, J. Grenzer, S. Brandl, I. Lingott, A. A. Ignatov, K. F. Renk, D. G. Pavel'ev, Yu. Koschurinov, B. Ya. Melzer, V. M. Ustinov, S. V. Ivanov, A. Zhukov, and P. S. Kop'ev, Phys. Rev. B **58**, 4035 (1998).

[17] A. M. Bouchard and M. Luban, Phys. Rev. B **47**, R6815 (1993).

[18] To prevent charge domain formation that may be possible through negative differential conductance a superstructure of SLs of suitable lengths can be designed (Ref. 19).

[19] P. G. Savvidis, B. Kolasa, G. Lee, and S. J. Allen, Phys. Rev. Lett. **92**, 196802 (2004).

[20] N. Sekine, Y. Shimada, and K. Hirakawa, Appl. Phys. Lett. **83**, 4794 (2003).

[21] J. P. Reynolds and M. Luban, Phys. Rev. B **54**, R14301 (1996).

[22] G. von Plessen, T. Meier, J. Feldmann, E. O. Göbel, P. Thomas, K. W. Goossen, J. M. Kuo, and R. F. Kopf, Phys. Rev. B **49**, R14058 (1994).