Excitation of two-dimensional plasmon polaritons by an incident electromagnetic wave at a contact

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(Received 25 October 2006; published 17 January 2007)

We consider the scattering of an incident electromagnetic radiation on a two-dimensional (2D) electron layer with an imbedded metallic contact. We show that the incident wave excites in the system the 2D plasmon polaritons running along the 2D layer and localized near it, and electromagnetic waves reflected back from the system. The ratio of the energy transformed to the 2D plasmon polaritons and reflected back from the layer depends on the frequency and the value of a retardation parameter, which characterizes the importance of retardation effects. When the retardation parameter is large, the energy of the incident radiation is mainly reflected from the 2D electron system and the excitation of the 2D plasmon polaritons is less effective. The results obtained are discussed in connection with recent experiments on the microwave response of the 2D electron systems.

DOI: 10.1103/PhysRevB.75.045328 PACS number(s): 73.20.Mf, 73.40.Sx, 78.70.Gq

I. INTRODUCTION

The physics of plasma oscillations in low-dimensional electron systems (ES)—quantum wells, wires, and dots—has been experimentally studied over decades[1–25] and attracted much theoretical attention.[26–40] Experimental studies have been done using far-infrared transmission and emission spectroscopy, as well as Raman scattering technique. In most of these experiments the frequency of two-dimensional (2D) plasmons lied in the terahertz range. More recently, the focus of plasma-wave studies has shifted to much lower, microwave frequencies.[41–46] The microwave spectra of plasma waves have been studied in macroscopic disks,[41,42] stripes,[44,45] and rings.[43,46] Observation of the 2D plasmons at so low frequencies has become possible due to substantial improvement of quality of quantum-well samples. The 2D plasmons with the wavevector q and the frequency

$$\omega_p(q) = \sqrt{\frac{2\pi n_s e^2}{m^\star \epsilon} q} \tag{1}$$

can be observed only if $\omega_p(q)$ exceeds the momentum relaxation rate $\gamma=1/\tau=e/m^\star\mu$. Here n_s, μ, e, $m^\star$, and τ are the density, the mobility, the electric charge, the effective mass, and the momentum relaxation time of 2D electrons, respectively, and ϵ is the dielectric constant of the surrounding medium. In the first 2D-plasmon experiments in semiconductors,[2–7] the electron mobility was rather low, and the linewidth of plasma resonances often exceeded 100 GHz. In modern semiconductor GaAs/AlGaAs structures, the mobility achieves $\sim 10^6 - 10^7$ cm^2/Vs, which makes it possible to observe the 2D plasmons (1) at frequencies as low as and below $\sim$10 GHz.

At low frequencies response of the 2D electron systems has its specific features. First, one can observe[41,42] the influence of retardation effects on the spectrum of the 2D plasmons, theoretically predicted many years ago[26,28] (for a detailed theoretical treatment of the experiments[41,42] see Ref. 47). Another feature is related to the methods of excitation of the 2D plasma waves by the incident electromagnetic wave. At terahertz frequencies, the coupling of the longitudinal 2D plasmons to the transverse electromagnetic waves in a macroscopic sample had to be done by means of a grating placed on the top of the structure.[5] The grating period a (typically $\lesssim 1$ μm) determined the plasmon wavevector $q \simeq 2\pi/a$ and its frequency $\omega_p(q)$, Eq. (1). At microwave frequencies the incident electromagnetic radiation directly excites the 2D plasmons, due to finite dimensions of the 2D ES. The wavevector q is then determined by the inverse lateral size of the sample. At low frequencies plasmons can be also excited at any inhomogeneity of the sample or of the sample boundary, for instance at a contact to the 2D electron gas (EG). In the experiment of Refs. 48 and 49, for example, the excitation of edge magnetoplasmons[50] at the potential contacts to the 2D EG has been explored.

In this paper we theoretically study excitation of the 2D plasmon polaritons by an incident electromagnetic wave at a contact imbedded in the 2D electron layer. We assume that the lateral dimensions of the 2D layer are much bigger than the size of the contact and the wavelength of radiation. We calculate the absorption spectra of the system and the distribution of electric fields and energy flows in and around the 2D layer. We show that, in the quasistatic regime, when the retardation effects in the 2D gas are not important, the incident radiation excites the 2D plasmons in the electron-gas layer, which then propagate along the 2D gas away from the contact. In the opposite limit, when the retardation effects are essential, the incident wave excites the 2D plasmon-polaritons running along the 2D electron layer, as well as electromagnetic waves, propagating away from the system in the perpendicular direction. The part of the energy emitted from the 2D layer increases when the retardation effects become stronger, which may be interpreted as the radiative decay[47,51] of the 2D plasmon polaritons.

II. THEORY

We consider an infinite 2D electron layer, lying in the plane $z=0$, with an imbedded stripe metallic contact of the

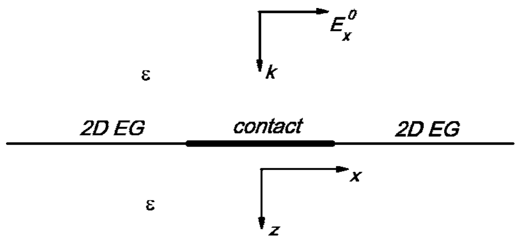

FIG. 1. Geometry of the considered structure. The width of the contact stripe is W.

width W, lying at the same plane at $|x|<W/2$ (see Fig. 1). The dielectric constant of the surrounding medium equals ϵ and is uniform in all space. An electromagnetic wave

$$E_x^{\rm ext}(x,z) = E_x^0 e^{i\omega\sqrt{\epsilon}z/c - i\omega t}, \tag{2}$$

having the frequency ω and linearly polarized in the x direction, is incident upon the structure along the positive z direction. The fields induced by the incident wave, $\mathbf{E}^{\rm ind}$ and $\mathbf{H}^{\rm ind}$, obey the Maxwell equations

$$\nabla \times \mathbf{H}^{\rm ind} = \frac{4\pi}{c}\mathbf{j}\,\delta(z) + \frac{\epsilon}{c}\frac{\partial \mathbf{E}^{\rm ind}}{\partial t}, \tag{3}$$

$$\nabla \times \mathbf{E}^{\rm ind} = -\frac{1}{c}\frac{\partial \mathbf{H}^{\rm ind}}{\partial t}, \tag{4}$$

where $\mathbf{j}=j_x\hat{\mathbf{x}}$ is the sheet current in the plane $z=0$. Due to the symmetry in the y direction, the only nonzero components of the electric and magnetic fields are $E_x^{\rm ind}$, $E_z^{\rm ind}$, and $H_y^{\rm ind}$. Solutions are searched for in the form

$$E_x^{\rm ind}(x,z,t) = \int_{-\infty}^{\infty} dq e^{iqx - i\omega t} E_x^{\rm ind}(q,z) \tag{5}$$

for the electric field $E_x^{\rm ind}$ and similarly for other field components. From Eqs. (3)–(5) we get the differential equation for $E_x^{\rm ind}$,

$$\frac{\partial^2 E_x^{\rm ind}}{\partial z^2} - \kappa_q^2 E_x^{\rm ind} = \frac{4\pi i \kappa_q^2}{\omega\epsilon} j_x(q)\,\delta(z), \tag{6}$$

which gives

$$E_x^{\rm ind}(q,z) = -\frac{2\pi i\kappa_q}{\omega\epsilon} j_x(q) e^{-\kappa_q|z|}. \tag{7}$$

Here $\kappa_q^2 = q^2 - \omega^2\epsilon/c^2$ and $j_x(q)$ is the Fourier component of the current. Substituting Eq. (7) into Eq. (5), we obtain the induced field in the real space

$$E_x^{\rm ind}(x,z) = -\frac{i}{\omega\epsilon}\int_{-\infty}^{\infty} dq\,\kappa_q e^{iqx-\kappa_q|z|}\int_{-\infty}^{\infty} dx' j_x(x') e^{-iqx'}. \tag{8}$$

The branch of the square root in κ_q must be chosen so that Eq. (8) correctly describes both the evanescent field and the scattered outgoing wave, that is, $\kappa_q = -i\sqrt{\omega^2\epsilon/c^2 - q^2}$ for $|q|<\omega\sqrt{\epsilon}/c$ and $\kappa_q = \sqrt{q^2-\omega^2\epsilon/c^2}$ for $|q|>\omega\sqrt{\epsilon}/c$, where the square roots take positive real values.

The current j_x in Eq. (8) can be described as

$$j_x(x) = \sigma E_x(x,0) + (\sigma_m - \sigma)\,\theta(W/2 - |x|) E_x(x,0), \tag{9}$$

where $E_x = E_x^{\rm ind} + E_x^{\rm ext}$ is the total electric field and σ and σ_m are the frequency-dependent local conductivities of the 2D ES and the contact, respectively. The conductivities are given by $\sigma \equiv \sigma(\omega) = \sigma_0/(1 - i\omega/\gamma) = i n_s e^2/m^\star(\omega + i\gamma)$ and $\sigma_m \simeq \sigma_{m0}$, where σ_0 and σ_{m0} are the static conductivities of the 2D ES and the contact. Here we assume that the contact conductivity is independent of the frequency, as is really the case at microwave frequencies, and that there are no nonlinear (Schottky) effects at the interface between the 2D gas and the contacts.

Combining Eqs. (8) and (9) we obtain the following equation for the total electric field $E_x(x,0)$ at $z=0$:

$$E_x(x,0) = \frac{1}{2\pi}\left(1 - \frac{\sigma}{\sigma_m}\right)\int_{-\infty}^{\infty} dq \frac{e^{iqx}}{\zeta(q,\omega)}\int_{-W/2}^{W/2} dx' e^{-iqx'} E_x(x',0) + \frac{\sigma}{\sigma_m}\frac{E_x^0}{\zeta(0,\omega)}, \quad -\infty < x < +\infty, \tag{10}$$

where

$$\zeta(q,\omega) = 1 + 2\pi i\sigma(\omega)\kappa_q/\omega\epsilon \tag{11}$$

is the dielectric function of the infinite and homogeneous 2D EG.[26] In Eq. (10) it describes the screening of the time- and space-dependent electric field, produced by the contact inhomogeneity. The spectrum of the 2D plasmon polaritons in the homogeneous 2D ES (Refs. 26 and 29) is given by zeros of the function $\zeta(q,\omega)$:

$$\zeta(q,\omega) = 0. \tag{12}$$

Equation (10) is valid at all $-\infty < x < +\infty$. If $|x|<W/2$, this is an integral equation for the unknown function $E_x(x,0)$. If $|x|>W/2$, Eq. (10) expresses the field everywhere outside the contact stripe via the (known) field inside the stripe.

To solve the integral equation (10) at $|x|<W/2$, we expand $E_x(x,0)$ in a complete set of cosine functions,

$$E_x(x,0) = \sum_{n=0}^{\infty} A_n \cos(2\pi nx/W), \quad |x|<W/2, \tag{13}$$

and reduce Eq. (10) to a system of algebraic equations. The latter can be solved numerically by truncating the matrix up to size N. Convergence of solutions was checked by increasing N, and we used the value $N=20$ for all calculations done in this paper. After calculating the coefficients A_n, we express the induced field $E_x^{\rm ind}$ in the whole space via the total field E_x in the contact:

$$E_x^{\mathrm{ind}}(x,z) = \frac{1-\zeta(0,\omega)}{\zeta(0,\omega)}E_x^0 e^{i\omega\sqrt{\epsilon}|z|/c} + \frac{1}{2\pi}\left(\frac{\sigma_m}{\sigma}-1\right)\int_{-\infty}^{\infty} dq \frac{1-\zeta(q,\omega)}{\zeta(q,\omega)} e^{iqx-\kappa_q|z|}\int_{-W/2}^{W/2} dx' E_x(x',0)e^{-iqx'}$$
$$= \frac{1-\zeta(0,\omega)}{\zeta(0,\omega)}E_x^0 e^{i\omega\sqrt{\epsilon}|z|/c} + \frac{W}{4\pi}\left(\frac{\sigma_m}{\sigma}-1\right)\sum_{n=0}^{\infty}(-1)^{n+1}A_n\int_{-\infty}^{\infty} dq \frac{1-\zeta(q,\omega)}{\zeta(q,\omega)}\frac{qW\sin(qW/2)}{(\pi n)^2-(qW/2)^2}e^{iqx-\kappa_q|z|}; \qquad (14)$$

the other components of electric and magnetic fields can be expressed in the similar way by using Eqs. (3) and (4).

The field (14) consists of two contributions. The first term describes the scattering of the incident electromagnetic wave by the uniform 2D ES,

$$E_{x,\mathrm{unif}}^{\mathrm{ind}}(x,z) = -\frac{2\pi\sigma(\omega)/c\sqrt{\epsilon}}{1+2\pi\sigma(\omega)/c\sqrt{\epsilon}}E_x^0 e^{i\omega\sqrt{\epsilon}|z|/c}$$
$$= -\frac{i\Gamma}{\omega+i(\gamma+\Gamma)}E_x^0 e^{i\omega\sqrt{\epsilon}|z|/c}. \qquad (15)$$

For the corresponding total electric field at $z=0$ we get

$$E_{x,\mathrm{unif}}(x,0) = \frac{\omega+i\gamma}{\omega+i(\gamma+\Gamma)}E_x^0. \qquad (16)$$

Here $\Gamma=2\pi n_s e^2/m^\star c\sqrt{\epsilon}$ is the radiative decay rate[52] and we have used that $\zeta(0,\omega)=1+2\pi\sigma(\omega)/c\sqrt{\epsilon}$. The fields (15) and (16) do not depend on x.

The second term in (14) represents the change of the field due to the presence of the contact. The integral over dq has poles in the q points, satisfying Eq. (12) and physically corresponding to the excitation of the 2D plasmon polaritons in the 2D gas. We will denote the second term in (14) as E_x^{2DP} hereafter and will study its coordinate dependence at different values of the frequency and the retardation parameter (18).

To quantitatively characterize the excitation of plasma waves in the system as a function of frequency, we need an integral characteristic like, for instance, the Joule heat. The integral $\int_{-\infty}^{\infty}dx j_x E_x^*/2$, however, diverges at the upper and lower limits, as at large distances from the contact the system absorbs energy as a uniform electron gas and $j_x E_x^*$ tends to a constant. To avoid this divergency we subtract the energy absorbed by the uniform 2D ES, and calculate the quantity

$$Q = \frac{1}{2}\,\mathrm{Re}\int_{-\infty}^{\infty} dx(j_x E_x^* - j_{x,\mathrm{unif}}E_{x,\mathrm{unif}}^*), \qquad (17)$$

where $E_{x,\mathrm{unif}}$ is given by Eq. (16) and $j_{x,\mathrm{unif}}=\sigma E_{x,\mathrm{unif}}$. The integral in Eq. (17) converges.

We will also calculate the Poynting vector of the induced electromagnetic wave $\mathbf{S}^{\mathrm{2DP}}=\mathrm{Re}[c(\mathbf{E}^{\mathrm{2DP}})^*\times\mathbf{H}^{\mathrm{2DP}}/8\pi]$, which shows the distribution of the energy flows around the 2D electron layer.

Response of the considered system is characterized by four dimensionless parameters: the normalized frequency ω/ω_0, the normalized collision frequency γ/ω_0, the ratio of the static conductivities of the contact and the 2D EG σ_{m0}/σ_0, and the retardation parameter[47]

$$\alpha = \frac{\omega_0\sqrt{\epsilon}W}{\pi c} \equiv \frac{\Gamma}{\omega_0}. \qquad (18)$$

Here

$$\omega_0 = \sqrt{\frac{2\pi^2 n_s e^2}{m\epsilon W}} \qquad (19)$$

is the frequency of the 2D plasmon (1) with the wavevector $q=\pi/W$. The α parameter is defined as the ratio of ω_0 to the frequency of light with the same wavevector $q=\pi/W$.

III. RESULTS AND DISCUSSION

Results of this section have been obtained at $\gamma/\omega_0=0.1$ and the conductivity of metal $\sigma_{m0}/\sigma_0=10$. A control study of the system response at $\sigma_{m0}/\sigma_0=50$ showed that the chosen value of this parameter is sufficiently high to represent the highly conducting contact.

Figure 2 shows the dependence of the normalized absorption cross section $Q/S_z^0 W$ on the frequency at different values of the retardation parameter α. Here Q is given by Eq. (17) and S_z^0 is the energy flow of the incident wave. One sees that the frequency dependence of the absorption is qualitatively different at $\alpha\lesssim 1$ and $\alpha\gtrsim 1$. At small α's, the system response is almost frequency independent in a broad frequency range, because the contact-stripe width W is much smaller than the wavelength of radiation $\lambda=2\pi c/\omega\sqrt{\epsilon}$,

$$\frac{W}{\lambda} = \frac{\omega\alpha}{2\omega_0} \ll 1, \qquad (20)$$

the contact stripe radiates as a point dipole, and for any given frequency ω the system emits the 2D plasmons with the wavevector q determined by Eq. (12). The coordinate dependence of the induced 2D plasmon electric field, Fig. 3, shows that the emitted wave is strongly localized near the plane $z=0$, and the plasmon propagates along the x direction experiencing a weak decay due to the scattering of electrons in the 2D gas by phonons and impurities.

At larger α's the cross section increases and acquires a quasiresonant shape. At $\alpha\gtrsim 1$ the width of the contact stripe becomes comparable with the wavelength of radiation, and the absorption spectrum becomes an oscillating function of frequency, Fig. 2(b), with the maxima at

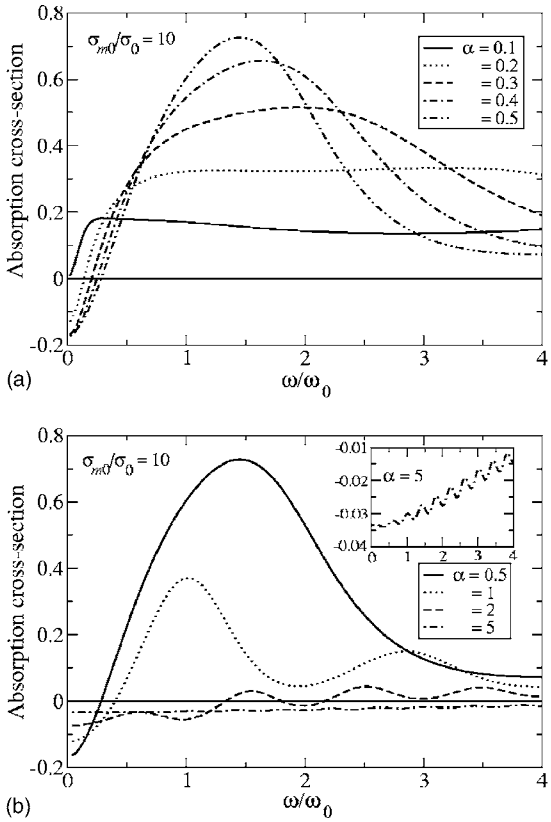

FIG. 2. The normalized absorption cross section vs the normalized frequency with (a) $\alpha \leqslant 0.5$ and (b) $\alpha \geqslant 0.5$. The inset in (b) depicts the curve for $\alpha = 5$.

$$W = \lambda(n + 1/2), \quad n = 0, 1, \dots , \tag{21}$$

corresponding to the maximum radiation efficiency of the contact-stripe antenna. The distribution of the induced electric field at ω points corresponding to the first absorption maxima ($W \simeq \lambda/2$) is shown in Fig. 4. One sees that in general the induced electromagnetic wave is emitted in both x and z directions, with the relative intensity dependent on the value of the retardation parameter. If $\alpha \lesssim 1$, the induced electromagnetic wave propagates mainly along the x axis (the 2D plasmon polariton). At larger α's the energy of the emitted wave mainly flows in the z direction. This reemission of electromagnetic waves from the contact stripe leads to a *radiative decay* of the 2D plasmon-polariton modes, which has been discussed from another viewpoint in Refs. 47 and 51.

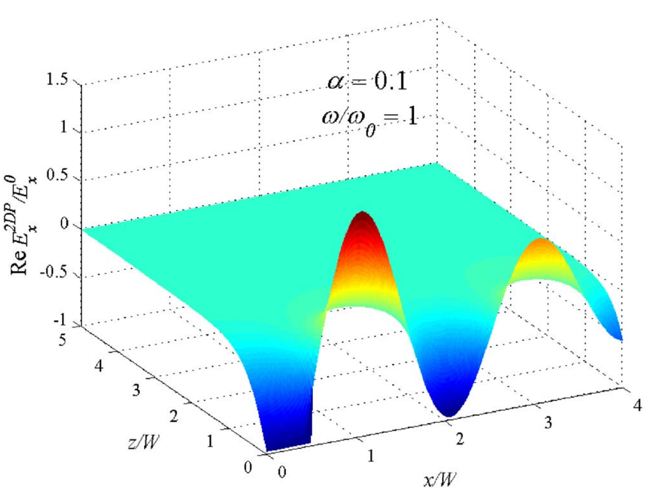

FIG. 3. (Color online) The distribution of the 2D plasmon-polariton electric field $\mathrm{Re}\, E_x^{2\mathrm{DP}}/E_x^0$ at $\alpha = 0.1$ and the frequency $\omega/\omega_0 = 1$.

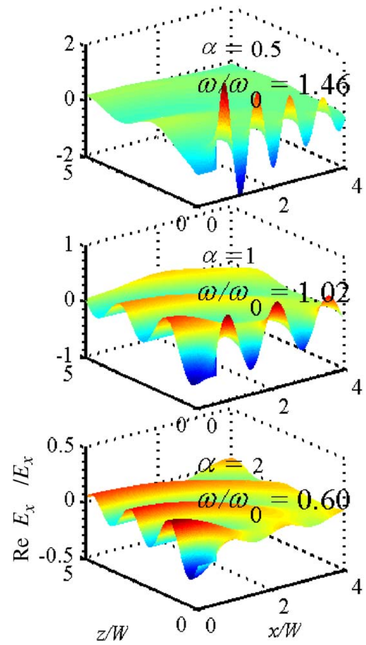

FIG. 4. (Color online) The distribution of the 2D plasmon-polariton electric field $\mathrm{Re}\, E_x^{2\mathrm{DP}}/E_x^0$ at different values of the retardation parameter with the frequency corresponding to the first maximum of the absorption cross section (Fig. 2).

The distribution of the energy flows around the contact stripe is shown in Figs. 5–7, where the Poynting vector of the induced electromagnetic wave $\mathbf{S}^{2\mathrm{DP}}/S_z^0$ is shown at different values of α. Figure 5 describes the nonretarded limit $\alpha \ll 1$. One sees that the energy of the induced electromagnetic wave is flowing along the x axis as it should be at the exci-

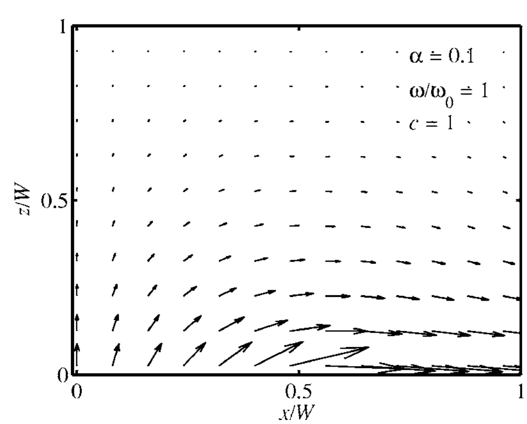

FIG. 5. The energy flow in the induced electromagnetic wave $\mathbf{S}^{2\mathrm{DP}}/S_z^0$ at $\alpha = 0.1$ and $\omega/\omega_0 = 1$. The quantity c is the scaling factor of vectors, so that the actual value of a vector is equal to the length in coordinates divided by c.

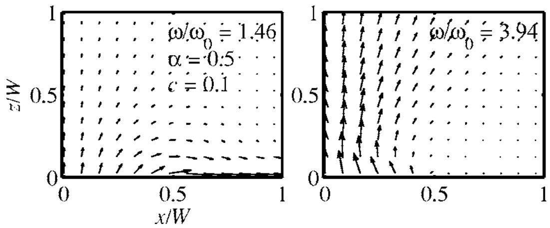

FIG. 6. The energy flow $\mathbf{S}^{2DP}/S_z^0$ at $\alpha=0.5$ and the frequencies corresponding to the maximum ($\omega/\omega_0=1.46$) and to the miminum ($\omega/\omega_0=3.94$) of the absorption cross section, see Fig. 2.

tation of the 2D plasmons in the quasistatic limit. Notice, that at large distances from the contact the Poynting vector has a (small) component directed toward the 2D EG, which means that the energy flows to the 2D layer and is dissipated in it.

In Fig. 6 we show the energy flow at a larger value of the retardation parameter $\alpha=0.5$. The left panel shows the distribution of the Poynting vector at the frequency $\omega/\omega_0=1.46$ corresponding to the maximum of the absorption spectrum (see Fig. 2). One sees that the $\mathbf{S}^{2DP}$ distribution is very similar to that shown in Fig. 5. In contrast, the right panel shows the energy flows around the 2D layer at the frequency $\omega/\omega_0=3.94$, corresponding to the minimum of the absorption. In this case the energy is mainly reflected back from the system in the z direction.

Figure 7 exhibits another feature of the energy-flow distribution at $\alpha=2$. The two left panels show the Poynting vector at frequencies corresponding to the first and the second maxima of the absorption cross section ($\omega/\omega_0=0.6$ and 1.54, respectively). In both cases there exists a visible energy flow along the x axis (the excitation of the 2D plasmon polaritons), but the reflection of waves at $\omega/\omega_0=1.54$ (the second harmonics) is much stronger than at $\omega/\omega_0=0.6$. This is due to the larger radiative decay of the higher modes. The right panels of Fig. 7 show the energy flows at $\omega/\omega_0=0.98$ and 1.98, which corresponds to the first and second minima of the absorption spectrum. At these frequencies, almost all the energy is reflected back in the z direction, and excitation of the 2D plasmon polaritons is weak. The energy-flow plots calculated for even larger values of α demonstrate similar behavior.

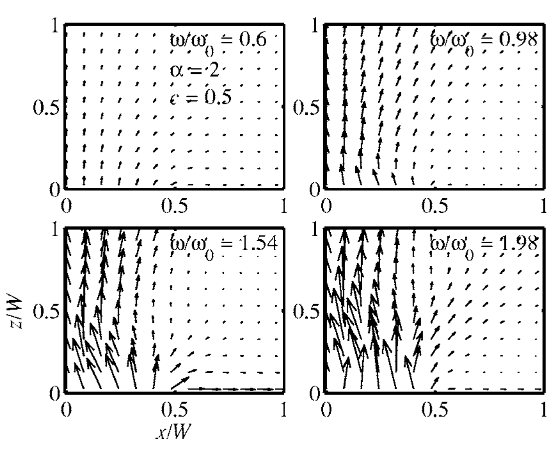

FIG. 7. The energy flow in the induced electromagnetic wave $\mathbf{S}^{2DP}/S_z^0$ at $\alpha=2$ and frequencies, corresponding to maxima (left panels) and minima (right panels) of the absorption cross-section (Fig. 2).

IV. SUMMARY

We have studied the scattering of electromagnetic waves incident upon a 2D electron-gas layer with a metallic contact stripe immersed in it. The absorption spectra of the system and the energy flows around the 2D ES have been studied at different frequencies and different values of the retardation parameter (18). We have shown that at small values of α (the electrostatic limit) the incident electromagnetic wave with the frequency ω excites the 2D plasmons with the wavevector determined by the dispersion relation (1). Response of the system at $\alpha \ll 1$ is almost independent of the frequency. At larger values of α the incident wave excites the 2D plasmon polaritons running along the 2D electron layer and electromagnetic waves propagating in the perpendicular direction away from it. The larger the retardation parameter, the larger the part of the energy emitted from the 2D gas in the form of electromagnetic waves. This can also be treated as the increase of the radiative decay of the 2D plasmon polaritons with the growing α. Absorption of the waves in the system becomes an oscillating function of frequency at $\alpha \gtrsim 1$, with maxima at $W/\lambda=n+1/2$ and minima at $W/\lambda=n$, with an integer n.

In the 2D plasmon experiments at terahertz frequencies[1–25] the retardation effects were not important and response of the system corresponded to the situation described by Fig. 2(a) (small values of α) and Fig. 3. In recent microwave experiments[41,42] the situation with $\alpha \gtrsim 1$ has been realized. At large α's the 2D plasmon-polariton resonances were shown to experience a substantial broadening of the linewidth, due to the radiative decay,[41,42,47,51] and results of our present work provide further illustration to the radiative decay phenomenon.

In this paper we have studied the excitation of *bulk* 2D plasmon polaritons in an infinite 2D electron system at zero magnetic field. In the experiments[48,49] the contacts served for coupling of the incident microwave radiation to the *edge* magnetoplasmons propagating along the boundaries of the 2D ES. An important and interesting extension of this work could be a study of the transformation efficiency of the energy of the incident microwave radiation to the energy of the edge magnetoplasmons in a bounded 2D ES in finite magnetic fields.

ACKNOWLEDGMENTS

The work in Sweden was supported by the Swedish Research Council (Vetenskapsrådet), the Swedish Foundation for International Cooperation in Research and Higher Education (STINT), and INTAS. This work has been started during a research visit of one of us (S.M.) to the University of Aizu, Japan. S.M. thanks Professor Victor Ryzhii for warm hospitality and the Japan Society for the Promotion of Science (JSPS) for financial support of this visit.

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