

**Build up and pinning of linear polarization in the Bose condensates of exciton polaritons**

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The linear polarization degree of the photoluminescence from a nonresonantly pumped CdTe microcavity exhibits a dramatic increase from about 0 to more than 85% once the threshold of stimulated scattering of the exciton polaritons toward the Bose-Einstein condensate is passed. The linear polarization direction is pinned to one of the crystallographic axes of the cavity. These effects are well accounted for within a model of localized condensate of interacting bosons.

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**I. INTRODUCTION**

Bose-Einstein condensation (BEC) of exciton polaritons in microcavities and polariton lasing have been the subject of numerous publications since the first proposal by Imamoglu in 1996.<sup>1</sup> Having a very light effective mass (on the order of  $10^{-5}$  of the free electron mass), the polaritons would condense at very high temperatures at thermal equilibrium.<sup>2,3</sup> On the other hand, due to their short radiative lifetime (on the order of  $10^{-12}$  s) the polaritons never achieve the thermal equilibrium with the crystal lattice, strictly speaking. The criterion for a BEC in an out-of-equilibrium bosonic system has been widely discussed in the literature.<sup>4,5</sup> While there are certainly multiple experimental evidences for macroscopic occupation numbers of single quantum states in resonantly excited microcavities,<sup>6,7</sup> we believe that only in the case of nonresonant excitation and further thermalization of the polariton gas one can speak about polariton BEC. In recent studies of a strong-coupling CdTe microcavity nonresonantly pumped above some critical value (threshold pumping  $P_{\text{thr}}$ ) we have demonstrated the formation of a condensate with an average occupation number  $\langle N \rangle$  of the order of 100.<sup>8</sup> Moreover, throughout the latest experiments it has been discovered that, in certain cases, polariton-polariton scatterings are fast enough to allow for thermalization of the polariton distribution at pumping slightly below ( $0.9P_{\text{thr}}$ ) the condensation threshold with the effective temperature of around 20 K.<sup>9</sup> The formation of exciton polariton condensate is expected to be accompanied by the appearance of an order parameter being the complex expectation value of the polariton annihilation operator  $\langle a \rangle$ .<sup>10</sup> The buildup of such quantity is usually hard to evidence since it requires specific photon counting experiments. Fortunately, in microcavities the order parameter of the condensate has two polarization components which can be accessed by conventional means of the

polarized optical spectroscopy. The recent theoretical works<sup>5,11</sup> have predicted that the appearance of an order parameter in the polariton system should be accompanied by the buildup of linear polarization of light emitted by the condensate. Refs. 9 and 12 reported significant linear polarization of light emission by a localized polariton condensates at cw and pulsed pumping, respectively. The polarization was found to be bound to one of the crystal axes. The theoretical model<sup>12</sup> shows that even a small splitting of the ground exciton polariton state may result in a high polarization degree of emission above the stimulation threshold.

In this paper, we experimentally study and theoretically describe the linear polarization dependence on the pumping power of the Bose condensate when the distribution of exciton polaritons is very close to the equilibrium Bose-Einstein function. As in Ref. 12, the orientation of the polarization plane is found to be pinned to one of the crystal axes, most probably due to the slight optical anisotropy of the cavity. The effect persists at different detunings between the cavity and exciton modes. It is quantitatively described within the original theoretical model of localized and weakly interacting spinor bosons. We show that while in the cw regime the buildup of linear polarization cannot be interpreted as an evidence for the spontaneous symmetry breaking, it is a fingerprint of formation of the Bose condensate of exciton polaritons.

**II. EXPERIMENT**

We study a CdTe/CdMgTe microcavity sample containing 16 quantum wells and characterized by a *vacuum field Rabi splitting* (i.e., splitting between the two exciton polariton modes at the anticrossing) of 26 meV. Electron-hole pairs are excited by a linearly or circularly polarized light at 1.77 eV provided by a Ti:sapphire laser operating in the cw

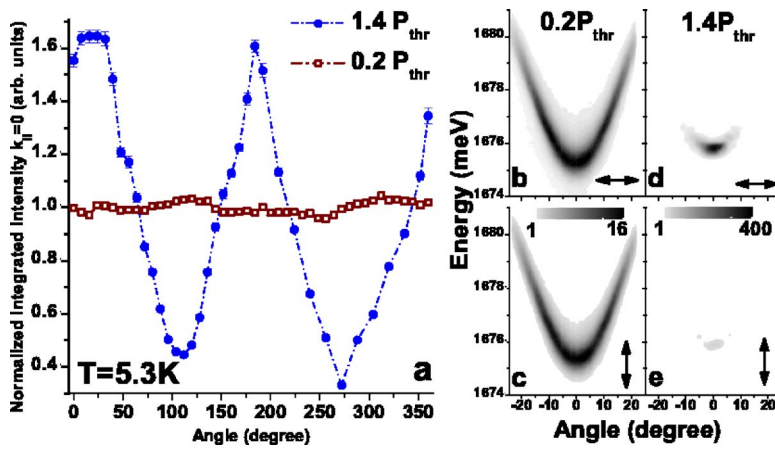


FIG. 1. (Color online) (a) Intensity of the ground state emission as a function of the angle of the linear analyzer, measured for below (open squares) and above the condensation threshold (solid circles). The lines are guides for eyes only. The angular and frequency dependencies of the polariton emission in orthogonal linear polarizations is shown below (b,c) and above the threshold (d,e). The same linear grey scale is used for each pumping power. All measurements were done at  $T=5.3$  K and for a positive detuning  $\delta = +5$  meV.

mode. The exciting laser beam is shaped into a “top-hat” intensity profile in order to provide the uniform excitation density over the entire excited area. The laser spot diameter is about  $25 \mu\text{m}$  on the sample surface. Both near-field and far-field time-integrated photoluminescence (PL) spectra are measured as a function of pumping intensity. The angular resolution is about  $0.4^\circ$  for the far-field emission. All the measurements have been done at 5 K. By a proper choice of the spot on the sample we are able to vary the detuning between cavity and photon modes. Supplementary experimental data on the PL behavior in the vicinity of the condensation threshold can be found in Refs. 8 and 9.

The angular and energy resolved PL spectra of the cavity below ( $P=0.2P_{\text{thr}}$ ) and above ( $P=1.4P_{\text{thr}}$ ) threshold are shown in Figs. 1(b)–1(e) for two orthogonal linear polarizations. The angle of emission  $\theta$  is related to the polariton in-plane wave vector  $k_{\parallel}$  by:  $k_{\parallel}=(E/\hbar c)\sin\theta$ , where  $E$  is the emitted photon energy. The analysis of these data shows that at threshold the polaritons follow the Boltzmann distribution function with  $T=20$  K. The emission from the ground state in this case appears to be completely unpolarized with an accuracy of 2%, as Fig. 1(a) shows (open squares). The situation drastically changes above the condensation threshold. The ground state average population increases at least two orders of magnitude for  $P=2P_{\text{thr}}$  as we judge from the pumping dependence of the integrated intensity of the PL line at

$k_{\parallel}=0$ .<sup>8</sup> The exciton polaritons in the excited states follow the thermalized distribution function with  $T=20$  K. The  $k_{\parallel}=0$  emission is blueshifted by 0.6 meV with respect to the bare  $k_{\parallel}=0$  energy for both polarizations. The polarization degree of the ground state emission achieves 85% (Fig. 4), whereas the excited polariton states remain unpolarized. The polarization of the emitted light is oriented along a well defined crystallographic direction ([110] direction, further referred to as  $x$  axis), with a standard deviation of  $15^\circ$  depending on the selected spot on the sample. Interestingly, the orientation of the ground state polarization is completely insensitive to the pumping polarization, as one can see from Figs. 2(c) and 2(f). It also remains the same at either circularly or linearly polarized pumping. Thus, the polarization is developed by the system itself and is not brought in by the excitation laser.

One more essential feature of the polariton condensate we observe is its spatial localization. We have investigated the spatial dependence of the polarization of polariton emission by means of the near-field spectroscopy. Below threshold, the emission is homogenous and unpolarized [Figs. 2(a) and 2(d)]. Above threshold, the emission mainly comes from several spots of about  $2\text{--}3 \mu\text{m}$  size [Figs. 2(b), 2(c), and 2(f)]. All spots emit light having the same linear polarization, which confirms that they emit light coherently as it has been previously suggested in Ref. 8 and recently proved in Ref. 9.

In order to reveal the build-up mechanism of the observed linear polarization, we have compared the dispersion of  $x$ -

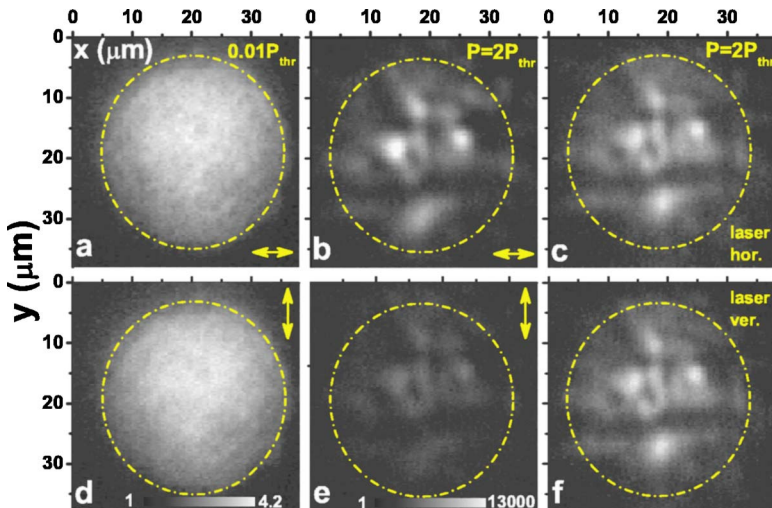


FIG. 2. (Color online) Near-field polarization characteristics of the polariton emission at  $T=5.3$  K; the upper and lower rows correspond to two orthogonal polarizations. Below condensation threshold (a,d) the emission spot is homogeneous and unpolarized. Above threshold (b,e) a strong and collective linear polarization is seen. The emission pattern above threshold remains unchanged when the polarization state of the pump is varied (c,f), confirming the spontaneous buildup of the linear polarization in the condensed state. The same linear grey scale is used for each column.

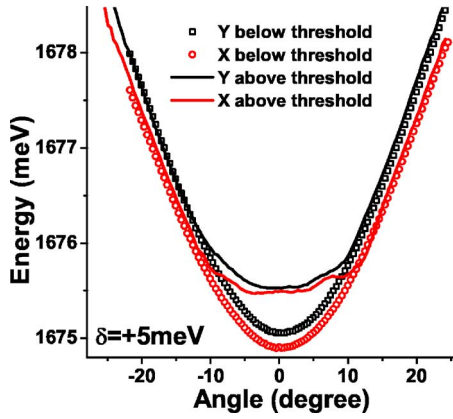


FIG. 3. (Color online) Dispersion of  $x$ - and  $y$ -polarized polariton branches below (open symbols) and above (solid lines) the condensation threshold for the detuning  $\delta = +5$  meV and  $T = 5.3$  K. The curves are extracted from the angular resolved PL spectra.

and  $y$ -polarized exciton polaritons extracted from the angle-resolved PL data measured below and above the stimulation threshold (Fig. 3). One can observe a small splitting of the ground polariton state  $k_{\parallel} = 0$  ( $\theta = 0$ ) into a linearly polarized doublet. The splitting persists at nonzero  $k_{\parallel}$ , where it is enhanced mostly due to the TE-TM splitting of the cavity modes. The value of the splitting at  $k_{\parallel} = 0$  is about 0.15 meV (much less than the polariton line width of 0.6 meV and temperature of 20 K). Most probably it is caused by the intrinsic anisotropy in the mirrors.<sup>17</sup> This splitting does not induce any significant polarization of the emission below the condensation threshold. Above the threshold, the polarization indeed appears, while the splitting seems to be reduced (see Fig. 3). One can clearly see the blueshift of the ground polariton state and the flattening of the dispersion for small  $k_{\parallel}$ , while the dispersion of both branches of excited states at large  $k_{\parallel}$  does not seem to be affected (Fig. 3). The spatial localization of the condensate is probably responsible for the flat part of the polariton dispersion observed above the condensation threshold. The blueshift and degree of linear polarization of the polariton ground state  $k_{\parallel} = 0$  measured as a function of the pumping power are displayed in Fig. 4(b) by crosses and solid circles, respectively. A marked increase of the linear polarization is observed above threshold, rapidly saturating at a value of about 85%.

### III. THEORY

Exciton polaritons are spinor bosons, which means that they can be described by a polarization two-dimensional (2D) complex vector  $\boldsymbol{\psi} = \{\psi_x, \psi_y\}$  where  $x$  and  $y$  are  $[110]$  and  $[\bar{1}10]$  in-plane axes, respectively. This vector describes the photon component of the polariton state, which can be different from the exciton component for a localized condensate. When many polaritons are collected in one localized state, the polarization vector can be treated as a classical variable and the free energy of the condensate reads<sup>13</sup>

$$F(\mathbf{S}) = -2\mu S - \varepsilon S_x + (\alpha_1 + \alpha_2)S^2 + (\alpha_1 - \alpha_2)S_z^2. \quad (1)$$

Here we introduced the components of the condensate *pseudospin*  $\mathbf{S}$  defined as  $S_x = (1/2)(|\psi_x|^2 - |\psi_y|^2)$ ,  $S_y = (1/2)(\psi_x \psi_y^* + \psi_x^* \psi_y)$ ,  $S_z = (1/2)(\psi_x \psi_y^* - \psi_x^* \psi_y)$ , so that  $S^2 = S_x^2 + S_y^2 + S_z^2 = (N/2)^2$ , where  $N = (\boldsymbol{\psi} \cdot \boldsymbol{\psi}^*) = |\psi_x|^2 + |\psi_y|^2$  is the occupation number of the condensate,  $\mu$  stands for the chemical potential,  $\alpha_1$  and  $\alpha_2$  are interaction constants for the polaritons with parallel and antiparallel spins, respectively, where  $\alpha_2 < 0 < \alpha_1$  and  $\alpha_1 > |\alpha_2|$ .<sup>14</sup> The energy  $\varepsilon$  describes a small splitting between bare  $x$ - and  $y$ -polarized states which might appear due to the optical anisotropy of the cavity ( $x$ -polarized state is assumed having the lowest energy).

In conditions of *cw* excitation the chemical potential  $\mu$  is proportional to the difference of the income and outcome rates of polaritons into the condensate. While the outcome rate is related mainly to the finite polariton life-time, the income rate is controlled by the pump. The threshold value of the pump corresponds to  $\mu = 0$ . Below the threshold, where  $\mu < 0$  and the polariton interactions can be neglected, the polariton distribution is thermal [the second-order coherence parameter  $g^{(2)}(0) = 2$ ]. Above the threshold, where  $\mu > 0$ , the polariton interactions results in the second-order coherence buildup. The polariton distribution is given by the Gibbs expression, but in general it is characterized by an effective temperature, which depends on the pump rate.<sup>15</sup> Note also that the positive chemical potential accounts for the blueshift of the emission line of the polariton condensate, equal to  $\mu + \varepsilon/2$  in our case.

If the condensate is localized by in-plane potential fluctuations in the cavity (which is exactly the case in our experiment), the in-plane polariton dispersion appears like a flat straight line (the energy is independent on the wave vector). Then, because of the blueshift, the condensate energy level is pushed deeply inside the band of free polariton states, crossing their dispersion curve. As a result, the overall dispersion curve looks like a truncated parabola at low  $k_{\parallel}$ , as it can be seen in Fig. 3. The size of the flat part  $k_0$  defines the polariton localization radius  $r_0 = k_0^{-1}$ . The flat part extending to angles  $\theta = \pm 5^\circ$ , as seen in Fig. 3, corresponds to  $r_0 \approx 1.4 \mu\text{m}$ , which is consistent with the localization radius seen in the images of Fig. 2. Note also that recently a flat polariton dispersion has been predicted by Szymańska *et al.*<sup>4</sup> for strongly dissipative polariton systems. This effect is unlikely to play a leading role in our cavity, where the in-plane localization of polaritons is present and the dissipation is dominated by the fast polariton relaxation above the stimulation threshold.

Above the threshold, the minimum of the free energy (1) is reached at  $S_y = S_z = 0$  and

$$S_x = \frac{\mu + (\varepsilon/2)}{\alpha_1 + \alpha_2}. \quad (2)$$

This means that the condensate is preferentially polarized along the  $x$  direction, which is the consequence of the anisotropic second term in (1). It is the splitting  $\varepsilon$  between  $x$  and  $y$  polarizations that provides the pinning to the  $x$  direction. If  $\varepsilon = 0$  the condensate remains linearly polarized ( $S_z = 0$ ), but its polarization direction is chosen spontaneously by the system and not fixed at a given crystal axis. At any finite tem-



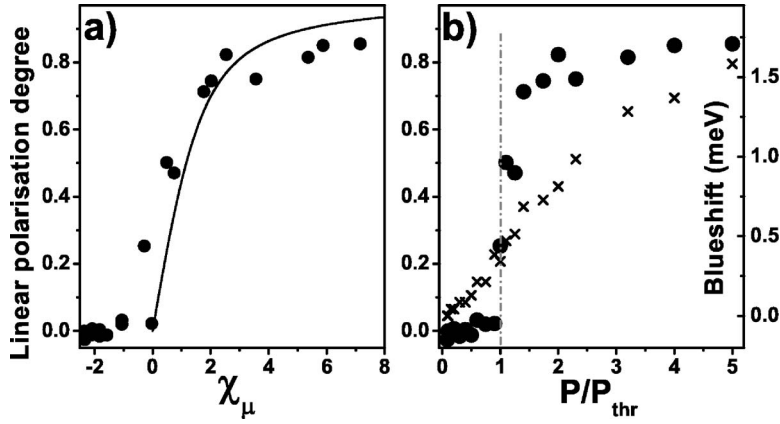


FIG. 4. Dependence of the linear polarization degree of the polariton ground state emission on  $\chi_\mu = \frac{\mu\varepsilon}{(\alpha_1 + \alpha_2)k_B T^*}$  (a): theory (solid line) and experiment (solid circles). Linear polarization degree (solid circles) and blueshift (crosses) of the polariton ground state as a function of the pump power  $P$  (b).

perature, the polarization would randomly change its direction with time. Therefore, in the *cw* regime the averaged polarization of the condensate emission would be zero in the  $\varepsilon=0$  case.

Different average values of physical parameters are related to the partition function  $Z$  of the polariton condensate. In particular, to calculate the linear polarization, we will need the average occupation  $\langle N \rangle = 2\langle S \rangle = k_B T^* \partial \ln Z / \partial \mu$  and the average projection of the pseudospin on the  $x$  axis  $\langle S_x \rangle = k_B T^* \partial \ln Z / \partial \varepsilon$ . In our classical spin approximation the partition function can be written as a three-dimensional (3D) integral in the pseudospin space

$$Z = \int d^2\psi_x d^2\psi_y \exp\left\{-\frac{F(\mathbf{S})}{k_B T^*}\right\} = \int d^3S \frac{\pi}{|\mathbf{S}|} \exp\left\{-\frac{F(\mathbf{S})}{k_B T^*}\right\}, \quad (3)$$

where  $T^*$  is the effective temperature. The partition function (3) can be evaluated analytically in the limit of strong pump and high average occupation number of the condensate. As usual, the method of steepest descent can be used. In our case, however, the small value of the anisotropic splitting  $\varepsilon$  results in large azimuthal fluctuations of the pseudospin in the  $xy$  plane. The main contribution to the integral (3) comes from the narrow ring around the circumference with radius  $S_0 = \mu / (\alpha_1 + \alpha_2)$ . Near this circumference the free energy (1) can be approximated as

$$F \approx -(\alpha_1 + \alpha_2)S_0^2 - \varepsilon S_0 \cos \varphi + (\alpha_1 + \alpha_2)s^2 + (\alpha_1 - \alpha_2)S_z^2. \quad (4)$$

Here  $\varphi$  is the azimuthal angle in the  $xy$  plane and  $s = \sqrt{S_x^2 + S_y^2} - S_0$ . Writing (4) we have kept only quadratic terms in  $|s|$  and  $|S_z|$ , which are small compared to  $S_0$  in the case  $k_B T^* \ll (\alpha_1 \pm \alpha_2)S_0^2$ , and omitted the small term  $-\varepsilon s \cos \varphi$ . Substitution of (4) into (3) gives

$$Z \approx Z_0(T^*) I_0\left(\frac{\mu\varepsilon}{(\alpha_1 + \alpha_2)k_B T^*}\right) \exp\left\{\frac{\mu^2}{(\alpha_1 + \alpha_2)k_B T^*}\right\}, \quad (5)$$

where  $Z_0(T^*) = 2\pi^3 k_B T^* / (\alpha_1^2 - \alpha_2^2)^{1/2}$  and  $I_n(x)$  denotes the modified Bessel functions ( $n=0, 1$ ). Using this equation and the property  $dI_0(x)/dx = I_1(x)$  we obtain the polarization degree

$$P_L = \frac{\langle S_x \rangle}{\langle S \rangle} \approx \frac{I_1(\chi_\mu)}{I_0(\chi_\mu)}, \quad (6)$$

where  $\chi_\mu = \mu\varepsilon / (\alpha_1 + \alpha_2)k_B T^*$ , and the mean number of polaritons

$$\langle N \rangle \approx \frac{2\mu}{\alpha_1 + \alpha_2}. \quad (7)$$

Figure 4(a) shows  $P_L$  plotted as a function of  $\chi_\mu$  (solid line) in comparison with the experimental data (solid circles). To determine  $\chi_\mu$  as a function of the pumping power  $P$ , we used the pump power dependencies of the polarization degree and the blueshift of the PL line shown in Fig. 4(b). The chemical potential has been deduced from the blueshift, assuming that  $\mu=0$  at the threshold. The effective temperature  $T^*$  was experimentally estimated as 20 K.<sup>9</sup> The value of  $\alpha_1 + \alpha_2 \approx 0.012$  meV was estimated from the mean number of polaritons in the condensate  $\langle N \rangle \approx 100$  (Ref. 8) and from the blueshift (0.6 meV) at  $P \approx 2P_{thr}$ . One can see that the analytical formula (6) yields a good quantitative agreement with the experimentally measured linear polarization degree reproducing its buildup and saturation. Note that Eq. (6) can only be used at the region of big positive chemical potentials, which corresponds to the pump far above the threshold. The numerical evaluation of the partition function (3) is necessary near the threshold. Note also that far below the threshold, the thermal equilibrium is not achieved and a kinetic modeling is required to calculate the polarization degree of the PL.<sup>16</sup>

#### IV. DISCUSSION

We interpret the observed build up of linear polarization of the PL from our microcavity as a supplementary evidence for the formation of a polariton Bose condensate. This condensate is localized as shown by the near-field spectroscopy data and the flat polariton dispersion above threshold. The direction of the linear polarization of the condensate is pinned to [110] crystallographic axis of our sample due to some small optical anisotropy. The degree of its polarization depends on its average population,  $xy$  splitting, polariton-polariton interaction constants and temperature.

The far-field and near-field data indicate the observation of a Bose condensation in a finite 2D system rather than a superfluid phase transition. A superfluid transition would be accompanied by the appearance of linear dispersions of  $x$ - and  $y$ -polarized polaritons in the vicinity of  $k_{\parallel}=0$  characterized by sound velocities proportional to  $\sqrt{(\alpha_1+\alpha_2)\langle N \rangle}$  and  $\sqrt{(\alpha_1-\alpha_2)\langle N \rangle}$ , respectively.<sup>11</sup> This is definitely not observed in our experiments. We believe that it is the disorder and induced potential fluctuations which lead to the localization of our condensate and prevent the superfluid transition. The transition to superfluidity would occur when the chemical potential reaches the percolation energy of the fluctuation potential in our system. At this point the condensate would cover 50% of the area where the polaritons were excited. This would require much higher pumping power than the one we disposed.

## V. CONCLUSIONS

In conclusion, our experiments have confirmed the theoretical prediction for the buildup of linear polarization in the emission spectra of microcavities due to the Bose-Einstein condensation of the exciton polaritons. The polarization orientation is pinned to one of the crystal axes ([110] axis) due to the small splitting of the polariton ground state. We do not observe the superfluid phase transition because of the in-plane disorder present in our cavity. What we see is rather the formation of a localized Bose condensate.

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