Antiferromagnetic metal to heavy-fermion metal quantum phase transition in the Kondo lattice model: A strong-coupling approach

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(Received 17 August 2006; revised manuscript received 17 October 2006; published 16 January 2007)

We study the quantum phase transition from an antiferromagnetic metal to a heavy fermion metal in the Kondo lattice model. Based on the strong-coupling approach we *first* diagonalize the Kondo coupling term. Since this strong-coupling approach makes the resulting Kondo term *relevant*, the Kondo hybridization persists even in the antiferromagnetic metal, indicating that fluctuations of Kondo singlets are not critical in the phase transition. We find that the quantum transition in our strong coupling approach results from *softening of antiferromagnetic spin fluctuations of localized spins*, driven by the Kondo interaction. Thus, the volume change of the Fermi surface becomes continuous across the transition. Using the boson representation of the localized spin $\vec{n}_i = \frac{1}{2} z_{i\sigma}^{\dagger} \vec{\tau}_{\sigma\sigma'} z_{i\sigma'}$ with the *spin-fractionalized* excitation $z_{i\sigma}$, we derive an effective U(1) gauge Lagrangian in terms of renormalized conduction electrons and fractionalized local-spin excitations interacting via U(1) *gauge fluctuations*, where the renormalized conduction electrons are given by *composites* of the conduction electrons and fractionalized spin excitations. Performing a mean field analysis based on this effective Kondo action, we find a mean field phase diagram as a function of J_K/D with various densities of conduction electrons, where J_K is the Kondo coupling strength and D the half-bandwidth of conduction electrons. The phase diagram shows a quantum transition, resulting from *condensation of the spin-fractionalized bosons*, from an antiferromagnetic metal to a heavy fermion metal away from half-filling. We show that beyond the mean field approximation our critical field theory characterized by the dynamic critical exponent $z=2$ can explain the observed non-Fermi liquid physics such as the specific heat coefficient $\gamma \equiv C_v/T \sim -\ln T$ near the quantum critical point. Furthermore, we argue that if our scenario is applicable, there can exist a narrow region of an anomalous metallic phase with the spin gap near the quantum critical point.

DOI: 10.1103/PhysRevB.75.035117 PACS number(s): 71.10.Hf, 71.27.+a, 75.30.Mb, 75.50.Ee

I. INTRODUCTION

The nature of non-Fermi liquid physics near quantum critical points in heavy fermion compounds is one of the central interests in modern condensed matter physics.[1] A standard theoretical framework is the Hertz-Moriya-Millis (HMM) theory in the Landau-Ginzburg-Wilson framework.[2] Using a Hubbard-Stratonovich (HS) transformation for an appropriate interactional channel, a local order parameter can be introduced. Integrating out fermion degrees of freedom and expanding the resulting logarithmic action based on the noninteracting itinerant fermion ensemble, one obtains an effective action of the local order parameter with dissipation that results from gapless electrons near the Fermi surface. Based on this order parameter action, a self-consistent mean field analysis and a renormalization group study can be performed to find non-Fermi liquid physics near the quantum critical point, characterized by critical fluctuations of the local order parameter.[3]

However, since the expansion in deriving HMM theory from the Fermi liquid action is basically a weak coupling approach, the effective theory will break down in a strong coupling limit. In addition, the presence of gapless electrons can cause nonlocal interactions between order parameters, making a conventional treatment unreliable in a local effective action.[4] Furthermore, HMM theory is not fully self-consistent because feedback effects to the fermion degrees of freedom by order parameter fluctuations are not taken into account. Actually, it is now believed that the critical field theory of order parameter fluctuations, that is, HMM theory cannot explain the observed non-Fermi liquid physics in thermal and electrical properties in the heavy fermion metals.[1]

In this study, using a strong-coupling approach, we derive an effective action in terms of both bosonic and fermionic excitations associated with order parameter fluctuations and gapless electrons, respectively. The weak-coupling approach solves the kinetic energy term first, and treats the interaction term perturbatively, implying that the theory is based on the noninteracting itinerant fermion ensemble. On the other hand, in the atomic limit of electrons the strong-coupling approach solves the interaction term first and then, treats the kinetic energy term perturbatively. In the present strong-coupling approach we take into account critical order parameter fluctuations and gapless electron excitations near the Fermi surface *on an equal footing*. In other words, we incorporate both bosonic and fermionic excitations in the effective action *without integrating out the gapless fermion degrees of freedom*. Hence, our strong coupling approach goes beyond the conventional treatment for the quantum phase transition in interacting itinerant electrons.

We start from an antiferromagnetic phase of localized spins in the Kondo lattice model, where the Kondo interaction term describes the coupling between the localized spins $\vec{n}_i$ and conduction spins $\vec{s}_i = \frac{1}{2} c_{i\sigma}^{\dagger} \vec{\tau}_{\sigma\sigma'} c_{i\sigma'}$. In the strong-coupling approach the Kondo interaction term can be diagonalized by using the CP^1 representation $\vec{n}_i \cdot \vec{\tau} = U_i \tau_3 U_i^{\dagger}$ with an SU(2) matrix $U_i = \begin{pmatrix} z_{i\uparrow} & -z_{i\downarrow}^{\dagger} \\ z_{i\downarrow} & z_{i\uparrow}^{\dagger} \end{pmatrix}$ for the localized spins and introducing renormalized electrons $\psi_{i\sigma}$ as composites of the conduction electrons and spin-fractionalized bosons, $\psi_{i\sigma}$

$=U^{\dagger}_{i\sigma\sigma'}c_{i\sigma'}$.[5–7] Then, the Kondo lattice model can be rewritten in terms of the spin-fractionalized bosons $z_{i\sigma}$ and renormalized conduction fermions $\psi_{i\sigma}$. Integrating out the fermion fields $\psi_{i\sigma}$ and performing the gradient expansion in the resulting logarithmic action, this conventional strong-coupling theory leads to an effective action of the $z_{i\sigma}$ bosons, which is well known in the context of the nonlinear σ model.[5] In the present study, however, we take into account both the bosons and fermions simultaneously.

The boson representation for the localized spin instead of its fermion representation is necessary for explaining the quantum phase transition involving an antiferromagnetic phase. The U(1) slave-boson representation has been used conventionally for the study of the heavy fermion phase in the Kondo lattice problem. If the localized spins are represented by fermions and an order parameter (slave-boson) corresponding to the Kondo hybridization is introduced, a critical theory can be obtained in terms of both order parameter fluctuations and fermion excitations by integrating out the conduction electrons.[8,9] Fermions and bosons then interact via long range interactions mediated by slave-boson U(1) gauge fields. This critical theory successfully explains the non-Fermi liquid physics such as the specific heat coefficient $\gamma=C_v/T\sim-\ln T$ near the quantum critical point. However, it is difficult to explain the observed antiferromagnetic long range order that begins to appear at the transition point where the heavy fermion phase disappears. Furthermore, the order parameter in the slave-boson representation is "hidden"; no symmetry breaking associated with lattice translations or spin rotations is involved. Thus, the transition driven by condensation of the hidden order parameter may not be physical.

II. EFFECTIVE ACTION FOR THE KONDO LATTICE MODEL

In the Hamiltonian of the Kondo lattice model

$$H_{KLM}=-t\sum_{ij\sigma}c^{\dagger}_{i\sigma}c_{j\sigma}+J_K\sum_{i\sigma\sigma'}\vec{S}_i\cdot c^{\dagger}_{i\sigma}\vec{\tau}_{\sigma\sigma'}c_{i\sigma'}, \tag{1}$$

the first term describes dynamics of conduction electrons $c_{i\sigma}$ and the second term represents antiferromagnetic exchange couplings between conduction electrons and localized spins $\vec{S}_i$, where t is the hopping integral of the conduction electrons and J_K is the Kondo coupling strength. Using the coherent state representation for the conduction electrons and localized spins,[10] the partition function of the Kondo lattice model in the path-integral representation can be given by

$$Z=\int Dc_{i\sigma}D\vec{n}_i\exp\Bigg[iS\sum_i\int_0^{\beta}d\tau\int_0^1 du\vec{n}_i\cdot\left(\frac{\partial\vec{n}_i}{\partial u}\times\frac{\partial\vec{n}_i}{\partial\tau}\right)-\int_0^{\beta}d\tau\Bigg(\sum_{i\sigma}c^{\dagger}_{i\sigma}(\partial_\tau-\mu)c_{i\sigma}-t\sum_{ij\sigma}c^{\dagger}_{i\sigma}c_{j\sigma}+J_KS\sum_{i\sigma\sigma'}\vec{n}_i\cdot c^{\dagger}_{i\sigma}\vec{\tau}_{\sigma\sigma'}c_{i\sigma'}\Bigg)\Bigg], \tag{2}$$

where we use $\vec{S}_i=S\vec{n}_i$ with $S=1/2$ and $|\vec{n}_i|=1$, and μ is the chemical potential. The first term corresponds to the Berry phase that comes from the path-integral quantization of spin with an additional parameter u in a unit sphere.[10]

In order to study the antiferromagnetic phase with collinear ordering of localized spins, we set

$$\vec{n}_i\to(-1)^i\vec{n}_i. \tag{3}$$

Then, Eq. (2) reads

$$Z=\int Dc_{i\sigma}D\vec{n}_i\exp\Bigg[iS\sum_i(-1)^i\int_0^{\beta}d\tau\int_0^1 du\vec{n}_i\cdot\left(\frac{\partial\vec{n}_i}{\partial u}\times\frac{\partial\vec{n}_i}{\partial\tau}\right)-\int_0^{\beta}d\tau\Bigg(\sum_{i\sigma}c^{\dagger}_{i\sigma}(\partial_\tau-\mu)c_{i\sigma}-t\sum_{ij\sigma}c^{\dagger}_{i\sigma}c_{j\sigma}+J_KS\sum_{i\sigma\sigma'}(-1)^ic^{\dagger}_{i\sigma}(\vec{n}_i\cdot\vec{\tau})_{\sigma\sigma'}c_{i\sigma'}\Bigg)\Bigg]. \tag{4}$$

Based on this partition function we investigate two closely related interesting problems; how the antiferromagnetic ordering of local spins is affected by dynamics of conduction electrons as varying the Kondo coupling J_K, and how the dynamics of the conduction electrons is influenced by the change of antiferromagnetic fluctuations of the local spins.

We apply the strong coupling approach to Eq. (4) where we first solve the Kondo coupling term. Although the HMM theory results in a firm and successive quantum theory for quantum phase transitions,[2] the naive perturbation for the Kondo coupling term based on the itinerant fermion ensemble do not work well in the case of strong couplings. Using the identity

$$\vec{n}_i\cdot\vec{\tau}=U_i\tau_3U_i^{\dagger},\quad U_i=\begin{pmatrix}z_{i\uparrow} & -z^{\dagger}_{i\downarrow}\\ z_{i\downarrow} & z^{\dagger}_{i\uparrow}\end{pmatrix}, \tag{5}$$

where $z_{i\sigma}$ is a boson field carrying a spin σ in the SU(2) matrix field U_i and satisfies the unimodular constraint $\Sigma_\sigma|z_{i\sigma}|^2=1$, and performing the gauge transformation

$$\psi_{i\sigma}=U^{\dagger}_{i\sigma\sigma'}c_{i\sigma'}, \tag{6}$$

the Kondo coupling term, $J_KS\Sigma_{i\sigma\sigma'}(-1)^ic^{\dagger}_{i\sigma}(\vec{n}_i\cdot\vec{\tau})_{\sigma\sigma'}c_{i\sigma'}$, can be represented as $J_KS\Sigma_{i\sigma\sigma'}(-1)^i\psi^{\dagger}_{i\sigma}\tau^3_{\sigma\sigma'}\psi_{i\sigma'}$. As a result, the two-body Kondo interaction term is represented by a one body term. We call $z_{i\sigma}$ and $\psi_{i\sigma}$ bosonic spinon and fermionic chargon, respectively.

In the strong coupling approach an antiferromagnetic spin fluctuation $\vec{n}_i$ carrying spin quantum number 1 fractionalizes into bosonic spinons $z_{i\sigma}$ with spin 1/2, which seems to occur through the screening of conduction electrons due to the strong Kondo interaction. The components of $\psi_{i\sigma}$ field are given by

$$\psi_{i\sigma}=\begin{pmatrix}\psi_{i\uparrow}\\ \psi_{i\downarrow}\end{pmatrix}=\begin{pmatrix}z^{\dagger}_{i\uparrow}c_{i\uparrow}+z^{\dagger}_{i\downarrow}c_{i\downarrow}\\ -z_{i\downarrow}c_{i\uparrow}+z_{i\uparrow}c_{i\downarrow}\end{pmatrix},$$

where $\psi_{i\uparrow}$ field represents the usual Kondo hybridization, and $\psi_{i\downarrow}$ field the polarization of the bosonic spinon and the conduction electron. The fermions $\psi_{i\sigma}$ can be considered to express Kondo resonances. Another way to describe this fractionalization is that the conduction electrons fractionalize into the bosonic spinons $U_{i\sigma\sigma'}$ and the fermionic chargons $\psi_{i\sigma}$, i.e., $c_{i\sigma}=U_{i\sigma\sigma'}\psi_{i\sigma'}$. The resulting partition function is obtained in terms of new field variables ψ_i and U_i,

$$Z=\int D\psi_{i\sigma}DU_{i\sigma\sigma'}\delta(U^{\dagger}_{i\sigma\sigma''}U_{i\sigma''\sigma'}-\delta_{\sigma\sigma'})\exp\Bigg[-S_B -\int_0^{\beta}d\tau\Bigg\{\sum_{i\sigma\sigma'}\psi^{\dagger}_{i\sigma}([\partial_\tau-\mu]\delta_{\sigma\sigma'}+[U^{\dagger}_i\partial_\tau U_i]_{\sigma\sigma'})\psi_{i\sigma'} -t\sum_{ij}\psi^{\dagger}_{i\sigma}U^{\dagger}_{i\sigma\alpha}U_{j\alpha\sigma'}\psi_{j\sigma'}+J_KS\sum_{i\sigma\sigma'}(-1)^i\psi^{\dagger}_{i\sigma}\tau^3_{\sigma\sigma'}\psi_{i\sigma'}\Bigg\}\Bigg], \tag{7}$$

where S_B is the Berry phase action. Note that the integration measure $\int Dc_{i\sigma}D\vec{n}_i$ in Eq. (4) is changed into $\int D\psi_{i\sigma}DU_{i\sigma\sigma'}\delta(U^{\dagger}_{i\sigma\sigma''}U_{i\sigma''\sigma'}-\delta_{\sigma\sigma'})$ in Eq. (7). Note that the chargons (renormalized conduction electrons) and spinons (fractionalized local spins) are now coupled in the kinetic energy term instead of the Kondo coupling between the conduction electrons and localized spins in Eq. (4).

A standard way to treat this nontrivial kinetic energy term is to integrate out the chargon fields,

$$Z=\int DU_{i\sigma\sigma'}\delta(U^{\dagger}_{i\sigma\sigma''}U_{i\sigma''\sigma'}-\delta_{\sigma\sigma'})\exp[-S_B+\mathbf{tr}\ln\{[\partial_\tau-\mu]\times\delta_{\sigma\sigma'}+J_KS(-1)^i\tau^3_{\sigma\sigma'}+[U^{\dagger}_i\partial_\tau U_i]_{\sigma\sigma'}-t_{ij}U^{\dagger}_{i\sigma\alpha}U_{j\alpha\sigma'}\}], \tag{8}$$

where $t_{ij}=t$ is the nearest neighbor hopping. An effective action of the spinons can be obtained by expanding the logarithmic term for the bosonic spinons. One important difference from the HMM theory is that the expansion parameter is t/J_K instead of J_K/t. However, this formulation has a serious defect. Metallic physics of the conduction electrons is not introduced since this treatment is valid in the atomic limit. Actually, expanding the logarithmic term in the expansion parameter t/J_K, the resulting effective action is known to be the O(3) nonlinear σ model appropriate to an *insulating* antiferromagnet.[3] Because an additional Berry phase term appears in the effective σ model, the two Berry phase terms cancel each other and the contribution of Berry phase vanishes.[6] In the following, although we develop a formulation different from the above standard approach, we can also exclude the Berry phase term.

An important issue of this study is how to introduce physics of the Fermi surface of the conduction electrons in the strong coupling approach. One possible route is to decouple the "interacting" kinetic energy into the conventional "noninteracting" one via the HS transformation. Unfortunately, there is a difficulty in performing the HS transformation for the time-derivative term in Eq. (7). We consider discrete-time steps and rewrite the partition function

$$Z\approx\int DU^{\sigma\sigma'}_{i\tau}D\psi^{\sigma}_{i\tau}\exp\Bigg[-\Bigg\{-\sum_{i\tau\tau'}\psi^{\dagger\sigma}_{i\tau}U^{\dagger\sigma\alpha}_{i\tau}U^{\alpha\sigma'}_{i\tau'}\psi^{\sigma'}_{i\tau'} -\frac{t}{J}\sum_{ij\tau}\psi^{\dagger\sigma}_{i\tau}U^{\dagger\sigma\alpha}_{i\tau}U^{\alpha\sigma'}_{j\tau}\psi^{\sigma'}_{j\tau}+\frac{J_KS}{J}\sum_{i\tau}(-1)^i\psi^{\dagger\sigma}_{i\tau}\tau^3_{\sigma\sigma'}\psi^{\sigma'}_{i\tau} -\frac{\mu}{J}\sum_{i\tau}\psi^{\dagger\sigma}_{i\tau}\psi^{\sigma}_{i\tau}\Bigg\}\Bigg], \tag{9}$$

where J is an energy scale associated with a time step $J=1/\Delta\tau$. This discrete-time expression can be reduced to the original one of Eq. (7) in the limit of $\Delta\tau\to 0$.[11]

Performing the HS transformation for the "hopping" terms in Eq. (9), we obtain the following expression:

$$\exp\Bigg[-\Bigg\{-\sum_{i\tau\tau'}\psi^{\dagger\sigma}_{i\tau}U^{\dagger\sigma\alpha}_{i\tau}U^{\alpha\sigma'}_{i\tau'}\psi^{\sigma'}_{i\tau'}-\frac{t}{J}\sum_{ij\tau}\psi^{\dagger\sigma}_{i\tau}U^{\dagger\sigma\alpha}_{i\tau}U^{\alpha\sigma'}_{j\tau}\psi^{\sigma'}_{j\tau}\Bigg\}\Bigg]$$
$$=\int DF^{\sigma\sigma'}_{\mu\nu}DE^{\sigma\sigma'}_{\mu\nu}\exp\Bigg[-\sum_{i\tau\tau'}\{E^{\dagger\sigma\sigma'}_{i\tau\tau'}F^{\sigma'\sigma}_{i\tau\tau'}+\text{H.c.} -U^{\dagger\sigma\alpha}_{i\tau}U^{\alpha\sigma'}_{i\tau'}F^{\sigma'\sigma}_{i\tau\tau'}-E^{\dagger\sigma\sigma'}_{i\tau\tau'}\psi^{\sigma'}_{i\tau'}\psi^{\dagger\sigma}_{i\tau}-\text{H.c.}\} -\frac{t}{J}\sum_{ij\tau}\{E^{\dagger\sigma\sigma'}_{ij\tau}F^{\sigma'\sigma}_{ij\tau}+\text{H.c.}-U^{\dagger\sigma\alpha}_{i\tau}U^{\alpha\sigma'}_{j\tau}F^{\sigma'\sigma}_{ij\tau} -E^{\dagger\sigma\sigma'}_{ij\tau}\psi^{\sigma'}_{j\tau}\psi^{\sigma\dagger}_{i\tau}-\text{H.c.}\}\Bigg]$$
$$=\int DF_{\mu\nu}DE_{\mu\nu}\exp\Bigg[-\sum_{i\tau\tau'}\mathbf{tr}\{E^{\dagger}_{i\tau\tau'}F_{i\tau\tau'}+\text{H.c.} -U_{i\tau'}F_{i\tau\tau'}U^{\dagger}_{i\tau}-\psi^{\dagger}_{i\tau}E^{\dagger}_{i\tau\tau'}\psi_{i\tau'}-\text{H.c.}\} -\frac{t}{J}\sum_{ij\tau}\mathbf{tr}\{E^{\dagger}_{ij\tau}F_{ij\tau}+\text{H.c.} -U_{j\tau}F_{ij\tau}U^{\dagger}_{i\tau}-\psi^{\dagger}_{i\tau}E^{\dagger}_{ij\tau}\psi_{j\tau}-\text{H.c.}\}\Bigg], \tag{10}$$

where $E_{i\tau\tau'}$, $F_{i\tau\tau'}$ and $E_{ij\tau}$, $F_{ij\tau}$ are HS matrix fields associated with hopping of $\psi_{i\tau}$ fermions and $z_{i\tau}$ bosons in time and space, respectively.

We make an ansatz for the hopping matrix fields

$$E_{i\tau\tau'}\equiv E_te^{ia_{i\tau\tau'}\tau_3},\quad E_{ij\tau}\equiv E_re^{ia_{ij\tau}\tau_3},$$
$$F_{i\tau\tau'}\equiv F_te^{ia_{i\tau\tau'}\tau_3},\quad F_{ij\tau}=F_re^{ia_{ij\tau}\tau_3}, \tag{11}$$

where E_t, F_t and E_r, F_r are longitudinal modes (amplitudes) of the hopping parameters, and $a_{i\tau\tau'}$, $a_{ij\tau}$ their transverse modes (phase fluctuations) which are time and spatial components of U(1) gauge fields. In fact, the transverse modes should be represented by SU(2) gauge fields generally, but we limit our discussion in the U(1) case for simplicity. The hopping parameters will be determined self-consistently. Inserting Eq. (11) into Eq. (10), and using the explicit representation [Eq. (5)] of the SU(2) matrix U_i, we obtain an effective U(1) gauge theory of the Kondo lattice model

$$Z = \int Dz_{i\tau\sigma}D\psi_{i\tau\sigma}Da_{\mu\nu}D\lambda_{i\tau}DE_tDF_tDE_rDF_r \times \exp\Bigg[- \Bigg\{ \sum_{\tau\tau'}\sum_{i} E_tF_t + \frac{t}{J}\sum_{\tau}\sum_{ij} E_rF_r - E_t\sum_{\tau\tau'}\sum_{i\sigma} \psi^{\dagger}_{i\tau\sigma}e^{i\sigma a_{i\tau\tau'}}\psi_{i\tau'\sigma} - \frac{t}{J}E_r\sum_{\tau}\sum_{ij\sigma} \psi^{\dagger}_{i\tau\sigma}e^{i\sigma a_{ij\tau}}\psi_{j\tau\sigma} + \frac{J_KS}{J}\sum_{\tau}\sum_{i\sigma}(-1)^i\sigma\psi^{\dagger}_{i\tau\sigma}\psi_{i\tau\sigma} - \frac{\mu}{J}\sum_{\tau}\sum_{i\sigma}\psi^{\dagger}_{i\tau\sigma}\psi_{i\tau\sigma} - F_t\sum_{\tau\tau'}\sum_{i\sigma} z^{\dagger}_{i\tau\sigma}e^{ia_{i\tau\tau'}}z_{i\tau'\sigma} - \frac{t}{J}F_r\sum_{\tau}\sum_{ij\sigma} z^{\dagger}_{i\tau\sigma}e^{ia_{ij\tau}}z_{j\tau\sigma} + i\sum_{\tau}\sum_{i}\frac{\lambda_{i\tau}}{J}\Big(\sum_{\sigma}|z_{i\tau\sigma}|^2 - 1\Big)\Bigg\}\Bigg], \quad (12)$$

where the τ_3 matrix is replaced with $\sigma=\pm$. λ_i is a Lagrange multiplier field to impose the unimodular constraint.

The last step in deriving an effective U(1) gauge theory of the Kondo lattice model is to perform the limit of $\Delta\tau\to 0$. Ignoring the time component $a_{i\tau\tau'}$ of the U(1) gauge field for the time being, the time-derivative term of the bosonic spinons becomes in the tight-binding approximation for the discrete time

$$- F_t\sum_{\tau\tau'}\sum_{i\sigma} z^{\dagger}_{i\tau\sigma}z_{i\tau'\sigma} = -2F_t\sum_{\Omega_n}\sum_{i\sigma}\cos\Big(\frac{\Omega_n}{J}\Big)z^{\dagger}_{i\sigma}(\Omega_n)z_{i\sigma}(\Omega_n) \approx -2F_t\sum_{\Omega_n}\sum_{i\sigma}\Bigg(1-\frac{1}{2}\Big(\frac{\Omega_n}{J}\Big)^2\Bigg) \times z^{\dagger}_{i\sigma}(\Omega_n)z_{i\sigma}(\Omega_n) = \int_0^{\beta} d\tau\sum_{i\sigma}\Big(\frac{F_t}{J}|\partial_\tau z_{i\sigma}|^2 - 2JF_t|z_{i\sigma}|^2\Big). \quad (13)$$

The last term can be absorbed into the Lagrange multiplier term, and thus has no physical effects. The derivation of Eq. (13) is based on the relativistic invariance for the bosonic spinons. This is reasonable because spin excitations in the antiferromagntic phase have the $\omega_n \sim k$ dispersion, exhibiting the relativistic invariance. On the other hand, the relativistic assumption for the bosonic spinons is not appropriate for the fermionic chargons because the chargons have Fermi surface. In this case it is natural to perform the $\Delta\tau\to 0$ limit naively. Then the time-derivative term for the fermionic chargons is given by

$$- E_t\sum_{\tau\tau'}\sum_{i\sigma}\psi^{\dagger}_{i\tau\sigma}\psi_{i\tau'\sigma} = -E_t\sum_{\tau}\sum_{i\sigma}\psi^{\dagger}_{i\tau\sigma}\Big(\psi_{i\tau\sigma} - \frac{\Delta\psi_{i\tau\sigma}}{\Delta\tau}\Delta\tau\Big) = \int_0^{\beta} d\tau\sum_{i\sigma}(E_t\psi^{\dagger}_{i\sigma}\partial_\tau\psi_{i\sigma} - JE_t\psi^{\dagger}_{i\sigma}\psi_{i\sigma}), \quad (14)$$

where $\Delta\tau=1/J$ is used. The last term is also absorbed into the chemical potential term.

Based on the above discussion, we find the effective U(1) gauge action for the Kondo lattice model

$$Z = \int Dz_{i\sigma}D\psi_{i\sigma}Da_{\mu\nu}D\lambda_iDE_tDF_tDE_rDF_re^{-S_{eff}},$$

$$S_{eff} = S_0 + S_\psi + S_z,$$

$$S_0 = \int_0^{\beta} d\tau\Big(J\sum_i E_tF_t + t\sum_{ij}E_rF_r\Big),$$

$$S_\psi = \int_0^{\beta} d\tau\Big(E_t\sum_{i\sigma}\psi^{\dagger}_{i\sigma}(\partial_\tau - i\sigma a_{i\tau})\psi_{i\sigma} - tE_r\sum_{ij\sigma}\psi^{\dagger}_{i\sigma}e^{i\sigma a_{ij}}\psi_{j\sigma} + J_KS\sum_{i\sigma}(-1)^i\sigma\psi^{\dagger}_{i\sigma}\psi_{i\sigma} - \mu\sum_{i\sigma}\psi^{\dagger}_{i\sigma}\psi_{i\sigma}\Big),$$

$$S_z = \int_0^{\beta} d\tau\Bigg(\frac{F_t}{J}\sum_{i\sigma}|(\partial_\tau - ia_{i\tau})z_{i\sigma}|^2 - tF_r\sum_{ij\sigma}z^{\dagger}_{i\sigma}e^{ia_{ij}}z_{j\sigma} + i\sum_i\lambda_i\Big(\sum_\sigma|z_{i\sigma}|^2 - 1\Big)\Bigg). \quad (15)$$

Here, both the conduction electrons and localized spins are taken into account on an equal footing *in the strongcoupling regime*, and the bosonic effective action S_z associated with order parameter fluctuations is derived without integrating out the gapless conduction electrons *explicitly*. The fermionic effective action S_ψ has essentially the same structure as the action of the conduction electrons in Eq. (4), considering that antiferromagnetic spin fluctuations are frozen to be $\vec{n}_i\cdot\vec{\tau}=\tau_3$ in Eq. (4), and gauge fluctuations are ignored in Eq. (15). When the spinons are condensed, the effective action in Eq. (15) is reduced to Eq. (4) with $\vec{n}_i\cdot\vec{\tau}=\tau_3$. In the condensed phase gauge fluctuations can be ignored in the low energy limit because they are gapped due to the Anderson-Higgs mechanism. The spinon action S_z is equivalent to the CP^1 action of the O(3) nonlinear σ model.[5] Hence, the effective action of Eq. (15) can recover the insulating antiferromagnet at half filling of the conduction electrons with Fermi-nesting.

III. MEAN FIELD ANALYSIS

Now we perform the saddle-point analysis to obtain possible phases of the Kondo lattice model. The mean field phases will be determined by condensation of the bosonic spinons. We ignore the U(1) gauge fluctuations in the mean field analysis. Later, these gauge excitations will be allowed beyond the mean field approximation. Then, we consider the effective mean field action

$$Z_{MF} = \int Dz_{i\sigma} D\psi_{i\sigma} \exp\Bigg[- \Bigg\{ \int_0^\beta d\tau \Bigg(E_t \sum_{i\sigma} \psi_{i\sigma}^\dagger \partial_\tau \psi_{i\sigma} - tE_r \sum_{ij\sigma} \psi_{i\sigma}^\dagger \psi_{j\sigma} + J_K S \sum_{i\sigma} (-1)^i \sigma \psi_{i\sigma}^\dagger \psi_{i\sigma} - \mu \sum_{i\sigma} \psi_{i\sigma}^\dagger \psi_{i\sigma} + \frac{F_t}{J} \sum_{i\sigma} |\partial_\tau z_{i\sigma}|^2 - tF_r \sum_{ij\sigma} z_{i\sigma}^\dagger z_{j\sigma} + \lambda \sum_i \left(\sum_\sigma |z_{i\sigma}|^2 - 1 \right) + J \sum_i E_t F_t + t \sum_{ij} E_r F_r \Bigg) \Bigg\} \Bigg] \tag{16}$$

with $\lambda \equiv i\lambda_i$. The hopping parameters E_t, F_t, E_r, F_r, and the effective chemical potentials λ, μ can be written from the saddle-point equations as follows:

$$JE_t = \frac{1}{J} \sum_\sigma \langle |\partial_\tau z_{i\sigma}|^2 \rangle, \quad JF_t = \sum_\sigma \langle \psi_{i\sigma}^\dagger \partial_\tau \psi_{i\sigma} \rangle,$$

$$E_r = \sum_\sigma \langle z_{i\sigma}^\dagger z_{j\sigma} \rangle, \quad F_r = \sum_\sigma \langle \psi_{i\sigma}^\dagger \psi_{j\sigma} \rangle,$$

$$1 = \sum_\sigma \langle z_{i\sigma}^\dagger z_{i\sigma} \rangle, \quad 1 - \delta = \sum_\sigma \langle c_{i\sigma}^\dagger c_{i\sigma} \rangle = \sum_\sigma \langle \psi_{i\sigma}^\dagger \psi_{i\sigma} \rangle, \tag{17}$$

where δ is hole concentration.

Integrating out fermions $\psi_{i\sigma}$ and bosons $z_{i\sigma}$ in Eq. (16), we obtain the following expression for the free energy:

$$F_{MF} = -\frac{1}{\beta} \sum_{\omega_n} \sum_{k\sigma} \mathbf{tr} \ln \begin{pmatrix} iE_t\omega_n - \mu + E_r \epsilon_k^\psi & \sigma J_K S \\ \sigma J_K S & iE_t\omega_n - \mu + E_r \epsilon_{k+Q}^\psi \end{pmatrix} + \frac{1}{\beta} \sum_{\Omega_n} \sum_{k\sigma} \ln\left(\frac{F_t}{J} \Omega_n^2 + F_r \epsilon_k^z + \lambda \right) + \sum_k (JE_t F_t + DE_r F_r) + \sum_k (\mu[1-\delta] - \lambda). \tag{18}$$

Here ϵ_k^ψ and ϵ_k^z are the bare dispersions of chargons and spinons, respectively. ω_n (Ω_n) is the fermionic (bosonic) Matsubara frequency. Minimizing the free energy in Eq. (18) with respect to E_t, F_t, E_r, F_r, λ, and μ, we obtain the self-consistent mean field equations

$$JE_t = -\int_{-D}^{D} d\epsilon D(\epsilon) \frac{2}{\beta} \sum_{\Omega_n} \frac{\Omega_n^2 / J}{\frac{F_t}{J} \Omega_n^2 + F_r \epsilon + \lambda},$$

$$JF_t = \sum_k{}' \frac{2}{\beta} \sum_{\omega_n} \frac{i\omega_n [(iE_t\omega_n - \mu + E_r \epsilon_{k+Q}^\psi) + (iE_t\omega_n - \mu + E_r \epsilon_k^\psi)]}{(iE_t\omega_n - \mu + E_r \epsilon_k^\psi)(iE_t\omega_n - \mu + E_r \epsilon_{k+Q}^\psi) - (J_K S)^2},$$

$$DE_r = -\int_{-D}^{D} d\epsilon D(\epsilon) \frac{2}{\beta} \sum_{\Omega_n} \frac{\epsilon}{\frac{F_t}{J} \Omega_n^2 + F_r \epsilon + \lambda},$$

$$DF_r = \sum_k{}' \frac{2}{\beta} \sum_{\omega_n} \frac{(iE_t\omega_n - \mu)(\epsilon_k^\psi + \epsilon_{k+Q}^\psi) + 2E_r \epsilon_{k+Q}^\psi \epsilon_k^\psi}{(iE_t\omega_n - \mu + E_r \epsilon_k^\psi)(iE_t\omega_n - \mu + E_r \epsilon_{k+Q}^\psi) - (J_K S)^2},$$

$$1 = \int_{-D}^{D} d\epsilon D(\epsilon) \frac{2}{\beta} \sum_{\Omega_n} \frac{1}{\frac{F_t}{J} \Omega_n^2 + F_r \epsilon + \lambda},$$

$$1 - \delta = -\sum_k{}' \frac{2}{\beta} \sum_{\omega_n} \frac{[(iE_t\omega_n - \mu + E_r \epsilon_{k+Q}^\psi) + (iE_t\omega_n - \mu + E_r \epsilon_k^\psi)]}{(iE_t\omega_n - \mu + E_r \epsilon_k^\psi)(iE_t\omega_n - \mu + E_r \epsilon_{k+Q}^\psi) - (J_K S)^2}. \tag{19}$$

Here Σ_k is replaced with $\int_{-D}^{D} d\epsilon D(\epsilon)$ in the bosonic equations, where $D(\epsilon)$ is the density of states for the bosonic spectrum ϵ_k^z. Σ_k' in the fermionic equations means sum over the folded Brillouin zone. The factor 2 in the $1/\beta$ terms comes from the spin degeneracy. The chargon spectrum ϵ_k^ψ is given by the electron bare dispersion in the tight binding approximation, i.e., $\epsilon_k^\psi = -2t(\cos k_x + \cos k_y)$. Furthermore, a constant density of states $D(\epsilon) = 1/2D$ will be used, where $D = 4t$ is half of the bandwidth.

In Eq. (19) we should introduce energy cutoff in the frequency integrals for E_t and F_t. Note that the usual momentum cutoff D is introduced in the integrals for E_r and F_r to prevent the divergence. Quite similarly, we also introduce an energy cutoff J in the integrals for E_t and F_t because it corresponds to the inverse of lattice spacing in the frequency space. When evaluating the frequency integrals for E_t and F_t, we first divide the integrals into two parts, divergent and divergent-free parts. We calculate the divergent parts within the energy cutoff, but for the divergent-free integrals we perform the Matsubara summation without the energy cutoff.

A. Kondo insulator

We solve the mean field equations [Eq. (19)] at half-filling of the conduction electrons, where the chargon chemical potential μ is zero due to the particle-hole symmetry. The Fermi-nesting induces a gap corresponding to the Kondo coupling in the chargon spectrum. The excitation spectrum of the chargons is given by $E_k^\psi = \sqrt{(E_r\epsilon_k^\psi)^2 + (J_K S)^2}$. Performing the Matsubara summation in the frequency integrals in Eq. (19), we obtain

$$JE_t = -\frac{2J}{\pi F_t} + \frac{1}{J}\int_{-D}^{D} d\epsilon D(\epsilon)\frac{\sqrt{F_r\epsilon+\lambda}}{\left(\frac{F_t}{J}\right)^{3/2}}\coth\left(\frac{\beta}{2}\sqrt{\frac{F_r\epsilon+\lambda}{\frac{F_t}{J}}}\right) =_{(T\to 0)} -\frac{2J}{\pi F_t} + \frac{1}{2DJ}\int_{-D}^{D} d\epsilon \frac{\sqrt{F_r\epsilon+\lambda}}{\left(\frac{F_t}{J}\right)^{3/2}},$$

$$JF_t = \frac{2J}{\pi E_t} + \frac{2}{E_t^2}{\sum_k}' E_k^\psi \tanh\left(\frac{\beta E_k^\psi}{2E_t}\right) =_{(T\to 0)} \frac{2J}{\pi E_t} + \frac{1}{2DE_t^2}\int_{-D}^{D} d\epsilon \sqrt{E_r^2\epsilon^2 + (J_K S)^2},$$

$$DE_r = -\int_{-D}^{D} d\epsilon D(\epsilon)\frac{\epsilon}{\frac{F_t}{J}\sqrt{\frac{F_r\epsilon+\lambda}{\frac{F_t}{J}}}}\coth\left(\frac{\beta}{2}\sqrt{\frac{F_r\epsilon+\lambda}{\frac{F_t}{J}}}\right) =_{(T\to 0)} -\frac{1}{2D}\int_{-D}^{D} d\epsilon \frac{\epsilon}{\frac{F_t}{J}\sqrt{\frac{F_r\epsilon+\lambda}{\frac{F_t}{J}}}},$$

$$DF_r = 2E_r{\sum_k}' \epsilon_k^{\psi 2}\frac{\tanh\left(\frac{\beta E_k^\psi}{2E_t}\right)}{E_t E_k^\psi} =_{(T\to 0)} \frac{E_r}{2D}\int_{-D}^{D} d\epsilon \frac{\epsilon^2}{E_t\sqrt{E_r^2\epsilon^2 + (J_K S)^2}},$$

$$1 = \int_{-D}^{D} d\epsilon D(\epsilon)\frac{1}{\frac{F_t}{J}\sqrt{\frac{F_r\epsilon+\lambda}{\frac{F_t}{J}}}}\coth\left(\frac{\beta}{2}\sqrt{\frac{F_r\epsilon+\lambda}{\frac{F_t}{J}}}\right) =_{(T\to 0)} \frac{1}{2D}\int_{-D}^{D} d\epsilon \frac{1}{\frac{F_t}{J}\sqrt{\frac{F_r\epsilon+\lambda}{\frac{F_t}{J}}}}, \qquad (20)$$

where we use a constant density of states, $D(\epsilon) = 1/2D$. The first terms in the equations for E_t and F_t come from the divergent parts.

Performing the momentum integrals in Eq. (20), we obtain the following expressions for the self-consistent mean field equations of E_t, F_t, E_r, F_r, and λ:

$$JE_t = -\frac{2J}{\pi F_t} + \frac{(J[\lambda + DF_r])^{3/2} - (J[\lambda - DF_r])^{3/2}}{3DJF_r F_t^{3/2}},$$

$$JF_t = \frac{2J}{\pi E_t} + \frac{DE_r\sqrt{D^2E_r^2 + (J_K S)^2} + (J_K S)^2\sinh^{-1}(DE_r/J_K S)}{2DE_t^2 E_r},$$

$$DE_r = \frac{(2\lambda - DF_r)\sqrt{J(\lambda + DF_r)} - (2\lambda + DF_r)\sqrt{J(\lambda - DF_r)}}{3DF_r^2\sqrt{F_t}},$$

$$DF_r = \frac{DE_r\sqrt{D^2E_r^2 + (J_K S)^2} - (J_K S)^2\sinh^{-1}(DE_r/J_K S)}{2DE_t E_r^2},$$

$$1 = \frac{\sqrt{J(\lambda + DF_r)} - \sqrt{J(\lambda - DF_r)}}{DF_r\sqrt{F_t}}. \qquad (21)$$

Although it is not easy to obtain analytic expressions for the mean field parameters as a function of J_K/D and J/D, we can find the quantum critical point where the bosonic spinons begin to be condensed. The spinon condensation occurs at $\lambda_c = DF_{rc}$, where c denotes "critical." Inserting this condition into Eq. (21), we find the quantum critical point defined by

$$F_{tc}^3\sqrt{1 + \left(\frac{3J_{Kc}S}{D}\right)^2} = \frac{24}{\pi^2}\frac{J}{D}F_{tc}^2 - \frac{28}{\pi}\frac{J}{D}F_{tc} + 8\frac{J}{D},$$

$$F_{rc} = \frac{2}{F_{tc}}\frac{J}{D}, \quad E_{tc} = -\frac{2}{\pi F_{tc}} + \frac{4}{3F_{tc}^2}, \quad E_{rc} = \frac{1}{3},$$

$$F_{tc}^3\left(\frac{3J_{Kc}S}{D}\right)^2\sinh^{-1}\left(\frac{1}{3J_{Kc}S/D}\right) = \frac{24}{\pi^2}\frac{J}{D}F_{tc}^2 - \frac{20}{\pi}\frac{J}{D}F_{tc} + \frac{8}{3}\frac{J}{D}. \qquad (22)$$

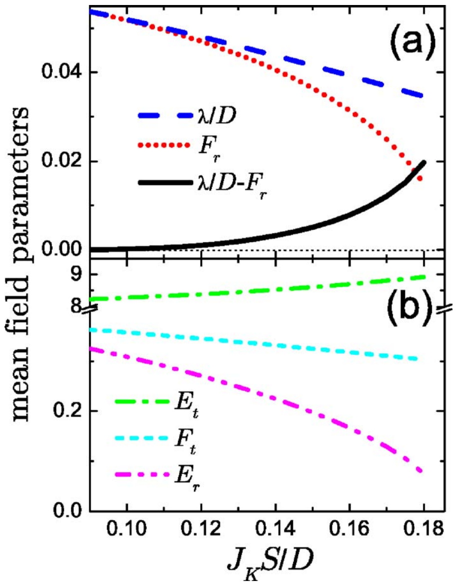

FIG. 1. (Color online) The mean field parameters at half-filling are shown as varying J_KS/D with $J/D=0.01$. The quantum critical point defined by $\lambda_c/D-F_{rc}=0$ in (a) coincides exactly with the analytic result $J_{Kc}S/D\approx 0.09$. The effective hopping parameters F_r and E_r become zero at a certain value of the Kondo coupling beyond its critical point, which implies that our spin decomposition cannot cover the whole range of the Kondo lattice model.

Solving Eq. (22), we find that the quantum critical point $J_{Kc}S/D$ depends on the value of J/D. In the case of $J/D=0.01$ we obtain $J_{Kc}S/D\approx 0.09>0$ that completely coincides with the result obtained by solving Eq. (21) numerically as shown in Fig. 1. Decreasing J/D from this value, the critical value becomes larger. In the case of $J/D=0.1$ we find $J_{Kc}S/D\approx -0.10<0$, indicating that there is no phase transition at half-filling, and only the Kondo insulating phase (see below) appears. Increasing J/D further, the critical value gets more negative. The condition for this quantum transition can be obtained from the boson sector of the mean field action in Eq. (16) *without any detailed calculations*. Since the boson Lagrangian coincides with the rotor model, more precisely, the CP^1 Lagrangian of the O(3) nonlinear σ model, one can read the transition condition $(D/J)F_tF_r\approx 1$ from the boson Lagrangian itself. Actually, it can be seen from Eq. (22) that the mean field condition for the spinon condensation given by $\lambda_c=DF_{rc}$ coincides with $(D/J)F_{tc}F_{rc}=2$ exactly.

The mean field parameters in this effective Kondo action at half-filling is shown in Fig. 1, where $J/D=0.01$. For $J_K<J_{Kc}$ the condensation of bosonic spinons occurs, indicating the emergence of an antiferromagnetic order for the localized spins. On the other hand, for $J_K>J_{Kc}$ the spinons become gapped, implying that the localized spins are disordered and the antiferromagnetic order vanishes. Increasing the Kondo coupling strength, the localized spins are strongly affected by the conduction electrons. Thus the effective hopping parameters F_t and F_r decrease as J_K increases (Fig. 1). As F_t and F_r decrease further so that $(D/J)F_tF_r\ll 1$ for large Kondo couplings, quantum fluctuations of spinons get stronger, destructing the antiferromagnetic long range order of the localized spins.

TABLE I. Quantum phases at half-filling in the Kondo lattice model.

$J_K<J_{Kc}$ Antiferromagnetic insulator	$J_K>J_{Kc}$ Paramagnetic insulator
$\langle z_{i\sigma}\rangle\neq 0$	$\langle z_{i\sigma}\rangle=0$
$\langle c^\dagger_{i\alpha}\tau_{3\alpha\beta}c_{i\beta}\rangle\neq 0$	$\langle c^\dagger_{i\alpha}\tau_{3\alpha\beta}c_{i\beta}\rangle=0$
$\langle \psi^\dagger_{i\alpha}\tau_{3\alpha\beta}\psi_{i\beta}\rangle\neq 0$	$\langle \psi^\dagger_{i\alpha}\tau_{3\alpha\beta}\psi_{i\beta}\rangle\neq 0$

Meanwhile the Kondo hybridization $\langle \vec{n}_i\cdot(c^\dagger_{i\sigma}\vec{\tau}_{\sigma\sigma'}c_{i\sigma'})\rangle$ is nonzero in both phases because

$$\langle c^\dagger_{i\sigma}(\vec{n}_i\cdot\vec{\tau})_{\sigma\sigma'}c_{i\sigma'}\rangle=\langle c^\dagger_{i\sigma}U_{i\sigma\alpha}\tau_{3\alpha\beta}U^\dagger_{i\beta\sigma'}c_{i\sigma'}\rangle=\langle \psi^\dagger_{i\alpha}\tau_{3\alpha\beta}\psi_{i\beta}\rangle$$
$$=\langle \psi^\dagger_{i\uparrow}\psi_{i\uparrow}-\psi^\dagger_{i\downarrow}\psi_{i\downarrow}\rangle\neq 0, \quad (23)$$

which is the hallmark of the present strong-coupling approach. In a different angle one may view this as an assumption in our strong-coupling theory. At half-filling, due to the Fermi-nesting the chargon excitations are gapped, thus both phases are insulators.

When the localized spins form an antiferromagnetic order in $J_K<J_{Kc}$ with the condensation of bosonic spinons, $\langle z_{i\sigma}\rangle\neq 0$, the conduction electrons also exhibit an antiferromagnetic order through the Kondo couplings with the localized spins. One can see this antiferromagnetic order from

$$\langle c^\dagger_{i\sigma}\tau_{3\sigma\sigma'}c_{i\sigma'}\rangle=\langle \psi^\dagger_{i\alpha}U^\dagger_{i\alpha\sigma}\tau_{3\sigma\sigma'}U_{i\sigma'\beta}\psi_{i\beta}\rangle$$
$$\approx\langle \psi^\dagger_{i\alpha}\psi_{i\beta}\rangle\langle U^\dagger_{i\alpha\sigma}\tau_{3\sigma\sigma'}U_{i\sigma' a}\rangle$$
$$=\langle \psi^\dagger_{i\uparrow}\psi_{i\uparrow}-\psi^\dagger_{i\downarrow}\psi_{i\downarrow}\rangle\langle z^\dagger_{i\uparrow}z_{i\uparrow}-z^\dagger_{i\downarrow}z_{i\downarrow}\rangle\neq 0, \quad (24)$$

if the easy axis anisotropy is assumed. In the easy plane limit one finds $\langle c^\dagger_{i\sigma}\tau_{1\sigma\sigma'}c_{i\sigma'}\rangle\neq 0$ or $\langle c^\dagger_{i\sigma}\tau_{2\sigma\sigma'}c_{i\sigma'}\rangle\neq 0$. Since the conduction electrons form an insulator due to the Fermi-nesting at half-filling, the phase of the conduction electrons is an antiferromagnetic insulator.

When the localized spins are in a disordered phase, the antiferromagnetic order of the conduction electrons also vanishes as $\langle z_{i\sigma}\rangle=0$. As a result, for $J_K>J_{Kc}$ the phase of the conduction electrons is identified as a Kondo insulator because the conduction electrons are still gapped due to the Fermi-nesting. In the Kondo insulator the origin of the excitation gap is the Kondo hybridization, not the antiferromagnetic ordering. Figure 1 shows that mean field analysis for the effective action of the Kondo lattice model exhibits a second order phase transition from an antiferromagnetic insulator to a Kondo insulator, as increases the Kondo coupling strength J_K. The possible mean field phases at half-filling are summarized in Table I.

Although the continuous quantum transition between the two insulating phases was obtained in the mean field approximation, it should be considered as an artifact of the mean field analysis because instanton excitations of compact U(1) gauge fields[12] cause confinement of the massive spinons and chargons in the Kondo insulating phase beyond the mean field level. From the seminal work of Fradkin and Shenker[13] we know that there can be no phase transition between the Higgs and confinement phases. The order parameter discriminating the Higgs phase from the confinement one has not been known yet.[14] In this respect only a crossover behavior is expected. In this study the antiferromagnetic state corresponds to the Higgs phase because the phase is characterized by the spinon condensation, while the Kondo insulating state coincides with the confinement phase. Applying Fradkin and Shenker's result to the present problem, we conclude that the second order phase transition turns into a crossover between the antiferromagnetic insulator and the Kondo insulator. This crossover picture is reasonable, considering that the chargon excitations are gapped in both phases.

Note that the present spin decomposition is not allowed for all values of J_K/D because the renormalized hopping integrals tF_r and tE_r for the spinons and chargons, respectively, become zero above a certain value of J_K/D (Fig. 1) in the Kondo insulator. Solving the mean field equations (21) in the limit of $E_r \to 0$ and $F_r \to 0$, one can determine the value of $J_K S/D$ resulting in $E_r=0$ and $F_r=0$ from the following conditions:

$$\left(F_t E_t + \frac{2}{\pi}\right) F_t = 1, \quad \left(F_t E_t - \frac{2}{\pi}\right) E_t = \frac{J_K S}{J},$$

$$18 \frac{J_K S J}{D^2} = \frac{F_t}{E_t}, \quad \lambda = \frac{J}{F_t}.$$

These equations give $J_K S/D=0.19$ and $E_t=8.99$ for $J/D=0.01$, and $J_K S/D=0.13$ and $E_t=2.27$ for $J/D=0.1$. Both the spinon and chargon bands become flat above this Kondo coupling strength, causing these particles localized with $\langle z_{i\sigma}\rangle=0$. We believe that this localization originates from our strong coupling approach. We interpret the localization as the breakdown of our spin decomposition.

B. Heavy fermion metal

Now we consider a hole-doped case where the Fermi-nesting is destroyed. We can expect the metallic behavior of chargons. Introducing the electron chemical potential, we obtain the self-consistent mean field equations for the chargon sector

$$\begin{aligned} JF_t &= \sum_k{}' \left[\frac{2E_k^\psi}{E_t^2}\left(n_f\left(-\frac{E_k^\psi - \mu}{E_t}\right) - n_f\left(\frac{E_k^\psi + \mu}{E_t}\right)\right) + \frac{2\mu}{E_t^2}\left(2 - n_f\left(-\frac{E_k^\psi - \mu}{E_t}\right) - n_f\left(\frac{E_k^\psi + \mu}{E_t}\right)\right)\right] + \frac{2J}{\pi E_t} \\ &=_{(T\to 0)} \frac{1}{4D}\int_{-D}^{D} d\epsilon \left[\frac{2\sqrt{E_r^2\epsilon^2 + (J_K S)^2}}{E_t^2}\left(\Theta\left(\frac{\sqrt{E_r^2\epsilon^2 + (J_K S)^2} - \mu}{E_t}\right) - \Theta\left(-\frac{\sqrt{E_r^2\epsilon^2 + (J_K S)^2} + \mu}{E_t}\right)\right)\right. \\ &\quad \left. + \frac{2\mu}{E_t^2}\left(2 - \Theta\left(\frac{\sqrt{E_r^2\epsilon^2 + (J_K S)^2} - \mu}{E_t}\right) - \Theta\left(-\frac{\sqrt{E_r^2\epsilon^2 + (J_K S)^2} + \mu}{E_t}\right)\right)\right] + \frac{2J}{\pi E_t}, \end{aligned}$$

$$\begin{aligned} DF_r &= \frac{2E_r}{E_t}\sum_k{}' \frac{\epsilon_k^{\psi 2}}{E_k^\psi}\left(n_f\left(-\frac{E_k^\psi - \mu}{E_t}\right) - n_f\left(\frac{E_k^\psi + \mu}{E_t}\right)\right) \\ &=_{(T\to 0)} \frac{E_r}{2DE_t}\int_{-D}^{D} d\epsilon \frac{\epsilon^2}{\sqrt{E_r^2\epsilon^2 + (J_K S)^2}}\left(\Theta\left(\frac{\sqrt{E_r^2\epsilon^2 + (J_K S)^2} - \mu}{E_t}\right) - \Theta\left(-\frac{\sqrt{E_r^2\epsilon^2 + (J_K S)^2} + \mu}{E_t}\right)\right), \end{aligned}$$

$$1 - \delta = 2\sum_k{}' \left(2 - n_f\left(-\frac{E_k^\psi - \mu}{E_t}\right) - n_f\left(\frac{E_k^\psi + \mu}{E_t}\right)\right) =_{(T\to 0)} \frac{1}{2D}\int_{-D}^{D} d\epsilon \left(2 - \Theta\left(\frac{\sqrt{E_r^2\epsilon^2 + (J_K S)^2} - \mu}{E_t}\right) - \Theta\left(-\frac{\sqrt{E_r^2\epsilon^2 + (J_K S)^2} + \mu}{E_t}\right)\right). \tag{25}$$

The mean field equations in the spinon sector remains the same as those in Eq. (20). One can recover Eq. (20) for half filling by setting $\delta=0$ and $\mu=0$ in Eq. (25).

In the doped case two kinds of phase transitions are expected to appear. One occurs in the spinon sector, characterized by the spinon condensation, thus associated with an antiferro- to paramagnetic transition of the localized spins. The other can appear in the chargon sector, not understood by condensation of an order parameter since there is no order parameter in this fermion part. The phase transition is an insulator to metal transition of the chargon excitations, occurring when the gap in the chargon spectrum vanishes.

TABLE II. Quantum phases away from half filling in the Kondo lattice model.

$J_K < J_{Kc}$ Antiferromagnetic metal	$J_K > J_{Kc}$ Heavy fermion metal
$\langle z_{i\sigma}\rangle \neq 0$	$\langle z_{i\sigma}\rangle = 0$
$\langle c^\dagger_{i\alpha}\tau_{3\alpha\beta}c_{i\beta}\rangle \neq 0$	$\langle c^\dagger_{i\alpha}\tau_{3\alpha\beta}c_{i\beta}\rangle = 0$
$\langle \psi^\dagger_{i\alpha}\tau_{3\alpha\beta}\psi_{i\beta}\rangle \neq 0$	$\langle \psi^\dagger_{i\alpha}\tau_{3\alpha\beta}\psi_{i\beta}\rangle \neq 0$

From the last equation in Eq. (25) one can see how the chemical potential changes as a function of δ and J_K, given by

$$\mu = -\sqrt{(E_r D)^2\delta^2 + (J_K S)^2} \tag{26}$$

for $\delta > 0$ and $\mu = 0$ for $\delta = 0$. This means that as soon as holes are doped in the conduction band, the chemical potential that lies between the upper and lower "conduction" bands jumps to the lower band, thus metallic properties of the conduction electrons appear.

The most important question in this study is how the antiferro- to paramagnetic transition of the localized spins arises when the conduction electrons become metallic away from half-filling. This quantum transition is driven by the spinon condensation. Performing the momentum integrals in the first and second equations in Eq. (25), we obtain the following expressions for the mean field equations:

$$\begin{aligned} JF_t &= \frac{2J}{\pi E_t} + \frac{\mu}{E_t^2}\left(1 - \frac{\sqrt{\mu^2 - (J_K S)^2}}{DE_r}\right) \\ &\quad + \frac{1}{2DE_t^2E_r}\big(DE_r\sqrt{D^2E_r^2 + (J_KS)^2} + \mu\sqrt{\mu^2 - (J_KS)^2} \\ &\quad + (J_KS)^2[\sinh^{-1}(DE_r/J_KS) \\ &\quad - \sinh^{-1}(\sqrt{\mu^2 - (J_KS)^2}/J_KS)]\big), \end{aligned}$$

$$\begin{aligned} DF_r &= \frac{1}{2DE_tE_r^2}\big(DE_r\sqrt{D^2E_r^2 + (J_KS)^2} + \mu\sqrt{\mu^2 - (J_KS)^2} \\ &\quad - (J_KS)^2[\sinh^{-1}(DE_r/J_KS) \\ &\quad - \sinh^{-1}(\sqrt{\mu^2 - (J_KS)^2}/J_KS)]\big). \end{aligned} \tag{27}$$

Using the equations for E_t, E_r, λ in Eq. (21) and Eq. (27) with Eq. (26), we can find the quantum critical point associated with the magnetic transition, given by

$$\begin{aligned} &F_{tc}^3\left(\sqrt{1 + \left(\frac{3J_{Kc}S}{D}\right)^2} - \sqrt{\delta^2 + \left(\frac{3J_{Kc}S}{D}\right)^2}\right) \\ &\quad = \frac{24}{\pi^2}\frac{J}{D}F_{tc}^2 - \frac{28}{\pi}\frac{J}{D}F_{tc} + 8\frac{J}{D}, \end{aligned}$$

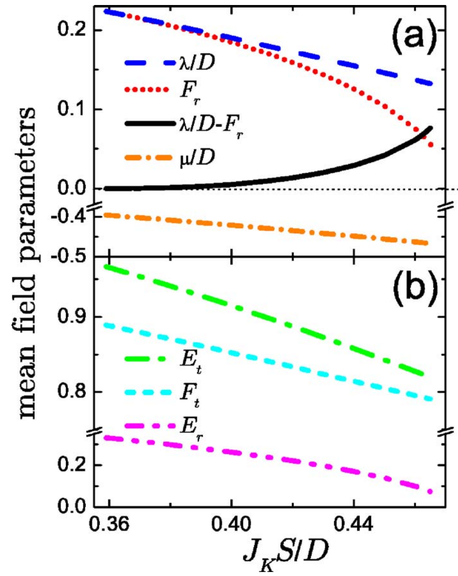

FIG. 2. (Color online) The mean field parameters away from half-filling (δ=0.5) are shown as varying J_KS/D with J/D=0.1. The quantum critical point defined by $\lambda_c/D - F_{rc} = 0$ in (a) coincides exactly with the analytic result $J_{Kc}S/D \approx 0.36$. The chemical potential agrees well with the analytic result $\mu = -\sqrt{(E_rD)^2\delta^2 + (J_KS)^2}$. Even away from half-filling the effective hopping parameters F_r and E_r also vanish at a certain value of the Kondo coupling beyond its critical point.

$$\begin{aligned} &F_{tc}^3\left[\left(\frac{3J_{Kc}S}{D}\right)^2\left(\sinh^{-1}\left(\frac{1}{3J_{Kc}S/D}\right) - \sinh^{-1}\left(\frac{\delta}{3J_{Kc}S/D}\right)\right) \right. \\ &\quad \left. - (1-\delta)\sqrt{\delta^2 + \left(\frac{3J_{Kc}S}{D}\right)^2}\right] \\ &\quad = \frac{24}{\pi^2}\frac{J}{D}F_{tc}^2 - \frac{20}{\pi}\frac{J}{D}F_{tc} + \frac{8}{3}\frac{J}{D}. \end{aligned} \tag{28}$$

Note that Eq. (28) is not reduced to Eq. (22) in the $\delta \to 0$ limit because there is a chemical potential jump at half-filling.

Let us consider the doped case with δ=0.5. For J/D =0.1 we find $J_{Kc}S/D \approx 0.36$, indicating the existence of the phase transition from an antiferromagnetic metal to a paramagnetic metal away from half-filling. Remember that there is no phase transition at half-filling when J/D=0.1. We identify this antiferromagnetic metal to paramagnetic metal transition as the quantum phase transition in the Kondo lattice model. Possible mean field phases away from half-filling are summarized in Table II, basically the same as Table I except that the phases are metallic rather than insulating. The mean field parameters away from half-filling is shown in Fig. 2, where J/D=0.1. Note that the transition point in Fig. 2, obtained by solving the mean field equations (21), (26), and (27) numerically, agrees completely with the analytic calculation.

A standard way of interpreting the quantum transition uses the ground state wave function. For $J_K < J_{Kc}$, since $\langle z_{i\sigma}\rangle \neq 0$ and $\langle \psi^\dagger_{i\alpha}\tau_{3\alpha\beta}\psi_{i\beta}\rangle \neq 0$, the ground state $|\vec{n}_i\rangle$ for a localized spin at site i can be written as $|\vec{n}_i\rangle = n_{AF}|AF\rangle$

$+n_{KS}|KS\rangle$, where $|AF\rangle$ is the antiferromagnetic state with weight n_{AF}, and $|KS\rangle$ the Kondo singlet state with weight n_{KS}. We emphasize again that the Kondo hybridization still exists for $J_K < J_{Kc}$. The ground state of the conduction electrons for $J_K < J_{Kc}$ is given by $|c_{i\sigma}\rangle = c_{AF}|AF\rangle + c_{KS}|KS\rangle$ in the same way, where the antiferromagnetism and Kondo hybridization result from the Kondo interaction. Increasing the Kondo coupling strength, the antiferromagnetic long range order vanishes due to the Kondo hybridization, thus the ground state for the localized spin at site i turns into $|\vec{n}_i\rangle = |KS\rangle$. In the same way the ground state for the conduction electrons is given by $|c_{i\sigma}\rangle = |KS\rangle$.

This discussion implies that fluctuations of the Kondo singlets are not critical in this transition, and only antiferromagnetic spin fluctuations are critical to drive the quantum transition via the Kondo interaction. This picture is consistent with the single impurity problem, where the Kondo coupling always causes the Kondo singlet ground state. The presence of the Kondo singlets in both phases originates from the strong coupling approach, where the Kondo coupling term is solved first, thus allowing the Kondo singlets in both phases. This leads us to conclude that the volume change of Fermi surface should be continuous, which can be checked from the fact that the chemical potential varies continuously across the transition.

However, there can exist another solution for the mean field equations (21), (26), and (27). If we assume $E_r = 0$ and $F_r = 0$ for $J_K S/D$ larger than the quantum critical point $J_{Kc}S/D$, these mean field equations become

$$\left(F_t E_t + \frac{2}{\pi}\right) F_t = 1, \quad F_t E_t = \frac{2}{\pi}, \quad \lambda = \frac{J}{F_t},$$

yielding $F_t = \pi/4$, $E_t = 8/\pi^2$, and $\lambda = 4J/\pi$. This solution has an interesting physical interpretation although our spin-decomposed effective action for the Kondo lattice model is not available above the value of J_K where $E_r \to 0$ and $F_r \to 0$. Note that this solution cannot be compatible with the spinon condensation because the spinon condensation occurs when $(D/J)F_t F_r \geq 2$ is satisfied. Thus, it can be allowed only in the paramagnetic phase. Approaching the quantum critical point from the antiferromagnetic phase, the effective hopping parameters remain finite, i.e., $E_{rc} \neq 0$ and $F_{rc} \neq 0$ while they are zero approaching the quantum critical point from the paramagnetic phase. There is discontinuity for E_r and F_r, given by $\Delta E_r = E_r(J_K \to J_{Kc} - 0) - E_r(J_K \to J_{Kc} + 0) = E_{rc}$ and $\Delta F_r = F_r(J_K \to J_{Kc} - 0) - F_r(J_K \to J_{Kc} + 0) = F_{rc}$. We expect that this abrupt change in the hopping parameters may be related with discontinuity in the volume change of the Fermi surface[1] even if the quantum transition is the second order described by the spinon condensation. Although the delocalized solution ($E_r \neq 0$, $F_r \neq 0$) near the quantum critical point is expected to be a genuine solution in the mean field level, it will be meaningful to consider this localization solution, considering gauge fluctuations beyond the mean field approximation. Later, we will comment on this issue. Since we do not have any clear physical picture for this localization, we do not go further based on this mean field solution.

IV. BEYOND THE MEAN FIELD APPROXIMATION

A. Effective field theory

To examine non-Fermi liquid physics near the quantum critical point, it is necessary to obtain an effective continuum action of Eq. (15). It is important to introduce nonperturbative effects of Kondo interactions in the continuum action. We rewrite the mean field action for the fermion sector in Eq. (16), and diagonalize it as follows:

$$\begin{aligned} S_\psi^{MF} &= \sum_{\omega_n}\sum_{k\sigma} (\psi^\dagger_{\omega_n k\sigma} \ \psi^\dagger_{\omega_n k+Q\sigma}) \\ &\quad \times \begin{pmatrix} iE_t\omega_n - \mu + E_r\epsilon_k^\psi & \sigma J_K S \\ \sigma J_K S & iE_t\omega_n - \mu + E_r\epsilon_{k+Q}^\psi \end{pmatrix} \begin{pmatrix} \psi_{\omega_n k\sigma} \\ \psi_{\omega_n k+Q\sigma} \end{pmatrix} \\ &= \sum_{\omega_n}\sum_{k\sigma} (\eta^\dagger_{+\omega_n k\sigma} \ \eta^\dagger_{-\omega_n k\sigma}) \\ &\quad \times \begin{pmatrix} iE_t\omega_n - \mu + E_k^\eta & 0 \\ 0 & iE_t\omega_n - \mu - E_k^\eta \end{pmatrix} \begin{pmatrix} \eta_{+\omega_n k\sigma} \\ \eta_{-\omega_n k\sigma} \end{pmatrix}, \end{aligned} \quad (29)$$

where the $\eta_{\pm\omega_n k\sigma}$ fermions are given by the unitary transformation of the $\psi_{\omega_n k\sigma}$ fermions in the following way:

$$\begin{pmatrix} \eta_{+\omega_n k\sigma} \\ \eta_{-\omega_n k\sigma} \end{pmatrix} = \begin{pmatrix} \cos\vartheta_{\omega_n k} & -\sigma\sin\vartheta_{\omega_n k} \\ \sigma\sin\vartheta_{\omega_n k} & \cos\vartheta_{\omega_n k} \end{pmatrix} \begin{pmatrix} \psi_{\omega_n k\sigma} \\ \psi_{\omega_n k+Q\sigma} \end{pmatrix}. \quad (30)$$

Here $E_k^\eta = \sqrt{(E_r\epsilon_k^\psi)^2 + (J_K S)^2}$ is the quasiparticle energy obtained before, and $\cos\vartheta_{\omega_n k}$, $\sin\vartheta_{\omega_n k}$ are coherence factors, given by $\cos^2\vartheta_{\omega_n k} = \frac{1}{2}\left[1 + \frac{E_r\epsilon_k^\psi}{E_k^\eta}\right]$ and $\sin^2\vartheta_{\omega_n k} = \frac{1}{2}\left[1 - \frac{E_r\epsilon_k^\psi}{E_k^\eta}\right]$.

Expanding the quasiparticle band in the long wavelength limit, we obtain

$$\begin{aligned} E_k^\eta &= J_K S\sqrt{1 + \left(\frac{E_r\epsilon_k^\psi}{J_K S}\right)^2} \approx J_K S\left\{1 + \frac{1}{2}\left(\frac{E_r\epsilon_k^\psi}{J_K S}\right)^2\right\} \\ &\approx J_K S + \frac{2(E_r t)^2}{J_K S} - \frac{2(E_r t)^2}{J_K S}(k_x^2 + k_y^2) + \mathcal{O}(k^4), \end{aligned} \quad (31)$$

where the terms beyond the fourth order are ignored in the long wavelength limit. Inserting the above into Eq. (29) and performing the Fourier transformation, one can obtain a low energy continuum action for the $\eta_{-\sigma}$ fermions

$$\begin{aligned} S_\psi = \int_0^\beta d\tau \int d^2r \Bigg[\sum_\sigma \Bigg(& \eta^\dagger_{-\sigma}(E_\eta[\partial_\tau - i\sigma a_\tau] - \mu - iA_\tau)\eta_{-\sigma} \\ & + \frac{1}{2M_\eta}|(\vec{\nabla} - i\sigma\vec{a} - i\vec{A})\eta_{-\sigma}|^2 \Bigg) \Bigg], \end{aligned} \quad (32)$$

where $M_\eta^{-1} \equiv (DE_r)^2/8J_K S$ and E_t is replaced by E_η. Note that the effective mass M_η of the renormalized conduction electrons $\eta_{-\sigma}$ is proportional to the Kondo coupling. The U(1) gauge field a_μ is introduced by shifting the three momentum k_μ as $k_\mu - \sigma a_\mu$. The empty high energy band for the $\eta_{+\sigma}$ fermions is ignored in the low energy limit.

Performing the continuum approximation for the boson sector in Eq. (15), the resulting effective field theory is given by

$$S_{\text{eff}} = S_\eta + S_z,$$

$$S_\eta = \int_0^\beta d\tau \int d^2r \left[\sum_\sigma \left(\eta_\sigma^\dagger (E_\eta[\partial_\tau - i\sigma a_\tau] - \mu - iA_\tau)\eta_\sigma + \frac{1}{2M_\eta} |(\vec{\nabla} - i\sigma\vec{a} - i\vec{A})\eta_\sigma|^2 \right) \right],$$

$$S_z = \int_0^\beta d\tau \int d^2r \left[\sum_\sigma \left(\frac{F_z}{J} |(\partial_\tau - ia_\tau)z_\sigma|^2 + \frac{1}{2M_z} |(\vec{\nabla} - i\vec{a})z_\sigma|^2 + m_z^2 |z_\sigma|^2 \right) + \frac{u_z}{2} \left(\sum_\sigma |z_\sigma|^2 \right)^2 + w_z |z_\uparrow|^2 |z_\downarrow|^2 \right], \tag{33}$$

where $M_z^{-1} \equiv tF_r/2$ and F_t is replaced by F_z in the spinon part. Here the "$-$" symbol in $\eta_{-\sigma}$ field is omitted for a simple notation. The unimodular constraint in the spinon sector is softened via their local interactions u_z. The w_z term is phenomenologically introduced, associated with the easy-plane anisotropy.

The above effective action can be simplified via the following scale transformation:

$$\tau' = \frac{\tau}{\sqrt{F_z/J}}, \quad \vec{r}' = \sqrt{2M_z}\vec{r}. \tag{34}$$

Performing the field-transformation for the boson sector accordingly,

$$a'_\tau = \sqrt{F_z/J} a_\tau, \quad \vec{a}' = \frac{\vec{a}}{\sqrt{2M_z}}, \quad z'_\sigma = \frac{(F_z/J)^{1/4}}{\sqrt{2M_z}} z_\sigma, \tag{35}$$

the effective spinon action is given by

$$S_z = \int_0^{\beta'} d\tau' \int d^2r' \left[\sum_\sigma (|(\partial_{\tau'} - ia'_\tau)z'_\sigma|^2 + |(\vec{\nabla}' - i\vec{a}')z'_\sigma|^2 + m_z'^2 |z'_\sigma|^2) + \frac{u'_z}{2} \left(\sum_\sigma |z'_\sigma|^2 \right)^2 + w'_z |z'_\uparrow|^2 |z'_\downarrow|^2 \right] \tag{36}$$

with

$$\beta' = \frac{\beta}{\sqrt{F_z/J}}, \quad m_z'^2 = m_z^2, \quad u'_z = \frac{2M_z}{(F_z/J)^{1/2}} u_z,$$

$$w'_z = \frac{2M_z}{(F_z/J)^{1/2}} w_z.$$

The effective chargon action can also be obtained as follows:

$$S_\eta = \int_0^{\beta'} d\tau' \int d^2r' \left[\sum_\sigma \left(\eta_\sigma'^\dagger (E'_\eta[\partial_{\tau'} - i\sigma a'_\tau] - \mu' - iA'_\tau)\eta'_\sigma + \frac{1}{2M'_\eta} |(\vec{\nabla}' - i\sigma\vec{a}' - i\vec{A}')\eta'_\sigma|^2 \right) \right] \tag{37}$$

with the scale transformation for the fermion sector

$$E'_\eta = \frac{E_\eta}{\sqrt{F_z/J}}, \quad M'_\eta = \frac{M_\eta}{2M_z}, \quad \vec{A}' = \frac{\vec{A}}{\sqrt{2M_z}}, \quad \eta'_\sigma = \frac{(F_z/J)^{1/4}}{\sqrt{2M_z}} \eta_\sigma,$$

$$\mu' = \mu, \quad A'_\tau = A_\tau. \tag{38}$$

As a result, we find the effective field theory

$$\mathcal{S}_{eff} = \int_0^\beta d\tau \int d^2r \left[\sum_\sigma \left(\eta_\sigma^\dagger (E_\eta[\partial_\tau - i\sigma a_\tau] - \mu - iA_\tau)\eta_\sigma + \frac{1}{2M_\eta} |(\vec{\nabla} - i\sigma\vec{a} - i\vec{A})\eta_\sigma|^2 \right) \right] + \int_0^\beta d\tau \int d^2r \left[\sum_\sigma (|(\partial_\tau - ia_\tau)z_\sigma|^2 + |(\vec{\nabla} - i\vec{a})z_\sigma|^2 + m_z^2 |z_\sigma|^2) + \frac{u_z}{2} \left(\sum_\sigma |z_\sigma|^2 \right)^2 + w_z |z_\uparrow|^2 |z_\downarrow|^2 \right] + \frac{1}{2} \sum_{q,\omega_n} \left(\frac{q^2}{g} + \frac{E_\eta}{v_\eta} \frac{|\omega_n|}{q} \right) \left(\delta_{ij} - \frac{q_i q_j}{q^2} \right) a_i a_j, \tag{39}$$

where the prime symbol is omitted for a simple notation. In the gauge action the former with an internal gauge charge g of the η_σ and z_σ particles is the Maxwell term resulting from high energy fluctuations of the η_σ and z_σ particles.[15,16] The latter with the quasiparticle "renormalization" E_η and the Fermi velocity of the chargons v_η is the Landau damping term representing dissipative dynamics of gauge fluctuations, which come from particle-hole excitations of the η_σ fermions near the Fermi surface.[15] Since the time component of the gauge field mediates local interactions due to the η_σ polarization[15] which are irrelevant in the renormalization group sense, it can be ignored in low energy limit.

B. Antiferromagnetic metal

For weak Kondo couplings $J_K < J_{Kc}$ ($m_z^2 < 0$) the bosonic spinons become condensed, leading to an antiferromagnetic order of the localized spins. An antiferromagnetic metal appears for the conduction electrons away from half filling. Gauge fluctuations are massive due to the Anderson-Higgs mechanism, thus safely ignored in the low energy limit. Although the gauge fluctuations are irrelevant in the renormalization group sense, they play an important role in confining the fermionic chargons with the bosonic spinons to make usual conduction electrons. This can be seen from the unitary gauge. If the easy plane limit $w_z<0$ is considered in Eq. (39), the spinons can be treated as $z_\sigma=(1/\sqrt{2})e^{i\phi_\sigma}$. The unitary gauge means $\tilde{a}_\mu=a_\mu-\partial_\mu\phi_\uparrow$, causing an excitation gap for the $\tilde{a}_\mu$ fields. In this unitary gauge the phase degrees of freedom appear in the chargon sector, and these phase fields can be gauged away from the gauge transformation $c_\sigma=e^{-i\sigma\phi_\uparrow}\eta_\sigma$. Thus, low energy excitations are antiferromagnons $e^{i(\phi_\uparrow-\phi_\downarrow)}$ in the localized spins and electron excitations c_σ in the renormalized conduction band instead of the fermionic chargons η_σ. These conduction electrons feel weak staggered magnetic fields due to the antiferromagnetic ordering of the localized spins.

C. Quantum critical point

As the Kondo coupling increases, the antiferromagnetic metal approaches the quantum critical point where the antiferromagnetic order vanishes. Critical boson fluctuations renormalize gauge dynamics[17] in the critical field theory

$$\mathcal{S}_c=\int_0^\beta d\tau\int d^2r\left[\sum_\sigma\left(\eta_\sigma^\dagger(E_\eta\partial_\tau-\mu-iA_\tau)\eta_\sigma+\frac{1}{2M_\eta}|(\vec{\nabla}-i\sigma\vec{a}-i\vec{A})\eta_\sigma|^2\right)\right]+\frac{1}{2}\sum_{q,\omega_n}\left(\frac{E_\eta}{v_\eta}\frac{|\omega_n|}{q}+\frac{N_z}{8}q\right)\left(\delta_{ij}-\frac{q_iq_j}{q^2}\right)a_ia_j, \tag{40}$$

where $N_z=2$ is the flavor number of the spinons. Integration for the critical spinons should be understood in the renormalization group sense. Since critical boson fluctuations yield a term linearly proportional to momentum q in the gauge action, the dynamical critical exponent is obtained as $z=2$.[18]

In the random phase approximation[19,20] the free energy is given by

$$\begin{aligned}\frac{F}{V}&=\int\frac{d^2q}{(2\pi)^2}\int\frac{d\omega}{2\pi}\coth\left[\frac{\omega}{2T}\right]\tan^{-1}\left[\frac{\text{Im}\,D(q,\omega)}{\text{Re}\,D(q,\omega)}\right]\\&\approx\frac{1}{4\pi^2}\int_T^{\omega_c}d\omega\int_0^\infty dqq\tan^{-1}\left[\frac{8E_\eta}{v_\eta N_z}\frac{\omega}{q^2}\right]\\&=\frac{1}{4\pi^2}\frac{E_\eta}{v_\eta N_z}\left[\left(3-2\ln\frac{8E_\eta}{v_\eta N_z}-2\ln\omega_c\right)\omega_c^2\right.\\&\quad\left.-\left(3-2\ln\frac{8E_\eta}{v_\eta N_z}-2\ln T\right)T^2\right],\end{aligned} \tag{41}$$

where $D(q,\omega)=\left(-i\frac{E_\eta}{v_\eta}\frac{\omega}{q}+\frac{N_z}{8}q\right)$ is the gauge kernel in the real frequency ω, and ω_c is an energy cutoff. The specific heat is obtained to be

$$\begin{aligned}C_V&=-T\left(\frac{\partial^2F}{\partial T^2}\right)_V=-\frac{1}{\pi^2}\frac{E_\eta}{v_\eta N_z}\ln\frac{8E_\eta}{v_\eta N_z}T-\frac{1}{\pi^2}\frac{E_\eta}{v_\eta N_z}T\ln T\\&=-\frac{1}{8\pi^2}\frac{T}{T_0}\ln\frac{T}{T_0}\end{aligned} \tag{42}$$

with an energy scale $T_0=\left(\frac{8E_\eta}{v_\eta N_z}\right)^{-1}$. Thus, the specific heat coefficient γ has a singular dependence $\gamma=C_V/T\propto-\ln T$ in the $T\to0$ limit. The logarithmic divergence also can be seen in the two dimensional itinerant antiferromagnet, where its critical field theory is characterized by the dynamical exponent $z=2$.[20]

In the one-loop level the imaginary part of the fermion self-energy is given by

$$\begin{aligned}\Sigma_\eta''(k,\epsilon_k^\eta)&=\int_0^\infty d\omega\int\frac{d^Dk'}{(2\pi)^D}[n(\omega)+1][1-f(\epsilon_{k'}^\eta)]\\&\quad\times\frac{(k+k')_\alpha(k+k')_\beta}{(2M_\eta)^2}\left(\delta_{\alpha\beta}-\frac{q_\alpha q_\beta}{q^2}\right)\\&\quad\times\text{Im}\,D(q,\omega)\delta(\epsilon_k^\eta-\epsilon_{k'}^\eta-\omega)\\&=\frac{N_\eta}{2\pi M_\eta^2}\int_0^\infty d\omega\int d\epsilon'd\theta\delta(\epsilon_k^\eta-\epsilon'-\omega)[n(\omega)+1]\\&\quad\times[1-f(\epsilon')]|\mathbf{k}\times\hat{\mathbf{q}}|^2\frac{v_\eta q\omega/E_\eta}{\omega^2+\left(\frac{N_zv_\eta}{8E_\eta}\right)^2q^4}\\&=\frac{k_FN_\eta}{2\pi M_\eta^2}\int_0^{\epsilon_k^\eta}d\omega\int_0^\infty dq\frac{v_\eta q\omega/E_\eta}{\omega^2+\left(\frac{N_zv_\eta}{8E_\eta}\right)^2q^4}\\&=\frac{k_F}{M_\eta^2}\frac{N_\eta}{N_z}\epsilon_k^\eta,\end{aligned} \tag{43}$$

where $\epsilon_k^\eta=k^2/2M_\eta$ is the energy dispersion of the η_σ fermions, and N_η is the density of states at the Fermi energy. The scattering rate can be obtained from the self-energy expression with an additional q^2 factor in the integrand.[15] Since the imaginary part of the fermion self-energy is linearly proportional to the fermion dispersion, the dc conductivity[21] is given by $\sigma_c\sim T^{-2}$.

D. Region of strong Kondo couplings

Increasing J_K further from the quantum critical point, an anomalous metallic phase appears. Integrating out the gapped z_σ excitations in Eq. (39), we obtain the Maxwell term for gauge fluctuations

$$S_{NFL} = \int_0^\beta d\tau \int d^2r \left[\sum_\sigma \left(\eta_\sigma^\dagger (E_\eta \partial_\tau - \mu - iA_\tau) \eta_\sigma + \frac{1}{2M_\eta} |(\vec{\nabla} - i\sigma\vec{a} - i\vec{A}) \eta_\sigma|^2 \right) \right] + \frac{1}{2} \sum_{q,\omega_n} \left(\frac{q^2}{g} + \frac{E_\eta}{v_\eta} \frac{|\omega_n|}{q} \right) \left(\delta_{ij} - \frac{q_i q_j}{q^2} \right) a_i a_j, \tag{44}$$

where the dynamical critical exponent is $z=3$. The effective field theory of Eq. (44) is well known to cause non-Fermi liquid physics due to scattering with massless gauge fluctuations. The imaginary part of the fermion self-energy is given by $\omega^{2/3}$ at the Fermi surface, implying that its real part also has the same frequency dependence via the Kramer's Kronig relation, thus giving rise to a non-Fermi liquid behavior.[22] The coefficient γ of the specific heat is proportional to $-\ln T$ in three spatial dimensions and $T^{-1/3}$ in two dimensions.[8] The dc conductivity is proportional to $T^{-5/3}$ in three dimensions and $T^{-4/3}$ in two dimensions.[15] Note that the dynamical exponent z changes from $z=2$ at the quantum critical point to $z=3$ in the non-Fermi liquid phase. Physical responses in the Fermi liquid to non-Fermi liquid transition for two spatial dimensions are summarized in Table III.

E. How to recover the Fermi liquid phase

When the spinon excitations are gapped, they can be ignored in the low energy limit. Thus, if the gauge fluctuations are suppressed in Eq. (44), Fermi liquid physics can be obtained. As the Kondo coupling constant increases, the effective chargon mass $M_\eta = 8J_K S/(DE_r)^2$ becomes heavier and gauge fluctuations are suppressed because $1/v_\eta \sim M_\eta$ in Eq. (44). This may give rise to the Fermi liquid physics in the case of large Kondo couplings. In this scenario the non-Fermi liquid is expected to turn into the Fermi liquid continuously. In our mean field analysis $M_z \to \infty$ and $M_\eta \to \infty$ were found at the point where $E_r=0$ and $F_r=0$. Hence, the spin decomposition scheme cannot cover the whole range of the phase diagram so that we were not able to recover the Fermi liquid phase.

There is another possibility associated with the confinement-deconfinement transition due to the compactness of the U(1) gauge field in the present problem. Note that we did not take into account instanton excitations in the previous discussion. In two space and one time dimensions there is no deconfined phase owing to the proliferation of instanton excitations when only gapped fermion or boson excitations exist.[12] However, the presence of gapless matter fields was recently argued to allow a deconfined phase.[16,23–28] Non-Fermi liquid phase corresponds to the deconfined phase which gapless fermion (η_σ) excitations make stable against instanton excitations.[16,23] The present quantum critical point is identified as the deconfined quantum critical point[24] that can be stable due to critical boson (z_σ) excitations[25] and gapless fermion excitations.[16,23] On the other hand, Fermi liquid corresponds to the confinement phase. Instanton condensation leads to confinement between the η_σ fermion and the z_σ boson to make an electron $c_\sigma = U_{\sigma\sigma'} \eta_{\sigma'}$. We don't know whether the non-Fermi liquid phase is stable against instanton excitations or not. If it is stable, the confinement-deconfinement transition corresponding to the non-Fermi liquid to Fermi liquid transition would occur in the strong Kondo coupling region. The nature of this transition may be KT-like (Kosterlitz-Thouless).[29] On the contrary, if the non-Fermi liquid phase is unstable against the confinement, the parameter region of the non-Fermi liquid phase would shrink to vanish. Then, the quantum critical point will coincide with the point where localization occurs with $M_z \to \infty$ and $M_\eta \to \infty$.

TABLE III. Physical response in the Fermi liquid to non-Fermi liquid transition.

	$J_K < J_{Kc}$ Antiferromagnetic Fermi liquid	$J_K \approx J_{Kc}$ Quantum critical point	$J_K > J_{Kc}$ Paramagnetic non-Fermi liquid
$\gamma = C_v/T$	Const.	$-\ln T$	$T^{-1/3}$
σ_{dc}	T^{-2}	T^{-2}	$T^{-4/3}$

V. DISCUSSION AND PERSPECTIVES

Our approach has some analogies with that of Ref. 30 where bosonic spinons are used for the localized spins, resulting in charged fermions for the Kondo resonances. However, there are several important differences between our approach and that of Ref. 30. Reference 30 starts from the Kondo-Heisenberg lattice model

$$H_{KHM} = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J_K \sum_{i\sigma\sigma'} b_{i\sigma}^\dagger b_{i\sigma'} c_{i\sigma^{prime}}^\dagger c_{i\sigma} + J_H \sum_{\langle ij \rangle \sigma\sigma'} b_{i\sigma}^\dagger b_{i\sigma'} b_{j\sigma'}^{gger} b_{j\sigma}, \tag{45}$$

where $c_{k\sigma}$ represents a conduction electron with momentum k and spin σ, and $\vec{S}_i = \frac{1}{2} b_{i\sigma}^\dagger \vec{\tau}_{\sigma\sigma'} b_{i\sigma'}$ is the boson representation of the localized spin $\vec{S}_i$. Performing the HS transformation for the particle-hole channel in the Kondo coupling term and the particle-particle channel in the Heisenberg interaction term, Eq. (45) reads

$$H_{eff} = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_{i\sigma} (b_{i\sigma}^\dagger \chi_i^\dagger c_{i\sigma} + \text{H.c.}) - \sum_i \frac{\chi_i^\dagger \chi_i}{J_K} + \sum_{\langle ij \rangle \sigma} (|\Delta_{ij}| e^{i\pi(i-j)} b_{i\sigma}^\dagger b_{j-\sigma}^\dagger + \text{H.c.}) - \sum_{\langle ij \rangle} \frac{|\Delta_{ij}|^2}{J_H}, \tag{46}$$

where the onsite bond variable χ_i is a Grassman field associated with the Kondo resonance and the bond variable Δ_{ij} is introduced to keep short range antiferromagnetic correlations.

The crucial difference between the two approaches lies in the HS decoupling scheme of the Kondo interaction term; Ref. 30 allows three kinds of matter fields that correspond to two fermions $c_{k\sigma}$, χ_i, and one boson $b_{i\sigma}$ while our decomposition introduces only two kinds of matter fields, one fermion $\psi_{i\sigma}$ and one boson $z_{i\sigma}$. In fact, χ_i fermion corresponds to the $\psi_{i\uparrow}$ fermion while the $\psi_{i\downarrow}$ fermion is not allowed in Ref. 30. Since the χ_i field follows fermion statistics and the condensation of fermions is not possible, the conventional mean field analysis in the slave-boson approach[3] is not applicable. Recently, there has been progress in this spin-boson approach for the single impurity problem based on the noncrossing approximation scheme of the U(1) slave-boson theory,[3] although its extension to the Kondo lattice model has not been reported yet.[31]

A schematic phase diagram based on the effective Kondo action in Eq. (15) is shown in Fig. 3, where the horizontal axis is the Kondo coupling strength and vertical is temperature. "AF" represents the antiferromagnetic metal and "HF" the heavy fermion metal. The dashed line denoted by T_K represents the Kondo temperature for the Kondo singlets to form, where $\langle\sigma\psi^\dagger_{i\sigma}\psi_{i\sigma}\rangle\neq 0$ below T_K while $\langle\sigma\psi^\dagger_{i\sigma}\psi_{i\sigma}\rangle=0$ above T_K. The solid line shows the second order antiferromagnetic transition related with the spinon condensation in the present theory. This phase diagram is quite similar to that of the HMM theory since the Kondo hybridization always exists in both the antiferromagnetic and heavy fermion phases below the Kondo temperature, and antiferromagnetic ordering is associated with the quantum transition. However, an important difference between these two theories can be found near the antiferromagnetic quantum critical point.

In the HMM theory low energy elementary excitations at the quantum critical point are critical antiferromagnetic fluctuations with spin quantum number 1, described by

$$S_{HMM}=\frac{1}{2}\sum_{q,\omega_n}(\Gamma|\omega_n|+q^2)\vec{n}(q,\omega_n)\cdot\vec{n}(-q,-\omega_n)+V(|\vec{n}|), \tag{47}$$

where the damping term with a damping coefficient Γ comes from gapless electron excitations near the Fermi surface and $V(|\vec{n}|)=\int_0^\beta d\tau\int d^2r\frac{u_n}{2}|\vec{n}|^4+\cdots$. is an effective potential for the spin-fluctuation order parameter $\vec{n}$.[2] On the other hand, at the quantum critical point of the present approach the critical antiferromagnetic fluctuations are fractionalized into critical spinon excitations with spin quantum number 1/2 due to strong Kondo interactions. The critical field theory is given by

$$\begin{aligned}S_{DQCP}=&\int_0^\beta d\tau\int d^2r\Bigg[\sum_\sigma(|(\partial_\tau-ia_\tau)z_\sigma|^2+|(\vec{\nabla}-i\vec{a})z_\sigma|^2)\\&+\frac{u_z}{2}\Big(\sum_\sigma|z_\sigma|^2\Big)^2+w_z|z_\uparrow|^2|z_\downarrow|^2\Bigg]\\&+\frac{1}{2}\sum_{q,\omega_n}\left(\frac{E_\eta}{v_\eta}\frac{|\omega_n|}{q}+\frac{N_z}{8}q\right)\left(\delta_{ij}-\frac{q_iq_j}{q^2}\right)a_ia_j,\end{aligned} \tag{48}$$

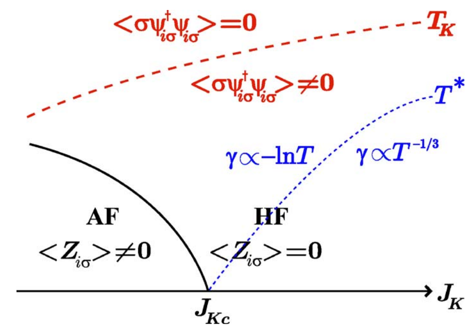

FIG. 3. (Color online) A schematic phase diagram of the Kondo lattice model based on our effective theory.

where the gapless renormalized fermions η_σ are integrated out, causing dissipation in gauge dynamics. Here "DQCP" means the deconfined quantum critical point[24,25] discussed in the previous section.

It is interesting to see that both critical theories in Eqs. (47) and (48) are characterized by the same dynamic critical exponent $z=2$. Thus, two critical theories show similar critical physics although the low lying excitations are completely different. Actually, the HMM theory can also explain the logarithmic divergence in the specific heat coefficient in two spatial dimensions.[1] Since two spatial dimensions lie in the upper critical dimension due to the dynamic critical exponent, universal scaling for the spin susceptibility does not appear in both critical theories. Thus, it is impossible to compare an anomalous critical exponent in one theory with that of the other. However, there exists one crucial difference; if we have the deconfined quantum critical point, the non-Fermi liquid metal as a deconfined critical phase can appear above the deconfined quantum critical point ($J_K>J_{Kc}$) while this non-Fermi liquid phase cannot be allowed in the HMM theory. According to our effective field theory, it is clear that there should be a crossover between the quantum critical region and non-Fermi liquid phase as temperature decreases in $J_K>J_{Kc}$ (Fig. 3). The crossover temperature T^* depends on the spin gap $\lambda-DF_r$ as $T^*\sim\lambda-DF_r$. This crossover should appear in the upturn behavior of the specific heat coefficient from $\gamma\sim-\ln T$ to $\gamma\sim T^{-1/3}$ since the effective field theory changes from $z=2$ to $z=3$ during the crossover. The upturn behavior was also discussed in Ref. 30, but the mechanism is different.

Let us discuss dissipation in both the weak coupling critical theory of Eq. (47) and the strong coupling one of Eq. (48). The strong coupling theory may be derived from the weak coupling one using the CP^1 representation. Remember that the O(3) nonlinear σ model can be mapped onto the U(1) gauge theory within the CP^1 representation, as discussed before. The main point of this CP^1 decomposition is how dissipative dynamics of spin fluctuations $\vec{n}(r,\tau)$ in the HMM theory [Eq. (47)] are transferred into that of gauge fluctuations $\vec{a}(r,\tau)$ in the CP^1 gauge theory [Eq. (48)]. In the context of the standard weak coupling theory order parameter fluctuations directly couple to gapless fermion excitations near the Fermi surface. As a result, dissipation effects in order parameter fluctuations appear in the kinetic energy term. In our strong coupling approach fractionalized order parameter fluctuations do not couple to the gapless fermion excitations directly. Instead, their couplings are realized indirectly via gauge fluctuations. Dissipative dynamics of fractionalized order parameter fluctuations are induced by damped gauge fluctuations that result from the gapless fermion excitations near the Fermi surface. The damped gauge fluctuations play an important role in quantum critical physics. If fermion excitations are gapped and the damping effects in gauge fluctuations are not taken into account in Eq. (48), the quantum phase transition belongs to the inverted *XY* (I*XY*) universality class in the case of large flavors of fractionalized boson excitations.[17] However, the presence of dissipation in gauge excitations due to gapless fermions changes the I*XY* universality class completely. Since the dissipation results in the dynamic critical exponent $z=2$, the spacial dimension $d=2$ becomes the upper critical dimension and the nature of the quantum transition would be a mean fieldlike type with logarithmic corrections.

Inserting the CP^1 representation $\vec{n}=\frac{1}{2}z_\sigma^\dagger\vec{\tau}_{\sigma\sigma'}z_{\sigma'}$ into Eq. (47),[32] Eq. (47) can be written in terms of the bosonic spinons interacting with gauge fluctuations. Unfortunately, the $|\omega_n|$ linear (damping) term prevents obtaining a complete expression. Performing the HS transformation for the damping term, dissipation in order parameter fluctuations would appear in gauge fluctuations although the dissipative gauge action is not given by that in Eq. (48). This implies that damping effects due to gapless fermions are imposed in a different way for the weak and strong coupling theories.

It is valuable to apply our spin decomposition to the one dimensional Kondo lattice model in order to confirm that the non-Fermi liquid metal with spin gap can be allowed. The effective field theory in Eq. (39) will be applicable to the one dimensional case. An important difference from the two dimensional case is that dissipative dynamics in gauge fluctuations do not appear because the gapless conduction fermions are described by massless Dirac fermions near the Fermi points. In one dimensional effective field theory strong quantum fluctuations coming from low dimensionality do not allow the spinon condensation. Furthermore, the spinon excitations are gapped because the Berry phase contribution disappears due to the Kondo coupling. The most crucial point in the one dimensional effective theory is that the gapless Dirac fermions make gauge fluctuations massive,[33] which can be seen using the bosonization technique. As a result, the gapped spinon excitations are deconfined[34] although the mechanism is different from the two dimensional case. Moreover, the gapless fermion excitations exhibit strong superconducting correlations as the two dimensional case. Our spin-gauge theory in Eq. (39) allows superconducting instability because the spin-gauge fields mediate attractive interactions between $\eta_\uparrow$ and $\eta_\downarrow$ fermions. The σ symbol in the gauge coupling shows that the gauge charge of the $\eta_\uparrow$ fermion is opposite to that of the $\eta_\downarrow$ fermion. In this respect the non-Fermi liquid phase with spin gap is the two dimensional analogue of the one dimensional spin-gapped phase.

If SU(2) gauge fluctuations are taken into account in Eq. (11) instead of U(1) gauge fluctuations, the non-Abelian nature of SU(2) gauge fluctuations may not allow the deconfined quantum criticality and non-Fermi liquid phase.[12] However, there is no consensus for the confinement problem in the context of the SU(2) gauge theory, as far as we know. If the deconfined non-Fermi liquid phase turns into a confinement state, spinons should be confined with chargons via SU(2) gauge fluctuations. Instead, electron excitations are allowed and the resulting phase may be the Fermi liquid. The spinons and chargons are not meaningful objects in the low energy limit. However, the spinon excitations may emerge as broad spin spectrum (particle-hole continuum) at high energies beyond multiparamagnon scattering according to the asymptotic freedom of the SU(2) gauge theory.[12]

In this SU(2) gauge theoretic description the quantum critical point would lie between the Higgs phase (antiferromagnetism) and the confinement one (Fermi liquid), while it is between the Higgs phase and the deconfinement one (non-Fermi liquid) in the present U(1) gauge theory. It was argued that there is no phase transition between the Higgs and confinement phases and they are smoothly connected.[13] Then, the spin-decomposition method in the context of the SU(2) gauge theory cannot describe the quantum phase transition of the Kondo lattice model. In this case another order parameter should be considered to study the quantum phase transition of the Kondo lattice model, for example, the hybridization order parameter in the context of the slave-boson theory.[8,9]

In summary, we investigated the quantum phase transition from an antiferromagnetic metal to a heavy fermion metal in the Kondo lattice model. First, we diagonalized the Kondo coupling term in the strong coupling approach. Then, we derived the effective Kondo action [Eq. (15)] and performed the mean field analysis [Eq. (19)] to obtain the mean field phase diagram, showing the quantum phase transition from the antiferromagnetic metal to the heavy fermion metal. The Kondo term is always relevant so that the Kondo hybridization persists even in the antiferromagnetic metal, which means that fluctuations of the Kondo singlets are not critical in the phase transition. The volume change of Fermi surface thus is expected to be continuous across the transition. We found that softening of antiferromagnetic spin fluctuations leads to the quantum transition driven by the Kondo interaction in the strong coupling approach. Beyond the mean field level we derived the effective U(1) gauge theory [Eq. (39)] in terms of the renormalized conduction electrons η_σ and the spin-fractionalized excitations z_σ interacting via the U(1) spin-gauge fields a_μ. Our critical field theory characterized by the critical exponent $z=2$ can explain the non-Fermi liquid physics such as $\gamma\sim-\ln T$ near the quantum critical point. Furthermore, we showed that if our scenario is applicable, there can exist a narrow region of the non-Fermi liquid phase with the spin gap near the quantum critical point. We also discussed how the present theory can recover the Fermi liquid phase, but this issue should be clarified near future. Lastly, we commented on the superconducting instability near the quantum critical point. This interesting possibility remains as a future study.

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[11] The time derivative term in the discrete-time becomes in the limit of $\Delta\tau\rightarrow 0$

$$
\begin{aligned}
&-\sum_{\tau\tau' i} \psi_{i\tau}^{\dagger\sigma} U_{i\tau}^{\dagger\sigma\alpha} U_{i\tau'}^{\alpha\sigma'} \psi_{i\tau'}^{\sigma'} \\
&\approx -\sum_{\tau i} \psi_{i\tau}^{\dagger\sigma} U_{i\tau}^{\dagger\sigma\alpha}\left(U_{i\tau}^{\alpha\sigma'} - \frac{\Delta U_{i\tau}^{\alpha\sigma'}}{\Delta\tau}\Delta\tau\right)\left(\psi_{i\tau}^{\sigma'} - \frac{\Delta\psi_{i\tau}^{\sigma'}}{\Delta\tau}\Delta\tau\right) \\
&= \sum_{\tau i}\Bigg(-\psi_{i\tau}^{\dagger\sigma}\psi_{i\tau}^{\sigma} + \psi_{i\tau}^{\dagger\sigma} U_{i\tau}^{\dagger\sigma\alpha}\frac{\Delta U_{i\tau}^{\alpha\sigma'}}{\Delta\tau}\Delta\tau\psi_{i\tau}^{\sigma'} + \psi_{i\tau}^{\dagger\sigma}\frac{\Delta\psi_{i\tau}^{\sigma'}}{\Delta\tau}\Delta\tau \\
&\quad - \psi_{i\tau}^{\dagger\sigma} U_{i\tau}^{\dagger\sigma\alpha}\frac{\Delta U_{i\tau}^{\alpha\sigma'}}{\Delta\tau}\Delta\tau\frac{\Delta\psi_{i\tau}^{\sigma'}}{\Delta\tau}\Delta\tau\Bigg) \\
&\approx -J\sum_{\tau i}\Delta\tau\psi_{i\tau}^{\dagger\sigma}\psi_{i\tau}^{\sigma} \\
&\quad + \sum_{\tau i}\Delta\tau\psi_{i\tau}^{\dagger\sigma} U_{i\tau}^{\dagger\sigma\alpha}\frac{\Delta U_{i\tau}^{\alpha\sigma'}}{\Delta\tau}\psi_{i\tau}^{\sigma'} + \sum_{\tau i}\Delta\tau\psi_{i\tau}^{\dagger\sigma}\frac{\Delta\psi_{i\tau}^{\sigma'}}{\Delta\tau} \\
&= \int_0^\beta d\tau\sum_i \left(\psi_{i\tau}^{\dagger\sigma}(\partial_\tau\delta_{\sigma\sigma'} + U_{i\tau}^{\dagger\sigma\alpha}\partial_\tau U_{i\tau}^{\alpha\sigma'})\psi_{i\tau}^{\sigma'} - J\psi_{i\tau}^{\dagger\sigma}\psi_{i\tau}^{\sigma}\right),
\end{aligned}
$$

where $\Sigma_\tau\Delta\tau$ is replaced with $\int_0^\beta d\tau$ with $J=1/\Delta\tau$, and the last J term can be absorbed in the chemical potential term.

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$$S_a = \int_0^\beta d\tau \int d^2r \frac{N_z}{16}(\partial\times a)\frac{1}{\sqrt{-\partial^2}}(\partial\times a),$$

where N_z is the flavor number of spinons. In energy-momentum space the effective gauge action is

$$S_a = \frac{N_z}{16}\sum_{q,\omega_n}\sqrt{q^2+\omega_n^2}\left(\delta_{ij} - \frac{q_i q_j}{q^2}\right)a_i a_j,$$

where the time component a_τ can be ignored since their fluctuations are gapped, as mentioned in the text. Thus, the total gauge action at the quantum critical point is given by

$$S_a = \frac{1}{2}\sum_{q,\omega_n}\left(\frac{q^2}{g} + \frac{E_\eta}{v_\eta}\frac{|\omega_n|}{q} + \frac{N_z}{8}\sqrt{q^2+\omega_n^2}\right)\left(\delta_{ij} - \frac{q_i q_j}{q^2}\right)a_i a_j.$$

To determine the dynamical exponent z, we find the dispersion relation for gauge fluctuations from the pole of the above gauge kernel, given by in real frequency

$$\frac{q^2}{g} - i\frac{E_\eta}{v_\eta}\frac{\omega}{q} + \frac{N_z}{8}\sqrt{q^2-\omega^2} = 0.$$

Solving this equation, we obtain

$$\omega = \frac{-i\frac{E_\eta}{v_\eta g}q^3 \pm \sqrt{-\left(\frac{E_\eta}{v_\eta g}\right)^2 q^6 - \left[\left(\frac{E_\eta}{v_\eta}\right)^2 - \left(\frac{N_z}{8}\right)^2 q^2\right]\left[\left(\frac{N_z}{8}\right)^2 q^4 - \frac{q^6}{g^2}\right]}}{\left(\frac{E_\eta}{v_\eta}\right)^2 - \left(\frac{N_z}{8}\right)^2 q^2}.$$

In the long wavelength limit $(q\rightarrow 0)$ the dispersion becomes

$$\omega \approx \pm i\frac{1}{\left(\frac{E_\eta}{v_\eta}\right)}\frac{N_z}{8}q^2.$$

This implies that the dynamical exponent is $z=2$ at the quantum critical point. The corresponding gauge action is that in Eq. (40).

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$$
\begin{aligned}
Z &= \int Da \exp\left[-\frac{1}{2}\frac{\delta^2 S_z}{\delta a^2}a^2 - \frac{1}{2}\left(\frac{\delta^2 S_\psi}{\delta a^2}a^2 + 2\frac{\delta^2 S_\psi}{\delta a\,\delta A}aA + \frac{\delta^2 S_\psi}{\delta A^2}A^2\right)\right] \\
&= Z_0 \exp\left[-\frac{1}{2}\frac{\delta^2 S_\psi}{\delta A^2}A^2 + \frac{1}{2}\frac{\frac{\delta^2 S_\psi}{\delta a\,\delta A}}{\frac{\delta^2 S_z}{\delta a^2} + \frac{\delta^2 S_\psi}{\delta a^2}}A^2\right],
\end{aligned}
$$

where S_z (S_ψ) is the boson (fermion) action. Differentiating the above effective action by an electromagnetic field A_μ twice, one can obtain the conductivity expression. The presence of $\sigma=\pm$ results in $\delta^2 S_\psi/\delta a\,\delta A=0$ because correlations between the charge current and the spin current are zero. As a result, the conductivity is given by the fermion contribution only.

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