Unusual field and temperature dependence of the Hall effect in graphene

L. A. Falkovsky
L.D. Landau Institute for Theoretical Physics, Moscow 117334, Russia
and Institute of the High Pressure Physics, Troitsk 142190, Russia
(Received 10 November 2006; published 23 January 2007)

We calculate the classic Hall conductivity and mobility of the undoped and doped (or at the gate voltage) graphene as a function of temperature, magnetic field, and carrier concentration. Carrier collisions with defects and acoustic phonons are taken into account. The Hall resistivity varies almost linearly with temperature below the Debye temperature. The magnetic-field dependence of resistivity and mobility is anomalous in weak fields at low gate voltage: there is a square-root field contribution in the resistivity and the Hall mobility diverges logarithmically with the field.

DOI: 10.1103/PhysRevB.75.033409 PACS number(s): 73.63.Bd, 71.70.Di

Recent investigations[1,2] of a monotomical layer of graphite with a honeycomb lattice (graphene) have attracted widespread attention. The single layer of graphite can be wrapped into zero-dimensional fullerenes, rolled into one-dimensional nanotubes, and stacked into three-dimensional (3D) graphite.[3] Therefore, one has the possibility of studying the dimensionality effects for this unique substance.[4] Graphene has a very simple band structure in the domain of several eV around the Fermi level. As has long been known from studies of graphite,[5,6] the energy bands near the Fermi level (of its single layer, i.e., graphene) are cones $\varepsilon_{1,2}(\mathbf{p})=\pm vp$ at the corners K of the two-dimensional (2D) Brillouin zone with $v=10^8$ cm/s. Thus, the Fermi surface shrinks to points. Such degeneracy is the result of symmetry because the small group C_{3v} of the K points has a two-dimensional irreducible representation.

According to this symmetry consideration, the electron spectrum (of the Dirac type but massless and two dimensional) is stable with respect to the Coulomb interaction, as was shown in Ref. 7 for the case of 3D dimensionality. In more recent work,[8,9] the authors argue that under doping, the behavior of graphene is altered from a marginal Fermi liquid to an ordinary 2D Fermi liquid. Graphene exhibits Shubnikov–de Haas oscillations with a standard Fermi-liquid temperature dependence. The weak-localization corrections to the conductivity can either have a different sign,[10] depending on the interaction range of impurity potentials, or be strongly suppressed,[11] due to the Dirac singularity of the spectrum.

Much theoretical effort[12–18] using different approaches has been devoted to understand the minimal conductivity discovered in Refs. 1 and 2. The finite values of conductivity observed at low temperature means that 2D graphene is a metal (or a semimetal), in contradiction to a recent theoretical analysis.[19]

The challenging task of evaluating the carrier interaction with defects in graphene and with the underlying substrate was undertaken in Refs. 14 and 20–22. In order to simplify the transport problem, we have considered[23] the quantum conductivity of graphene in the collisionless limit, in the case where the electric-field frequency ω or the spatial dispersion k is more important than the collision rate, i.e., $(\omega,kv)\gg\tau^{-1}$. We found the conductivity consisting of two terms: one of the Drude type and another describing the interband carrier transitions. Since there is no gap between the conduction and valence bands, these two terms can compete and the interband contribution becomes larger at high frequencies $\omega>T$; otherwise, the intraband contribution plays the leading role and the quantum considerations give the same result as the semiclassical Boltzmann equation.

The Hall effect was studied in Refs. 22, 24, and 25 in both quantum and classic regimes at zero temperature, with the gap due to interaction-induced phenomena[26] or interactions with chemical adsorbents[27] being taken into account.

In this Brief Report, we consider the classic Hall effect in both undoped and doped (or at gate voltage) graphenes for finite temperatures, including two independent types of scattering mechanisms, namely, slowly varying static disorder and electron-phonon scattering. We show that quasiparticles in graphene display intriguing properties: (i) a linear temperature dependence of resistivity below the Debye temperature and (ii) a singular behavior of the Hall mobility and resistivity as functions of the magnetic field at low carrier concentrations, where the Boltzmann statistics is used in the region of the gapless density of states.

A fundamental difficulty in the calculations of graphene transport properties occurs because modern methods (for instance, the diagrammatic approach) are restricted by the requirement that the mean free path $\ell=v\tau$ of carriers must be much larger than the electron wavelength $\lambda=h/p_F$, i.e., $\ell p_F\gg 1$. This condition evidently cannot be satisfied in the case of undoped graphene, where $p_F=0$. One can avoid the difficulty by considering the problem at finite temperatures, when $T\gg\tau^{-1}$, or at the gate voltage, when the carrier concentration is finite. In the first case, the temperature appears instead of the Fermi energy and electrons obey the Boltzmann statistics.

At low temperatures, collisions with defects are important. Using the Fermi golden rule, one can find the collision rate for scattering by defects as follows:

$$\tau_{\mathrm{imp}}^{-1}(\varepsilon)=\frac{n_{\mathrm{imp}}}{2\pi}\int d^2\mathbf{p}'|u(\mathbf{p}-\mathbf{p}')|^2(1-\cos\theta)\delta[\varepsilon-\varepsilon(\mathbf{p}')], \tag{1}$$

where n_{imp} is the defect concentration per unit area and $u(\mathbf{p}-\mathbf{p}')$ is the Fourier component of the defect potential. As

noted previously, we are interested in the carrier energy of the order of temperature, $\varepsilon \simeq T$, whereas the defect potential varies on interatomic distances. Consequently, for the intravalley scattering, the potential in the integrand of Eq. (1) can be considered as a constant $u(\mathbf{p}-\mathbf{p}')=u_0 \simeq \varepsilon_0 a^2$ (u_0 and a are the energy and the distance of the atomic scale, respectively). In integrating over angles, $\cos\theta$ cancels. For the intervalley processes, carriers view the neighboring valley in small angles, which gives a small factor Ta/v, so the intervalley scattering can be ignored. Integration of Eq. (1) gives

$$\tau_{\mathrm{imp}}^{-1}(\varepsilon) = n_d|\varepsilon|,$$

where $n_d=n_{\mathrm{imp}}(u_0/v)^2$ is the defect concentration per lattice unit. It is important that the collision rate is proportional to the energy. Since the electron density of states is also proportional to the energy, one obtains the residual resistance independent of temperature.

The temperature dependence arises from scattering by phonons. At temperatures below the Debye temperature, the electron collisions with acoustic phonons are important. Extending the result of Ref. 28 to the 2D electron system, we obtain

$$\tau_{el\text{-}ph}^{-1} = \alpha|\varepsilon|T/T_D, \tag{2}$$

where α is a factor of the order of the unity and T_D is the Debye temperature (around 2000 K for graphene). For the essential values of energy $\varepsilon \simeq T$, Eq. (2) gives $\tau^{-1} \simeq T^2/T_D$, in comparison with $\tau^{-1} \simeq T^3/T_D^2$ for 3D systems. Notice that all scattering angles are important now, since the Fermi surface (or the chemical potential) is assumed to be small.

As the processes of electron-defect and electron-phonon scattering are independent, the total scattering rate can be written in accordance with Matthiessen's rule in the form

$$\tau^{-1} = \tau_{\mathrm{imp}}^{-1}(\varepsilon) + \tau_{el\text{-}ph}^{-1}(\varepsilon) = |\varepsilon|n_d^*,$$

where the notation

$$n_d^* = n_d + \alpha T/T_D$$

is introduced.

In the τ approximation, the solution to the Boltzmann equation gives the electrical conductivity

$$\sigma_{\alpha\beta} = -\frac{e^2v^2}{\pi}\sum_{\mathrm{bands}}\int d\varepsilon \frac{df_0}{d\varepsilon}\frac{m\tau}{1+(\Omega\tau)^2}\begin{pmatrix} 1 & \Omega\tau \\ -\Omega\tau & 1 \end{pmatrix},$$

where the factor 4 results from the summation over spin and the two K points per Brillouin zone. The integration is performed over ε from 0 to ∞ in the conduction band and from $-\infty$ to 0 in the valence band; $m=\varepsilon/v^2$ is the cyclotron mass, $\Omega=eH/mc$ is the cyclotron frequency, the magnetic field H is assumed to be normal to the graphene layer, and $f_0(\varepsilon-\mu)$ is the Fermi function.

As a result, the Hall conductivity tensor takes the form

$$\sigma_{xx} = \frac{e^2}{4\pi\hbar n_d^*}\int_{-\infty}^{\infty}\frac{\varepsilon^4 d\varepsilon}{\varepsilon^4+\eta^2}\,\mathrm{sech}^2\,\frac{\varepsilon-\varphi}{2}, \tag{3}$$

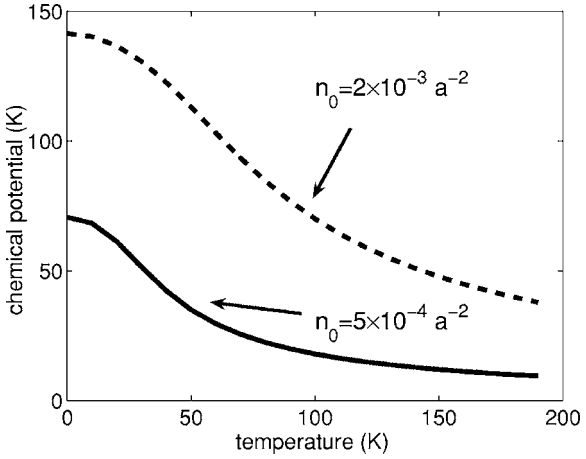

FIG. 1. Chemical potential versus temperature for two doping values n_0 per the lattice unit.

$$\sigma_{xy} = \frac{e^2\eta}{4\pi\hbar n_d^*}\int_{-\infty}^{\infty}\frac{\varepsilon^2\,\mathrm{sign}(\varepsilon)d\varepsilon}{\varepsilon^4+\eta^2}\,\mathrm{sech}^2\,\frac{\varepsilon-\varphi}{2}, \tag{4}$$

where we have restored the Planck constant and introduced the dimensionless magnetic field η and chemical potential φ as follows:

$$\eta = |e|\hbar Hv^2/cn_d^*k_B^2T^2, \quad \varphi = \mu/k_BT.$$

The longitudinal resistivity ρ and Hall angle θ_H (or Hall mobility μ_H),

$$\rho = \frac{\sigma_{xx}}{\sigma_{xx}^2+\sigma_{xy}^2}, \quad \tan\theta_H = \frac{\sigma_{xy}}{\sigma_{xx}}, \quad \mu_H = \frac{\sigma_{xy}}{H\sigma_{xx}},$$

can be obtained from measurements.

The chemical potential $\mu=0$ for undoped graphene in the absence of the gate voltage. In the field effect experiment, i.e., at the gate voltage (or for the doped material), μ is determined by the fixed carrier concentration

$$n_0 = \frac{2}{\pi v^2}\int_0^{\infty}\varepsilon[f_0(\varepsilon-\mu)-f_0(\varepsilon+\mu)]d\varepsilon.$$

As follows from Fig. 1, the chemical potential tends to the value $\mu=0$ for the undoped graphene as temperature is increased.

One can see from Eqs. (3) and (4) that, as it must be, the diagonal component of conductivity is an even function of the chemical potential and the off-diagonal one is an odd function. With the help of these equations, the Hall mobility and longitudinal resistivity are plotted as functions of temperature and magnetic field in Figs. 2–5. They both can be analytically calculated in various limiting cases.

For a case of the nondegenerate carrier statistic ($\varphi \ll 1$), an expansion of conductivity tensor components in terms of the magnetic field ($\eta \ll 1$) takes the form

$$\sigma_{xx} = \sigma_0[1-(\pi/4\sqrt{2})\sqrt{\eta}],$$

$$\sigma_{xy} = 0.25\sigma_0\eta\varphi\ln(1/\eta), \tag{5}$$

where the zero-field conductivity

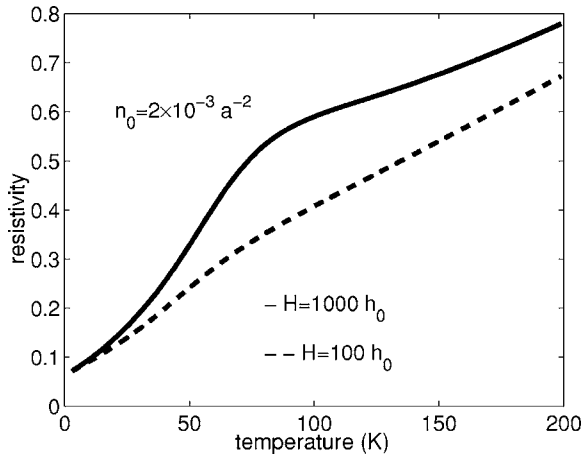

FIG. 2. Resistivity (in units of $\hbar/e^2$) versus temperature for two values of magnetic field H (in units of $h_0=ck_B^2K^2/e\hbar v^2=0.11$ G), and the doping level n_0 as indicated in the figure. The electron collisions with uncharged defects (of the concentration 0.02 per lattice unit in this and all other figures) and with acoustic phonons are taken into account.

$$\sigma_0 = \frac{e^2}{\pi\hbar n_d^*}$$

is dependent on temperature by means of n_d^*. Therefore, the resistivity in the absence of a field grows linearly with temperature (see Fig. 2), and is independent of chemical potential μ, i.e., of doping.

We emphasize that the resistivity corresponding to conductivity [Eq. (5)] grows as the square root of the magnetic field (also see the upper curve in Fig. 3), in contrast to the ordinary squared field dependence (see, for instance, Ref. 29).

As mentioned above, the Hall conductivity σ_{xy} does not vanish only in the doped material and changes sign if the sign of the chemical potential $\mu=\varphi T$ changes, while electrons are replaced by holes. In weak fields, the Hall conductivity reveals (similarly to the longitudinal conductivity) the

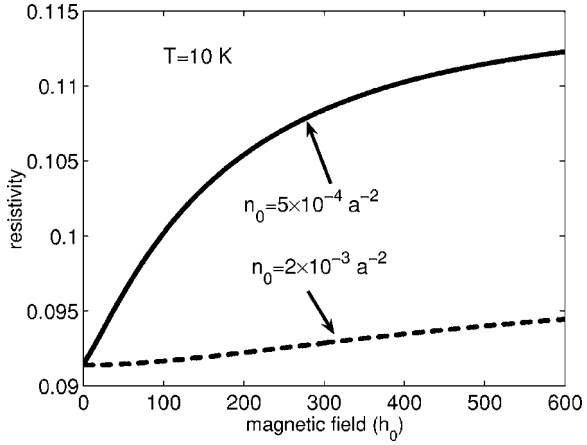

FIG. 3. Resistivity (in units of $\hbar/e^2$) as a function of the magnetic field (in units of $h_0=0.11$ G) at given temperature 10 K and doping levels n_0 as indicated on the curves.

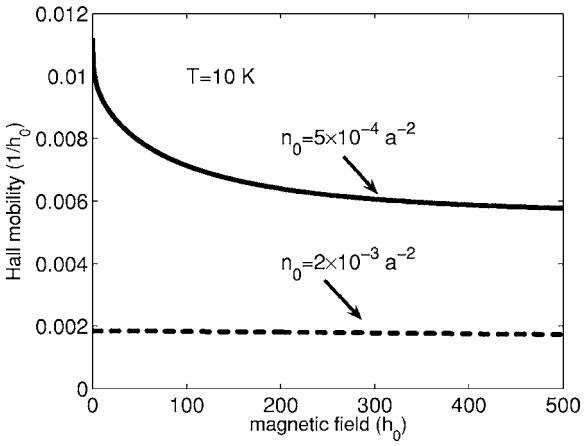

FIG. 4. Hall mobility (in units of $1/h_0$) as a function of magnetic field (in units of $h_0=0.11$ G) for two doping levels n_0 as indicated on the curves.

anomalous field dependence, that is the Hall mobility is logarithmically divergent (see Fig. 4, the upper curve).

The increase of the Hall mobility is cut off in weak fields at sample sizes r, which must be larger than the Larmor radius $r_c=v/\Omega$. This condition restricts the magnetic field to $\eta > \hbar v/rk_BTn_d^*$.

For the nondegenerate carrier statistic ($\varphi \ll 1$), in the opposite limit of high fields ($\eta \gg 1$), the conductivity tensor has the form

$$\sigma_{xx} = \sigma_0[(7/15)\pi^4 + 1.23 \times 2^4\varphi^2]/\eta^2,$$

$$\sigma_{xy} = 2.77\sigma_0\varphi/\eta. \quad (6)$$

In high magnetic fields, σ_{xy} is inversely proportional to the field, so the Hall coefficient is independent of the field. Since in high fields $\sigma_{xx} \propto H^{-2}$, the longitudinal resistivity tends to "saturate" (see Fig. 3).

For the degenerate statistics ($\varphi \gg 1$) from Eqs. (3) and (4), we obtain

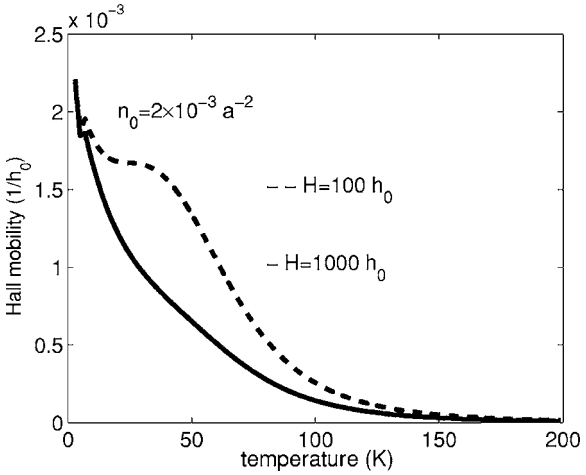

FIG. 5. Hall mobility (in units of $1/h_0$) versus temperature for two values of magnetic field H (in units of $h_0=0.11$ G).

$$\sigma_{xx} = \sigma_0 \varphi^4/(\eta^2 + \varphi^4),$$

$$\sigma_{xy} = \sigma_0 \eta \varphi^2/(\eta^2 + \varphi^4). \quad (7)$$

Equations (7) lead to the widely known behavior of the Hall effect: the resistivity $\rho = 1/\sigma_0$ is independent of the field (see the lower curve in Fig. 3) and the Hall coefficient $R = 1/ecn_0$ gives the carrier concentration n_0. In high magnetic fields ($\eta \gg \varphi$), Eqs. (7) give the same dependence on the field as Eqs. (6) in the case of the nondegenerate statistics, but they differ dramatically in the dependence on the carrier concentration determined by φ.

The Hall mobility as a function of temperature is shown in Fig. 5. In relatively high magnetic fields, the mobility is governed by resistivity, but it has more complicated behavior in weak fields.

In conclusion, considering electrons and holes in graphene as ordinary Fermi liquids, we found that the longitudinal resistivity mainly varies linearly with temperature at low temperatures. Hall resistivity and mobility vary anomalously in weak magnetic fields for the case of low carrier concentrations when the degenerate statistic holds. There is a square-root contribution from the field to the resistivity. The Hall mobility diverges logarithmically with the field for low dopings. This anomalous behavior results from the linear dependence of both the electron density of state and collision rate for the Dirac fermions in graphite.

I thank S. M. Stishov and Rex P. Hjelm from the Los Alamos National Laboratory for critical comments. This work was supported by the Russian Foundation for Basic Research.

[1] K. S. Novoselov *et al.*, Science **306**, 666 (2004); K. S. Novoselov *et al.*, Nature (London) **438**, 197 (2005).
[2] Y. Zhang, J. P. Small, M. E. S. Amory, and P. Kim, Phys. Rev. Lett. **94**, 176803 (2005).
[3] R. Saito, G. Dresselhaus, and M. S. Dresselhaus, *Physical Properties of Carbon Nanotubes* (Imperial College Press, London, 1998).
[4] E. Fradkin, Phys. Rev. B **33**, 3263 (1986).
[5] P. R. Wallace, Phys. Rev. **71**, 622 (1947).
[6] J. C. Slonczewski and P. R. Weiss, Phys. Rev. **109**, 272 (1958).
[7] A. A. Abrikosov and S. D. Beneslavsky, Sov. Phys. JETP **32**, 699 (1971).
[8] D. V. Khveshchenko, Phys. Rev. B **74**, 161402(R) (2006).
[9] S. Das Sarma, E. H. Hwang, and Wang-Kong Tse, cond-mat/0610581 (unpublished).
[10] H. Suzuura and T. Ando, Phys. Rev. Lett. **89**, 266603 (2002).
[11] D. V. Khveshchenko, Phys. Rev. Lett. **97**, 036802 (2006).
[12] P. A. Lee, Phys. Rev. Lett. **71**, 1887 (1993).
[13] A. W. W. Ludwig, M. P. A. Fisher, R. Shankar, and G. Grinstein, Phys. Rev. B **50**, 7526 (1994).
[14] N. M. R. Peres, F. Guinea, and A. H. Castro Neto, Phys. Rev. B **73**, 125411 (2006).
[15] J. Tworzydlo, B. Trauzettel, M. Titov, A. Rycerz, and C. W. J. Beenakker, Phys. Rev. Lett. **96**, 246802 (2006).
[16] K. Ziegler, cond-mat/0604537 (unpublished).
[17] K. Nomura and A. H. MacDonald, cond-mat/0606589 (unpublished).
[18] M. I. Katsnelson, cond-mat/0606611 (unpublished).
[19] I. L. Aleiner and K. B. Efetov, cond-mat/0607200 (unpublished).
[20] V. V. Cheianov and V. I. Falko, cond-mat/0608228 (unpublished).
[21] P. M. Ostrovsky, I. V. Gornyi, and A. D. Mirlin, cond-mat/0609617 (unpublished).
[22] Y. Zheng and T. Ando, Phys. Rev. B **65**, 245420 (2002).
[23] L. A. Falkovsky and A. A. Varlamov, cond-mat/0606800 (unpublished).
[24] N. A. Sinitsyn, A. H. MacDonald, T. Jungwirth, V. K. Dugaev, and J. Sinova, cond-mat/0608682 (unpublished).
[25] Y. Hasegawa and M. Kohmoto, Phys. Rev. B **74**, 155415 (2006).
[26] V. P. Gusynin and S. G. Sharapov, Phys. Rev. B **73**, 245411 (2006); V. P. Gusynin, S. G. Sharapov, and J. P. Carbotte, Phys. Rev. Lett. **96**, 256802 (2006).
[27] E. H. Hwang, S. Adam, S. Das Sarma, and A. K. Geim, cond-mat/0610834 (unpublished).
[28] B. M. Askerov, B. I. Kuliev, S. R. Figarova, and I. R. Gadirova, J. Phys.: Condens. Matter **7**, 843 (1995).
[29] A. A. Abrikosov, *Introduction to the Theory of Normal Metals: Solid State Physics Supplement* (Academic, New York, 1972), Vol. 12.