Universal non-near-field focus of acoustic waves through high-symmetry quasicrystals

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The focus behaviors of acoustic waves through high-symmetry quasicrystals (QC) have been investigated by using exact multi-scattering numerical simulation. We have found that eightfold, tenfold, and twelvefold two-dimensional (2D) QC slabs possess some universal features for non-near-field focus of the acoustic wave. The non-near-field focus of the acoustic wave can be realized by these QC slabs from very low packing density (filling fraction β=10%) to close packing (filling fraction β=40%) at large ranges of the frequency. The physical origin for such a superior feature has also been analyzed. Thus, it is anticipated that such a phenomenon can be potentially applied to phononic devices.

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I. INTRODUCTION

During the past decade, a significant effort has been devoted to the study of phononic crystals.[1–14] The existence of gaps in such materials provides an opportunity to confine and control the propagation of acoustic waves. It has profound signification for the applications in sound shields acoustic filters and other areas. Thus, much attention has been focused on the existence and properties of the phononic band gaps.[1–13] The band structures of many kinds of two-dimensional and three-dimensional phononic crystals have been calculated.[1–13] The properties of defect modes and waveguides in phononic crystals have also been discussed.[10] A method to engineer acoustic band gaps has been proposed,[11] and many other interesting results related to the acoustic band gaps have also been obtained.[13,14]

Recently, there has been a great deal of interest in studying unusual transmission features of acoustic wave in a wide range of frequencies outside the band gaps.[15–21] In particular, the focus of the acoustic wave has been realized by using the phononic crystal. In general, there are two kinds of method to realize the focus of the acoustic wave. One is in the long-wave range (the wavelengths well below the first acoustic band gap), where the dispersion relation in the phononic crystal is linear. For such a case, the media seems homogeneous for the wave propagation, in analogy with wave propagation in normal media. Consequently, the phononic crystal can be used for the design of acoustic lens through controlling the shape of the sample.[15–18] The other way to realize the focus of acoustic wave is through the negative refraction of flat lens.[19,20] For such a case, the frequency is not necessary to be confined in the long-wave range. The above discussions about the focus of acoustic wave were all focused on the periodic phononic crystal.

In this paper, we investigate the focus features of the acoustic wave through high-symmetry phononic quasicrystal (QC) by using exact numerical simulations. Our simulations show that the high-symmetry phononic QC slabs possess some unique features for non-near-field focus of the acoustic wave, which is different from those in periodic phononic crystal.

The rest of this paper is arranged as follows. In Sec. II, we define the system and introduce the method of numerical simulation. The results and discussion are described in Sec. III. A conclusion is given in Sec. IV.

II. SYSTEM AND METHOD

According to the level of symmetry, the QCs can be divided into fivefold, eightfold, tenfold, and twelvefold structures. In this paper, we consider eightfold, tenfold, and twelvefold high-symmetry phononic QC systems. The schemes of these QC structures are shown in Figs. 1(a)–1(c), respectively. In previous studies, large phononic band gaps have been found in these phononic QC systems.[12] In order to gain understanding of the band and gap regions for the acoustic waves transport in these QC structures, we first cal-

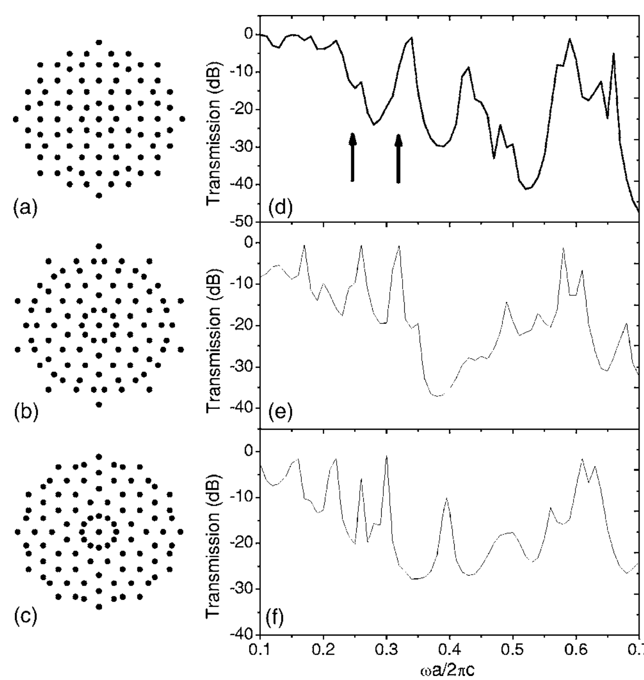

FIG. 1. Schemes of the basic QC structures and the corresponding transmission coefficients as a function of frequency for the QC slabs consisting of the steel cylinders with R=0.35a. (a) and (d) correspond to the eightfold QC; (b) and (e) to tenfold QC; (c) and (f) to twelvefold QC.

culate the transmission spectrums. The calculations are performed by using the multiple-scattering method.[17,21] The corresponding results for three kinds of structure are plotted in Figs. 1(d)–1(f), respectively. Solid lines in the figures represent the transmission coefficients of the acoustic wave passing through rectangular QC slabs with $6a$ thickness and $40a$ width. Here a represents the distance between the neighboring cylinders in the QC lattice. The phononic QC slabs consist of a number of steel cylinders embedded in air background. The radius (R) of the steel cylinder is $0.35a$. The ratios of density and velocity of the steel to that of the air are taken as 7800 and 17.9,[11,12] respectively. It is shown clearly that the band gaps of the acoustic wave exist in these structures. Because we aim at focus and imaging by using flat lens consisting of the above QCs, in the following discussion we focus on the band regions.

The properties of wave transport in the periodic phononic crystals can well be described by analyzing the equifrequency surface of the band structures. However, such a method is not applicable for the QC systems, due to the absence of the Bloch theorem. Thus, the study of quasiperiodic composites is much more difficult than that of periodic composites. In this paper, we adopt the multiple-scattering Korringa-Kohn-Rostoker method[11,12] to perform numerical simulations for wave propagating in these systems. The multiple-scattering method is a very efficient method of handling the scattering problem of a finite sample containing cylinders of circular cross sections, and it is capable of reproducing accurately the experimental transmission data, which should be regarded as exact numerical simulation.

III. NUMERICAL RESULTS AND DISCUSSION

In order to investigate the focus features of the acoustic wave, we design the flat lenses by using the above QC systems. Thus, we take the phononic QC slabs with $40a$ width and $7a$ thickness. A continuous-wave point source is placed at a distance $4.0a$ from the left surface of the slab. For the incident wave emitting from such a point source, we perform numerical simulations of wave propagating at various frequencies by using the multiple-scattering method. According to the previous studies on the periodic phononic crystal,[20] we have known that the non-zero-order diffractions are weak and nearly single-beam behavior is maintained when the frequencies are below $0.5(2\pi a/c)$. Thus, in the present calculations we limit the frequencies below $0.5(2\pi a/c)$.

We first consider the case of the eightfold phononic QC. At $\omega=0.25(2\pi a/c)$ and $\omega=0.32(2\pi c/a)$ [marked by arrows in Fig. 1(d)], we actually find the non-near-field focus of the acoustic wave. The result for $\omega=0.25(2\pi c/a)$ is shown in Fig. 2(a). Figure 2(a) shows the intensity distributions of pressure field of a sonic point source and its image across such a QC slab. X and Y present vertical and transverse direction of wave propagating. The pressure fields in figures are over $30a\times 30a$ region around the center of the sample. The geometry of the eightfold QC slab is also displayed. One can find that the focus is formed in the opposite side of the slab. The image distance is approximately equal to the object distance, which are about the half thickness of the slab. If we

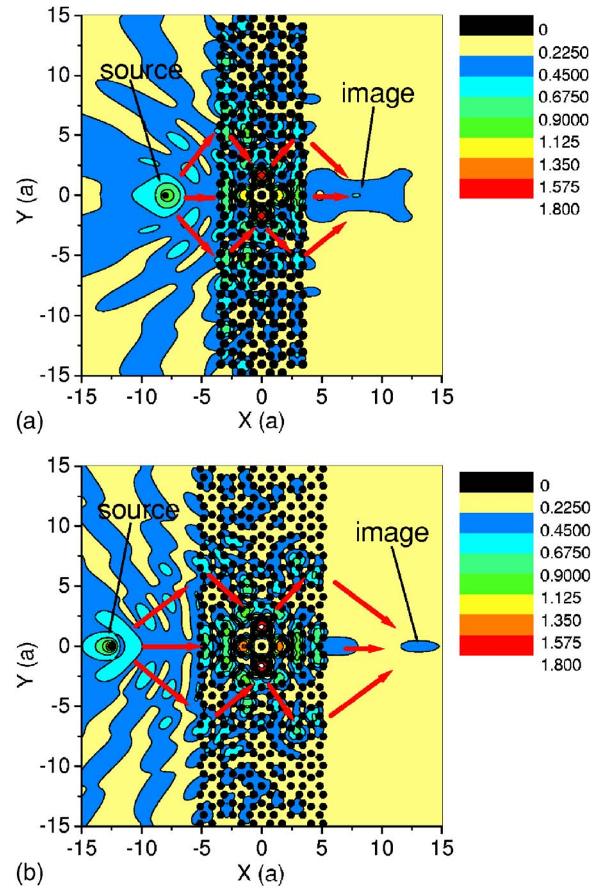

FIG. 2. (Color online) The intensity distributions of pressure wave of a point source and its image across a slab with $7a$ thick eightfold QC at frequency $\omega=0.25(2\pi c/a)$. The radii of the steel cylinders are taken as $0.35a$. (b) The corresponding case for a slab with $11a$ thickness.

change the thickness of the samples, similar features of the focus can also be found. Figure 2(b) represents the case with $11a$ thickness slab. With the increase of the sample thickness, the image distance and the object distance increase simultaneously. It means that the focus is non-near-field.

Next we shall show such a focus as a function of packing density, R/a. The dependence of the focus frequencies on R/a is plotted by dotted lines in Fig. 3. It is interesting to find that the non-near-field focus of the acoustic wave can be realized even at very low packing density (filling fraction $\beta=10\%$ or $R/a=0.15$) and close packing (filling fraction $\beta=40\%$ or $R/a=0.48$). Within the whole packing range from $\beta=10\%$ to $\beta=40\%$, we can always find the corresponding frequency, in which the focus appears. The focus frequency can also cover a large range. That is to say, the property of the non-near-field focus provided by the eightfold QC is "universal."

Not only the position of the focus depends on the thickness of the QC slab, but also the image resolution (full width at half maximum of the focus spot) is related to it. Figure 4 shows that the intensity distributions along the transverse (Y) direction at the image plane for eightfold QC slabs with different thickness. Solid line, dashed line and dotted line correspond to the case of $7a$, $9a$, and $11a$ thickness, respec-

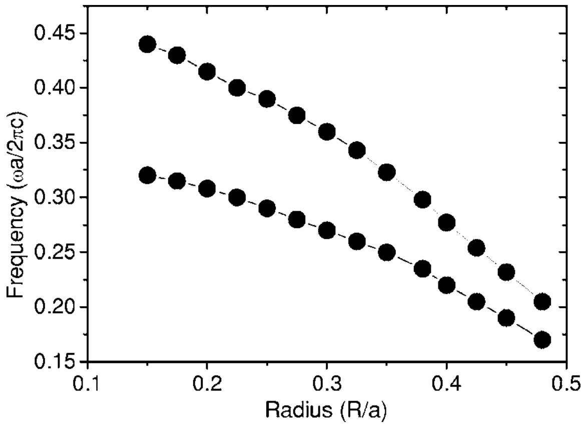

FIG. 3. The frequency of focus as a function of packing density R/a for eightfold QC.

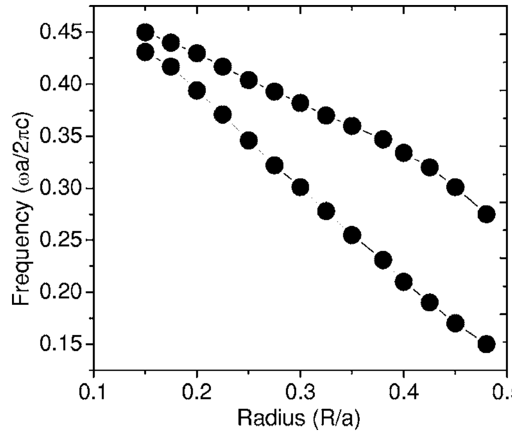

FIG. 5. The frequency of focus as a function of packing density R/a for tenfold QC. A schematic view of the tenfold QC is shown on top of the figure.

tively. With the increase of the thickness, the focus intensity decreases and the image resolution becomes worse due to the loss of the scattering. In general, the half maximum width of the focus spot are about 0.5λ and can not be lower than the diffraction limit. Only at the case of thinner slabs with large packing density, the focus size can be well below the conventional diffraction limit. For example, the full width at half maximum of the focus spot is about 0.38λ for $4a$ thickness slab with $R/a=0.45$. Since superlens effect involves evanescent waves, the higher resolution being able to achieve in thinner slabs is a general property of negative refraction materials. This is similar to the case of metal photonic crystal for electromagnetic wave.[26]

The above results are only for eightfold QC. In fact, similar focus properties can also be observed in tenfold and twelvefold high-symmetric QC. Figure 5 shows the focus frequency as a function of R/a for tenfold QC slab which consists of steel cylinders embedded in air background, and Fig. 6 gives the corresponding results for twelvefold QC structure. The parameters for steel cylinders are also identical to those of eightfold QC. The universal features for non-near-field focus of the acoustic wave in these high-symmetry structures are obtained again.

The origin of such a universal property of the focus can be explained from the following two aspects. First, the appearance of the focus through the high-symmetry QC can be explained similar to the cases in the periodic phononic crystals. The focus principle of acoustic wave through the flat slabs consisting of the periodic phononic crystals is due to the negative refraction and complex Bragg scattering effects, which can be described in terms of the equifrequency surface of the band structures.[18–24] Strictly speaking, the equifrequency surface and the band structures in QC do not exist. However, recent experiments[27] showed that analogous concepts of Bloch-like functions and Bloch-like states in the periodic structures could be applied approximately to some quasicrystals. This means that the complex Bragg scattering effects still exist in some QCs. Thus, the existence of the negative refraction and the focus for the acoustic wave in QC slabs becomes understandable.

In addition, the high rotational symmetry of the high-symmetry QC plays important role for such a non-near-field focus. Although the non-near-field focus can also be realized by periodic phononic crystal,[20] it can only be realized by the crystal with very large filling rate. It is very difficult to con-

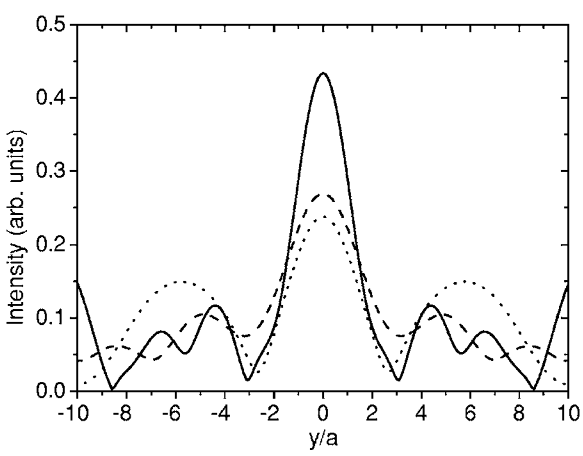

FIG. 4. The intensity distributions along the transverse (Y) direction at the image plane for eightfold QC slabs with $7a$ (solid line), $9a$ (dashed line), and $11a$ (dotted line) thickness.

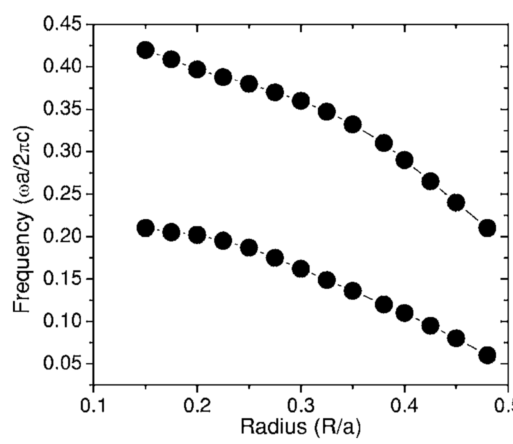

FIG. 6. The frequency of focus as a function of packing density R/a for twelvefold QC. A schematic view of the twelvefold QC is shown on top of the figure.

struct a flat lens by using steel cylinder of periodic structure to realize the non-near-field focus for the crystal with low filling rate. This is because that the position of the image depends on the homogeneity of the materials. In general, the homogeneous dispersion can be obtained easily in the high-symmetry structures. However, the highest level of symmetry that can be found in the periodic lattice is only six. In contrast, the high geometric symmetry of QCs can reach 8, 10, and 12. The motivation for using the high-symmetry QC is to maintain the periodic scattering of light while reducing the orientational order of the system to get more isotropy around the symmetric point. This can be embodied from the distribution of the field. Red arrows in Figs. 2(a) and 2(b) mark the directions of energy flux of the pressure wave. The focus of the field around the symmetric center (also the center of the slab) can be observed clearly. Such a center focus caused by the high rotational symmetric structure and the negative refraction can be observed in the crystals with various filling rates, which makes the QC slab similar to the case of a point source being put on the front of left-handed slab, which was predicted by Veselago for electromagnetic wave.[24,25] These superior features of the focus make high-symmetry phonon QC promising for application in a range of acoustic devices.

IV. CONCLUSION

Based on the exact multi-scattering numerical simulations, we have demonstrated that the high-symmetric QCs possess some universal features for non-near-field focus of the acoustic wave due to their high-symmetry. The non-near-field focus of the acoustic waves has been realized at whole packing range from very low packing density (filling fraction $\beta=10\%$) to close packing (filling fraction $\beta=40\%$) by eightfold, tenfold and twelvefold 2D QCs. Thus extensive applications of such a phenomenon to phononic devices are anticipated.

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