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Transverse rectification of disorder-induced fluctuations in a driven system

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We study numerically the overdamped motion of particles driven in a two-dimensional ratchet potential. In the proposed design, of the so-called geometrical-ratchet type, the mean velocity of a single particle in response to a constant force has a transverse component that can be induced by the presence of thermal or other unbiased fluctuations. We find that additional quenched disorder can strongly enhance the transverse drift at low temperatures, in spite of reducing the transverse mobility. We show that, under general conditions, the rectified transverse velocity of a driven particle fluid is equivalent to the response of a one-dimensional flashing ratchet working at a drive-dependent effective temperature, defined through generalized Einstein relations.

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The idea of generating a directed dissipative transport in a system kept out of thermal equilibrium only by unbiased perturbations has motivated an outburst of experimental and theoretical works in the last years. The ratchet effect is indeed of interest, for both applications and modeling, in very diverse systems, ranging from biological motors,[1] colloidal matter,[2] granular matter,[3] vortex matter in superconductors,[4] Josephson junction arrays,[5] atoms in optical traps,[6] and electrons in semiconductor heterostructures[7] to gambling games.[8] One of the simplest models is the "flashing ratchet," where a directed motion of a Brownian particle (i.e., breaking of the detailed balance condition) is obtained by coupling it to a pulsating asymmetric-periodic potential. The identification of the essential physical ingredients for the effect shows that a large variety of ratchets and rectification mechanisms can be realized.[9]

Recently, there has been a growing interest in the so-called *geometrical ratchets* since they can be used as continuous "molecular sieves" to separate particles experimentally (such as macromolecules or mesoscopic objects), according to their physical properties. These devices are typically two-dimensional systems containing a periodic array of asymmetric obstacles. By driving the particles through the array an average lateral drift appears, as transverse diffusive motion is rectified by collisions with the asymmetric obstacles. Different types of geometrical ratchets have been analyzed in the literature, both experimentally[10] and theoretically.[11] The effect of additional quenched disorder in these two-dimensional systems has not been discussed yet, though interesting anomalous transport properties of one-dimensional disordered ratchet systems were reported.[12] Such a study is not only relevant for applications where disorder cannot be avoided, but it is also an interesting and challenging issue. The driven motion of particles in a disordered substrate yields a nontrivial hydrodynamics. The current-driven motion of vortices in type-II superconductors is a prominent example, where disorder, apart from reducing dissipation, is responsible for marked nonequilibrium transport and magnetic properties.[13] On the other hand, already the simpler case of driven noninteracting Brownian particles in two dimensions has displayed complex phenomena. While diffusion is anomalous at equilibrium,[14] under a finite drive diffusion becomes normal in the comoving frame, with anisotropic and velocity-dependent diffusion constants and mobilities.[15] Moreover, a disordered substrate can provide alone a local or global ratchet effect, such as the generation of large-scale vorticity in the probability current by driving particles with a uniform alternate drive[16] and the net directed motion produced by driving the particles with crossed ac drives.[17]

In this paper we investigate the effect of quenched disorder in a simple geometrical ratchet design under a uniform and constant driving force. We find that disorder can strongly enhance the transverse drift at low temperatures both for noninteracting and interacting particles, thus improving the performance of the device for applications. We show that the transverse velocity of a driven fluid is equivalent to the response of a one-dimensional flashing ratchet working at a drive-dependent effective temperature, defined through generalized fluctuation-dissipation relations.

Let us consider the overdamped motion of particles in a two-dimensional (2D) potential like the one depicted in Fig. 1. The equation of motion of a particle in position $\mathbf{R}_i$ is

$$\eta\frac{d\mathbf{R}_i}{dt} = -\nabla_i\left[\sum_{j\neq i} V(R_{ij}) + U(\mathbf{R}_i)\right] + \mathbf{F} + \boldsymbol{\zeta}_i(t), \qquad (1)$$

where $R_{ij}=|\mathbf{R}_i-\mathbf{R}_j|$ is the distance between particles i and j, R_{ip} is the distance between the particle i and a site at $\mathbf{R}_p$, η is the friction, and $\mathbf{F}=F_y\hat{y}$ is the driving force. The effect of a thermal bath at temperature T is given by the stochastic force $\zeta_i(t)$, satisfying $\langle\zeta_i^\mu(t)\rangle=0$ and $\langle\zeta_i^\mu(t)\zeta_j^{\mu'}(t')\rangle =2\eta k_B T\delta(t-t')\delta_{ij}\delta_{\mu\mu'}$, where $\langle\cdots\rangle$ denotes the average over the ensemble of ζ_i. For concreteness we consider a logarithmic repulsive particle-particle interaction $V(r)=-A_v\ln(r)$ which corresponds, for instance, to the vortex-vortex interaction in 2D thin-film superconductors.[19] Particles interact with the quenched potential $U(\mathbf{R}) =U_R(\mathbf{R})+U_p(\mathbf{R})$. U_R is a ratchet potential with the form $U_R(\mathbf{R})=\frac{a}{2\pi}F_R(Y)G_R(X)$, where $X\equiv\mathbf{R}\cdot\hat{\mathbf{x}}$, $Y\equiv\mathbf{R}\cdot\hat{\mathbf{y}}$, $G_R(X)=\sin(2\pi X/a)+0.25\sin(4\pi X/a)$, and $F_R(y) =U_0\cos(2\pi y/b)\Theta[\cos(2\pi y/b)]$, with Θ the Heaviside function. This ratchet potential is similar to a periodic array of obstacles, asymmetrical around the x axis but symmetrical around the y axis, as the ones considered in Ref. 11. Disorder is short-range correlated, and it is modeled as a random dis-

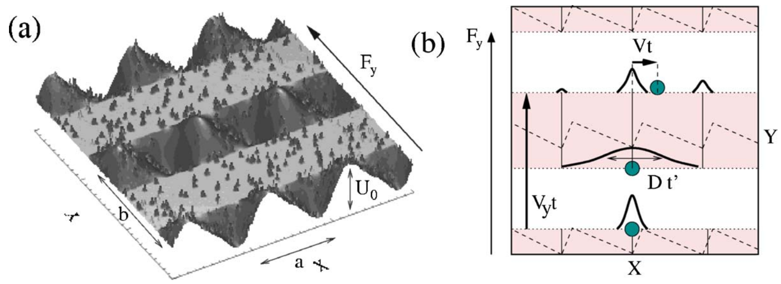

FIG. 1. (Color online) (a) Ratchet potential with disorder. (b) Schematics of the transverse rectification mechanism (top view). Particles move with an average velocity V_y in the direction of the applied force F_y. In the white regions the interaction with the (attractive or repulsive) centers and with the thermal bath induces diffusion in the nondriven direction. In the shaded regions a periodic-asymmetric potential tends to localize particles at its minima. An average transverse shift (see circles) is produced at a rate V.

tribution of centers such that $U_p(\mathbf{R}_i)=\Sigma_p A_p e^{-(R_{ip}/r_p)^2}$, where $R_{ip}=|\mathbf{R}_i-\mathbf{R}_p|$ is the distance between particle i and a center at $\mathbf{R}_p$. Centers can be either attractive $A_p<0$ (wells) or repulsive $A_p>0$ (humps) or a combination of both. We solve Eq. (1) numerically by using the method of Ref. 18. Length is normalized by r_p, energy by $2A_p$, and time by $\tau=\eta r_p^2/|2A_p|$. We consider $N=60$ particles and N_p pinning centers in a rectangular box of size $L_x\times L_y$ and periodic boundary conditions, with $L_y=100$, $L_x=20\sqrt{3}L_y$, $b=33$, and $a=20$. We average calculated properties over 500 disorder realizations.

We start by discussing the simplest case of noninteracting particles, $A_v=0$, without disorder, $N_p=0$, and with a ratchet potential of amplitude $U_0=1$. The dashed lines of Fig. 2(a) show the transverse drift rate $V\equiv\langle\frac{1}{N}\Sigma_i\frac{dX_i}{dt}\rangle$ at $T=0.05$ as a function of the longitudinal velocity $V_y\equiv\langle\frac{1}{N}\Sigma_i\frac{dY_i}{dt}\rangle$. We see that the transverse velocity V increases from zero, has a maximun $V\sim 0.0075$ at $V_y\sim 0.5$, and decays to zero at large longitudinal velocity. Since $V=0$ at $T=0$, the average directed transverse motion observed is induced by the thermal noise. This rectification effect is easy to understand, and Fig. 1(b) illustrates the mechanism, where the transverse diffusion constant is $D=2T$ if there is no disorder or interparticle interactions. For our discussion it is useful to make explicit the connection between the type of response shown in Fig. 2(a) and the one of a flashing ratchet. If F_y is large, the mean velocity in the driven direction, $V_y\equiv\langle\frac{dY}{dt}\rangle$, is $V_y\sim F_y-O(F_y^{-1})$ and longitudinal fluctuations are much smaller [by a factor of $O(F_y^{-1})$] than transverse fluctuations.[15] At $T=0$, the equation of motion for the coordinate X of a particle located at $\mathbf{R}=X\hat{x}+Y\hat{x}$ can be thus written as

$$\frac{dX}{dt}\approx -F_R(V_y t)G_R'(X). \tag{2}$$

Since $F_R(V_y t)=U_0\cos(2\pi V_y t/b)\Theta[\cos(2\pi V_y t/b)]$, X feels the ratchet potential $G_R(X)$ switching on and off periodically with time periods $\tau_{\mathrm{on}}=\tau_{\mathrm{off}}=b/2V_y$. At small drives the mechanism is the same although τ_{on} becomes increasingly larger than τ_{off} since the wells of $U_R(X,Y)$ delay the motion in the y direction (see Fig. 1). The mapping to a flashing ratchet explains the observed directed transverse motion with $V>0$ when $T>0$ and can be thus used as an effective model to explain all the features of the response shown in Fig. 2(a).

The effect discussed so far is similar to the one described in Ref. 11. Let us now add disorder by putting $N_p=2000$ randomly located pinning sites. The resulting response is shown in Fig. 2(a) (symbols). As we can see, disorder strongly enhances the rectification at intermediate and large longitudinal velocities and also broadens the range of V_y where the response is appreciable with respect to the clean case. In addition, we find that at intermediate and large V_y, V is finite even in the $T=0$ limit, since disorder induces transverse diffusion when $V_y>0$, even in the absence of thermal fluctuations.

Finally, let us now turn on the repulsive interaction between particles. In Fig. 2(b) we show V as a function of V_y for different values of the repulsion strength A_v. In the inset of Fig. 2(b) we see that the maximun response $V_{\max}$ is almost

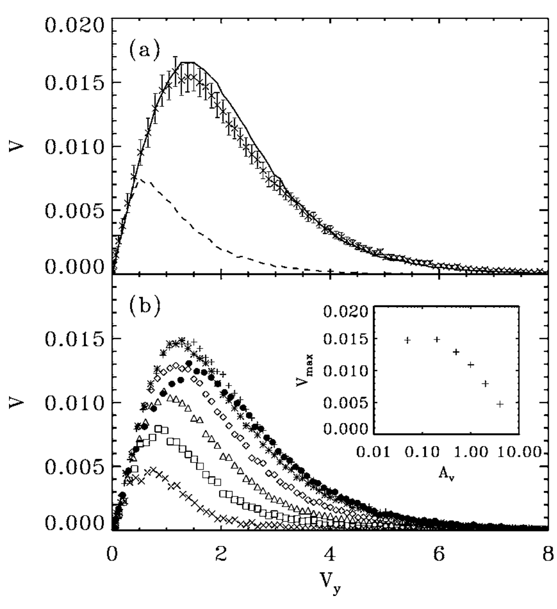

FIG. 2. Transverse velocity V vs longitudinal velocity V_y. (a) Symbols correspond to the disordered system at $T=0.05$, the dashed line to the clean system at $T=0.05$, and the solid line to the clean system at an effective temperature $T_{\mathrm{eff}}(V_y,T)$. (b) V vs V_y for different interaction strengths A_v: $A_v=0.05$ (+), $A_v=0.2$ (*), $A_v=0.5$ (◇), $A_v=1$ (△), $A_v=2$ (□), and $A_v=5$ (×). (●) symbols correspond to purely attractive pinning centers, with $A_p=-0.5$, at $T=0.05$. Inset: maximun rectification as a function of A_v.

constant with A_v up to values $A_v \sim 0.2$ where a slow decay starts, but it is larger than the response of the noninteracting clean system up to $A_v=2$. The decay of the response at large A_v is explained by the decrease of transverse wandering due to increasingly correlated collective motion.[19] This effect is, however, stronger in the absence of disorder. In Fig. 2(b) we show that the response for purely attractive pinning centers, $A_p=-1$, is smaller than for repulsive centers, $A_p>1$, for small values of V_y, but indistinguishable for larger values of V_y. This is due to the fact that, at the density of centers considered, attractive centers are more effective to pin particles than humps, since the latter can provide two-dimensional pinning only by forming rare geometric traps. However, at a density $N_p/L_xL_y \sim 1/r_p^2$ all these differences disappear completely.

In order to understand the rectification characteristics described above it is instructive to study, separately, the motion of particles in the purely disordered case without the ratchet potential (i.e., $U_0=0$). For simplicity we consider only the case of noninteracting particles, but we expect similar results for interacting particles in the dynamical regimes where transverse diffusion is nonzero.[18,20] We analyze in detail the nonequilibrium transverse fluctuations as a function of F_y, since they affect directly the rectification in the presence of the ratchet potential. Following Ref. 18 we define the observables $O(t)=\frac{1}{N_v}\sum_{i=1}^{N_v} s_i X_i(t)$ and $\tilde{O}(t)=\sum_{i=1}^{N_v} \underline{s_i X_i}(t)$, where $s_i=-1,1$ are random numbers with $\overline{s_i}=0$ and $\overline{s_i s_j}=\delta_{ij}$. The quadratic mean displacement can be written as $\Delta(t,t_0) \equiv \frac{1}{N_v}\sum_{i=1}^{N_v}\langle \underline{|X_i(t)-X_i(t_0)|^2}\rangle = C(t,t)+C(t_0,t_0)-2C(t,t_0)$, with $C(t,t_0)=\langle O(t)\tilde{O}(t_0)\rangle$. The integrated response function χ for the observable O is obtained by applying a perturbative force $\mathbf{f}_i=\epsilon s_i \hat{x}$ at time t_0 and keeping it constant for all subsequent times on each particle, $\chi(t,t_0)=\lim_{\epsilon\to 0}\frac{1}{\epsilon}[\overline{\langle O(t)\rangle_\epsilon}-\overline{\langle O(t)\rangle_{\epsilon=0}}]$. In the steady state $\Delta(t,t_0)=\Delta(t-t_0)$ and $\chi(t,t_0)=\chi(t-t_0)$ and in particular at equilibrium the fluctuation-dissipation theorem (FDT) imposes $\chi(t)=\Delta(t)/2T$. When $F_y>0$ the system is out of equilibrium and the FDT does not hold. We will show, however, that generalized fluctuation-dissipation relations can still be defined for our system. In the long-time limit we find $\Delta(t)\sim Dt$ and $\chi(t)\sim \mu t$, thus allowing us to define the transverse diffusion constant D and the transverse mobility μ. These two quantities depend on the longitudinal driving force as shown in Fig. 3(b). D is nonmonotonic, has a peak at $F_y\sim 1.5$, and decays approximately as a power law towards the equilibrium value without disorder, $2T$, for large forces. This behavior can be understood by considering the effective transverse random walk induced only by collisions with the pinning centers at $T=0$, and by simple heuristic arguments it is possible to find the asymptotic forms $D\sim n_p r_p^3 V_y$ at small V_y and $D\sim n_p r_p A_p^2/V_y$ at large V_y,[15] indicated in Fig. 3(b). At large F_y the transverse mobility μ approaches the equilibrium value without disorder, $\mu=1$ (independent of T). At small F_y, μ decreases due to trapping and its value at the limit $F_y\to 0$ is controlled by T. In the inset of Fig. 3(c) we show the parametric plot of $\chi(t)$ vs $\Delta(t)/2T$ for $F_y=0$ (equilibrium) and $F_y=4.0$ (out of equilibrium). We see that the equilibrium FDT holds for $F_y=0$ as expected. For $F_y=4.0$ (and in general for $F_y>0$) we observe instead that the FDT holds only at very short time scales, $t\lesssim r_p/V_y$, but it is violated at long times. The type of violation observed can be quantified using the notion of time-scale-dependent "effective temperatures" introduced by Cugliandolo, Kurchan, and Peliti.[21] Following Ref. 18 we define a velocity-dependent transverse effective temperature $T_{\rm eff}$ from the slope shown in the inset of Fig. 3(c). At long times this implies the generalized Einstein relation $T_{\rm eff}\sim D/2\mu$ in the nondriven direction. In Fig. 3(c) we see that $T_{\rm eff}$ follows closely D except at low forces where μ decreases towards the value $\mu\sim 0.5$ at very low forces. For interacting particles similar results for $T_{\rm eff}$ were obtained at low and intermediate forces in the plastic and smectic regimes of motion.[19] At large forces and small temperatures, however, the formation of pinned rough channels for particle motion[13] leads to a transverse freezing of the moving vortex fluid[19] at $T=0$. At this dynamical transition D and μ are strongly reduced and vanish at $T=0$ strictly.[15,18]

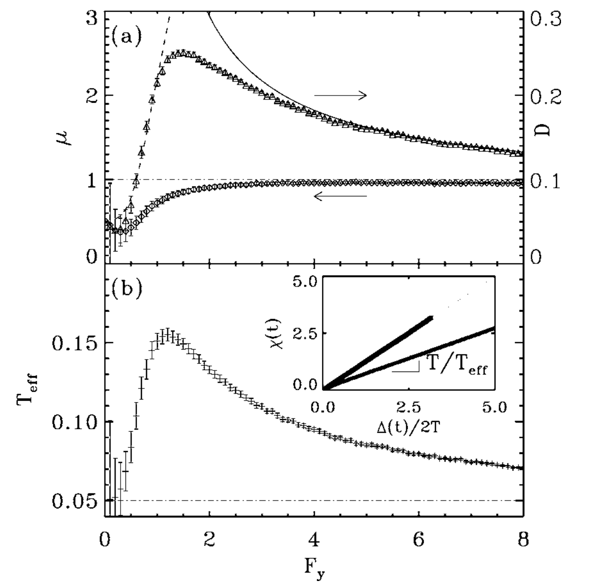

FIG. 3. Motion in a purely disordered potential. (a) Transverse diffusion constant D and mobility μ. Dashed and solid lines indicate asymptotic forms of D. (b) Effective temperature $T_{\rm eff}\sim D/2\mu$. The dash-dotted line indicates the bath temperature $T=0.05$. The inset shows that $T_{\rm eff}$ satisfies a generalized fluctuation-dissipation relation between the integrated response $\chi(t)$ and the quadratic mean displacement $\Delta(t)$. The dashed line corresponds to the equilibrium fluctuation-dissipation relation, valid only at short times $t\lesssim \xi/V_y$. Upper symbols correspond to $F_y=0.0$ and lower symbols to $F_y=4.0$.

The fluctuating motion in a purely disordered substrate and the connection between the response of a geometrical and a flashing ratchet discussed above motivate a simple model that can be used to describe the rectification response of the disordered geometrical ratchet. We propose the following equation for the transverse motion of noninteracting particles:

$$\frac{1}{\mu(V_y)}\frac{dX_i}{dt} \approx -F_R(V_y t)G_R'(X_i) + \zeta_{\rm eff}^i(t), \tag{3}$$

where we have replaced $Y_i\sim V_y t$ as before, the pinning force by an effective thermal noise $\zeta_{\rm eff}^i(t)$, and the bare mobility by

$\mu(V_y)$. We use that $\langle \zeta^i_{\rm eff}(t)\rangle = 0$ and $\langle \zeta^i_{\rm eff}(t)\zeta^j_{\rm eff}(t')\rangle = [2T_{\rm eff}(V_y)/\mu(V_y)]\delta(t-t')\delta_{ij}$ where $T_{\rm eff}(T,V_y)$ and $\mu(T,V_y)$ are the ones shown in Fig. 3(a). Equation (3) therefore models transverse motion in a coarse-grained way (in time and space), satisfying the generalized fluctuation-dissipation relation shown in the inset of Fig. 3(b) (in the absence of the ratchet potential). By construction, the assumptions of the model are that (i) transverse forces are small compared with the longitudinal drive F_y and (ii) the particle motion is incoherent at the length scales of the ratchet potential. In Fig. 2(a) we see that the transverse drift generated by this model is close to the one of the full model for the parameters analyzed in this paper, which assure that the particle has many independent collisions with the pinning centers between the rectifying regions. The rectification characteristics of the two-dimensional geometrical ratchet are therefore well described by the one-dimensional flashing ratchet described by Eq. (3) working at the effective temperature $T_{\rm eff}(T,V_y)$ and friction $\mu^{-1}(T,V_y)$, determined by the disorder and the longitudinal velocity. Using this model the enhancement of the rectification observed in Fig. 2(a) can be simply attributed to the fact that $T_{\rm eff} > T$, but with $T_{\rm eff}$ still smaller than the optimal temperature for rectification of the effective pulsating ratchet. Equation (3) is also expected to work for interacting particles by using the respective $T_{\rm eff}(T,V_y)$ and $\mu(T,V_y)$ except at low T and large F_y where condition (ii) can be violated since particles become correlated over long times and distances.[20]

In conclusion, we have studied numerically the effect of quenched disorder in a geometrical ratchet. We find that disorder enhances the transverse-rectified velocity of a driven fluid. If particle motion is incoherent at the scale of the ratchet potential, the response can be simply described by a one-dimensional flashing ratchet working at a disorder-induced, drive-dependent effective mobility and temperature $T_{\rm eff}$, satisfying generalized Einstein relations. This effect can be used experimentally to enhance and control the performance of geometrical ratchets at low temperatures, and conversely, it can be used as a "thermometer" to access $T_{\rm eff}$ in this kind of driven disordered systems.

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