Thermal expansion of Josephson junctions as an elastic response to an effective stress field

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By introducing a concept of thermal expansion (TE) of a Josephson junction as an elastic response to an effective stress field, we studied (both analytically and numerically) the temperature and magnetic field dependences of the TE coefficient α in a single small junction and in a square array. In particular, we found that in addition to *field* oscillations due to Fraunhofer-type dependence of the critical current, α of a small single junction also exhibits strong flux driven *temperature* oscillations near T_C . We also numerically simulated stress-induced response of a closed loop with finite self-inductance (a prototype of an array) and found that α of a 5×5 array may still exhibit temperature oscillations provided the applied magnetic field is strong enough to compensate for the screening-induced effects.

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I. INTRODUCTION

Inspired by possibilities offered by the cutting-edge nanotechnologies, the experimental and theoretical physics of increasingly sophisticated mesoscopic quantum devices (heavily based on Josephson junctions and their arrays) is becoming one of the most exciting and rapidly growing areas of modern science [see, e.g., Refs. 1–4 for the recent reviews on charge and spin effects in mesoscopic two-dimensional (2D) Josephson junctions and quantum-state engineering with Josephson-junction devices]. In particular, a remarkable increase of the measurement technique resolution made it possible to experimentally detect such interesting phenomena as flux avalanches⁵ and geometric quantization⁶ as well as flux dominated behavior of heat capacity⁷ in Josephson junctions (JJs) and their arrays (JJAs).

At the same time, given the rather specific magnetostrictive⁸ and piezomagnetic⁹ response of Josephson systems, one can expect some nontrivial behavior of the thermal expansion (TE) coefficient in JJs as well. Of special interest are the properties of TE in an applied magnetic field. For example, some superconductors such as $Ba_{1-x}K_xBiO_3$, $BaPb_xBi_{1-x}O_3$, and $La_{2-x}Sr_xCuO_4$ were found¹⁰ to exhibit anomalous temperature behavior of both magnetostriction and TE, which were attributed to the field-induced suppression of the superstructural ordering in the oxygen sublattices of these systems.

By introducing a concept of TE of Josephson contact (as an elastic response of JJ to an effective stress field), in the present paper we consider the temperature and magnetic field dependences of the TE coefficient $\alpha(T,H)$ in a small single JJ and in a single plaquette (a prototype of the simplest JJA). In a short contact, the field-induced $\alpha(T,H)$ is found to exhibit strong temperature oscillations near T_C . At the same time, in an array (described via a closed loop with finite self-inductance), for these oscillations to manifest themselves, the applied field should be strong enough to overcome the screening-induced self-field effects.

II. THERMAL EXPANSION OF A SMALL JOSEPHSON CONTACT

Since the thermal expansion coefficient $\alpha(T, H)$ is usually measured using mechanical dilatometers,¹¹ it is natural to introduce TE as an elastic response of the Josephson contact to an effective stress field σ .^{9,12} We define the TE coefficient (TEC) $\alpha(T, H)$ as follows:

$$\alpha(T,H) = \frac{d\epsilon}{dT},\tag{1}$$

where an appropriate strain field ϵ in the contact area is related to the Josephson energy E_J as follows (V is the volume of the sample):

$$\epsilon = -\frac{1}{V} \left[\frac{dE_J}{d\sigma} \right]_{\sigma=0}.$$
 (2)

For simplicity and to avoid self-field effects, we start with a small Josephson contact of length $w < \lambda_J$ ($\lambda_J = \sqrt{\Phi_0/\mu_0 dj_c}$ is the Josephson penetration depth) placed in a strong enough magnetic field (which is applied normally to the contact area) such that $H > \Phi_0/2\pi\lambda_J d$, where $d=2\lambda_L+t$, λ_L is the London penetration depth, and *t* is an insulator thickness.

The Josephson energy of such a contact in an applied magnetic field is governed by a Fraunhofer-type dependence of the critical current:¹³

$$E_J = J \left(1 - \frac{\sin \varphi}{\varphi} \cos \varphi_0 \right), \tag{3}$$

where $\varphi = \pi \Phi / \Phi_0$ is the frustration parameter with $\Phi = Hwd$ being the flux through the contact area, φ_0 is the initial phase difference through the contact, and $J \propto e^{-t/\xi}$ is

the zero-field tunneling Josephson energy with ξ being a characteristic (decaying) length and *t* the thickness of the insulating layer. The self-field effects (screening) neglected here will be treated in the next section for an array.

Notice that in a nonzero applied magnetic field H, there are two stress-induced contributions to the Josephson energy E_J , both related to the decrease of the insulator thickness under pressure. Indeed, according to the experimental data,¹² the tunneling dominated critical current I_c in granular high- T_C superconductors was found to exponentially increase under compressive stress, viz., $I_c(\sigma) = I_c(0)e^{\kappa\sigma}$. More specifically, the critical current at $\sigma=9$ kbars was found to be three times higher its value at $\sigma=1.5$ kbars, clearly indicating a weak-link-mediated origin of the phenomenon. Hence, for small enough σ , we can safely assume that $t(\sigma) = t(0)(1 - \beta\sigma/\sigma_0)$, with σ_0 having some characteristic value (the parameter β is related to the so-called ultimate stress σ_m as $\beta = \sigma_0/\sigma_m$). As a result, we have the following two stressinduced effects in Josephson contacts:

(i) amplitude modulation leading to the explicit stress dependence of the zero-field energy,

$$J(T,\sigma) = J(T,0)e^{\gamma\sigma/\sigma_0},$$
(4)

with $\gamma = \beta t(0) / \xi$, and

(ii) phase modulation leading to the explicit stress dependence of the flux,

$$\Phi(T,H,\sigma) = Hwd(T,\sigma), \tag{5}$$

with

$$d(T,\sigma) = 2\lambda_L(T) + t(0)(1 - \beta\sigma/\sigma_0).$$
(6)

Finally, in view of Eqs. (1)–(6), the temperature and field dependences of the small single junction TEC read (the initial phase difference is conveniently fixed at $\varphi_0 = \pi$)

$$\alpha(T,H) = \alpha(T,0)[1+F(T,H)] + \epsilon(T,0)\frac{dF(T,H)}{dT}, \quad (7)$$

where

$$F(T,H) = \left[\frac{\sin\varphi}{\varphi} + \frac{\xi}{d(T,0)} \left(\frac{\sin\varphi}{\varphi} - \cos\varphi\right)\right], \quad (8)$$

with

$$\varphi(T,H) = \frac{\pi \Phi(T,H,0)}{\Phi_0} = \frac{H}{H_0(T)},$$
(9)

$$\alpha(T,0) = \frac{d\epsilon(T,0)}{dT},$$
(10)

and

$$\boldsymbol{\epsilon}(T,0) = -\left(\frac{\Phi_0}{2\pi}\right) \left(\frac{2\gamma}{V\sigma_0}\right) I_C(T). \tag{11}$$

Here, $H_0(T) = \Phi_0 / \pi w d(T, 0)$ with $d(T, 0) = 2\lambda_L(T) + t(0)$. For the explicit temperature dependence of $J(T, 0) = \Phi_0 I_C(T) / 2\pi$, we use the well-known¹⁴ analytical approximation of the BCS gap parameter (valid for all tempera-



FIG. 1. (Color online) Temperature dependence of the normalized flux driven strain field $\epsilon(T,f)/\epsilon(0,0)$ in a single short contact for different values of the frustration parameter $f=H/H_0(0)$ according to Eqs. (1)–(13).

tures), $\Delta(T) = \Delta(0) \tanh[2.2\sqrt{(T_C - T)/T}]$, with $[\Delta(0) = 1.76k_BT_C$, which governs the temperature dependence of the Josephson critical current,

$$I_C(T) = I_C(0) \left[\frac{\Delta(T)}{\Delta(0)} \right] \tanh\left[\frac{\Delta(T)}{2k_B T} \right], \tag{12}$$

while the temperature dependence of the London penetration depth is governed by the two-fluid model:

$$\lambda_L(T) = \frac{\lambda_L(0)}{\sqrt{1 - (T/T_C)^2}}.$$
(13)

From the very structure of Eqs. (1)–(9), it is obvious that the TEC of a single contact will exhibit *field* oscillations imposed by the Fraunhofer dependence of the critical current I_C . Much less obvious is its temperature dependence. Indeed, Fig. 1 presents the temperature behavior of the contact area strain field $\epsilon(T, f)$ [with $t(0)/\xi=1$, $\xi/\lambda_L(0)=0.02$, and β =0.1] for different values of the frustration parameter f= $H/H_0(0)$. Notice the characteristic flux driven temperature oscillations near T_C , which are better seen on a semilogarithmic plot shown in Fig. 2, which depicts the dependence of the properly normalized field-induced TEC $\alpha(T, f)$ as a function of $1-T/T_C$ for the same set of parameters.

III. THERMAL EXPANSION IN THE PRESENCE OF SCREENING CURRENTS

To answer an important question on how the screening effects neglected in the previous section will affect the above-predicted oscillating behavior of the field-induced TEC, let us consider a more realistic situation with a junction embedded into an array (rather than an isolated contact), which is realized in artificially prepared arrays using photo-



FIG. 2. (Color online) Temperature dependence of the flux driven normalized TEC $\alpha(T,f)T_C/|\epsilon(0,0)|$ in a single small contact for different values of the frustration parameter $f=H/H_0(0)$ (for the same set of parameters as in Fig. 1) according to Eqs. (1)–(13).

lithographic technique (that today allows for controlled manipulations of the junction parameters¹⁵). Besides, this is also a good approximation for a granular superconductor (if we consider it as a network of superconducting islands connected with each other via Josephson links¹³). Our goal is to model and simulate the elastic response of such systems to an effective stress σ (described in the previous section for an isolated contact). For simplicity, we will consider an array with a regular topology and uniform parameters (such approximation already proved useful for describing highquality artificially prepared structures⁶).

A. Model equations for a planar square array

Let us consider a planar square array as shown in Fig. 3. The total current includes the bias current (flowing through the vertical junctions) and the induced screening currents (circulating in the plaquette¹⁶). This situation corresponds to the inclusion of screening currents only into the nearest neighbors, thus neglecting the mutual inductance terms.¹⁷ Therefore, the equation for the vertical contacts will read (horizontal and vertical junctions are denoted by superscripts *h* and *v*, respectively)

$$\frac{\hbar C}{2e} \frac{d^2 \phi_{i,j}^v}{dt^2} + \frac{\hbar}{2eR} \frac{d \phi_{i,j}^v}{dt} + I_c \sin \phi_{i,j}^v = \Delta I_{i,j}^s + I_b, \quad (14)$$

where $\Delta I_{i,j}^s = I_{i,j}^s - I_{i-1,j}^s$ and the screening currents I^s obey the fluxoid conservation condition:

$$-\phi_{i,j}^{v}+\phi_{i,j+1}^{v}-\phi_{i,j}^{h}+\phi_{i+1,j}^{h}=2\pi\frac{\Phi^{ext}}{\Phi_{0}}-\frac{2\pi L I_{i,j}^{s}}{\Phi_{0}}.$$
 (15)

Recall that the total flux has two components (an external contribution and the contribution due to the screening cur-



FIG. 3. Sketch of an array. The junctions perpendicular (parallel) to the bias are called horizontal (vertical). (a) The node (i, j) is shown as a circle in the left bottom corner of a plaquette; (b) a single plaquette (the elementary unit of the circuit) along with the circulating current; and (c) the lumped element circuit for a small junction.

rents in the closed loop) and it is equal to the sum of the phase differences describing the array. It is important to underline that the external flux in Eq. (15), $\eta = 2\pi\Phi^{ext}/\Phi_0$, is related to the frustration of the whole array (i.e., this is the flux across the void of the network^{18,19}), and it should be distinguished from the previously introduced applied magnetic field *H* across the junction barrier, which is related to the frustration of a single contact $f=2\pi H dw/\Phi_0$ and which only modulates the critical current $I_C(T,H,\sigma)$ of a single junction while inducing a negligible flux into the void area of the array.

B. Stress-induced effects

For simplicity, in what follows we will consider the elastic effects due to a uniform (homogeneous) stress imposed on the array. With regard to the geometry of the array, the deformation of the loop is the dominant effect with its radius a deforming as follows:

$$a(\sigma) = a_0(1 - \chi \sigma / \sigma_0). \tag{16}$$

As a result, the self-inductance of the loop $L(a) = \mu_0 a F(a)$ [with F(a) being a geometry-dependent factor] will change accordingly:

$$L(a) = L_0(1 - \chi_g \sigma / \sigma_0). \tag{17}$$

The relationship between the coefficients χ and χ_g is given by

$$\chi_g = (1 + a_0 B_g) \chi, \tag{18}$$

where $B_g = [1/F(a)](dF/da)_{a_0}$.

It is also reasonable to assume that in addition to the critical current, the external stress will modify the resistance of the contact,

$$R(\sigma) = \frac{\pi\Delta(0)}{2eI_C(\sigma)} = R_0 e^{-\chi\sigma/\sigma_0},$$
(19)

as well as the capacitance (due to the change in the distance between the superconductors),

$$C(\sigma) = \frac{C_0}{1 - \chi \sigma / \sigma_0} \simeq C_0 (1 + \chi \sigma / \sigma_0).$$
 (20)

To simplify the treatment of the dynamic equations of the array, it is convenient to introduce the standard normalization parameters such as the Josephson frequency,

$$\omega_J = \sqrt{\frac{2\pi I_C(0)}{C_0 \Phi_0}},$$
 (21)

the analog of the superconducting quantum interference device (SQUID) parameter,

$$\beta_L = \frac{2\pi I_C(0)L_0}{\Phi_0},$$
(22)

and the dissipation parameter,

$$\beta_C = \frac{2\pi I_C(0)C_0 R_0^2}{\Phi_0}.$$
(23)

By combining Eqs. (14) and (15) with the stress-induced effects described by Eqs. (19) and (20) and by using the normalization parameters given by Eqs. (21)–(23), we can rewrite the equations for an array in a rather compact form. The equations for vertical junctions read

$$\frac{1}{1 - \chi \sigma / \sigma_0} \ddot{\phi}_{i,j}^v + \frac{e^{-\chi \sigma / \sigma_0}}{\sqrt{\beta_C}} \dot{\phi}_{i,j}^v + e^{\chi \sigma / \sigma_0} \sin \phi_{i,j}^v$$

$$= \frac{1}{\beta_L (1 - \chi_g \sigma / \sigma_0)} (\phi_{i,j-1}^v - 2\phi_{i,j}^v + \phi_{i,j+1}^v + \phi_{i,j}^h - \phi_{i-1,j}^h$$

$$+ \phi_{i+1,j-1}^h - \phi_{i,j-1}^h) + \gamma_b.$$
(24)

Here, an overdot denotes the time derivative with respect to the normalized time (inverse Josephson frequency), and the bias current is normalized to the critical current without stress, $\gamma_b = I_b/I_c(0)$.

The equations for the horizontal junctions will have the same structure safe for the explicit bias related terms:

$$\frac{1}{1 - \chi \sigma / \sigma_0} \ddot{\phi}_{i,j}^h + \frac{e^{-\chi \sigma / \sigma_0}}{\sqrt{\beta_C}} \dot{\phi}_{i,j}^h + e^{\chi \sigma / \sigma_0} \sin \phi_{i,j}^h \\
= \frac{1}{\beta_L (1 - \chi_g \sigma / \sigma_0)} (\phi_{i,j-1}^h - 2\phi_{i,j}^h + \phi_{i,j+1}^h + \phi_{i,j}^v - \phi_{i-1,j}^v \\
+ \phi_{i+1,j-1}^v - \phi_{i,j-1}^v).$$
(25)

Finally, Eqs. (24) and (25) should be complemented with the appropriate boundary conditions,²⁰ which will include the normalized contribution of the external flux through the plaquette area $\eta = 2\pi (\Phi^{ext}/\Phi_0)$.

It is interesting to note that Eqs. (24) and (25) will coincide with their stress-free counterparts if we introduce the stress-dependent renormalization of the parameters:

$$\widetilde{\omega}_J = \omega_J e^{\chi \sigma/2\sigma_0}, \qquad (26)$$

$$\tilde{\beta}_C = \beta_C e^{-3\chi\sigma/\sigma_0},\tag{27}$$

$$\widetilde{\beta}_L = \beta_L (1 - \chi_g \sigma / \sigma_0) e^{\chi \sigma / \sigma_0}, \qquad (28)$$

$$\tilde{\eta} = \eta (1 - 2\chi \sigma / \sigma_0), \qquad (29)$$

$$\widetilde{\gamma}_b = \gamma_b e^{-\chi \sigma / \sigma_0}. \tag{30}$$

C. Numerical results and discussion

Turning to the discussion of the obtained numerical simulation results, it should be stressed that the main problem in dealing with an array is that the total current through the junction should be retrieved by solving self-consistently the array equations in the presence of screening currents. Recall¹³ that the Josephson energy of a single junction for an arbitrary current *I* through the contact reads

$$E_J(T, f, I) = E_J(T, f, I_C) \left[1 - \sqrt{1 - \left(\frac{I}{I_C}\right)^2} \right].$$
 (31)

The important consequence of Eq. (31) is that if no current flows in the array's junction, such junction will not contribute to the TEC (simply because a junction disconnected from the current generator will not contribute to the energy of the system).

We sketch the main steps of the numerical procedure used to simulate the stress-induced effects in the array below.

(1) A bias point I_b is selected for the whole array.

(2) The parameters of the array (screening, Josephson frequency, dissipation, etc.) are selected and modified according to the intensity of the applied stress σ .

(3) The array equations are simulated to retrieve the static configuration of the phase differences for the parameters selected in step 2.

(4) The total current flowing through the individual junctions is retrieved as

$$I_{i,i}^{v,h} = I_C \sin \phi_{i,i}^{v,h}.$$
 (32)

(5) The energy dependence upon stress is numerically estimated by using the value of the total current $I_{i,j}^{v,h}$ (which is not necessarily identical for all junctions) found in step 4 via Eq. (32).

(6) The array energy E_J^A is obtained by summing up the contributions of all junctions with the above-found phase differences $\phi_{i,j}^{v,h}$.

(7) The stress-modified screening currents $I_{i,j}^{s}(T,H,\sigma)$ are computed by means of Eq. (15) and inserted into the magnetic energy of the array $E_{M}^{A} = (1/2L) \sum_{i,j} (I_{i,j}^{s})^{2}$.

(8) The resulting strain field and TE coefficient of the array are computed by using numerical derivatives based on the finite differences:

$$\epsilon^{A} \simeq \frac{1}{V} \left[\frac{\Delta (E_{M}^{A} + E_{J}^{A})}{\Delta \sigma} \right]_{\Delta \sigma \to 0}, \tag{33}$$



FIG. 4. (Color online) Numerical simulation results for a 5×5 array (black solid line) and a small single contact (red dashed line). The dependence of the normalized TEC $\alpha(T,f)T_C/|\epsilon(0,0)|$ on the frustration parameter *f* (applied magnetic field *H* across the barrier) for the reduced temperature $T/T_C=0.95$. The parameters used for the simulations are $\eta=0$, $\beta=0.1$, $t(0)/\xi=1$, $\xi/\lambda_L=0.02$, $\beta_L=10$, $\gamma_b=0.95$, and $\chi_g=\chi=0.01$.

$$\alpha(T,H) \simeq \frac{\Delta \epsilon^A}{\Delta T}.$$
(34)

The numerical simulation results show that the overall behavior of the strain field and the TE coefficient in the array is qualitatively similar to the behavior of the single contact. In Fig. 4, we have simulated the behavior of both the small junction and the array as a function of the field across the barrier of the individual junctions in the presence of bias and screening currents. As seen, the dependence of $\alpha(T, f)$ is very weak up to $f \simeq 0.5$, showing a strong decrease of about 50% when the frustration approaches f=1. A much more profound change is obtained by varying the temperature for the fixed value of the applied magnetic field. Figure 5 depicts the temperature behavior of $\alpha(T, f)$ (on a semilogarithmic scale) for different field configurations, which include barrier field f frustrating a single junction and the flux across the void of the network η frustrating the whole array. First of all, by comparing Fig. 5(a) and Fig. 2, we notice that, due to substantial modulation of the Josephson critical current $I_C(T,H)$ given by Eq. (3), the barrier field f has similar effects on the TE coefficient of both the array and the single



FIG. 5. (Color online) Numerical simulation results for a 5×5 array. The influence of the flux across the void of the network η frustrating the whole array on the temperature dependence of the normalized TEC $\alpha(T, f)T_C/|\epsilon(0,0)|$ for different values of the barrier field *f* frustrating a single junction for γ_b =0.5 and the rest of parameters same as in Fig. 4.

contact including temperature oscillations. However, finite screening effects in the array result in the appearance of oscillations at higher values of the frustration f (in comparison with a single contact). On the other hand, Figs. 5(d), 5(c), and 5(d) represent the influence of the external field across the void η on the evolution of $\alpha(T, f)$ (recall that in the absence of stress, this field produces a well-defined magnetic pattern^{18–20}). As seen, in comparison with a field-free configuration [shown in Fig. 5(a)], the presence of external field η substantially reduces the magnitude of the TE coefficient of the array. Besides, with increasing η , the onset of temperature oscillations markedly shifts closer to T_C .

IV. CONCLUSION

We have investigated the influence of a homogeneous mechanical stress on a small single Josephson junction and on a plaquette (array of 5×5 junctions). We have shown how the stress-induced modulation of the parameters describing the junctions (as well as the connecting circuits) produces such an interesting phenomenon as a thermal expansion (TE) in a single contact and two-dimensional array (plaquette). We also studied the variation of the TE coefficient with an external magnetic field and temperature. In particular, near T_C (due to some tremendous increase of the effective "sandwich" thickness of the contact), the field-induced TE coefficient of a small junction exhibits clear *temperature* oscillations scaled with the number of flux quanta crossing the contact area. Our numerical simulations revealed that these oscillations may actually still survive in an array if the applied field is strong enough to compensate for finite screening-induced self-field effects. Finally, it is important to emphasize that our analysis refers to regular arrays with square geometry (similar to the already existing artificially prepared arrays^{6,18}). However, we can argue that the effects predicted here should also manifest themselves in granular

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superconductors, which exhibit quite pronounced stressdependent behavior upon mechanical loading.^{9,12}

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