Macroscopic quantum tunneling in globally coupled series arrays of Josephson junctions

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We present a quantitative analysis of an escape rate for switching from the superconducting state to a resistive one in series arrays of globally coupled Josephson junctions. A global coupling is provided by an external shunting impedance. Such an impedance can strongly suppress both the crossover temperature from the thermal fluctuation to quantum regimes, and the macroscopic quantum tunneling (MQT) in short Josephson junction series arrays. However, in large series arrays we obtain an enhancement of the crossover temperature, and a giant increase of the MQT escape rate. The effect is explained by excitation of a *spatial-temporal charge instanton* distributed over a whole structure. The model gives a possible explanation of recently published experimental results on an enhancement of the MQT in single crystals of high-T_c superconductors.

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Great attention has been devoted to an experimental and theoretical study of dc biased series arrays of Josephson junctions.[1–4] Such a system displays diverse fascinated nonlinear classical and macroscopic quantum-mechanical phenomena. For example a resistive state of Josephson junction series arrays can show synchronized behavior,[1,2] and this effect has been used in Josephson voltage standard devices.[3] As we turn to a region of small dc bias currents and low temperatures, the macroscopic quantum-mechanical phenomena start to play a role. Thus, a quantum phase superconductor-insulator transition has been observed in artificially prepared series arrays of a small size $Al/Al_2O_3/Al$ junctions.[4] All these effects strongly depend on the interaction between Josephson junctions.

This field of research, i.e., the macroscopic quantum phenomena in spatially extended superconducting systems, has been boosted even further by recent discovery of macroscopic quantum tunneling (MQT) in single crystals of layered high-T_c superconductors.[5,6] At low temperatures the MQT determines the escape rate of the switching from the superconducting state to a resistive one. Although the MQT of a single Josephson phase has been found a long time ago in low-T_c lumped Nb Josephson junctions,[7,8] MQT in layered high-T_c superconductors has shown many novel features. A most unexpected result is that the MQT escape rate Γ_{MQT} in high-T_c superconductors is *four orders of magnitude* larger than the MQT escape rate for a lumped Josephson junction having the same parameters.[6] Moreover, the crossover temperature T^* from the thermal fluctuation regime to the MQT regime increases in respect to a lumped Josephson junction. In these experiments it was also found that the escape rate Γ_T in the thermal fluctuation regime did not differ from the escape rate of a single Josephson junction.

Layered high-T_c superconductors can be modeled as a stack (a series array) of intrinsic Josephson junctions.[9] A modern fabrication technique allows one to prepare single crystals of layered high-T_c superconductors with an extremely homogeneous distribution of critical currents of intrinsic Josephson junctions, and a low level of dissipation.[5,6] Since in the model of independent Josephson junctions the escape rate Γ is just proportional to N, an enhancement of the MQT observed in layered high-T_c superconductors stems from an interaction between intrinsic Josephson junctions. All experimental observations receive a natural explanation in a simple model[10] of Josephson junctions series array with an intrinsic charge interaction between nearest-neighbor Josephson junctions.[11,12] Moreover, such a model allows quantitative comparison with experimental results, and a good agreement has been found as the Debye screening length is of the order of superconducting layer thickness,[12] i.e., a rather large charge interaction between nearest-neighbor Josephson junctions has to be assumed.

However, the authors of Ref. 6 proposed another model in order to explain a giant increase of the MQT escape rate. In this model intrinsic Josephson junctions are *globally coupled* through an electromagnetic environment. The authors of Ref. 6 argued that in globally coupled Josephson junction series arrays the MQT escape rate has to be proportional to N^2, where N is the number of Josephson junctions, but the quantitative analysis of MQT in globally coupled Josephson junction arrays has not been carried out.

An electromagnetic environment in experiments with tunnel junctions can be described by an external shunting impedance Z (see schematic in Fig. 1). An influence of a shunting impedance Z on the MQT in a lumped Josephson junction has been studied a long time ago in Refs. 13–15. It was shown that the presence of a small shunting impedance

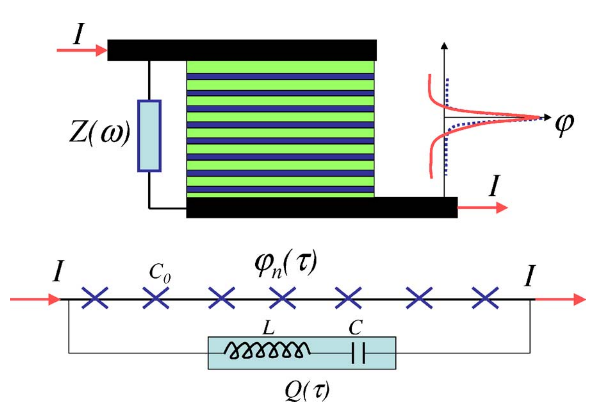

FIG. 1. (Color online) Schematic of a dc biased layered high-T_c superconductor and a series array of Josephson junctions. A strongly localized instanton (dashed line) and a charge instanton with long tails (solid line) are shown.

can lead to a strong *suppression* of the MQT in a lumped Josephson junction. Therefore, natural questions arise: what is a role of shunting impedance Z in the macroscopic quantum dynamics of large Josephson junction series arrays ($N \gg 1$) and what is a most appropriate model in order to explain a giant increase of the MQT escape rate in Josephson junction series arrays?

In order to answer these questions we carry out a quantitative analysis of the escape rate $\Gamma(I)$ in globally coupled Josephson junction series arrays. A global coupling is provided by an external shunting impedance Z (see Fig. 1). We obtain, and it is a main result of the paper, that if the electromagnetic environment strongly suppresses the MQT in a lumped Josephson junction, the role of an external shunting impedance is diminished in large Josephson junction arrays, and the standard quantum-mechanical behavior is recovered. Therefore, the globally coupled Josephson junction series arrays can show a giant increase of the MQT escape rate with a strong dependence on a number of junctions. Moreover, the MQT escape rate is tunable in a wide region by a simple change of Z. Such tuning of the MQT escape rate can be very promising for a modern field of quantum information processing.[16]

A Josephson junction series array is characterized by the set of Josephson phases $\varphi_n(\tau)$, where number n changes from 1 to N. Moreover, electrodynamics of a shunting impedance Z is described by a flowing charge $Q(\tau)$. We will consider a particular case as all resistive effects are small, i.e., the shunting impedance has only a reactive part, and it contains inductor L and capacitor C in series. In order to obtain the escape rate Γ we use an "instanton technique,"[14,17,18] and therefore, $\varphi_n(\tau)$ and $Q(\tau)$ are periodic functions of the imaginary time τ varying from 0 to $\hbar/(k_BT)$ (T is the temperature). The escape rate is determined by the action S as follows:

$$\Gamma \simeq \int Dq(\tau) D\varphi_n(\tau) \exp\left[-\frac{S\{q,\varphi_n\}}{\hbar}\right],$$

$$S = \int_0^{\hbar/k_BT} L(\tau) d\tau \qquad (1)$$

and the Lagrangian of a series array with the shunting impedance is written

$$L = \sum_n \frac{1}{2\omega_p^2}[\dot{\varphi}_n(\tau)]^2 + \frac{1}{2\omega_R^2}[\dot{q}(\tau)]^2 + \frac{1}{2}q(\tau)^2 + \sum_n U_n + i\frac{\alpha}{\omega_R}\sum_n q(\tau)\dot{\varphi}_n(\tau),$$

$$U_n(\varphi) = \cos\varphi_n(\tau) + j\varphi_n(\tau), \qquad (2)$$

where $j = I/I_c$ is the normalized external dc current, and I_c is the nominal value of the critical current of a single junction. Here, ω_p is the plasma frequency of a single Josephson junction in the absence of dc bias. The Lagrangian is expressed in units of E_J, where E_J is the Josephson energy of a single junction. The $q = Q/\sqrt{E_JC}$ is the normalized charge flowing through the impedance Z. The shunting impedance is characterized by the resonance frequency $\omega_R = 1/\sqrt{LC}$, where L and C are the impedance inductance and capacitance, accordingly. The coupling between the Josephson junction series array and the shunting impedance branch is described by dimensionless parameter $\alpha = \sqrt{2e/(\hbar I_c L)}$. In (2) we did not include the nondiagonal elements of the capacitance matrix, i.e., the intrinsic charge interaction between nearest-neighbor Josephson junctions that was a subject of Refs. 11 and 12, is neglected.

Integrating (1) over $q(\tau)$[19] we obtain the effective action S_{eff} that depends on the variables $\varphi_n(\tau)$ only

$$S_{eff}\{\varphi_n\} = \sum_n \int_0^{\hbar/k_BT} d\tau \frac{1}{2\omega_p^2}[\dot{\varphi}_n(\tau)]^2 + U_n + \frac{\alpha^2}{2}\int_0^{\hbar/k_BT}\int_0^{\hbar/k_BT} d\tau_1 d\tau_2 G_T(\tau_1-\tau_2) \times \left[\sum_n \dot{\varphi}_n(\tau_1)\right]\left[\sum_n \dot{\varphi}_n(\tau_2)\right], \qquad (3)$$

where the kernel $G_T(\tau)$ is determined as follows:

$$G_T(\tau) = \frac{k_BT}{\hbar}\sum_m \frac{e^{i\omega_m\tau}}{\omega_m^2 + \omega_R^2},$$

$$\omega_m = m(2\pi k_BT)/\hbar, \quad m = \pm 1, \pm 2 \cdots. \qquad (4)$$

Thus, the last term in Eq. (3) presents an effective global charge interaction, that is due to current fluctuations flowing through an external shunting impedance.

In the escape experiments $E_J \gg \hbar\omega_p$, and the switching to a resistive state occurs as the dc current I is close to I_c, and therefore $(j-1) \ll 1$. In this case the potential $U_n(\varphi)$ is written

$$U_n(\varphi) = (1-j)\varphi_n(\tau) - \frac{\varphi_n^3(\tau)}{6}. \qquad (5)$$

The escape rate is determined by the particular solution $\varphi_n(\tau)$ providing the extremum of effective action (3). At high temperatures such a solution is determined by extremum points of the potential U_n, and it is written

$$\varphi_n^T = 2\sqrt{2(1-j)}\delta_{nl} - \sqrt{2(1-j)}.$$

Here, l is a junction number where the fluctuation occurs. Since this solution does not depend on the time τ, we can immediately conclude that the last term in (3) does not give contribution to the escape rate exponent $\Gamma_T \simeq \exp(-S\{\varphi_n^T\}/\hbar)$. Note here that an absence of the dependence of the escape rate exponent in the thermal fluctuation regime on a number of junctions N is a generic property of Josephson junctions series arrays with a charge interaction.[12]

However, the crossover temperature from the thermal fluctuation regime to the MQT can be strongly enhanced by such a global coupling. Indeed, using the method elaborated in[14,17,18] we obtain that at high temperatures the optimal fluctuation $\varphi_n(\tau)$ around an extremum point has a form:

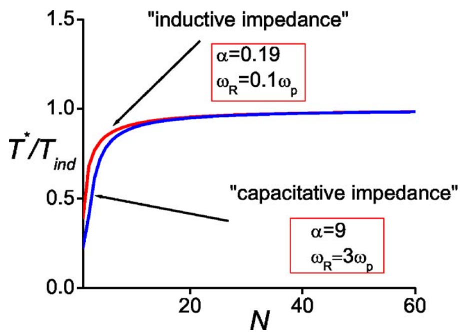

FIG. 2. (Color online) Typical dependencies of the crossover temperature $T^*(N)$ on a number of junctions N. Both cases of inductive (upper curve) and capacitative (lower curve) impedance for particular sets of parameters are shown.

$$\varphi_n(\tau) = e^{2\pi i k_B T \tau/\hbar}\phi_n, \tag{6}$$

where the eigenfunctions ϕ_n are the solution of the nonlocal and inhomogeneous equation:

$$\xi^2\phi_n + \frac{2\alpha^2\xi^2\omega_p^2}{\xi^2+\omega_R^2}\sum_n \phi_n - 2\omega_0^2\delta_{ln}\phi_n = (\lambda - \omega_0^2)\phi_n,$$

$$\xi = \frac{2\pi k_B T}{\hbar}. \tag{7}$$

Here, λ are the eigenvalues of the Eq. (7), $\omega_0 = \omega_p[2(1-j)]^{1/4}$ is the dc bias dependent frequency of oscillations on the bottom of potential well, $U_n(\varphi)$. The crossover temperature T^* is determined by the condition that there is the eigenvalue $\lambda=0$.[14,18] In a global coupling case [Eq. (7)] the crossover temperature is obtained as a solution of the particular transcendent equation:

$$\xi^{*4} + \frac{2N\alpha^2\omega_p^2\xi^{*2}}{\xi^{*2}+\omega_R^2}[\xi^{*2} - (1-2/N)\omega_0^2] = \omega_0^4, \tag{8}$$

where $\xi^* = \frac{2\pi k_B T^*}{\hbar}$. Thus, one can see that the crossover temperature T^* is strongly suppressed for short arrays ($N \simeq 1$) for both cases, namely, "inductive" ($\omega_R \ll \omega_0$, and $\alpha \simeq 1$) or "capacitive" ($\omega_R \gg \omega_0$, and $\alpha \gg 1$) types of an external impedance. However, T^* recovers to the value $T^* = \hbar\omega_0/(2\pi k_B)$ for long arrays ($N \gg 1$). Typical dependencies of $T^*(N)$ are shown in Fig. 2.

Now we turn to the MQT regime, where the extremum point of the action $S_{eff}\{\varphi_n\}$ is the "tau-dependent" instanton (bounce) solution. At zero temperature and in the presence of an external impedance Z, a spatial-temporal instanton solution satisfies the equation:

$$\frac{1}{\omega_p^2}\ddot{\varphi}_n(\tau) + \alpha^2\sum_m \int_0^\infty d\tau_1 G_0(\tau-\tau_1)\ddot{\varphi}_m - \frac{dU_n}{d\varphi} = 0, \tag{9}$$

where $G_0(\tau) = \frac{1}{2\omega_R}\exp(-\omega_R|\tau|)$. The solution of Eq. (9) has a following form: a large bounce solution localized on a particular junction l, $\varphi_l(\tau) = f(\tau)$, and a small spatial-temporal tail solution distributed over a whole array (see schematic in Fig. 1, solid line). The Fourier transform of the instanton tail is obtained as follows:

$$\sum_{n\neq l}\varphi_n(\omega) = -\frac{\alpha^2(N-1)\omega_p^2\omega^2 g_0(\omega)}{\omega^2+\omega_0^2+\alpha^2(N-1)\omega_p^2\omega^2 g_0(\omega)} f(\omega), \tag{10}$$

where $g_0(\omega)$ and $f(\omega)$ are the Fourier-transform of $G_0(\tau)$ and $f(\tau)$, respectively. The bounce solution $f(\tau)$ localized on the junction l is determined self-consistently from the equation:

$$\frac{1}{\omega_p^2}\ddot{f} + \alpha^2\int_0^\infty d\tau_1 G_1(\tau-\tau_1)\ddot{f}(\tau_1) - (1-j) + \frac{f^2}{2} = 0,$$

$$G_1(\tau) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{(\omega^2+\omega_0^2)e^{i\omega\tau}}{\omega^2+\omega_0^2+\alpha^2(N-1)\omega_p^2\omega^2 g_0(\omega)}. \tag{11}$$

In the absence of a global coupling the instanton solution is strongly localized on a particular junction (see schematic in Fig. 1, dashed line), i.e., Ref. 14,

$$\varphi_n(\tau) = f_0(\tau)\delta_{nl} = \frac{3\sqrt{2(1-j)}}{\cosh^2(\omega_0\tau/2)}\delta_{nl}. \tag{12}$$

Substituting (10) in the expression (3) for the effective action S_{eff} and using a perturbative approach (similarly to Refs. 13 and 15), i.e., $f(\tau) \simeq f_0(\tau)$, we obtain the MQT escape rate (in physical units) as written:

$$\Gamma_{MQT} \simeq \Gamma_0 \exp\left[-\frac{72E_J}{15\hbar\omega_p}2^{1/4}(1-j)^{5/4}(1+\chi)\right], \tag{13}$$

where

$$\chi = \frac{30\pi\alpha^2\omega_p^2}{\omega_0^2}\int_0^\infty dx \frac{x^4(x^2+1)g_0(x)\sinh^{-2}(\pi x)}{x^2+1+\frac{\omega_p^2\alpha^2}{\omega_0^2}(N-1)x^2 g_0(x)}, \tag{14}$$

where

$$g_0(x) = 1/[x^2 + (\omega_R/\omega_0)^2]. \tag{15}$$

Here, the parameter Γ_0 is just proportional to N. A parameter χ having a positive value, characterizes a suppression of the MQT due to the presence of the charge interaction between Josephson junctions of the array and an external shunting impedance. For short Josephson junction array ($N \simeq 1$), such a MQT suppression can be rather large for moderate values of α. However, as we turn to large Josephson junction arrays ($N \gg 1$) a standard MQT behavior is recovered. Quantitatively an enhancement of MQT depends strongly on parameters α and ω_R. The expression (14) can be simplified in two limits: $\omega_R \gg \omega_0$ (capacitative impedance) and $\omega_R \ll \omega_0$ (inductive impedance) as follows:

$$\chi = 5\alpha^2\omega_p^2\begin{cases}\dfrac{1}{\omega_R^2+\alpha^2\omega_p^2(N-1)} & \omega_R \gg \omega_0;\\[2ex] \dfrac{1}{\omega_0^2+\alpha^2\omega_p^2(N-1)} & \omega_R \ll \omega_0.\end{cases} \tag{16}$$

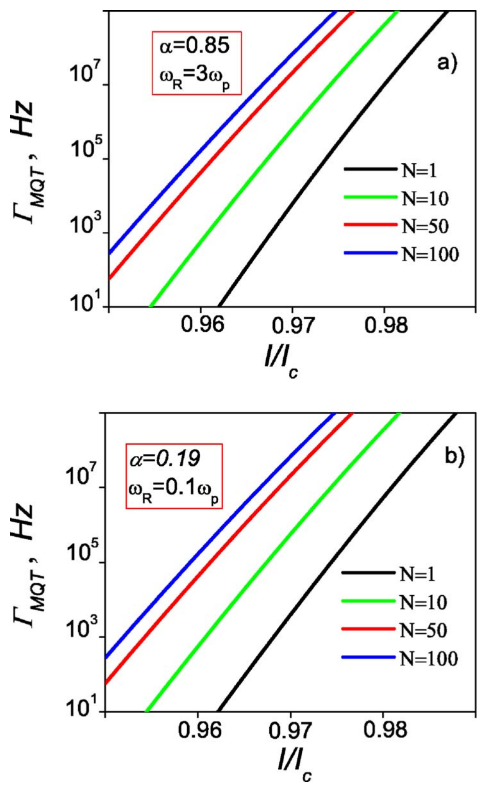

FIG. 3. (Color online) The dependence of the MQT escape rate Γ_{MQT} on the dc bias current I for various values of $N=1,50,100$. Both cases of capacitative (a) and inductive (b) impedance for particular sets of parameters are shown.

Typical dependencies of the MQT escape rate Γ_{MQT} on the dc bias current I for various values of N are presented in Fig. 3. One can see a giant enhancement of the MQT escape rate as we turn from short to long Josephson junction arrays. This enhancement results from a decrease of the slope of the bias current dependence escape rate. Comparing our theoretical predictions with the experimental curves published in Ref. 6 (see Fig. 5 in Ref. 6) we find a good agreement for both the crossover temperature T^* and the dependence of $\Gamma_{MQT}(I)$ for the *inductive* type of a shunting impedance. Therefore, in order to choose between two models, i.e., a nearest-neighbor intrinsic charge interaction or external global charge coupling, one needs additional independent measurements of Debye screening length[10] or to tune the MQT by variation of Z.

In conclusion we have shown that the dissipative (decoherence) effects can be strongly suppressed in long ($N \gg 1$) Josephson junction series arrays with a global charge interaction. Both the dissipation and global charge interaction can be introduced through an external shunting impedance. This effect manifests itself as a giant enhancement of the MQT escape rate for the switching from the superconductive state to a resistive one (see Fig. 3). A giant MQT enhancement is explained through an excitation of spatial-temporal charge instanton distributed over a whole array.

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