Casimir-energy-induced confinement and deconfinement of spinons in a two-dimensional anisotropic nonlinear σ model

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We consider a nonusual two-dimensional anisotropic nonlinear σ model. This model supports topological excitations (Skyrmions), which the configurations, at the classical level, resemble noninteracting vortex and antivortex defects. These vortices are spin-1/2 pseudoparticles (spinons). Quantization of these defects breaks the conformal invariance of the model, leading to an effective potential of interaction between the spinons that form the Skyrmion. The nature of this interaction depends on the anisotropy parameter λ and is attractive for $-1<\lambda<0$ and repulsive for $\lambda>0$. For the first case, the interaction grows considerably as the spinon separation is increased and leads to confinement. On the other hand, the case $\lambda>0$ may lead to deconfinement of spinons.

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I. INTRODUCTION

The theory of strongly correlated systems is an area of theoretical physics where the parallels between high-energy and condensed-matter physics are specially strong. One outstanding problem in high-energy physics is the problem of quark confinement (the fact that individual quarks are nonobservable but always exist only inside of other particles). It is a characteristic of a non-Abelian theory, which predicts that the bare charge does not induce a shielding charge but an "antishielding" one. In this paper we study a condensed-matter system with a similar phenomenon (i.e., confinement of spin-1/2 pseudoparticles). Nevertheless, as the range of the determined parameter in the model is changed, there is a deconfinement. For quantum spin liquid states of a certain class of spin-1/2 two-dimensional (2D) antiferromagnetic spin models, it has been conjectured that neutral spin-1/2 fermionic excitations are spinons.[1] The deconfinement of these structures is a characteristic of spin liquids. Recent experimental results[2] make the search for any signatures of spinons at low and high energies very meaningful and urgent. An interesting actual problem is to find a rigorous example of a 2D quantum spin liquid with spin 1/2. Following this line but not so rigorously, we consider the spectrum of a nonusual 2D model (referred to as anisotropic nonlinear σ model)[3,4] that keeps some similarities with antiferromagnets. This model has an additional "anisotropic" term added to the standard nonlinear σ model that does not break the scale invariance. Then, it also supports topological excitations, which, at the classical level, do not interact. Topological defects[5,6] are present in many condensed-matter materials, including superconductors, superfluids, magnetic systems, etc. These objects are extremely important because they cannot be made to disappear by any continuous deformation of the order parameter and therefore they are said to be topologically stable. The prototype examples of topological defects in the spin field are vortices and solitons (Skyrmions). The role of these structures in 2D quantum spin systems is not completely known. Here we argue that excitations with vortex-pair configurations and unit topological charge present in the anisotropic nonlinear σ model are two noninteracting spinons. Quantization of these defects breaks the scale invariance of the model, making the energy of the two-spinon configuration depend on the distance between them. Hence, our calculations show that the interplay between anisotropy and quantum fluctuations induces an effective interaction potential between spinons. The nature of such an interaction depends on the values of the anisotropy parameter λ, and it is attractive in the range $-1<\lambda<0$, while it is repulsive in the range $\lambda>0$. In the former case, the effective attractive potential between spinons increases linearly for large distances of separation, like quark confinement, in contrast to previous results for the isotropic case $\lambda=0$.[7] For the repulsive case our results indicate that there may be a critical Skyrmion size above which it is energetically favorable to make two spinons.

In the path-integration formulation of quantum mechanics we start with a quantum Hamiltonian but in the end we finish with a classical action $A=\int L dt$. This is very convenient for a nonperturbative approach where topological excitations are present because we have to treat classical nonlinear equations.[8] At low temperatures and long wavelengths, it is well known that the (2+1)-dimensional short-range isotropic Heisenberg antiferromagnets $H=J\Sigma_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j$ (where the sum $\langle i,j\rangle$ is over nearest-neighbor pairs, $J>0$ is the exchange constant, and $\vec{S}_i$ denotes the spin operator at site i) are described by the famous "isotropic" $O(3)$ nonlinear σ model augmented by Berry phases $Z=\int D\vec{n}\,\delta(\vec{n}^2-1)\exp(-iA)$, with

$$A=A_B+\frac{1}{2}\int_{t_i}^{t_f}dt\int d^2\vec{x}\rho_s[(\partial_\nu\vec{n})^2-(\partial_0\vec{n})^2], \qquad (1)$$

where the unit vector $\vec{n}$ denotes the Néel sublattice magnetization, $\rho_s\equiv JS^2/\hbar$ is the spin stiffness, $\nu=1,2$, $\partial_0=(1/c)\partial/\partial t$, and $c=2\sqrt{2}JSa/\hbar$ is the velocity of the longwavelength antiferromagnetic spin wave. Besides, A_B is the Berry phase given by

$$A_B = \frac{S}{\hbar}\sum_i \varepsilon_i \int_{t_i}^{t_f} dt \int_0^1 du \vec{n}_i \cdot \left(\frac{\partial \vec{n}_i}{\partial u} \times \frac{\partial \vec{n}_i}{\partial t}\right), \tag{2}$$

where, for a lattice divided in sublattices A and B of even and odd sites, respectively, we define $\varepsilon_i=1$ in A and $\varepsilon_i=-1$ in B (bipartite lattice). In addition, u is a geometrical invariant in such a way that the Berry-phase terms are sums of the areas swept by the vectors $\vec{n}_i(t,u)$ on the surface of a unit sphere as u evolves it from (0, 0, 1) to $\vec{n}_i(t)$ and then $\vec{n}_i(t)$ evolves in time from its direction at t_i to the direction at t_f. However, in two spatial dimensions these Berry-phase terms vanish[9–11] if the Néel order parameter is continuous as the contributions from each sublattice cancel. Indeed, the Berry-phase terms are important only in the disordered phase.[12] These results can be extended for the anisotropic model defined by the following action:[3]

$$A_\lambda = A_{B,\lambda} + \frac{1}{2}\rho_s \int_{t_i}^{t_f} dt \int d^2x[(\partial_\nu \vec{n})^2 + \lambda(\partial_\nu n_3)^2 - (\partial_0 \vec{n})^2], \tag{3}$$

where the total range of the anisotropy parameter is $\lambda \geqslant -1$ and $A_{B,\lambda}$ has an expression similar to Eq. (2), as we will see in next section, after a better analysis of the system. Note that the case $\lambda=0$ reproduces the usual nonlinear σ model, which supports the well-known Belavin-Polyakov (BP) multi-Skyrmion solutions[13] with classical energy $E_{BP} = 4\pi q\hbar\rho_s$, where $q=\pm1,\pm2,\ldots$ is the winding number. In condensed-matter physics, the action (3) could be viewed as an effective theory representing an approximation for a possible 2D anisotropic antiferromagnetic material.

As is known, there are fundamental differences between Abelian and non-Abelian models. Anisotropic easy-plane magnets, whose spins have three components, but they prefer to lie on the plane, have an $O(2)$ symmetry group and are Abelian, while all isotropic $O(N)$ models with $N>2$ are non-Abelian. Although the model considered here exhibits a type of anisotropy, it is non-Abelian. Really, such a model is not the complete continuum limit of the anisotropic easy-plane magnets since it is missing a term proportional to n_3^2 in Eq. (3) (such a term also breaks the scale invariance of the system). However, depending on the values of λ and on the excitations considered, the actual model may present some characteristics similar to $O(2)$ models. It is well known that there cannot exist a transition to a phase with long-range order in either of the $N=2$ or $N>2$ scenarios in low dimensions as stated by the Mermin-Wagner theorem.[14] In fact, in two dimensions, a continuous symmetry of the $O(N)$ type cannot be broken (at any finite temperature). However, in a 2D theory, topological defects of dimension d can exist if the $(1-d)$th homotopy group π_{1-d} of the order parameter is nontrivial. For $O(N)$ models, the only nontrivial group is $\pi_1(S^1)$ which is isomorphic to the set of integers under addition. This is the condition that gives rise to vortices (point defects) with integer charge[15] in the $N=2$ case or in the $N=3$ case with an easy-plane anisotropy. As is well known, the binding of these vortices at low temperature is the mechanism giving rise to the Berezinskii-Kosterlitz-Thouless (BKT) phase transition.[16] For isotropic cases and $N>2$, conditions are not supportive of the existence of these topological defects and the widely held belief is that there is no phase transition. Perturbation theory predicts that the $N>2$ isotropic models are asymptotically free. There is, however, no rigorous proof to this effect. This belief has been questioned, and some works[17,18] have given numerical evidence for the existence of phase transitions of the BKT type in these models as well as heuristic arguments of why such a transition should occur and also a rigorous proof that this would be incompatible with asymptotic freedom. On the other hand, some Monte Carlo and perturbative calculations for the $N=3$ and $N=8$ isotropic models[19] have shown that the BKT transition does not exist and instead are in agreement with the asymptotic freedom scenario. Nonetheless, the controversy has not entirely gone away.[20] Perhaps, with an additional small perturbation in the traditional nonlinear σ model $O(3)$ such an anisotropy may be also useful to shed some light to this problem. Here, we will show that, depending on the parameter λ, the particular anisotropic system defined by Eq. (3) may display the two behaviors described above. Really, if $-1<\lambda<0$, vortices, which are spinons in the model considered, must be confined and therefore are asymptotically free. It means that the nonlinear excitations of this anisotropic model are not really vortices like the ones defined in the usual XY model. On the other hand, for the other range of anisotropy ($\lambda>0$), these vortices may exist practically free. Then, the system keeps some similarities with the XY model. The isotropic nonlinear σ model should separate these two different phases. Such different characteristics of the two phases may be associated with quantum fluctuations about the different structures of the out-of-plane configurations of the Skyrmions in the two ranges of λ.

II. FIELD EQUATIONS AND STATIC SOLUTIONS

Using the constraint $\Sigma_{\alpha=1}^3 n_\alpha^2=1$, the static part of Lagrangian density in Eq. (3) can be rewritten as

$$\ell = \frac{1}{2}\rho_s\left\{\sum_{i=1}^{2}(\partial_\nu n_i)(\partial_\nu n_i) + (1+\lambda)\frac{\sum_{i=1}^{2}\sum_{j=1}^{2} n_i n_j(\partial_\nu n_i)(\partial_\nu n_j)}{1-\sum_{i=1}^{2} n_i n_i}\right\}. \tag{4}$$

Note that the existence of the "special" anisotropy defined in Eq. (3) does not change the $O(3)$ symmetry of the model as it is easy to see that the ground state of this model is any uniform configuration of the vector field $\vec{n}$. In fact, as we have already remarked in the Introduction, there is not any term breaking the scale invariance of the present system as required in more realistic continuum limits of the anisotropic Heisenberg models. The topological nature of the 2D anisotropic nonlinear σ model given by Eq. (4) was studied by Watanabe and Otsu (WO).[3] They have demonstrated that this model also supports nontrivial static metastable states producing local energy minima. Here, these topological defects will be referred to as WO Skyrmions. Equations (3) and (4) show us that solutions with nonzero but finite energy (Skyrmions) must satisfy the condition $\lim_{\vec{r}\to\infty}\vec{n}\to\text{const}$. Hence, like the usual isotropic $O(3)$ model, one must choose a particular direction for the vector $\vec{n}$ as $\vec{r}$ goes to infinity. To get these configurations it is convenient first to rewrite the field $\vec{n}=(n_1,n_2,n_3)$ as $\vec{v}=(n_1,n_2,(1+\lambda)^{1/2}n_3)$, obtaining the surface Σ^2 of a three-dimensional spheroid $(v_1^2+v_2^2)+v_3^2/b^2=1$, with $b^2=(1+\lambda)$. Further, $\vec{v}$ can be parametrized by the polar $\theta(\vec{r})$ and azimuthal $\phi(\vec{r})$ angles, $\vec{v}=(\sin\theta\cos\phi,\sin\theta\sin\phi,b\cos\theta)$. With the new vector field $\vec{v}$, the anisotropic nonlinear σ model has a form similar to the isotropic one and the action (3) is rewritten as $A_\lambda=A_{B,\lambda}+\frac{1}{2}\rho_s\int_{t_i}^{t_f}dt\int d^2x[(\partial_\nu\vec{v})^2-(\partial_0\vec{v})^2]$, where $A_{B,\lambda}$ is given by an expression like Eq. (2) with $\vec{n}$ substituted by $\vec{v}$. Now the Berry phase terms are sums of the areas swept by the vectors $\vec{v}_i(t,u)$ on the surface of the spheroid Σ^2 as u evolves it from $(0,0,b)$ to $\vec{v}_i(t)$, and then $\vec{v}_i(t)$ evolves in time from its direction at t_i to the direction at t_f.

Like the isotropic case, another useful way of describing the model (3) is through the stereographic projection $w=(v_1+iv_2)/(1+v_3/b)$.[13] Then we can write $w=w_1+iw_2$, where w_1 and w_2 describe the complex number plane with the point at infinity added to. In terms of these variables, the Lagrangian density becomes[4]

$$\ell=\frac{1}{2}\rho_s\sum_{i,j}g_{ij}(w)(\partial_\nu w_i)(\partial_\nu w_j)\quad(i,j=1,2),\tag{5}$$

where the metric g_{ij} is given by

$$g_{ij}=\begin{pmatrix}\dfrac{4[1+4\lambda w_1^2(1+|w|^2)^{-2}]}{(1+|w|^2)^2} & \dfrac{16\lambda w_1w_2}{(1+|w|^2)^4}\\ \dfrac{16\lambda w_1w_2}{(1+|w|^2)^4} & \dfrac{4[1+4\lambda w_2^2(1+|w|^2)^{-2}]}{(1+|w|^2)^2}\end{pmatrix}.\tag{6}$$

Thus Eq. (4) is rewritten as

$$\ell=\frac{1}{2}\rho_s\left\{\frac{4|\partial_\nu w|^2}{(1+|w|^2)^2}+\frac{4\lambda[w(\partial_\nu\bar{w})+\bar{w}(\partial_\nu w)]^2}{(1+|w|^2)^4}\right\}\tag{7}$$

and, therefore,

$$\begin{aligned}L={}&2\rho_s\int\frac{|\partial_0 w|^2}{(1+|w|^2)^2}d^2x-8\rho_s\int\frac{|\partial_{\bar{z}}w)|^2}{(1+|w|^2)^2}\\&\times\left[1+\frac{2\lambda|w|^2}{(1+|w|^2)^2}\right]d^2x\\&-8\lambda\rho_s\int\frac{[w^2(\partial_z\bar{w})(\partial_{\bar{z}}\bar{w})+\bar{w}^2(\partial_z w)(\partial_{\bar{z}}w)]}{(1+|w|^2)^4}d^2x\\&-QA(\lambda)\hbar\rho_s,\end{aligned}\tag{8}$$

where $\partial_z=(\partial_x-i\partial_y)/2$, $\partial_{\bar{z}}=(\partial_x+i\partial_y)/2$, $z=x+iy$, and $\bar{z}=x-iy$. Besides, Q is the winding number (for the anisotropic model) given by

$$Q=\frac{4}{A(\lambda)}\int\frac{(\partial_z w)(\partial_{\bar{z}}\bar{w})-(\partial_{\bar{z}}w)(\partial_z\bar{w})}{(1+|w|^2)^2}\left[1+\frac{2\lambda|w|^2}{(1+|w|^2)^2}\right]d^2x\tag{9}$$

and measures the number of times the spheroid Σ^2 of area $A(\lambda)$ is wrapped in the mapping spheroid plane.[3]

Static, finite-energy Skyrmion solutions to the field equations can be found by imposing the condition $\partial_{\bar{z}}w=0$. We will consider only the configurations with the lowest energy. Due to the $O(3)$ symmetry, even in this anisotropic model, one has to consider a more general class of these solutions; e.g., the lowest-energy Skyrmions are described by $w_{c,1}=\alpha(z-\xi_1)/(z-\xi_2)$, where α, ξ_i, and ξ_j are complex parameters. These solutions can easily be checked to have winding number $Q=1$ and energy $E_{c,1}=4\pi(3+\lambda)\hbar\rho_s/3$. Depending on the values of the complex parameters one can obtain, for any λ, Skyrmion configurations with vector $\vec{n}$ (or equivalently $\vec{v}$) at infinity pointing either along the z direction or along a particular direction in the XY plane. For example, if one lets ξ_1 go to infinity and α go to zero simultaneously, the solution describes a Skyrmion whose constituents are far apart and becomes proportional to $1/(z-\xi_2)$. It is equivalent to say that the configuration has only one center and the boundary conditions at $\vec{r}\to\infty$ imply $n_3\to1$. It is also easy to get an analogous configuration, but with $n_3\to-1$ at infinity, by making $\alpha\to\infty$ simultaneously with $\xi_2\to\infty$, leading to another "one-center" solution like $(z-\xi_1)$. These two solutions contain one locally ordered region of which one core spin is antiparallel to the direction of those on the boundary. On the other hand, if $\alpha=1$ and ξ_1 and ξ_2 are finite numbers, one has the simplest Skyrmions with two centers (see below) whose spins obey $n_1\to\pm1$ (or $n_2\to\pm1$) at $\vec{r}\to\infty$. Thus, changing the complex parameter α, we can get physically different configurations with the same classical energy, but having an arbitrary direction of the vector $\vec{n}$ (or $\vec{v}$) at infinity. Nevertheless, with the anisotropy term in mind, Watanabe and Otsu required the following boundary conditions at infinity $|\vec{r}|\to\infty$: $\vec{v}\to(\pm1,0,0)$ for "XY-like magnets" and $\vec{v}\to(0,0,\pm b)$ for "Ising-like magnets." However, the simple fact that $b<1$ $(b>1)$ does not ensure that only Skyrmions with spins confined to the XY plane (spins pointing along the z axis) at infinity exist in the system. Indeed, the $O(3)$ symmetry guarantees the two types of excitations for any value of λ, even for the isotropic case $(\lambda=0)$. Therefore, independently of λ, one has to choose the order parameter in a particular direction as it will always point in an arbitrary, but fixed direction. We can summarize the above picture as follows: it does not matter the direction one chooses; the excitations with the same winding number will have the same energy (for the same λ), at least at the classical level. But the choice with XY-like boundary conditions for the Skyrmion implies an excitation containing two centers (vortices) separated by a finite distance while the choice with Ising-like boundary conditions implies only one center. Arbitrary values of α may lead to boundary conditions with spins pointing along intermediary directions (between the plane and the z axis). These conclusions are very general and do not depend on the anisotropy. In particular, the simplest $|Q|=1$ Skyrmion with two centers is expressed as $(z-\xi_1)/(z-\xi_2)$. Using $\xi_1 = -\xi_2 = \xi$, this solution is centralized at the origin. Here ξ is a complex parameter and refers to the size of the Skyrmion (the magnitude and phase of the complex number ξ give the size and rotational orientation of the defect, respectively).

The above results show that the classical energy of a static WO-Skyrmion configuration depends only on the anisotropy λ via the area $A(\lambda)$ $[E_{c,1}=A(\lambda)\hbar\rho_s]$. Note particularly that, for the case $\alpha=1$, one has $w_{c,1}(z)\to\infty$ as $z\to\xi_2=-\xi$ while it tends to zero as $z\to\xi_1=\xi$. As already mentioned, it means that this WO Skyrmion has two centers separated by distance $2|\xi|$. In fact, Watanabe and Otsu showed that, irrespectively of the signs of Q, a vortex is found to appear rotating counterclockwise around the point $(0,-|\xi|)$ and to accompany an antivortex at $(0,|\xi|)$. The core of these vortices contains out-of-plane spins which are directed upward above the plane at point $(0,-|\xi|)$ and downward at $(0,|\xi|)$, while this is done inversely for $Q=-1$ (anti-Skyrmions, obtained by interchange $z\leftrightarrow\bar{z}$). This Skyrmion configuration looks like a noninteracting "out-of-plane" vortex-antivortex pair with a vortex centered at $(0,-|\xi|)$ and antivortex at $(0,|\xi|)$, which means that their centers are separated by the distance $R = 2|\xi|$. However, these structures must not be the standard vortex modes predicted by Kosterlitz and Thouless[16] since their energy $[E_{c,1}=A(\lambda)\hbar\rho_s]$ does not depend on the Skyrmion size (or vortices separation) R due to the scale invariance of the model. Of course, it is very different from the situation of the usual XY model (or easy-plane Heisenberg model), in which a vortex and an antivortex interact through a logarithmical potential $(\ln R)$, leading to the BKT transition. Hence, whenever the structures considered here have the same in-plane spin configuration of a vortex pair, the roles they play in the system properties are not completely understood. These conclusions apply equally to other finite values of α that permit two-center configurations. In the next sections we explore some possible behavior of these Skyrmions by considering their spins and how quantum fluctuations may affect their energy for determined anisotropy λ. Note that, classically, for the same charge $|Q|=1$, Skyrmions of systems with anisotropy in the range $-1<\lambda<0$ have smaller energy than their counterparts in the $\lambda>0$ case.

III. SPINONS

Now we discuss the possible spin of the neutral topological pseudoparticles containing two centers. More precisely, we discuss the spin of the constituent particles (vortices) of these excitations. Without loss of generality we employ configurations with $\alpha=1$ to exemplify the mains points. An interesting argument showing that a Skyrmion contains a pair of spinons was proposed in Ref. 7. Using the standard (isotropic) nonlinear σ model, Baskaran[7] has found an interesting phenomenon which was referred to as a "chain anomaly." Really, in a Berry-phase analysis the spin of the constituent particle (vortex) of a BP Skyrmion appears as a chain anomaly, which is not present in the Haldane's argument[9] for calculation of the Berry phase. Haldane suggested that if the field $\vec{n}(\vec{r},t)$ is continuous and nonsingular, the chain Berry phases should be all identical and as a result the staggered sum should be identically zero, for an even number of chains. However, Baskaran[7] has shown that a singular chain containing the coordinates of the center of the vortices is an exception to this and it contributes a Berry phase π and $-\pi$ to the two vortices. It is based on a global rotation of determined chain parallel to the y axis around the x axis (direction of sublattice magnetization at infinity). A global rotation simply amounts to giving the temporal dependence $(R/2)\exp(i2\pi t/T)$ and $-(R/2)\exp(i2\pi t/T)$ to the constituent particles of the Skyrmion (from $t=0$ to $t=T$). Such a phenomenon can also be applied here, which means that all, except the y-axis chain (which passes by the line joining the origin and vortex-antivortex centers), fail to wrap the Σ^2 spheroid completely. They all leave a hole, making the winding number (Berry phase) identically zero. The axis containing the vortex coordinates makes the winding number equal to 1. This can be easily seen by using the situation with anisotropy $\lambda\to-1$ for "XY-like magnets," which admits an exact solution for the spatial configuration of the Skyrmion representing a vortex-antivortex pair,[3]—i.e., $m_{q,1}(\vec{r}) = \cos\theta_{q,1}(\vec{r}) = (\sqrt{2R}y/\{|y|[x^2+(|y|+R/2)^2]\}^{1/2})$, $\phi_{q,1}(\vec{r}) = \arctan[(y+R/2)/x] - \arctan[(y-R/2)/x]$. In these expressions, $m_{q,1}(\vec{r})$ gives the structure of the out-of-plane spins (the projection of the spins along the z direction) in the Skyrmion while $\phi_{q,1}(\vec{r})$ is the angle that each spin makes with the x axis in the XY plane. Such a configuration represents a vortex centered at position $(0,-R/2)$ and an antivortex centered at $(0,R/2)$. Particularly, for this case, the projection $P_{y\to x}$ of all spins of the y axis into the x direction (sublattice magnetization direction) can be written as

$$P_{y\to x} = \sqrt{(1-m_{q,1}^2)}\cos\phi_{q,1} = \frac{|y|-R/2}{|y|+R/2}, \tag{10}$$

and therefore, a global rotation (GR) of all spins of the y axis about the direction of sublattice magnetization at infinity leads to a simple spin structure $m_{GR}(\lambda=-1)=(r-R/2)/(r+R/2)$, $\Phi_{GR}(\lambda=-1)=\arctan(y/x)$, which is nothing but a cylindrically symmetric spin configuration in the "YZ plane." Now, considering the "YZ plane," m_{GR} is simply the projection of the spins in the out-of-plane direction (x direction in this case) and Φ_{GR} is the angle that the in-plane components make with an axis in this plane. Note that the forms of $m_{GR}(\lambda=-1)$, $\Phi_{GR}(\lambda=-1)$ also imply a configuration with the spin vectors pointing in all different directions. Thus, it is easy to see that this global rotation leads to $|Q|=1$ and consequently the Berry phase does not vanish. Just for the effect of comparison, a Skyrmion with a vortex-pair configuration in the isotropic model (BV Skyrmion with boundary condition $n_1=1$ at $\vec{r}\to\infty$) leads to a GR with configuration (in the YZ plane) (Ref. 7): $m_{GR}(\lambda=0)=[r^2-(R/2)^2]/[r^2+(R/2)^2]$ and $\Phi_{GR}(\lambda=0)=\arctan(y/x)$. This is exactly the usual form of the BV Skyrmion when the boundary condition implies in the out-of-plane component equal to 1 at $\vec{r}\to\infty$. It is interesting to mention something about the main difference between $m_{GR}(\lambda\to-1)$ and $m_{GR}(\lambda=0)$: the first is related to the limiting situation of the mapping plane spheroid with $|Q| = 1$ while the second is related to the traditional mapping plane sphere $|Q|=|q|=1$. The expression for m_{GR} can be generalized for other values of λ based on a simple interpolation of the exactly known cases $\lambda \to -1$ and $\lambda = 0$. Then, for a general λ in the range $[-1,0]$, the GR of all spins of the y axis about the x direction leads to $m_{GR}(\lambda) = [r^{(2+\lambda)} - (R/2)^{(2+\lambda)}]/[r^{(2+\lambda)} + (R/2)^{(2+\lambda)}]$, $\Phi_{GR} = \arctan(y/x)$ with $|Q| = 1$ (such configurations are not solutions of the motion equations for $\lambda \neq 0$ and it can be visualized only by rotating the slice about the x axis—i.e., the direction of sublattice magnetization at infinity). Of course, a similar behavior of $m_{GR}(\lambda)$ for $\lambda > 0$ is expected. The y-axis chain just manages to wrap the Σ^2 spheroid, contributing a phase π for one half and $-\pi$ for the other half of the chain. Following Ref. 7, one can identify these two phases, which arise predominantly from the region of the vortex and antivortex cores, with the spin-1/2 Berry phases of the vortex cores. Then the two vortices that constitute the WO Skyrmions considered here carry spin-1/2 projections of value 1/2 and −1/2 along the x axis. They are, therefore, noninteracting spinons. This analysis goes through for any spin S and one can get spin-S spinons or the $O(3)$ vortices.

IV. SPINON-MAGNON INTERACTIONS AND CASIMIR ENERGY

Several authors, with different intentions, have considered the BP Skyrmions interacting with small amplitude oscillations[7,21–25] and also calculated the quantum corrections to their energies[21,22,24,26] using several approaches. By using the second-order Born terms to calculate the phase-shift matrix of the scattered magnons, Rodriguez[21] has proposed that quantum corrections reduce the classical BP-Skyrmion energy as Skyrmion size increases. A fully quantized field theory developed for Skyrmions of the $O(3)$-symmetric CP^1-nonlinear σ model confirms this tendency.[26] However, applying other approach, Walliser and Holzwarth[22] found the contrary: the magnitude of the quantum corrections decreases with increasing BP-Skyrmion size, and therefore, quantum effects should increase the Skyrmion energy as R increases. Of course, these discrepancies must not be associated with the method but with the approximations used in each calculation. For instance, in Ref. 21, it was applied the second-order Born approximation since the first-order terms do not contribute to the quantum corrections in the isotropic case. However, to get such second-order Born terms, which have more complicated expressions, several other approximations were applied. In fact, the calculations indicating the main conclusion are valid only in the limit of very small Skyrmions[21] $R \to 0$. Regardless of it, Baskaram[7] has used the approach of Refs. 21 and 26 and found that, for the isotropic model (BP Skyrmions with a vortex-pair shape), quantum fluctuations give rise to a repulsive interaction between " $O(3)$ vortices," causing a deconfinament of spinons. Here, we will also apply the approach of Ref. 21. Nevertheless, for the anisotropic model, the first-order Born terms are the dominant contributions for the quantum corrections and, therefore, our calculations involve a smaller amount of approximations and can be applied for relatively large excitations. To do this, we have to examine the time-dependent equation for small disturbances $\eta(\vec{r},t)$, which propagate on the classical background $w_{c,1}(z)$ (WO Skyrmion). The ansatz $w = w_{c,1} + \eta$ is used, where the deviation from the classical minimum $\eta \ll 1$ represents the spin-wave mode. By minimizing the action associated with Lagrangian (8) to second order in η, one gets a Schrödinger-like equation

$$\nabla^2 \eta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\eta = U_1 \eta + U_2 \bar{\eta}, \tag{11}$$

where the potentials are given by

$$\begin{aligned} U_1 = {} & 8\partial_z \ln(1 + |w_{c,1}|^2)\Bigg[\partial_{\bar{z}} - \frac{\lambda}{(1+|w_{c,1}|^2)}\partial_{\bar{z}} + \frac{4\lambda |w_{c,1}|^2}{(1+|w_{c,1}|^2)^2}\partial_{\bar{z}} \\ & + \frac{4\lambda w_{c,1}(\partial_{\bar{z}}\bar{w}_{c,1})}{(1+|w_{c,1}|^2)^2}\Bigg] + \frac{8\lambda \bar{w}_{c,1}(\partial_z w_{c,1})}{(1+|w_{c,1}|^2)^2}\partial_{\bar{z}} \\ & - \frac{8\lambda(\partial_z w_{c,1})(\partial_{\bar{z}}\bar{w}_{c,1})}{(1+|w_{c,1}|^2)^2} - \frac{8\lambda w_{c,1}(\partial_{\bar{z}}\bar{w}_{c,1})}{(1+|w_{c,1}|^2)^2}\partial_{\bar{z}} \\ & - \frac{2\lambda |w_{c,1}|^2}{(1+|w_{c,1}|^2)^2}\nabla^2 \end{aligned} \tag{12}$$

and

$$\begin{aligned} U_2 = {} & \frac{16\lambda w_{c,1}^2}{(1+|w_{c,1}|^2)^2}[\partial_z \ln(1+|w_{c,1}|^2)\partial_{\bar{z}} + \partial_{\bar{z}}\ln(1+|w_{c,1}|^2)\partial_z] \\ & - \frac{8\lambda w_{c,1}(\partial_z w_{c,1})}{(1+|w_{c,1}|^2)^2}\partial_{\bar{z}} - \frac{2\lambda w_{c,1}^2}{(1+|w_{c,1}|^2)^2}\nabla^2. \end{aligned} \tag{13}$$

These two potential operators have finite range (of the order of R) since the Skyrmion configurations relax back to the original Néel order at $|\vec{r}| \to \infty$. Writing $\eta(\vec{r},t) = \psi(\vec{r})\exp(i\omega t)$, one obtains, in the limit $|\vec{r}| \to \infty$, two magnon solutions to Eq. (11) with frequencies $\omega_q = (1+\lambda)^{1/2} qc$. It means that there exist usual linearized excitations (magnons) about the order parameter with relativistic dispersion law that vanish at long wavelengths as dictated by Goldstone's theorem. These magnons are, of course, spin-1 particles. They have only two polarizations as they are transverse to the Néel order. Hence, the lowest-order effect of an inhomogeneous Skyrmion background $w_{c,1}$ is to produce elastic scattering centers for magnons. The asymptotic solutions to the scattering equation are then given by phase-shifted cylindrical magnons. In addition to the scattered states, there may exist additional bound-state solutions of Eq. (11). These solutions would correspond to internal oscillation modes in which the Skyrmion structure undergoes a harmonically varying shape change localized about the Skyrmion center. However, we do not get to show if such local modes exist in the present system. Since their contributions to the quantum energy of the BV Skyrmion in the isotropic model are neglected,[21,22] we also assume that the same happens to the WO excitations in the anisotropic case. Consequently, considering the Casimir energy with contributions coming only from the continuum states, the Skyrmion energy is given by[5,21] $E_q^{xy} = E_{c,1} - (\hbar/\pi)\int_0^{1/a}(\partial\omega_q/\partial q)\mathrm{tr}\,\delta_{nm}(q)dq$. Therefore, we need to calculate only the diagonal elements of the phase-shift matrix δ_{nm}. It is clear that, for the general solution $w_{c,1}$, the potential operators are not cylindrically symmetric (two-center problem) and have complicated forms making it a hard task to get the exact expression to the phase shifts. Usually, in this case, the Born approximation is a useful approach. The potential terms that are not cylindrically symmetric will basically couple the different angular momentum channels n and m, being responsible for transitions between them. Thus, we have to worry only about cylindrically symmetric terms.

At infinity, η is a plane wave and then the first-order Born terms for the diagonal elements can be written as[21]

$$\delta_n^{(1)} = -\frac{\pi}{2}\int_0^\infty r dr \langle J_{|n|}(qr)\exp(-in\varphi)(U_1+U_2)\exp(in\varphi)J_{|n|} \times (qr)\rangle_\varphi. \tag{14}$$

To calculate the phase shifts we note first that, although the classical energy of the Skyrmion is independent of α, it is not so clear that the quantum corrections are also independent of this parameter in the anisotropic model. Then, in principle, it should be kept in the expressions since it may lead to nontrivial physical effects. As we have seen in Sec. II, when α changes from 1 to ≈ 0 (or to ∞), the problem of two centers transforms to a problem of only one center. Hence, the potential induced by the general configuration with unit winding number $w_{q,1}$ is continuously modified as α is varied. Therefore, there may exist a range of values of the parameter α in which only Skyrmions with two centers are possible, while outside this range, only Skyrmions with one center arise. However, it is not a simple task to find the transition between the two situations. We remark that nontrivial physical effects dependent on α could arise if there is a critical value of this parameter (inside the range in which it permits only excitations with two centers) above which the sign of the Casimir energy is changed (modifying the nature of a possible interaction between two spinons). We have verified analytically and numerically the effects of α on the phase shifts (in all its possible ranges). Calculations using Eqs. (12)–(14) show that the trace of the phase-shift matrix has analogous expressions for the extreme cases $\alpha=1$ (with configuration $[z-\xi_1]/[z-\xi_2]$) and $\alpha\to 0$ (with configuration $1/[z-\xi_2]$) or $\alpha\to\infty$ (with configuration $[z-\xi_1]$). Nevertheless, one term of the trace replaces its sign (from positive to negative) when one passes from the "two-particle" configuration to the "one-particle" configuration, indicating that some differences may exist in the behaviors of the quantum corrections for these two extreme cases. On the other hand, when we consider only the values of α inside the relevant range (containing Skyrmions with two centers), our analysis did not find any qualitative change of the physical picture as α varies. For the observed values, the Casimir energy did not change the sign. Therefore, we will consider only the ordinary case $\alpha=1$—i.e., configurations with spins at infinity pointing along a specific direction in the XY plane. It is the simplest configuration for two spinons and permits the obtaining of analytical results. All qualitative conclusions apply equally to other values of α that satisfy a problem of two vortices. Then, as we will see below, only variations in the anisotropy parameter λ can change the nature of the spinons interaction.

Most terms of the phase shifts have an expression like $(-\pi/2)\int_0^\infty dr g(\vec{r})J_{|n|}(qr)[dJ_{|n|}(qr)/dr]$, where $g(\vec{r})$ is a function of $\vec{r}$. Hence, almost all terms in the potentials contribute with a null trace due to the relation $\sum_{n=-\infty}^{\infty}J_{|n|}^2(qr)=1$. Only the fourth, sixth, and eighth terms of U_1 and the fourth term of U_2 have a nonzero trace. Besides, only two of them (the last terms of U_1 and U_2) have a nonconstant trace; i.e., their trace depends on the Skyrmion size R. It means that, differently from the isotropic case, the lowest-order contributions to the zero-point energy comes from the first-order Born terms. After a lengthy work one obtains

$$\mathrm{tr}\,\delta_{nm}(q) = -2\pi\lambda\left\{1-\sum_{n=1}^{L/a}\frac{qR}{2}(2n-1)K_{n-1}(qR/2)I_n(qR/2)\right\}, \tag{15}$$

where K_n and I_n are modified Bessel functions. In the above equation, we have also introduced an angular momentum cutoff for the trace sum $n_{max}=L/a$, where L is the system size. Note that in the limit $\lambda\to 0$, the trace vanishes as expected for the isotropic model.[21] The higher-order Born terms can be neglected for an appreciable range of λ if the Skyrmion size R is not large enough—i.e., $R\ll L$ (short-range potential). Using Eq. (15), the quantum energy of a Skyrmion ("vortex-antivortex" with separation R) is

$$E_q^{xy} = A(\lambda)JS^2\left\{1+\frac{2\sqrt{2}}{AS}\lambda(1+\lambda)^{1/2}\left[2-\frac{a}{\sqrt{\pi}R}G_{pq}^{jk}\left(\left\{\left\{1,1,\frac{3}{2}\right\},\{\,\}\right\},\left\{\left\{\frac{3}{2},\frac{3}{2}\right\},\left\{0,\frac{1}{2},\frac{1}{2}\right\}\right\},\frac{R}{2a},\frac{1}{2}\right)-\frac{a}{\sqrt{\pi}R}\sum_{n=1}^{L/a-1}(2n+1)G_{pq}^{jk}\left(\left\{\left\{1,1,\frac{3}{2}\right\},\{\,\}\right\},\left\{\frac{3}{2},\frac{2n+3}{2}\right\},\left\{\frac{1-2n}{2},0,\frac{1}{2}\right\},\frac{R}{2a},\frac{1}{2}\right)\right]\right\}, \tag{16}$$

where $G_{pq}^{jk}(\{\{a_1,\ldots,a_k\},\{a_{k+1},\ldots,a_p\}\},\{\{b_1,\ldots,b_j\},\{a_{j+1},\ldots,a_p\}\},h,f)$ is the generalized form of the Meijer G function defined as

$$\frac{f}{2\pi i}\int\left[\frac{\Gamma(1-a_1-fs)\cdots\Gamma(1-a_k-fs)\Gamma(b_1+fs)\cdots\Gamma(b_j+fs)}{\Gamma(a_{k+1}+fs)\cdots\Gamma(a_p+fs)\Gamma(1-b_{j+1}-fs)\cdots\Gamma(1-b_q-fs)}\right]h^{-s}ds, \tag{17}$$

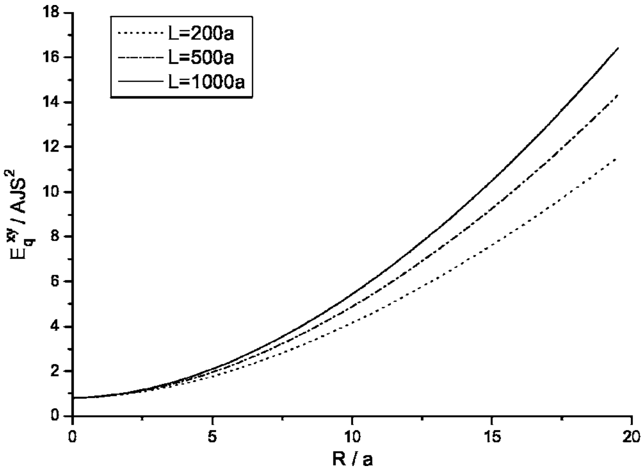

FIG. 1. Energy of a quantized Skyrmion as a function of its size R (the distance between the two spinons) for lattices with $L=200a$, $500a$, and $1000a$ (for $\lambda=-1/2$). Small Skymions are not very sensitive to the lattice size. We notice that this energy tends to increase linearly with R for large distances of separation, like the quark confinement.

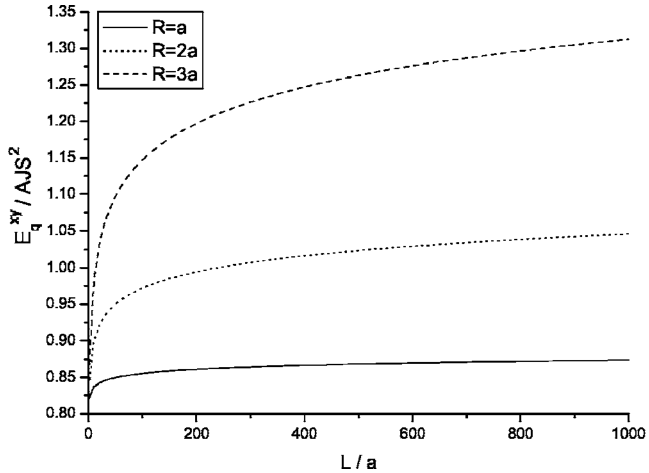

FIG. 2. Behavior of the Skyrmion energy as a function of the lattice size L for spinon separations $R=a$, $2a$, and $3a$. Note that this energy varies slowly with L, mainly for small separation between spinons.

and we have used $c=2\sqrt{2}JSa/\hbar$. We have to distinguish two different types of interaction of the two spinons as the anisotropy range changes from $-1\leq\lambda<0$ to $\lambda>0$. Clearly, as λ changes sign, the Casimir energy has opposite behaviors as R varies. Below we describe the effect of quantum fluctuations on the nature of the interactions between spinons and how its characteristics are completely modified as the parameter λ goes from negative to positive values.

V. RESULTS

First we discuss the case $-1\leq\lambda<0$. In Fig. 1 we plot the WO-Skyrmion energy as a function of the spinons separation R for lattice size $L=1000a$ (and $\lambda=-1/2$). We notice (not shown) that semiclassical quantum corrections lower the classical WO-Skyrmion energy for small Skyrmions ($R<1.76a$) while increase this energy for relatively large Skyrmions ($R>1.76a$). However, the Skyrmion energy always tends to increase as R increases. A simple comparison with the isotropic model shows that this result keeps more similarities with results of Ref. 22 than the ones of Refs. 7, 21, and 26. Particularly, since magnons are suppressed for $\lambda\to-1$, quantum corrections do not exist for this anisotropy and the system is essentially classical. Figure 2 shows the dependence of the soliton quantum energy on the lattice size L for Skyrmions with sizes $R=a$, $2a$, and $3a$ for the case $\lambda=-1/2$. Note that it varies very slowly with L. The main conclusion for the case of negative anisotropy ($-1<\lambda<0$) is that semiclassical quantum corrections induce an effective interaction between the constituents of the Skyrmion which is always attractive (Fig. 1). Now a vortex (spinon) and an antivortex (antispinon) attract themselves and the quantum energy increases as the separation between them increases. Note, however, that the effective potential of attraction between the two spinons is almost constant in the interval range between $R=0$ and $R\approx2a$, which implies that the force of attraction, $\vec{F}_R=-dE_q^{xy}/dR(\vec{R}/R)$, almost vanishes and the vortices are approximately free for small separations. The attractive force is not a constant, however, but varies with distance between the spinons. The dynamics of the vacuum enhances the force at large distances, while at short distances the interaction grows weaker. The notion that the force between spinons becomes vanishingly small as the spinons come close together leads to the so-called asymptotic freedom and, therefore, this anisotropic model does not support a BKT phase transition: its vortices (spinons) cannot be seen as free particles since the energy cost to separate them grows linearly with separation (for large R; see Fig. 1) instead of having a logarithmical dependence on the separation. The spinons are therefore confined, similarly to quarks in elementary particles. In contrast to the isotropic case,[7] no deconfinement transition is expected here. Hence, our calculations for negative λ suggest that only small Skyrmions must be present in the system and their constituents (spinons) may not exist as finite-energy excitations. We have mentioned earlier that in the continuous model the Skyrmion is a topological excitation and as such cannot dissipate. However, in the discrete lattice the continuity of the field $\vec{n}$ is lost and the very notion of topological excitation becomes inconsistent.[27] On a discrete lattice, the topological charge is not conserved and, then, Skyrmions will be unstable. Indeed, when the radius of a Skyrmion shrinks to the lattice scale, it will collapse. So, in the range $-1<\lambda<0$, the Skyrmions will only have sense as metastable excitations, with a finite lifetime. Spinons will be merely internal degrees of freedom of this metastable "pseudoparticle." A recent observation of the destruction of a Skyrmion with configuration like the one we are studying here was presented in Ref. 28. In this case, the vortex-antivortex annihilation was accomplished by a singularity and accompanied by a violent burst of spin waves. In fact, a change in the topological sector requires the injection of a magnetic monopole (a Bloch point).[29,30] While this process is strictly forbidden in continuum theories with a fixed length of magnetization, it is allowed in lattice models. For

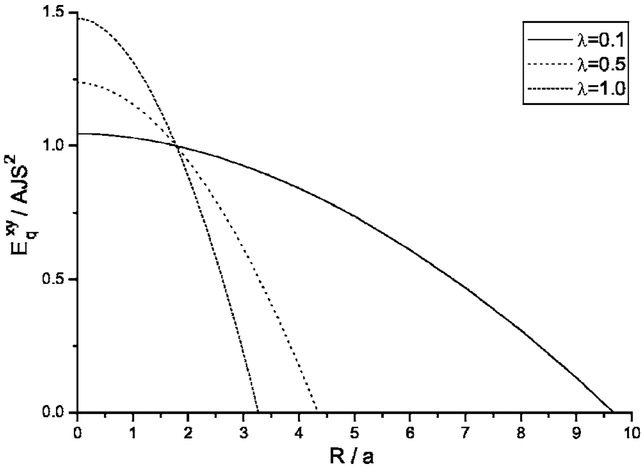

FIG. 3. Energy of quantized Skyrmions as a function of their sizes R (the distance between the two spinons) for positive values of λ and lattice size $L=1000a$. Note that, in contrast to the case $-1<\lambda<0$, here the energy decreases as R increases.

the model considered here (with negative λ), a similar phenomenon with emission of spin waves may occur when the spinons are annihilated.

Now we discuss the case $\lambda>0$. Here the behavior of the spinons is completely different from the preceding situation. In Fig. 3 we plott the Skyrmion energy as a function of R for three values of the anisotropic parameter $(\lambda=0.1,0.5,1)$ and it is easy to see that semiclassical quantum corrections raise the classical WO-Skyrmion energy for very small Skyrmions. However, the energy always tends to decrease as the spinons separation increases and, therefore, such excitations repel themselves. Note that the classical energy $A(\lambda)\hbar\rho_s$ is recovered at the same point $R\approx 2a$ for all anisotropies. For very short distances, the force of repulsion is small, but atypically, it increases with R until becoming almost constant, separating the spinons definitively. We notice that there may be a critical distance R_c above which it is energetically favorable to create two spinons—i.e., $E_q^{xy}<0$. Of course, such a critical size $R_c(\lambda)$ is a function of λ and decreases rapidly as λ increases [for instance, $R_c(0.1)\approx 9.7a$, $R_c(0.5)\approx 4.4a$, $R_c(1)\approx 3.3a$]. These results indicate that the two spinons are indeed deconfined for positive anisotropies. Concerning comparisons with the isotropic nonlinear σ model we note that, in contrast to the anterior case, the actual finding is more similar to results of Refs. 7, 21, and 26 than the ones of Ref. 22. Unfortunately our calculations cannot say anything about the spinons behavior for the isotropic case since the first-order Born phase shifts have a null trace for this particular situation. However, the opposite behaviors of the interaction between spinons in the two possible ranges of the anisotropy suggest that there may be a critical λ above which a transition occurs and this point is clearly $\lambda=0$. Perhaps, more rigorous calculations should be done to get a definitive conclusion. We remember that, for the isotropic model, the effect of quantum fluctuations on the Skyrmions is still a motive of discussions.[21,22,26] Baskaran[7] argues that, in addition to gapless magnon excitations, we should have deconfined, freely propagating spinons above a finite-energy gap, even in the ordered phase for the $\lambda=0$ case. It would imply that the transition "point" mentioned above should happen at a negative value of the anisotropy. We do not completely exclude this possibility because for very small values of λ ($|\lambda|\ll 1$), the first-order Born results become very small and then, probably, the contribution of the second-order terms [coming mainly from the isotropic term in Eq. (12)] to the quantum energy may be of the same order of the corrections obtained in Eq. (16), changing the picture of attraction. In addition, other small contributions (e.g., if there is a bound state or if one takes into account contributions from topological fluctuations like other Skyrmions, etc.) could also be comparable to Eq. (16) in the limit $|\lambda|\to 0$. However, our simple approach shows that the $\lambda=0$ case is the most probable (and natural) point of transition since the two-center Skyrmion configurations suffer a more dramatic modification exactly when λ passes from negative to positive values. For instance, the spin spheroid Σ^2 changes its shape from "oblate" ($b<1$) to "prolate" ($b>1$) as λ overtakes the value zero, causing fundamental changes in the out-of-plane structures of the Skyrmions, and hence, the behavior of the magnon scattering and quantum fluctuations should alter substantially. At this stage we would like to suggest that a quantum phase transition takes place as the parameter λ is varied: for $-1<\lambda<0$ the spinons are confined in the Néel state. The case $\lambda>0$ corresponds to a phase with deconfined spinons and, then, freely propagating vortices may be present, disordering the system significatively. We do not know, however, the nature of this phase transition, and it would be an interesting topic for future works.

VI. CONCLUSIONS

In summary, using first-order Born terms, we have argued that, in the anisotropic nonlinear σ model with $-1<\lambda<0$, vortices and antivortices exist only bound in pairs due to quantum fluctuations while they are deconfined for $\lambda>0$. Higher-order Born terms may not modify this picture, at least for anisotropy strength not too small. Such terms may cause only small quantitative changes in the calculations. In fact, in one of the few examples in which the problem vortex-magnon interactions was solved exactly[31] (in the continuum limit), the comparison between the exact results with the first-order Born approximation is really striking. That vortices studied here have spin 1/2 can be understood from the chain anomaly phenomenon[7] generalized to the anisotropic case. This effect implies a global rotation of the y axis about the x axis, leading to the configurations $m_{GR}(\lambda)$ and Φ_{GR}, which wrap the Σ^2 spheroid completely. For negative anisotropy, the effective potential experienced between the two spinons increases considerably as they try to separate. Essentially, spin-1 magnons seem to be the particles responsible for it, inducing an "antishielding charge" for spinons. For large spinon separations, the energy increases linearly with R. Nevertheless, a finite density of these structures with relatively small sizes must be excited at any temperature $T>0$, since they have finite energy. Extrapolating these results for excitations with $|Q|>1$, which contain $|Q|$ vortices and $|Q|$ antivortices,[3] the WO multi-Skyrmion solutions could be viewed as extremely strongly bound aggregates of spinons in the anisotropic nonlinear σ model with $-1<\lambda<0$. On the other hand, for positive anisotropies, the Skyrmion solutions have a structure that permits deconfinement of vortices due to quantum fluctuations. In this case, since large Skyrmions require lower energy to be created, it is conceivable that they would be the preferential excitations, nucleating everywhere and disordering the system considerably. This suggests that a quantum phase transition from a phase containing confined spinons to a phase containing deconfined spinons takes place as the parameter λ varies from negative to positive values. Indeed, due to quantum fluctuations, there are only vortex-antivortex bound states for $\lambda<0$ while vortices become free for $\lambda>0$. Finally, we point out that the model considered here and the results obtained may also have some relevancy for particle physics as a non-Abelian toy model.

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