Motions of quantized vortices attached to a boundary in alternating currents of superfluid ^{4}He

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The motions of superfluid vortices attached to a boundary are investigated in alternating currents by using a vibrating wire. The attached vortices appear to form a layer on the wire and enhance the mass of the wire, even for low velocity currents. In turbulence, chaotic motions of vortices such as entanglement and reconnection reduce the thickness of the layer in spite of the fact that the vortices unstably expand. When turbulence subsides, the attached vortices appear to shrink, with the degree of shrinking influenced by thermal excitations in the superfluid.

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Quantum turbulence is a fundamental topic in low temperature physics.[1] In superfluids, turbulence simply consists of a tangle of identical vortices, the structure of each being a quantized circulation of a superfluid flow around a nonsuperfluid string core. This structure allows for simple models of vortices and turbulence in fluids. To investigate quantum turbulence in superfluid ^{4}He, counter flows and second sound techniques, which generate and detect quantum turbulence, respectively, have been utilized experimentally for decades.[1] Although normal fluid component is present in the temperature ranges of these experiments, quantum turbulence generated by counter flows is well described by the vortex-filament model, which simulates motions of quantized vortices numerically.[2] In recent years, quantum turbulence has been investigated by using a grid and an oscillator, to extend the temperature range down to zero, in which a nonsuperfluid component is absent. A grid drawing can generate homogeneous and isotropic quantum turbulence in superfluid ^{4}He, and its decay reveals a classical picture with a Kolmogorov energy spectrum at the temperatures where normal fluid behaves hydrodynamically.[3] A Kolmogorov spectrum of superfluid turbulence has been predicted even at zero temperature by Tsubota *et al.*[4] using a vortex-filament model and a Gross-Pitaevskii equation. In recent studies,[5] this energy decay has been found in the turbulence of superfluid ^{3}He generated with an oscillating grid at very low temperatures where thermal quasiparticle excitations are almost absent. Thus the simple turbulence composed of identical vortices mimics classical turbulence complicated by eddies or vortices.

In quantum turbulence, the motions of vortices, especially reconnection between vortex strings, are expected to play an important role in decaying turbulence.[2,4] In investigating the vortex motions experimentally, it is extremely useful to observe the response of an oscillator in a superfluid. Remnant vortices are believed to be attached to an oscillator in superfluid ^{4}He.[6] Therefore, the oscillator causes turbulence when it moves at sufficiently high velocity.[7,8] The resonance curve of a grid oscillating in superfluid ^{4}He shows enhancement of the effective mass of the grid above a certain velocity.[7] This behavior can be explained by the motions of remnant vortices attached to the windows of the grid.[8] A simple shape oscillator such as a sphere and a wire has also been used to investigate quantum turbulence in superfluid helium.[9,10] The drag of a microsphere oscillating in superfluid ^{4}He indicates laminar flow and turbulent flow, and these two flow regimes are clearly separated at a critical velocity.[9] Even if many vortices are initially attached to an oscillator and a large drag force hides flow states, sufficiently high speed oscillations will eliminate the drag force and clarify the two regimes.[10] Although turbulent flow generates many vortices, the attached vortices seem to be confined to a finite size in the turbulent flow regime.

In the present paper, we concentrate on the resonance frequency of a vibrating wire, to investigate the motions of vortices in alternating superfluid currents. Since the resonance frequency is insensitive to thermally excited phonons at low temperatures,[11] the vortex motions should be reflected directly in the resonance frequency. We also discuss the temperature dependence of the critical velocity at which turbulent flow disappears.

A conventional vibrating metal wire formed into a semicircular shape was used as an oscillator. It was made from 2.5 μm NbTi wire, drawn from a commercial multifilament superconducting wire with a die. The distance between the legs fixed on a copper plate was trimmed so as to give a suitable resonance frequency. We prepared a 0.7 kHz vibrating wire, which has a distance of 2 mm between the legs. The wire was located in a cell in a magnetic field of 25 mT. The Lorentz force oscillates the wire carrying current of resonance frequency. The phase locked method enables resonance data to be taken as well as velocity data. Peak velocity data were taken in the present study, estimated as the maximum velocity of the top part of the vibrating wire.

The vibrating wire was cooled to 30 mK with a dilution refrigerator. Before measuring the wire in ^{4}He fluid, we measured the response of the vibrating wire as a function of temperature in vacuum. The quality factor Q of the wire grows even below 100 mK. The maximum Q value was 2800 at 30 mK measured at a cell wall; however, the temperature of the wire is expected to be higher than the measured value because a finite Q value leads to energy dissipation in the wire, causing a temperature difference. In the case of a wire velocity of 100 mm/s, the dissipation energy is estimated to be 0.4 pW. Hence, the Q value is believed to be higher in ^{4}He fluid at 30 mK than that measured in vacuum.

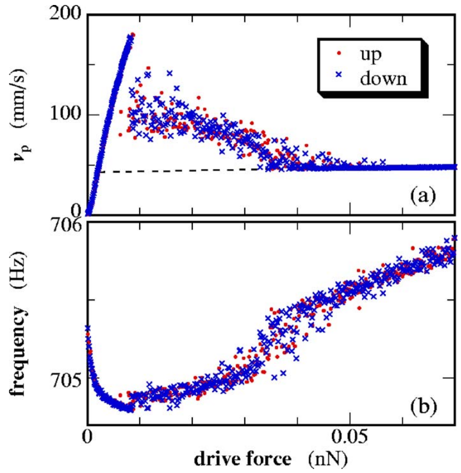

FIG. 1. (Color online) Responses of a vibrating wire in superfluid ^{4}He at 30 mK: (a) peak velocity v_p at the top of the vibrating wire, measured for up sweeps and down sweeps of the drive force; (b) resonance frequency. The dashed line is extrapolated from data in the turbulent flow regime.

Heat due to the vibration enters the superfluid ^{4}He as well as directly into the cell walls through the wire. We used packed silver powder as a heat exchanger to eliminate temperature differences between the helium fluid and the cell walls. The resonance frequency in vacuum was also measured as a function of wire velocity. It was found to decrease almost linearly with increasing velocity by 116 mHz in the range from 30 to 200 mm/s, being insensitive to temperature variation below 100 mK. We settled a 3.9 kHz vibrating wire made from the same 2.5 μm wire in the cell, used in a previous study.[12]

The response of a vibrating wire reveals two flow regimes. At low drive force, the velocity of the vibrating wire increases linearly with drive force. The linear response arises from drag force due to the intrinsic friction in the wire and phonon scattering, and consequently the flow around the wire is laminar. At high drive force, the wire velocity suddenly drops and then increases gradually with drive force. The nonlinear response indicates that a vortex attached to the wire expands and creates vortices, causing turbulent flow around the wire.[10] Typical responses have been described in previous papers[10,12] for vibrating wires at high temperature and for a 3.9 kHz vibrating wire at low temperature. The response of a 0.7 kHz vibrating wire, however, reveals a "mid" regime between that of the linear and the nonlinear regimes, as shown in Fig. 1(a). The velocity in the "mid" regime seems to be between that of the low and high drive force regimes with large uncertainties. To examine the behavior in the "mid" regime, we measured the response with a fixed drive as a function of time. At low and high drive forces, the velocity was constant with an expected value; however, in the mid regime we observed oscillations in the velocity of the vibrating wire. The velocity increases with a certain relaxation time from an expected value in the turbulent flow regime, and then suddenly drops to the initial value. These oscillations occur as often as 1.6 times per second. The increase in velocity seems to occur just after the flow around the wire changes from turbulent to laminar. A similar behavior has been observed in microsphere studies, known as intermittent switchings.[13] For a 3.9 kHz vibrating wire, the switchings do not occur as frequently.[12] The vibrating frequency of the wire might influence the switching behavior.

We also measured the resonance frequency of the vibrating wire simultaneously with the velocity by locking the phase to keep the quadrature at zero. The results are plotted in Fig. 1(b). The frequency decreases with increasing drive force in the laminar flow regime; however, it increases in the turbulent flow regime. The resonance frequency should relate directly to the motion of vortices attached to the wire at temperatures where thermal excitations are almost absent, because the resonance frequency is insensitive to thermally excited phonons in the ballistic regime.[11] The variation of the resonance frequency suggests that the motions of vortices attached to the wire vary the moment of inertia of the wire. A similar dependence has been observed for a 3.9 kHz vibrating wire.[12]

To examine the effect of the motions of the attached vortices, we compared the resonance frequencies with those obtained in vacuum. The ratio of the frequencies without a normal fluid component can be given by a simple equation,[7]

$$\frac{f_{\mathrm{He}}}{f_{\mathrm{vac}}} = \left(\frac{\rho_w d_w^2}{(\rho_w + \beta\rho_s) d_w^2} \right)^{1/2}, \tag{1}$$

where f_{He} and f_{vac} are the resonance frequencies with and without helium, ρ_w and ρ_s are the wire density and helium density, and d_w and β are the diameter and the geometrical constant of the wire. The geometrical constant β is derived from the cross section of an oscillator, given by $\beta=0.5$ for a sphere and $\beta=1$ for an infinite cylinder. In the present study, β is estimated to be 1.00 ± 0.04 from the shape of the wire. Using the values measured with and without helium for a 3.9 kHz vibrating wire, we estimated the density ρ_w of the wire to be 2.0 g/cm^3, which is much smaller than the expected value of 6.3 g/cm^3 for NbTi.[11] This result indicates that the wire thickens or becomes heavy in a superfluid. A vortex can enhance the mass of an object it is attached to by alternately expanding and shrinking with the oscillation of the superfluid current.[8] To study the effect of vortex motions, we assume a surrounding layer consisting of superfluid that moves together with the wire due to attached vortices. We represent this by replacing the denominator of the right-hand side term in Eq. (1) with $\rho_w d_w^2 + \beta\rho_s(d_{\mathrm{eff}}^2 - d_w^2) + \beta\rho_s d_{\mathrm{eff}}^2$. Here d_{eff} is the effective wire diameter, including a bare diameter and the thickness of the surrounding layer. For the case of a bare wire diameter of 2.5 μm, d_{eff} is estimated to be 3.6 μm. For a 0.7 kHz vibrating wire, the excess thickness due to attached vortices has been observed to be 0.08 μm, which is much thinner than that for a 3.9 kHz vibrating wire. Although the excess thickness is always observed for a vibrating wire, the origin of the different thicknesses has not been explained. In recent studies,[14,15] the excess thickness was not observed for a wire and a grid within experimental uncertainties. The number of vortices attached to an oscillator might be affected by the roughness of its surface;[15] however, the present study shows that the excess thickness varies widely under the same roughness. Further experimental work is re-

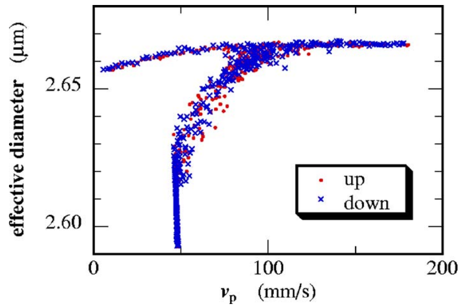

FIG. 2. (Color online) Effective diameter of a 0.7 kHz vibrating wire in superfluid helium. Upper and lower regimes represent the laminar and turbulent flow, respectively, and a middle corresponds to the regime of intermittent switchings.

quired to understand the origin of the different thicknesses.

Under the assumption of a surrounding superfluid layer, the effective diameter can be drawn as a function of wire velocity as shown in Fig. 2. We also plot the effective diameter for a 3.9 kHz vibrating wire in Fig. 3. In both cases, upper and lower regimes represent the laminar and turbulent flow, respectively, and a middle regime in Fig. 2 corresponds to the regime in which intermittent switchings occur. In the laminar flow regime, we find a curious velocity dependence of the effective diameter. If the attached vortices were extended only by a superfluid current, the excess thickness would go to zero at the zero velocity limit. The effective diameters, however, seem not to decrease to the bare diameter of 2.5 μm. Moreover, they saturate at high velocities. Thus the attached vortices seem to form a surrounding layer even for low velocity currents and the thickness is not greatly influenced by the current velocity. These behaviors imply the existence of collective motions of vortices attached to the wire. If a local current induced by a vortex can stabilize neighboring vortices and if vortex motion can prevent an external current from being applied to neighboring vortices to some degree, whole vortices might move together and form a layer with a finite thickness even at zero velocity. Microscopic images showing the motions of attached vortices are necessary to understand the excess thickness observed here.

In the turbulent flow regime, vortices attached to the wire expand unstably and cause energy dissipation.[7] The expansion of the vortices seems to increase the effective diameter

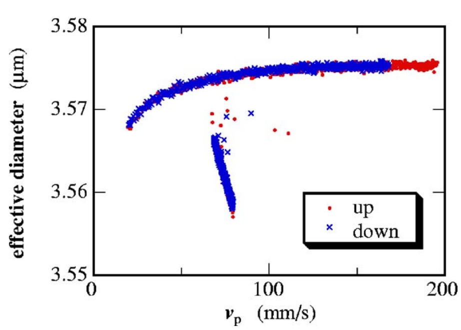

FIG. 3. (Color online) Effective diameter of a 3.9 kHz vibrating wire in superfluid helium.

of the wire. As shown in Figs. 2 and 3, however, the effective diameter decreases steeply in the turbulence regime with increasing velocity. This steep decrease may be caused by chaotic motions of vortices, such as entanglement and reconnection in turbulence. An unstably expanding vortex may easily entangle itself, and reconnection will then occur when one part of the vortex filament closely approaches another part.[2] Since the entanglement is likely to reduce drag on a vortex in a superfluid current, it may reduce the effective diameter. Reconnection can also reduce the effective diameter as the vortex ring is released from the attached vortex, shrinking its size. Thus competition between the expansion and the reconnection could determine the size of the attached vortices in turbulence. Vortices detached from an oscillator are predicted to cascade to small vortices and evaporate and escape.[16] Therefore a vibrating wire is unlikely to catch vortices detached from itself. An oscillator consisting of multiwires, however, may capture vortices again. The resonance of a vibrating grid, which consists of 106 μm square windows with 20.8 μm width frames, shows that the effective mass of the grid increases with increasing velocity in turbulence.[7] This result implies that vortices detached from one part of the frames are captured again by another part, causing an increase in the effective mass. The influence of detached vortices should relate to evolution and diffusion of the vortices.[16]

When the turbulence subsides, the wire velocity increases to high values in the laminar flow regime, as shown in Fig. 1(a). The turbulent flow is caused by unstable expansion of vortices because of Glaberson-Donnelly instability,[17] which is induced by a superfluid current at a velocity almost inversely proportional to the vortex size. In turbulent flow, therefore, large vortices are present and slow currents can continue to induce expansion. In contrast, rapid currents cannot extend a vortex in laminar flow, which means that large vortices vanish. It is conceivable that motions of vortices, such as reconnection, confine whole vortices into small sizes and stop expansion. The effective diameter, however, grows when laminar flow appears, as shown in Figs. 2 and 3. The vortex sizes do not seem to influence the effective diameter directly in turbulent flow, as mentioned above.

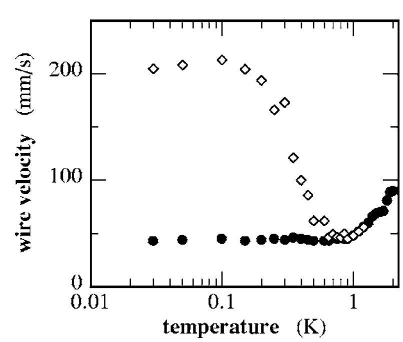

FIG. 4. Temperature dependence of velocities of a vibrating wire for which the flow state changes. The solid circles show the critical velocity v_c at which the turbulent flow completely disappears. The open circles show the maximum velocity $v_{\max}$ in the laminar flow regime.

The confinement size of vortices in turbulence is expected to equal the maximum size of attached vortices in the laminar flow regime, which should correspond to the maximum velocity $v_{\max}$ of the wire just after the turbulent flow subsides and the laminar flow stabilizes. The critical velocity v_c at which the turbulent flow completely disappears is expected to correspond to the size of a vortex just before expansion starts. Although this size is expected to equal the confinement size, $v_{\max}$ is much higher than v_c, as shown in Fig. 1(a). This result suggests the existence of a mechanism for shrinking whole vortices into small sizes to stop expansion. This mechanism must relate to reconnection, but evidently differs from the reconnection of vortices continuously occurring in turbulence. Figure 4 shows both velocities as a function of temperature. The velocity v_c is constant below 0.9 K, which corresponds to the temperature range in which thermal excitations are relatively rare and behave ballistically.[11] This result indicates that vortices in turbulence are insensitive to ballistic excitations. The velocity $v_{\max}$, however, increases from v_c with decreasing temperature below 0.7 K. This dependence indicates that the mechanism shrinking vortices is influenced by ballistic excitations. In the temperature range of hydrodynamic excitations, v_c increases with temperature, which indicates that the sizes of the vortices are suppressed by normal fluid component in helium fluid.

In summary, vortices attached to an object form a layer in alternating currents of superfluid ^{4}He. The thickness of the layer is not greatly sensitive to current velocity in the laminar flow. In turbulence, chaotic motions reduce the layer thickness drastically as the current velocity increases. The attached vortices appear to shrink when turbulence subsides, which suggests the existence of a mechanism for shrinking whole vortices into small sizes.

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