Diffusive magnetotransport in a two-dimensional electron gas in the presence of Rashba spin-orbit interaction

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(Received 27 December 2005; revised manuscript received 5 September 2006; published 29 December 2006)

An analytical approach to calculation of the conductivity tensor σ of a two-dimensional electron system with Rashba spin-orbit interaction (SOI) in an orthogonal magnetic field is proposed. The electron momentum relaxation is assumed to be due to electron scattering by a random field of short-range impurities, which is taken into account in the Born approximation. An exact expression for the one-particle Green's function of an electron with Rasba SOI in an arbitrary magnetic field is suggested. This expression allows us to obtain analytical formulas for the density of states and σ in the self-consistent Born and ladder approximations, respectively, which hold true in a wide range of magnetic fields, from the weak ($\omega_c\tau \ll 1$) up to the quantizing ($\omega_c\tau \gtrsim 1$) ones. It is shown that in the ladder approximation the Rashba SOI has no effect at all on the conductivity magnitude in the whole range of classical (nonquantizing) magnetic fields. The Shubnikov–de Haas oscillation period is shown to be related to the total charge carrier concentration by the conventional formula, irrespective of the SOI magnitude. A simple equation defining the location of the SdH oscillation beating nodes is obtained. The results are in good agreement with the experimental and recent numerical investigations.

DOI: 10.1103/PhysRevB.74.245333 PACS number(s): 73.20.At, 71.70.Ej, 72.15.Gd, 73.21.−b

I. INTRODUCTION

The growing interest in studying the spin-orbit interaction (SOI) in semiconductor two-dimensional (2D) structures is mostly due to its potential application to the spin-based electronic devices.[1] There are two main types of SOI in a quantum well based on zinc-blende-lattice semiconductors: first, the Dresselhaus interaction[2] which originates from the bulk inversion asymmetry; second, the Rashba interaction[3] induced by structural inversion asymmetry of the confining field of a quantum well. Both of these interactions lead to the momentum-dependent spin splitting of the electron energy spectrum and to the formation of quantum states with the hard linked spatial and spin degrees of freedom of the electrons. They are responsible for many interesting effects in the transport phenomena like beatings in the Shubnikov–de Haas (SdH) oscillations;[3,4] weak antilocalization;[5–8] current-induced nonequilibrium spin polarization;[9–11] spin Hall effect;[12,13] and so on.

At present there are some sufficiently well-developed theories of the kinetic and spin phenomena in 2D systems with SOI in zero or classical weak ($\omega_c\tau \ll 1$) orthogonal magnetic fields. Here $\omega_c = |e|B/mc$ is the cyclotron frequency, and τ is the electron scattering time. As for theoretical studies of the considered systems in strong, and especially in quantizing ($\omega_c\tau \gtrsim 1$) magnetic fields, there is still no satisfactory analytical description of the kinetic phenomena even in the usual diffusive regime (without quantum corrections). The complex form of the eigenspinors and energy spectrum of an electron in the presence of SOI and a strong magnetic field[3] is the main cause of such a situation. Direct employment of this basis forces one to proceed almost right from the start to the numerical analysis of very cumbersome expressions.[14–16,24]

The strong magnetic field is, however, one of the most efficient tools for investigation[4] of SOI and manipulation of the spin degrees of freedom in semiconductor 2D structures. Thus, a rather simple theoretical description of the kinetic phenomena in 2D systems with SOI in a strong orthogonal magnetic field becomes a necessity. In the present work, we consider the problem of calculation of the longitudinal and Hall resistances of a 2D Rashba system in the ladder approximation assuming that the electron momentum relaxation is due to elastic scattering by short-range impurities which is taken into account in the Born approximation.

We have found the *exact* relation between the one-particle Green's function (GF) of the Rashba 2D electron in an arbitrary orthogonal magnetic field and the well-known GF of an "ideal" electron, that is, an electron with the ideal value of the Zeeman coupling ($g_0=2$) and without SOI. This allows one to obtain analytical expressions for the density of states (DOS) in the self-consistent Born approximation (SCBA), and the conductivity tensor $\hat{\sigma}$ in the ladder approximation. The total DOS in the SCBA is defined as the sum of the partial DOS's of two spin-split subbands. At the same time, the conductivity in the ladder approximation looks as if the current were generated by charge carriers of one type with total concentration n and mobility μ. These expressions hold good in a wide range, from classically weak magnetic fields ($\omega_c\tau \ll 1$) up to quantizing ones ($\omega_c\tau \gtrsim 1$). On the basis of these results, we perform a numerical analysis of the beatings of the SdH oscillations of the considered kinetic coefficients, as well as of their behavior in the classical magnetic field region. The results are in good agreement with the experimental data,[18,19] and with the results of a recent numerical investigation.[17]

II. MODEL

Let us consider a two-dimensional ($||OXY$) degenerate gas of electrons with effective mass m and effective Zeeman

coupling g. The electrons move in an external orthogonal ($\mathbf{B}||OZ$) magnetic field $\mathbf{B}=\nabla\times\mathbf{A}$ in the presence of a random field $U(\mathbf{r})$ due to pointlike impurities distributed by the Poisson law in the sample. We assume the Rashba interaction to be the dominant mechanism of energy spin splitting in the absence of a magnetic field. This situation occurs, for example, in the narrow-gap semiconductor heterostructures, such as InAs/GaSb (Ref. 4) and $In_xGa_{1-x}As/In_yAl_{1-y}As$.[18,19] The one-particle Hamiltonian of the considered system has the form

$$\mathcal{H}+U=\frac{\boldsymbol{\pi}^2}{2m}+\alpha(\boldsymbol{\sigma}\times\boldsymbol{\pi})\cdot\mathbf{n}+\frac{1}{4}g\omega_c\sigma_z+U(\mathbf{r}) \qquad (1)$$

($\hbar=1$). Here $\boldsymbol{\pi}=\mathbf{p}-e\mathbf{A}/c=m\mathbf{v}$ is the operator of the kinematic electron momentum; $\boldsymbol{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ is the vector formed by the Pauli spin matrices; α is the Rashba spin-orbit coupling; g is the effective Zeeman coupling (g factor).

In the gauge $\mathbf{A}=(0,Bx,0)$, the components of the eigenspinors of the Hamiltonian $\mathcal{H}$ (1) of a free $[U(\mathbf{r})=0]$ Rashba electron are expressed through the Landau wave functions $\psi_{n,X}(\mathbf{r})$ depending on the Landau level number $n=0,1,2,\ldots$ and the X coordinate of the cyclotron orbit center $X=-k_y/m\omega_c$[3]

$$\hat{\Psi}_\alpha(\mathbf{r})=\frac{1}{\sqrt{1+C_{s,n}^2}}\begin{bmatrix}C_{s,n}\psi_{n-1,X}(\mathbf{r})\\ \psi_{n,X}(\mathbf{r})\end{bmatrix},\quad \alpha=(s,n,X). \qquad (2a)$$

The corresponding energy levels have the following form:

$$\mathcal{E}_{s,n}=\begin{cases}-\omega_c\delta, & n=0,\quad s=+1,\\ \omega_c\left(n+s\sqrt{\delta^2+2\gamma^2 n}\right), & n>0,\quad s=\pm 1.\end{cases} \qquad (2b)$$

Here $C_{s,n}=\gamma\sqrt{2n}/(s\sqrt{\delta^2+2\gamma^2 n}-\delta)$ is a normalizing coefficient; $\delta=(g-2)/4$ is the relative deviation of the effective Zeeman coupling from its ideal value $g_0=2$ (for definiteness, it is assumed that $\delta<0$ in these equations, but all the following results are valid for any sign of δ); and, finally, $\gamma=\alpha\sqrt{m/\omega_c}$ is the dimensionless Rashba spin-orbit coupling.

The quantum number $s=\pm 1$ describes the *helicity* of the Rashba electron eigenstate in the absence of a magnetic field.[11] Indeed, it can be verified immediately that $s=\pm 1$ is the eigenvalue of the operator

$$\nu=\frac{(\alpha\boldsymbol{\sigma}\times\boldsymbol{\pi}+\omega_c\delta\boldsymbol{\sigma})\cdot\mathbf{n}}{\sqrt{2m\alpha^2\mathcal{H}_0+\omega_c^2\delta^2}}, \qquad (3)$$

which is diagonal in the basis (2a) and approaches the helicity operator $(\boldsymbol{\sigma}\times\mathbf{p})\cdot\mathbf{n}/|\mathbf{p}|$ as $B\to 0$. Here $\mathbf{n}$ is the unit normal vector to the considered 2D system;

$$\mathcal{H}_0=\frac{\boldsymbol{\pi}^2}{2m}+\frac{1}{2}\omega_c\sigma_z \qquad (4)$$

is the Hamiltonian of the ideal ($g_0=2$) electron in a magnetic field, which commutes with $\boldsymbol{\sigma}\cdot\mathbf{n}$, $(\boldsymbol{\sigma}\times\boldsymbol{\pi})\cdot\mathbf{n}$, and $\mathcal{H}$ (1).

In spite of this analogy with the $B=0$ case, we cannot say that the Rashba electron has in the states (2a) the spin projection $\pm 1/2$ onto the direction $\alpha\boldsymbol{\pi}\times\mathbf{n}+\omega_c\delta\mathbf{n}$, because the components of the kinematic momentum operator $\boldsymbol{\pi}$ are not commuting motion integrals. Nevertheless, this interpretation makes sense in the quasiclassical limit, when one can speak about the electron path in a magnetic field; namely, the quantum number $s=\pm 1$ determines the value of the spin projection on the instant direction of $\alpha\boldsymbol{\pi}\times\mathbf{n}+\omega_c\delta\mathbf{n}$ which changes along the quasiclassical electron path. Thus, the spin configurations of the Rashba electron states form *vortices* in the XY plane with center at the origin.

The conductivity tensor $\hat{\sigma}$ of the considered system has just one independent circularly polarized component $\sigma=\sigma_{xx}+i\sigma_{yx}$. In the one-electron approximation, it has the form[20]

$$\sigma=\sigma^I+\sigma^{II}=\frac{e^2}{8\pi}\,\mathrm{Tr}\,V_+\left[\left(2\Phi_{EE}^{RA}-\Phi_{EE}^{RR}-\Phi_{EE}^{AA}\right)\Big|_{E=E_F}+\int_{-\infty}^{E_F}(\partial_E-\partial_{E'})\left(\Phi_{EE'}^{AA}-\Phi_{EE'}^{RR}\right)\Big|_{E'=E}dE\right]. \qquad (5)$$

Here, $\Phi_{EE'}^{XY}=\langle\hat{G}^X(E)V_-\hat{G}^Y(E')\rangle$ is the current vertex operator; $V_\pm=V_x\pm iV_y=v_\pm\pm 2i\alpha\sigma_\pm$ are circularly polarized components of the full velocity operator [the corresponding components $\boldsymbol{\sigma}$ are defined as $\boldsymbol{\sigma}_\pm=(\sigma_x\pm i\sigma_y)/2$], where the last term occurs due to SOI (1). $\hat{G}^{R(A)}(E)=1/(E-\mathcal{H}-U\pm i0)$ is the resolvent [retarded (R) or advanced (A)] of the Hamiltonian (1), and angular brackets $\langle\cdots\rangle$ denote averaging over the random field U configurations. Finally, the symbol ∂_E denotes the derivative with respect to energy E.

III. ONE-ELECTRON GREEN'S FUNCTION

By definition, the one-particle GF is the averaged resolvent of the Hamiltonian (1) $\langle\hat{G}^{R(A)}(E)\rangle=\langle 1/(E-\mathcal{H}-U\pm i0)\rangle$. It is connected with the electron self-energy operator $\hat{\Sigma}^{R(A)}(E)$ by the relation ($X=R,A$)

$$\langle\hat{G}^X(E)\rangle=\begin{bmatrix}\langle G_{\uparrow\uparrow}^X(E)\rangle & \langle G_{\uparrow\downarrow}^X(E)\rangle\\ \langle G_{\downarrow\uparrow}^X(E)\rangle & \langle G_{\downarrow\downarrow}^X(E)\rangle\end{bmatrix}=\frac{1}{E-\mathcal{H}-\hat{\Sigma}^X(E)}. \qquad (6)$$

The direct employment of the eigenspinors (2) for calculation of (6) or kinetic and thermodynamic properties of the Rashba system in a strong magnetic field leads to very complicated expressions. One is forced almost from the first steps either to turn to numerical calculations,[14,16,24] or to make simplifying approximations like a momentum-independent spin-splitting energy.[15] This makes more difficult the interpretation of the results obtained in such a way, as well as the understanding of the whole physical picture. But it turns out that the GF of the free ($U=0$) Rashba system is expressed *exactly* through the GF of the ideal electron in a magnetic field. This opens up new possibilities for analytical studies of the considered system. Indeed, it is easy to check that the Hamiltonian of the free Rashba systems can be presented in the following form:

$$\mathcal{H}=\mathcal{H}_0+\nu\sqrt{2m\alpha^2\mathcal{H}_0+\omega_c^2\delta^2}. \qquad (7)$$

Here ν is the helicity operator defined in Eq. (3) and $\mathcal{H}_0$ is the Hamiltonian of the ideal electron (4).

The substitution of the Hamiltonian (7) into the resolvent $\hat{G}(E)=(E-\mathcal{H})^{-1}$ gives, after some simple algebra, the following result (here and below, we drop superscripts R (A), if this does not lead to misunderstandings; sometimes, for brevity of notation, we shall not write explicitly the energy arguments of the resolvents or GF's):

$$\hat{G}(E)=\frac{E-\mathcal{H}_0+[\alpha(\boldsymbol{\pi}\times\mathbf{n})+\omega_c\delta\mathbf{n}]\cdot\boldsymbol{\sigma}}{(E+m\alpha^2-\mathcal{H}_0)^2-\frac{1}{4}\Omega_B^2}, \tag{8}$$

where

$$\Omega_B=2\sqrt{2m\alpha^2E+m^2\alpha^4+\omega_c^2\delta^2}=\sqrt{\Omega^2+4\omega_c^2\delta^2}. \tag{9}$$

The quantity Ω_B is equal to the magnetic-field-dependent frequency of the spin precession of the electron with energy E that is responsible for the Dyakonov-Perel spin relaxation mechanism;[21] Ω is the same frequency in the absence of a magnetic field. It should be noted that the same representation of the one-electron GF can be also obtained for a system with the momentum-linear Dresselhaus SOI. For example, in the case of a [001]-grown quantum well based on the $A_{\mathrm{III}}B_{\mathrm{V}}$ semiconductors, it is sufficient to replace $\boldsymbol{\pi}\to\tilde{\boldsymbol{\pi}}=(\pi_y,\pi_x)$ in the definition of the helicity operator (3), change the sign before the Zeeman term ($g_0=-2$) in the Hamiltonian of the ideal electron (4), and, finally, redefine the parameter $\delta\to\delta_D=(g+2)/4$.

The denominator of the right-hand side of Eq. (8) depends on the ideal electron Hamiltonian alone. Expanding this expression into partial fractions, we obtain the desired representation of the one-electron GF of the free Rashba system,

$$\hat{G}(E)=\frac{1}{2\Omega_B}\sum_{s=\pm1/2}\frac{\Omega_B+4s[m\alpha^2-\omega_c\delta\sigma_z-\alpha(\boldsymbol{\pi}\times\mathbf{n})\cdot\boldsymbol{\sigma}]}{E+m\alpha^2+s\Omega_B-\mathcal{H}_0}$$
$$=\sum_{s=\pm1/2}\left[\Phi_s-2s\frac{\alpha(\boldsymbol{\pi}\times\mathbf{n})\cdot\boldsymbol{\sigma}}{\Omega_B}\right]\hat{G}(E+m\alpha^2+s\Omega_B). \tag{10}$$

We use here the same notation ($\hat{G}$) for the GF of the Rashba electron and for the GF of the ideal electron. However, this does not lead to confusion since the latter depends always on energy arguments like $E+m\alpha^2+s\Omega_B$, etc.

It is important that the same representation can be obtained for the averaged resolvent of the Rashba system in the SCBA. We restrict ourselves here to an approximation in which the electron self-energy operator is diagonal in spin space. Then, the SCBA equation for $\Sigma^X(E)$ has the following form:

$$\hat{\Sigma}(E)=W\langle\mathrm{Sp}\hat{G}(E)\rangle=\begin{bmatrix}\Sigma_{\uparrow\uparrow}(E) & 0\\ 0 & \Sigma_{\downarrow\downarrow}(E)\end{bmatrix}. \tag{11}$$

Here Sp denotes the trace only over the spatial degrees of freedom; $W=n_IU_0^2$, where n_I is the impurity concentration, U_0 is the magnitude of the pointlike potential of an isolated impurity. Therefore, it is sufficient to make everywhere in Eq. (10) the following substitutions:

$$E\to E-\Sigma_e(E),\quad g\omega_c\to g\omega_c+4\Sigma_o(E) \tag{12}$$

to obtain the desired representations for the averaged GF's in the SCBA. Here $\Sigma_{e(o)}(E)=[\Sigma_{\uparrow\uparrow}(E)\pm\Sigma_{\downarrow\downarrow}(E)]/2$ are the even and odd parts of the electron self-energy. The first ($\Sigma_e=\Delta_e\pm i/2\tau_e$) describes the perturbation (shift Δ_e and broadening $1/\tau_e$) of the one-electron energy levels by a random field. The real part of $\Sigma_o=\Delta_o\pm i/2\tau_o$ defines the renormalization of the Zeeman coupling (12), while its imaginary part, proportional to $1/\tau_o$, makes a contribution to the overall broadening of the one-electron energy levels. As a result, we obtain a expression like Eq. (10) for the averaged GF, where

$$\hat{G}^{R(A)}(E+m\alpha^2+s\Omega_B)=\frac{1}{E+m\alpha^2+s\Omega_B-\mathcal{H}_0\pm\dfrac{i}{2\tau_s}} \tag{13}$$

is the averaged retarded (advanced) GF of the ideal electron, and

$$\Omega_B=\frac{1}{2}(\Omega_B^R+\Omega_B^A),$$
$$\frac{1}{\tau_s}=\frac{1}{\tau_e}-is(\Omega_B^R-\Omega_B^A)=\left(1+s\frac{4m\alpha^2}{\Omega_B}\right)\frac{1}{\tau_e}+s\frac{4\omega_c\delta}{\Omega_B}\frac{1}{\tau_o} \tag{14}$$

are the disorder-modified frequency of the spin precession (9) and the inverse lifetime of an electron in the sth spin-split subband. As usual, we do not take explicitly into consideration in (13) the one-electron energy level shift Δ_e that is absorbed by the normalization condition, but we mean here that the odd shift Δ_o is included in the definition of the effective g factor in accordance with (12). The explicit allowance for the Zeeman coupling renormalization is particularly important in the SdH oscillation regime.

IV. DENSITY OF STATES AND SELF-ENERGY

We first consider the calculation of the DOS $n(E)=\mathrm{Im}\langle\mathrm{Tr}\,\hat{G}^A(E)\rangle/\pi$ using the above-obtained expression for the one-particle GF (10). Here, the symbol Tr denotes the trace over the spatial and spin degrees of freedom. For the sake of simplicity, we shall deal with the case of large filling numbers ($E\gg\omega_c$). Calculating the trace of the resolvent (10) over the spatial and spin degrees of freedom, we obtain the following expression for the DOS:

$$n(E)=\sum_{s=\pm1/2}\frac{m_s}{m}n^{(0)}[E+m\alpha^2+s(\Omega_B\pm\omega_c)]=\sum_{s=\pm1/2}\frac{m_s}{m}n_s^{(0)}(E). \tag{15}$$

Here, we take into account that the DOS of a spinless electron in an orthogonal magnetic field $n^{(0)}(E)$ satisfies $n^{(0)}(E)=n^{(0)}(E\pm\omega_c)$ at large filling factors ($E\gg\omega_c$). The sign before ω_c is chosen in such a way as to ensure the right-hand limit $s(\Omega_B\pm\omega_c)\to\pm sg\omega_c/2$, as the spin-orbit coupling approaches

zero. The effective mass m_s in the sth subband is defined as

$$m_s = m\left(1 + s\frac{4m\alpha^2}{\Omega_B}\right) = m\partial_E(E + s\Omega_B). \tag{16}$$

In the considered case this expression coincides with the usual definition of the transport and cyclotron effective masses in the isotropic nonparabolic band.[22]

In full accordance with the two-subband model, the DOS in Eq. (15) is presented as a sum of partial contributions. Using this expression for the DOS, we can obtain the analytical form of the equation for the electron concentration $n = \int^{E_F} n(E)dE$ which is the normalization condition for the Fermi level determination. So, a more correct expression for the DOS is needed, which adequately describes its behavior not only in the vicinity of the Fermi level E_F (>0), but also near the lower boundary of the spectrum. For example, at $B=0$ we have

$$n(E) = \frac{m}{\pi}\begin{cases} \dfrac{m\alpha}{\sqrt{2mE + m^2\alpha^2}}, & -\dfrac{1}{2}m\alpha^2 < E < 0, \\ 1, & E \geq 0. \end{cases} \tag{17}$$

In the energy interval $-m\alpha^2/2 < E < 0$, the DOS is formed by the states of the lower spin-split subband and has the typical one-dimensional behavior. Integrating (17) between $-m\alpha^2/2$ and E_F, we obtain

$$n = \frac{m}{\pi}(E_F + m\alpha^2) = \frac{m}{\pi}E_0, \quad E_F > 0. \tag{18}$$

Thus, the energy $E_0 = E_F + m\alpha^2$ corresponds to the Fermi level in the absence of SOI. Notice that the partial electron concentrations $n_s = m(E_0 + s\Omega_B)/2\pi$ depend nonlinearly on the Fermi energy, in contrast to n (18). The correction to the Fermi energy in (18) comes from the low-energy tail of the DOS (17). Of course, the difference between E_0 and E_F is small for weak SOI ($m\alpha^2 \ll E_F$). However, as shown below, it is of crucial importance to take it into account for a correct interpretation of the spin-orbit interaction effect on both the conductivity in the absence of a magnetic field, and the SdH oscillations.

The representation (15) allows one to obtain a simple analytical expression for the DOS that holds good up to the quantizing fields region ($\omega_c\tau \gtrsim 1$). Indeed, the DOS of a spinless electron in the large filling factors region ($E \gg \omega_c$) has the form

$$n^{(0)}(E) = \frac{m}{2\pi}\frac{\sinh\dfrac{\pi}{\omega_c\tau}}{\cosh\dfrac{\pi}{\omega_c\tau} + \cos 2\pi\dfrac{E}{\omega_c}}. \tag{19}$$

Inserting Eq. (19) into Eq. (15), we obtain for the oscillating part of the DOS the following expression:

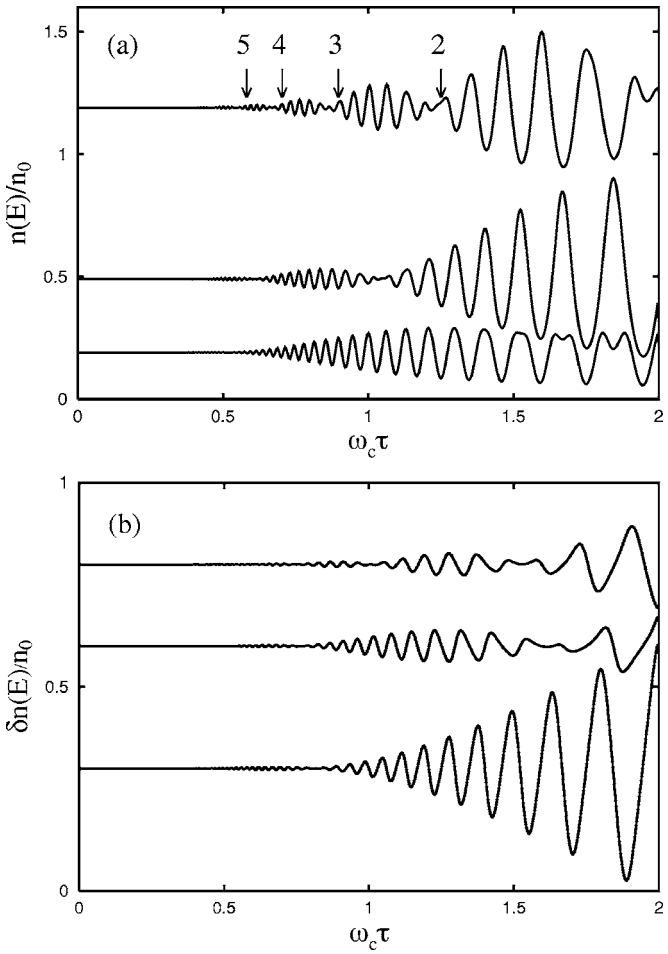

FIG. 1. Plots of the SdH oscillations of the total DOS (a) and of the difference of the partial DOS's (b) of the 2D Rashba system at fixed $g=2.8$ and $k_F l=35.0$, and different $\Omega\tau=3.0$, 1.5, 0.75 (top to bottom). The arrows point the node locations with their numbers $k=2,3,4,5$ that are calculated with Eq. (21).

$$\Delta n(E_F) = \frac{2m}{\pi}\exp\left(-\frac{\pi}{\omega_c\tau}\right) \times\left(\cos 2\pi\frac{E_0}{\omega_c}\cos\pi\frac{\Omega_B}{\omega_c} - \frac{2m\alpha^2}{\Omega_B}\sin 2\pi\frac{E_0}{\omega_c}\sin\pi\frac{\Omega_B}{\omega_c}\right) \tag{20}$$

which is valid in the magnetic field region under consideration. The second term in Eq. (20) appears due to the difference between the effective masses m_s (16).

It follows from (20) that the energy E_0 defines the main period of the SdH oscillations. In other words, in the large-filling-factor region ($E_F \gg \omega_c$) the SdH period is determined by the total electron concentration [see Eq. (18)], regardless of the spin-orbit interaction magnitude. On the other hand, the period of the SdH oscillation beatings (20) [see Fig. 1(a)] defines the spin precession frequency (9) which depends on the magnetic field even in the absence of Zeeman splitting ($g=0$).

In the case of weak SOI ($\Omega \ll E$), the oscillations of the DOS are determined completely by the first term in Eq. (20). Then, the location of the kth node of beatings is determined by the condition

$$B_k = \frac{2mc}{|e|} \frac{\Omega}{\sqrt{(2k+1)^2 - (g-2)^2}}. \quad (21)$$

This limit was considered in Ref. 23. Unlike the results of that work, the above-obtained equations still stand in the case of strong SOI, where it is important to take account of the difference E_0 and E_F for correctly defining the SdH oscillation period. In addition, we have taken into account the Zeeman splitting of the electron spectrum which allows us to describe more correctly the oscillation pattern. For example, Eq. (21) allows us to determine both the spin-orbit, α, and Zeeman, g, couplings by the measured locations of two different nodes [see the top curve in Fig. 1(a)]. On the other hand, the spin precession frequency Ω_B approaches $|\delta|\omega_c$ as the magnetic field B increases. Therefore, in this case a gradual transition from the beatings of the SdH oscillations to the familiar Zeeman splitting of the oscillating peaks should be observed. The beginning of this transition can be seen on the bottom curve in Fig. 1(a).

Another important characteristic of the one-electron states of the 2D Rashba system is the difference of the partial DOS's with opposite spin projections onto the OZ axis,

$$\delta n(E) = n_{\uparrow\uparrow}(E) - n_{\downarrow\downarrow}(E) = -\frac{4\omega_c \delta}{\Omega_B} \sum_{s=\pm 1/2} s n_s^{(0)}(E). \quad (22)$$

This quantity is proportional to the derivative of the transverse spin magnetization with respect to energy E and, therefore, it enters in the definition of the effective concentrations of current carriers in the dissipative part of the 2D Rashba system conductivity in an orthogonal magnetic field (see the next section).

Evidently, the difference of the partial DOS's (22) vanishes in the region of classical magnetic fields ($\omega_c \tau \ll 1$), but it plays an important role in the SdH oscillation regime. In the case of large filling factors, the oscillating behavior of this quantity is described by the following expression:

$$\delta n(E_F) = \frac{2m}{\pi} \frac{2\omega_c \delta}{\Omega_B} \exp\left(-\frac{\pi}{\omega_c \tau}\right) \sin 2\pi \frac{E_0}{\omega_c} \sin \pi \frac{\Omega_B}{\omega_c}. \quad (23)$$

Unlike the total DOS (20), this expression contains just one oscillating term, because $\delta n(E)$ does not depend on the effective masses m_s (16). Indeed, the difference of the partial DOS's $\delta n(E)$ is nonzero, which is entirely due to the spin degrees of freedom of the electrons. The typical SdH oscillation patterns of $\delta n(E)$ are depicted in Fig. 1(b).

Now, let us turn to the discussion of the electron lifetime τ_s in the sth spin-split subband which is defined, according to Eq. (14), by the imaginary parts of the even and odd self-energies $\Sigma_{e(o)}$. In other words, the total lifetime of the one-electron states τ_s is determined by the sum of the weighted relaxation rates of the orbital and spin degrees of freedom. The first term in this expression is proportional to the above-considered total DOS; hence its magnetic-field dependence coincides up to the scale factor with the patterns shown in Fig. 1(a). Of particular interest is the last term in Eq. (14) stemming from the Zeeman coupling renormalization. It is proportional to the difference of the partial DOS's (22) and, therefore, plays an important role in the SdH oscillation regime, as shown in Fig. 2. Notice that the beatings of the SdH oscillations are suppressed with the increase of the relative magnitude of the second term in Eq. (14). Indeed, Eq. (21) determines the location of the beating loops of the oscillation instead of the nodes. Thus, the broadening of the Zeeman levels leads to observable suppression of the beatings of the electron lifetime τ_s oscillations.

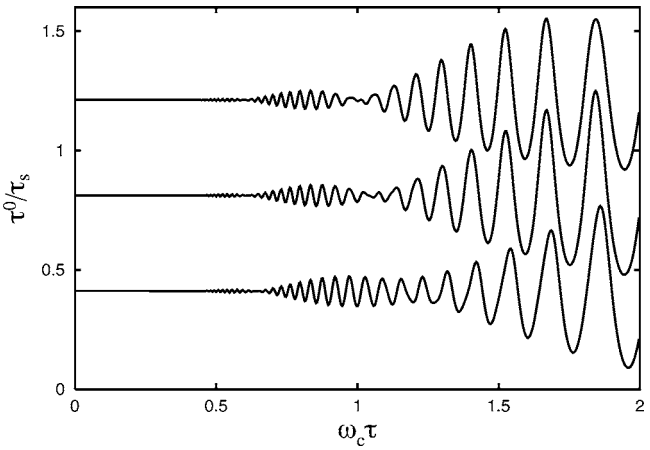

FIG. 2. Plots of the SdH oscillations of the inverse lifetime of one-electron states in the sth spin-split subband at different values of Zeeman factor $g=1.8$, 1.0, 0.2 (top to bottom), and fixed values of $k_F l=35.0$ and $\Omega\tau=1.5$.

V. CONDUCTIVITY

The general expression for the conductivity (5) consists of two different terms. The first of them describes the contribution of the electrons at the Fermi level; the second one contains the contributions of all filled states below the Fermi level. We begin the calculation of the conductivity with the last term of (5), σ^{II}. First of all, it is purely imaginary and, therefore, makes a contribution in the Hall conductivity alone. Středa and co-workers[26] were first to show that, for spinless electrons, this part of the conductivity is equal to

$$\sigma^{II} = i|e|c\left(\frac{\partial n}{\partial B}\right)_{E_F}, \quad (24)$$

where n is the electron concentration. It should be pointed out that Eq. (24) is *exact*, and with the thermodynamic Maxwell relation σ^{II} can be expressed through $(\partial M/\partial E)_B$, where M is the orbital magnetization of the electron gas. Detailed discussion of σ^{II} and its physical interpretation can be found in the survey.[27]

This result is extended immediately to electron systems with SOI. Following Středa *et al.*'s argument, it can be shown that the part σ^{II} of the 2D Rashba system conductivity is expressed as

$$\sigma^{II} = i|e|c\left[\left(\frac{\partial n}{\partial B}\right)_{E_F} - \left(\frac{\partial M_p}{\partial E}\right)_B\right], \tag{25}$$

where M_p is the spin magnetization of the electron gas. It follows that in the general case σ^{II} is determined only by the diamagnetic part of the electron gas magnetization. By direct differentiation of the electron concentration n with respect to the magnetic field induction B we obtain

$$\begin{aligned}\left(\frac{\partial n}{\partial B}\right)_{E_F} &= \frac{n}{B} + \int^{E_F} \mathrm{Tr}\,\frac{\partial}{\partial B}(G^A - G^R)\frac{dE}{2\pi i} \\ &= \frac{n}{B} - \int^{E_F} \mathrm{Tr}\left(\frac{\partial \mathcal{H}}{\partial B}\frac{\partial \hat{G}^A}{\partial E} + \frac{\partial \Sigma^A}{\partial B}\frac{\partial \hat{G}^A}{\partial E}\right. \\ &\quad \left. - \frac{\partial \Sigma^A}{\partial E}\frac{\partial \hat{G}^A}{\partial B} - \mathrm{H.c.}\right)\frac{dE}{2\pi i},\end{aligned} \tag{26}$$

where the symbol H.c. denotes the Hermitian conjugate of the terms. Considering that in the SCBA $\Sigma = W\,\mathrm{Tr}\,\hat{G}/2$, it is easily seen that the expression in large parentheses is the total derivative with respect to energy. As a result, integration by E in (26) is performed explicitly, and after some simple algebra the expression for σ^{II} takes on the form

$$\sigma^{II} = i\frac{|e|c}{B}(n - n_\perp), \tag{27}$$

where

$$n_\perp = \frac{1}{2\pi i}\,\mathrm{Tr}\left(E_0 - \frac{1}{4}g\omega_c\sigma_z\right)(\hat{G}^A - \hat{G}^R). \tag{28}$$

Equation (27) is a generalization of the known SCBA expression for σ^{II} in the case of spinless electrons.[27] In particular, the quantity $n_\perp$ (28) is the counterpart of the familiar parameter $n_\perp = En^{(0)}(E)$ which stands for the current carrier concentration in the dissipative part of the conductivity tensor of spinless 2D electrons in a magnetic field in the SCBA.[20] In the classical (i.e., nonquantizing, $\omega_c\tau \ll 1$) magnetic field region, it is equal to the total electron concentration n.

Now, we turn to the first term in the conductivity (5). It is quite easy to show, by identical transformations, that

$$\frac{e^2}{8\pi}\,\mathrm{Tr}\,V_+(\Phi^{AA}_{EE} + \Phi^{RR}_{EE}) = -\frac{e^2}{4\pi m}\,\mathrm{Tr}(\hat{G}^A + \hat{G}^R). \tag{29}$$

The main contribution to the dissipative part of the conductivity is proportional to the current vertex Φ^{RA}_{EE} in Eq. (5). If we accept the SCBA (11) for the electron self-energy $\hat{\Sigma}$, we must evaluate this part of the conductivity in the ladder approximation in order to satisfy the particle conservation law (see Fig. 3).

Replacing, as a first approximation, $\mathrm{Tr}\,V_+\Phi^{RA}_{EE} \to \mathrm{Tr}\,V_+\hat{G}^R V_-\hat{G}^A$, we obtain the "bare" conductivity

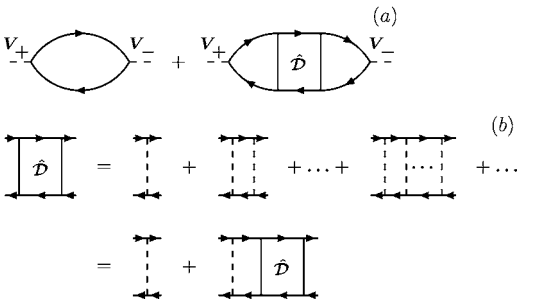

FIG. 3. (a) The diagrams depicting the conductivity in the ladder approximation; (b) the impurity ladder series in the particle-hole channel (diffusion).

$$\sigma^J_{\mathrm{bare}} = \frac{e^2}{4\pi}\,\mathrm{Tr}\left(V_+\hat{G}^R V_-\hat{G}^A + \frac{1}{m}(\hat{G}^A + \hat{G}^R)\right), \tag{30}$$

depicted by the first diagram in Fig. 3(a). In what follows, for simplicity, we neglect everywhere the odd part of the electron self-energy Σ_o (14). In this approximation one can obtain the following equations:

$$\begin{aligned}&\left(\omega_c + \frac{i}{\tau_e}\right)\hat{G}^R V_-\hat{G}^A \\ &\quad = V_-\hat{G}^A - \hat{G}^R V_- + 2i\alpha\left(\sigma_-\hat{G}^A - \hat{G}^R\sigma_- - \frac{i}{\tau_e}\hat{G}^R\sigma_-\hat{G}^A\right), \\ &\left(\omega_c + \frac{i}{\tau_e}\right)\hat{G}^A V_+\hat{G}^R \\ &\quad = \hat{G}^A V_+ - V_+\hat{G}^R - 2i\alpha\left(\hat{G}^A\sigma_+ - \sigma_+\hat{G}^R - \frac{i}{\tau_e}\hat{G}^A\sigma_+\hat{G}^R\right).\end{aligned} \tag{31}$$

Using these relations, we perform a series of transformations of the bare conductivity (30), neglecting the terms that are small in the parameter $1/E_F\tau$ and lying beyond the accuracy of the ladder approximation. Omitting intermediate transformations of purely technical character, the final result can be written as

$$\sigma^J_{\mathrm{bare}} = \frac{e^2\tau_e}{m}\frac{1}{1 - i\omega_c\tau_e}\left(n_\perp - \frac{2m\alpha^2 n_F}{1 - i\omega_c\tau_e}(1 - P)\right), \tag{32}$$

where $n_F = \mathrm{Tr}(\hat{G}^A - \hat{G}^R)/4\pi i$ is the DOS at the Fermi level per spin,

$$P = W\,\mathrm{Tr}\,\sigma_+\hat{G}^R\sigma_-\hat{G}^A. \tag{33}$$

The ladder correction to the conductivity shown by the second diagram in Fig. 3(a) is presented by the following analytical expression:

$$\Delta\sigma^J_{\mathrm{lad}} = \frac{e^2}{4\pi}\,\mathrm{tr}\left[\mathrm{Sp}(\hat{G}^A V_+\hat{G}^R)\hat{\mathcal{D}}\,\mathrm{Sp}(\hat{G}^R V_-\hat{G}^A)\right]. \tag{34}$$

The bare current vertices $\mathrm{Sp}(\hat{G}^{A(R)}V_\pm\hat{G}^{R(A)})$ involved in (34) can be expressed by the parameter P (33) and the relaxation

time τ_e, using relations (31). In the main order in the small parameter $1/E_F\tau$ they have the form

$$\mathrm{Sp}(\hat{G}^{A(R)}V_{\pm}\hat{G}^{R(A)}) = \mp i\sigma_{\pm}\frac{4\pi\alpha n_F\tau_e}{1-i\omega_c\tau_e}(1-P). \tag{35}$$

The diffusion $\hat{\mathcal{D}}$ depicted in Fig. 3(b) by a ladder series in the particle-hole channel is a 4×4 matrix in the representation of the total spin of the electron-hole pair. The Bethe-Salpeter equation for $\hat{\mathcal{D}}$ [see the second row in Fig. 3(b)] has the same structure. However, as seen from (34) and (35), the contribution to $\Delta\sigma^I_{\mathrm{lad}}$ comes from the scalar quantity $\mathcal{D}=\mathrm{tr}\,\sigma_+\hat{\mathcal{D}}\sigma_-$ which is the projection of the diffusion on the triplet state $|1,-1\rangle$ of the electron-hole pair. Projecting the Bethe-Salpeter equation on this state, we obtain a closed scalar equation for $\mathcal{D}$ with solution of the form

$$\mathcal{D} = \frac{W}{1-P}, \tag{36}$$

where P is defined in (33).

Substitution of (36) and (35) into (34) yields the expression for the ladder correction to the conductivity which exactly cancels the second term in the large parentheses in (32). As a result, the final expression for the conductivity of the Rashba system in a transverse magnetic field in the ladder approximation takes on the form

$$\sigma = i\frac{|e|c}{B}\left(n - \frac{n_{\perp}}{1-i\omega_c\tau_e}\right). \tag{37}$$

Equation (37) looks as if the current were generated by charge carriers of one type with mobility $\mu=|e|\tau_e/m$ and concentration n. This would be expected, because the conductivity tensor in the absence of a magnetic field is diagonal in the original spin space ($\sigma_{\uparrow\downarrow}=\sigma_{\downarrow\uparrow}\equiv0$) by virtue of the momentum parity of the GF's, and the full conductivity is equal to $\sigma=\sigma_{\uparrow\uparrow}+\sigma_{\downarrow\downarrow}$.[28] In the classical (nonquantizing) magnetic fields this property is retained, since the difference of the partial DOS's with opposite spin projections onto the OZ axis (22) is equal to zero in this region (see Fig. 1).

Of course, the simple structure of Eq. (37) for the conductivity breaks down in the region of sufficiently strong magnetic fields, $\omega_c\tau\gg1$, where the contribution $1/\tau_o$ to the inverse lifetime of the one-electron states (14) cannot already be neglected. As shown in the previous section, this may lead to flattening of the SdH oscillation beatings (see Fig. 2). Therefore we believe the approximation used in this section, $1/\tau_o=0$, to be correct in the magnetic field region where well-defined beating nodes of magneto-oscillation are observed.

VI. RESULTS AND DISCUSSION

First of all, let us summarize briefly the main results obtained in this work. We have shown that the eigenstates of the 2D Rashba electron in an orthogonal magnetic field are characterized by a special motion integral (3) that generalizes the notion of *helicity*.[11] Using this fact, we have found the relation (10) between the GF's of the 2D Rashba electron and the ideal one that holds good for arbitrary orthogonal magnetic fields as well as for strong spin-orbit coupling. With the help of this relation, we have obtained, in contrast to Refs. 14–16, analytical expressions for the DOS in the SCBA (15) and for the magnetoconductivity in the ladder approximation (37) of the 2D Rashba system that are valid in a wide range from classical magnetic fields up to quantizing ones ($\omega_c\tau\gtrsim1$). The spin-orbit as well as the Zeeman splitting of the electron energy are properly allowed for in these expressions, unlike the results of Refs. 23 and 24. In particular, we have obtained a simple expression (21) for the node locations of the SdH oscillation beatings. We have shown that the competition of the relaxation rates of the orbital and spin degrees of freedom in the total inverse lifetime $1/\tau_s$ of the one-electron states in the sth subband leads to the partial suppression of beatings of the $1/\tau_s$ SdH oscillations.

We start the discussion of the results with the conductivity in the classical magnetic field region ($\omega_c\tau_e\ll1$). In this case, it follows immediately from Eqs. (28) and (37) that the conductivity of a 2D Rashba system takes the usual Drude-Boltzmann form

$$\sigma = \frac{\sigma_D}{1-i\omega_c\tau_e}, \tag{38}$$

where

$$\sigma_D = \frac{e^2\tau_e}{\pi}(E_F+m\alpha^2) = \frac{e^2n\tau_e}{m} \tag{39}$$

is the Drudian conductivity in the absence of a magnetic field; $\tau_e=1/mW$ is the lifetime of a one-electron state at $B=0$. It immediately follows that in the ladder approximation the classical magnetoresistance of a 2D Rashba system is zero, $\rho(B)=\rho_D=1/\sigma_D$, and the Hall coefficient $R_H=-1/|e|nc$.

Thus, in the ladder approximation the Rashba spin-orbit interaction has no effect at all on the conductivity magnitude everywhere over the region of classical magnetic fields, $\omega_c\tau_e\ll1$ (including the case of $B=0$). Note that the first relation for the Drudian conductivity (39) formally coincides with that obtained in Ref. 28. However, as mentioned above, the correction $m\alpha^2$ to the Fermi energy does not lead to an observable change in σ, since it is absorbed by the normalization condition (18).

Let us proceed now to the discussion of the magnetotransport in the 2D Rashba system in the large filling factors ($E\gg\omega_c$) region, where the SCBA and the ladder approximation are applicable to the description of the one-electron states and kinetic phenomena, respectively. As usual, we extract in the linear approximation the oscillating parts of the conductivity that enter through DOS into the effective concentration $n_{\perp}$ (28) and mobility μ. As in the ladder approximation the conductivity (37) has the form characteristic of a conductor with one type of charge carriers, we can immediately use the expression obtained in Ref. 25 for the oscillating parts of the longitudinal resistance ρ and the Hall coefficient R_H,

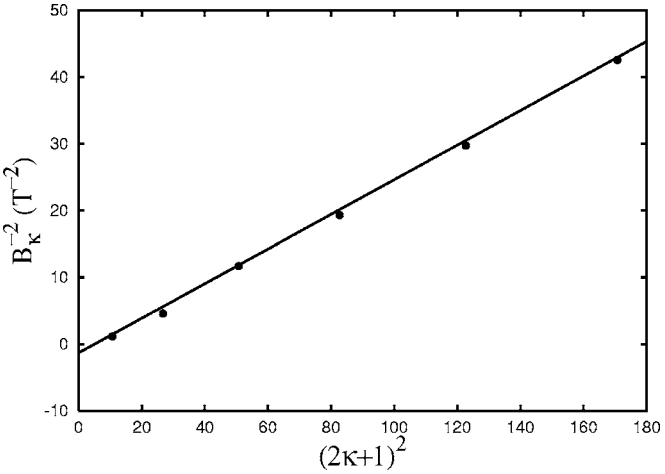

FIG. 4. Measured locations B_k of the SdH oscillation beating nodes as compared to the behavior predicted by Eq. (21). The points correspond to the values B_1=0.873 T, B_2=0.460 T, B_3=0.291 T, B_4=0.227 T, B_5=0.183 T, and B_6=0.153 T, measured for the $In_{0.65}Ga_{0.35}As/In_{0.52}Al_{0.48}As$ heterostructure (Ref. 19). The straight line is the result of least–squares fitting of Eq. (21).

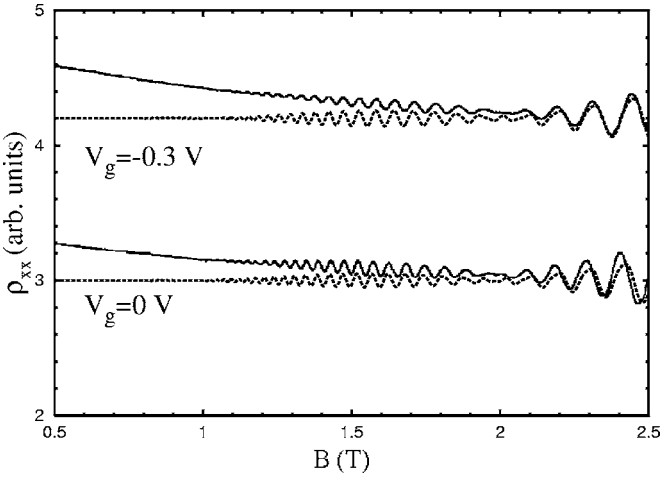

FIG. 5. The theoretical curves of the ρ magnetooscillation as compared with the measured (Ref. 18) results for the $In_xGa_{1-x}As/In_yAl_{1-y}As$ heterostructure at gate voltages V_g=0 and −0.3 V. The experimental data are denoted by the solid lines and the theoretical ones by dashed lines.

$$\frac{\Delta\rho(B)}{\rho_0} = 2\frac{\Delta n(E_F)}{n_F^{(0)}}, \tag{40a}$$

$$\frac{\Delta R_H(B)}{R_H^0} = \frac{1}{\omega_c^2\tau_e^2}\frac{\Delta n(E_F)}{n_F^{(0)}}. \tag{40b}$$

Here, $\rho_0=1/\sigma_D$ and $R_H^0=-1/|e|nc$ are the resistance [see Eq. (39)] and Hall coefficient in zero magnetic field, respectively. Thus, in the linear approximation the SdH oscillations ρ and R_H are entirely determined by the magnetic–field dependence of the DOS, $\Delta n(E_F)$, the first harmonics of which are of the form (20). From this we can draw two conclusions that are of great importance for an adequate interpretation of the SdH oscillation pattern in systems with spin-orbit interaction.

First, the period of the SdH oscillations of ρ and R_H is defined by energy $E_0=E_F+m\alpha^2$, and not by the Fermi energy E_F, as was stated in Refs. 23 and 24. From this and the normalization condition (18) it follows that the SdH oscillation period is related to the total charge carrier concentration n by the well-known formula

$$\Delta\left(\frac{1}{B}\right) = \frac{|e|}{\pi cn}, \tag{41}$$

which holds true irrespective of the magnitude of the spin-orbit interaction constant.

Second, the SdH oscillation beating period is defined by the magnetic-field-dependent spin precession frequency $\Omega_B=\sqrt{\Omega^2+(g-2)^2\omega_c^2/4}$ [see Eq. (9)]. From this follows a simple equation for the beating node location B_k (21) that allows the constants α and g to be found from the measured B_k values. This is illustrated in Fig. 4, which presents the results of least-squares fitting of Eq. (21) in variables B^{-2}, $(2k+1)^2$ to the measured[19] locations of the SdH oscillation beating nodes of the longitudinal resistance of an $In_{0.65}Ga_{0.35}As/In_{0.52}Al_{0.48}As$-type heterostructure. The slope of the fitted straight line and the point of its intersection with the ordinate axis yield the values Ω=2.47 meV and g=4.25 for the spin precession frequency in the absence of magnetic field and the g factor, respectively. This values are in good agreement with the results Ω=2.46 meV and g=4.4±0.2 obtained in Ref. 19.

It should be stressed that $\Omega_B=\Omega\approx 2k_F\alpha$ only at g=2. Only in this case are the beating node locations strictly periodic in the reverse magnetic field and are described by the condition $2k_F\alpha=(k+1/2)\omega_c$ obtained in Refs. 23 and 24.

Using Eq. (37), we have performed numeric analysis of the SdH oscillations of the longitudinal resistance ρ (Fig. 5) and the Hall coefficient R_H (Fig. 6) for parameters (g=4, E_0=108.93 meV, Ω=5.13 meV, and g=3.8, E_0=98.85 meV, Ω=5.59 meV) corresponding to the gate voltages V_g=0 and −0.3 V for the $In_xGa_{1-x}As/In_yAl_{1-y}As$ het-

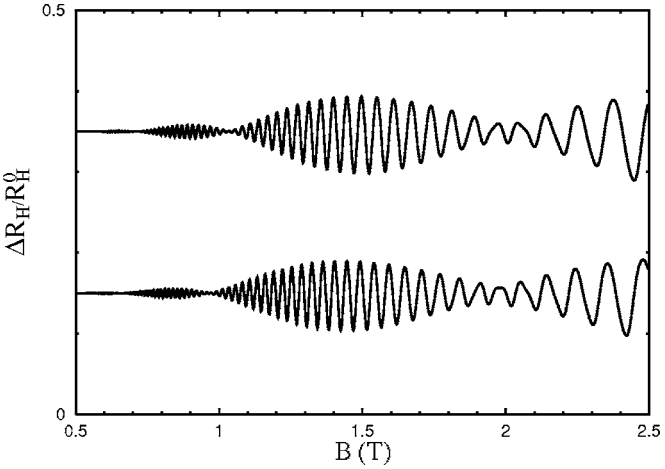

FIG. 6. Plots of the SdH oscillations of the Hall coefficient of the 2D Rashba system calculated for the parameters corresponding to the $In_xGa_{1-x}As/In_yAl_{1-y}As$ heterostructure (Ref. 18) for gate voltages V_g=0 and −0.3 V (from the bottom upward).

erostructure investigated in Ref. 18. The parameters g and Ω were calculated from the positions of two successive nodes B_1 and B_2, using Eq. (21), while E_0 was adjusted to the SdH oscillation period. At $V_g=0$ V, the calculated value $\Omega=5.13$ meV is close to the result $\Omega=5.4$ meV of Ref. 18, whereas at $V_g=-0.3$ V, the calculated value $\Omega=5.59$ meV is in excellent agreement with the value $\Omega=5.6$ meV obtained in Ref. 18. The results of ρ calculation are compared in Fig. 5 to the experimental curves from Ref. 18. It can be seen that the theoretical results reproduce well the period and beating node location of the measured magnetoresistance oscillations. Some difference in oscillation amplitude is due to the fact that the temperature smearing of the Fermi level was not taken into account in our analysis for simplicity. The negative magnetoresistance observed in Ref. 18 lies outside the ladder approximation.

In conclusion it should be stressed once more that the results of this work have been obtained with rigorous account taken of both the spin-orbit and the Zeeman splitting of the energy levels. Up till now this was done by using numeric analysis only.[14,16,17] Our results reproduce quantitatively all the peculiarities of the magneto-oscillation curves obtained in this way for models with Rashba spin-orbit interaction,[14,17] except for the anomalously large positive magnetoresistance obtained in Ref. 14. Our conclusion about the absence of positive magnetoresistance is in drastic contradiction with Ref. 14, but agrees with the results of analytical[23,24] and recent numerical[17] studies.

ACKNOWLEDGMENTS

We thank A. K. Arzhnikov, A. V. Germanenko, G. I. Kharus, G. M. Minkov, and V. I. Okulov for helpful discussions of results of this work. This work was supported by the RFBR, Grant No. 04-02-16614.

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