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Use of dynamical coupling for improved quantum state transfer

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We propose a method to improve quantum state transfer in transmission lines. The idea is to localize the information on the last qubit of a transmission line by dynamically varying the coupling constants between the first and the last pair of qubits. The fidelity of state transfer is higher then in a chain with fixed coupling constants. The effect is stable against small fluctuations in the system parameters.

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Efficient short-distance quantum state transfer is an important problem in the field of quantum computing. One of the most promising solutions is to use chains constructed from qubits that are statically coupled to each other. The idea to use quantum spin chains was initially put forward by Bose[1] and then developed in a number of papers. These proposals exploit the unitary time evolution governed by the system Hamiltonian. The state is initialized/encoded at the sender part of the chain and then, after a certain time, measured/decoded at the receiving part of the chain. The major advantage of this method is its simplicity: it does not require controllable coupling constants between the qubits or complicated gating schemes. It was shown[1] that for short-length chains the fidelity of state transfer is high, i.e., close to one. But the fact that it is substantially reduced with the length of the chain triggered the search of methods that allow one to increase the fidelity or even to obtain perfect state transfer, in the absence of decoherence and relaxation processes.

The main reason for imperfect transfer is the dispersion of the initial information over the whole chain. Therefore it was proposed to use spatially varying coupling constants to "refocus" the information at the receiving part of the chain.[2–4] Another possibility is to encode the information in Gaussian wave packets (with low dispersion) spread over several spins.[5] Chains where the first and the last qubits are only weakly coupled to the rest of the chain provide a very high fidelity,[6] because the intermediate spins are only slightly excited, which means that dispersion is small. This method has the major disadvantage that the time required for the transfer is long compared to the qubit decoherence/relaxation times in present experimental setups. The idea of so-called conclusive transfer, providing perfect state transfer using parallel quantum channels,[7,8] is very promising. It can be realized using almost any spin chain and it is stable against fluctuations of the chain parameters.[9]

Almost all the proposals mentioned above have one common disadvantage: the time interval for which the fidelity is high is very small for physical qubits and realistic qubit coupling parameters. For example, for a chain of flux qubits[10] with realistic experimental parameters,[11] the half-width of the first fidelity maximum is about 0.2 ns. At these time scales state readout and manipulation is impossible using current experimental technology. Here we show that by dynamically varying the coupling constants only between the first and the last pair of qubits we can solve this problem and also increase the fidelity of state transfer.

In real chains the state to be transmitted is initialized in the first qubit, and this process must not influence the fidelity and dynamics of the chain. The most natural idea for a full transferring protocol is as follows: initialize the state in the first qubit, that is decoupled from the rest of the chain, then adiabatically couple it, wait a certain time and then adiabatically decouple the last qubit from the chain. This method requires two controllable gates like one of the proposals for achieving perfect state transfer.[12] In this paper, the main purpose of the gates is to localize the state on the last qubit where it can be manipulated during times that are comparable to the decoherence/relaxation times.

In the following we use the terms spin and qubit as equivalent. State $|1\rangle$ in qubit language (which we will also call "excitation") corresponds to spin up in spin language, and state $|0\rangle$ corresponds to spin down.

We consider the *XXZ* Hamiltonian with time-dependent coupling constants between the first and the last pair of qubits:

$$H(t) = -J_{xy1}(t)(\sigma_2^+\sigma_1^- + \sigma_2^-\sigma_1^+) - J_{xy}\sum_{i=3}^{N-1}(\sigma_i^+\sigma_{i-1}^- + \sigma_i^-\sigma_{i-1}^+) - J_{xyN}(t)(\sigma_N^+\sigma_{N-1}^- + \sigma_N^-\sigma_{N-1}^+) - J_z\sum_{i=2}^{N}\sigma_i^z\sigma_{i-1}^z - B\sum_{i=1}^{N}\sigma_i^z. \quad (1)$$

This type of Hamiltonian or some of its special cases is used in most of the papers mentioned above. The *XX* part of the Hamiltonian describes the tunneling of the excitation from one site to another and is a necessary requirement for quantum state transfer.

The physical systems described by this type of Hamiltonian include Josephson arrays of charge[13] and persistent-current[10,14] (flux) qubits, connected by Josephson-junctions/capacitors. The time-dependent coupling constants can be realized by varying the gate voltages on the first/second and $(N-1)$th/Nth qubits for the flux qubit chain, or by replacing the Josephson junction between the charge qubits with a superconducting quantum interference device (SQUID) and varying the flux through it.

As a model we use "Fermi-functionlike" coupling constants:

$$J_{xy1}(t) = J_{xy}f(t_i,t),$$

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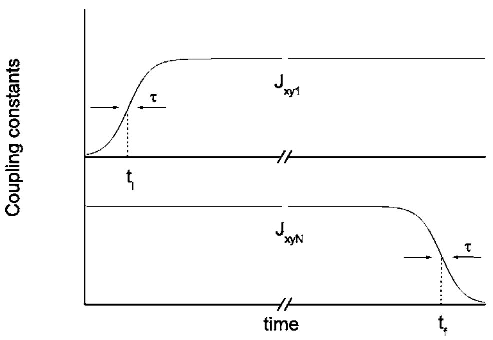

FIG. 1. Coupling constants J_{xy1} and J_{xyN} as functions of time and coupling parameters.

$$J_{xyN}(t) = J_{xy} f(t,t_f), \tag{2}$$

with

$$f(t,t') = \frac{1}{1 + \exp \dfrac{t-t'}{\tau}}. \tag{3}$$

These are smooth functions that vary from 0 (no coupling) to J_{xy} (full coupling) and vice versa, see Fig. 1. The time scale of the coupling/decoupling procedure is determined by τ. Instant coupling/decoupling corresponds to $\tau = 0$.

Our goal is to calculate the fidelity of state transfer, the quantity that characterizes the quality of the transmission line. We assume that the chain is initialized in the state $|00\cdots00\rangle$. Then, the first qubit is prepared in the state $|\psi_{in}\rangle$, i.e., the total state of the array is $|\psi_{in},00\ldots00\rangle$. This is not an eigenstate of the Hamiltonian (1), therefore the system will evolve in time. After a time t the state of the last qubit is read out. Following Bose,[1] we average the fidelity over all pure input states on the Bloch sphere, $F(t) = \int \langle \psi_{in} | \rho_{out}(t) | \psi_{in} \rangle d\Omega / (4\pi)$ to obtain a quantity $1/2 \leqslant F(t) \leqslant 1$ that measures the quality of transmission independent of $|\psi_{in}\rangle$. Here ρ_{out} is the reduced density matrix of the last qubit. Fidelity 1 corresponds to perfect state transfer.

By numerically solving the Schrödinger equation for the time-dependent Hamiltonian (1) we get the fidelity of the state transfer as a function of time and the coupling parameters τ, t_i, and t_f. The fidelity has a complex oscillating behavior. Our goal is to find the coupling parameters that allow us to localize the state at the last qubit by decoupling it from the rest of the chain such that the fidelity is maximal. In comparing this fidelity with the static case, we concentrate on the first maximum: higher maxima appear only after times much longer than the time at which the first one occurs.[10,13] The typical behavior of $F(t)$ for the static chain in the vicinity of the first maximum is shown in Fig. 2 (dashed line).

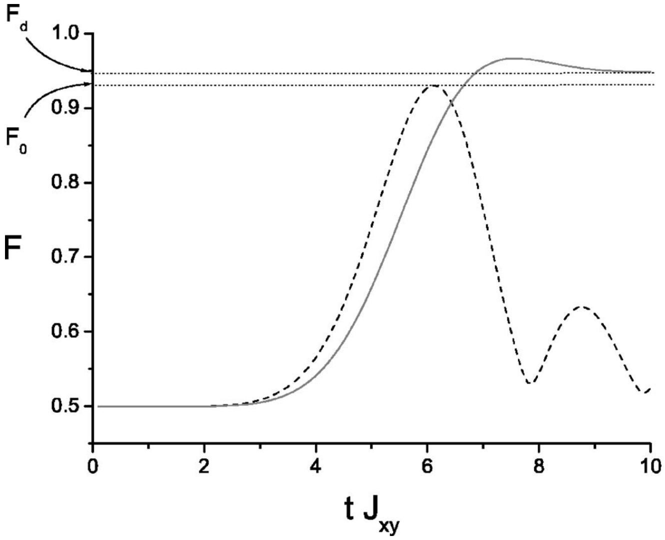

FIG. 2. Fidelity as a function of time (in units of J_{xy}^{-1}) for a chain with constant coupling parameters (dashed line) and time-dependent coupling parameters (solid line), $N=10$, $t_i=0$, $t_f=6.2/J_{xy}$, $\tau=1/J_{xy}$.

Figure 2 also shows the fidelity in the presence of time-dependent coupling constants (solid line). At large times, the state is localized at the last qubit with a fidelity F_d that is higher than for static coupling constants. The time at which the maximum is achieved is slightly larger. This is natural since in the presence of the coupling/decoupling procedure the transmission of the information from the first qubit to the chain and then to the last qubit is slower. After decoupling, the localized state can be manipulated during a time interval comparable with the decoherence and relaxation times for the qubit, which are several orders of magnitude longer than the half-width of the first fidelity maximum in the static case in present experimental setups. We would like to mention that the first fidelity maximum in the case of dynamical coupling constants is even higher than the stationary value of the fidelity after decoupling. Numerical calculations show that it can exceed the value 0.99 (but, in this case, after the full decoupling the fidelity will go down to about 0.9).

Figure 3 shows the fidelity of the state transfer after completely decoupling the last qubit from the rest of the chain for $t \to \infty$ as a function of the parameters τ and t_f (for $t_i=0$). There is a region where the fidelity for the localized state is higher than in the time-independent case (up to 4%).

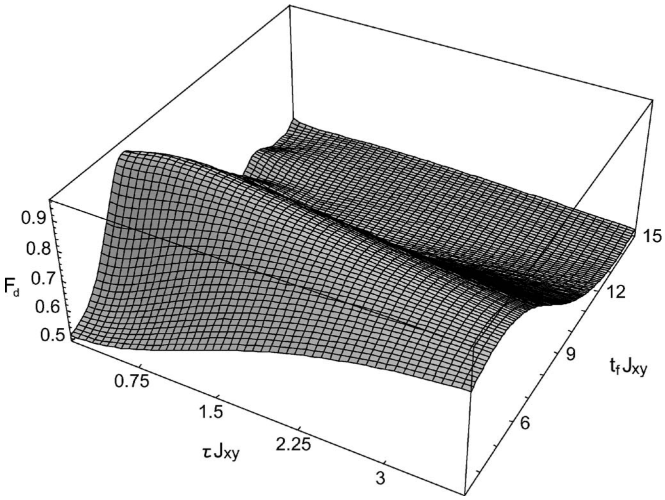

FIG. 3. Stationary value of the fidelity after decoupling as a function of τ and t_f, $N=10$, $t_i=0$.

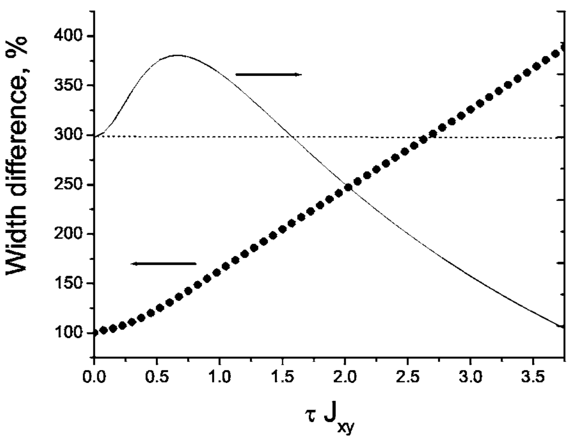

FIG. 4. Dots: relative increase of the width of the first fidelity maximum in Fig. 3. Solid line: fidelity of the maximum compared to F_0.

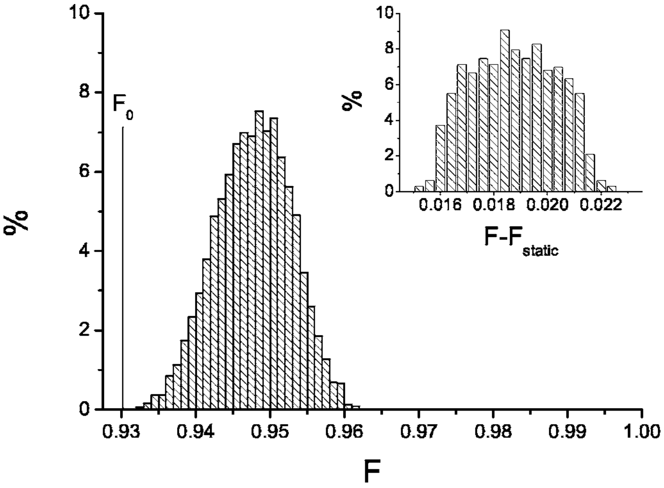

FIG. 5. Fidelity distribution in the presence of small disorder in the coupling constants J_{xy}, $N=10$, $t_i=0$, $\tau=0.325/J_{xy}$, $t_f=6.2/J_{xy}$. F_0 is the first fidelity maximum for the ideal chain with static coupling constants. Inset: distribution of the fidelity difference between the dynamical and statical cases in the presence of equal disorder.

The origin of this phenomenon is similar to the effect described in Ref. 12. By dynamically varying the coupling constant between the first and the second qubit, the information about the state enters the chain as a wave packet that has small dispersion. This corresponds to some sort of filtering, an interpretation in agreement with the fact that the fidelity is higher in the case of equal "profiles" for the coupling and decoupling functions. If we use dynamical decoupling only at the end of the chain and employ instant coupling to initialize the chain, the maximal possible fidelity for a chain of $N=10$ qubits drops from about 0.99 to 0.95 (but it is still higher than the fidelity for the time-independent case, which is around 0.93). Apparently, during the dynamical decoupling, the information that is still dispersed in the chain will arrive at the last qubit. Therefore, slow decoupling allows more information to be gathered before the full decoupling occurs.

Figures 3 and 4 also show that adiabatic coupling requires a less precise definition of t_f to achieve the same quality of the state transfer, compared to instantaneous coupling.

Experimental qubit arrays are always inhomogeneous, so in the rest of the paper we will discuss the effect of static disorder in J_{xy} and dynamical fluctuations in the coupling/decoupling functions. For charge qubit arrays, the most important source of inhomogeneity is the variance of the Josephson energies of the junctions (about 5%). In the case of the flux-qubit chain with capacitive coupling, J_{xy} is a complicated function of the Josephson and charging energies as well as the capacitance of the coupling capacitor, see Ref. 11. A rough estimate using realistic parameters leads to a variance of 10%.

We have performed numerical simulations to evaluate the time evolution of the system. As a result we find that the phenomena described above, are stable to static disorder and dynamical fluctuations in the coupling functions, see Figs. 5 and 6. Figure 5 shows the distribution of the fidelity after complete decoupling in the presence of disorder in the coupling constants. Its half-width is quite small: even in the worst case the fidelity is higher than the fidelity of the ideal chain without disorder. The graph was constructed using a numerical simulation for an ensemble of 10 000 chains where the coupling constants were of the form $J_{xyi}\rightarrow J_{xyi}(1+r_i)$, $i=1\cdots N$. The quantity r_i was a random number with uniform distribution in the interval [0; 0.07].

The inset of Fig. 5 shows the difference between the fidelities for different realizations of the chains with constant and time-dependent couplings. This difference is around 2%, so the effect of increased fidelity persists. In each realization both chains have the same randomized coupling constants and the only difference is that J_{xy1} and J_{xyN} are not multiplied by coupling functions for the time-independent chain.

Figure 6 shows the influence of fluctuations in the coupling/decoupling functions. Here the coupling constants J_{xy} are the same for all realizations and the coupling/decoupling functions are of the form

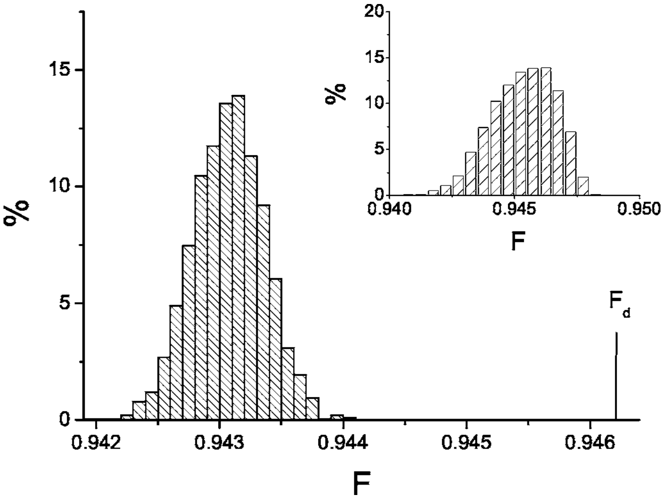

FIG. 6. Fidelity distribution in the presence of fluctuations in the coupling/decoupling function, all other coupling constants are fixed and equal. $t_i=0$, $\tau=0.325/J_{xy}$, $t_f=6.2/J_{xy}$. F_d is the fidelity after decoupling in the absence of fluctuations. Inset: fidelity distribution in the presence of site energy fluctuations ($\delta B=5\%$).

$$J_{xy1}(t) = J_{xy}\left(1 + \exp\frac{t_i - t}{\tau}\right)^{-1}(1 + r_1(t)),$$

$$J_{xyN}(t) = J_{xy}\left(1 + \exp\frac{t - t_f}{\tau}\right)^{-1}(1 + r_N(t)). \tag{4}$$

The quantities $r_{1,N}(t)$ are stepwise stochastic processes of step width 0.036τ, the step heights are uniformly distributed in the interval $[0; 0.02]$. The influence of these fluctuations is small. The fidelity in the presence of dynamical fluctuations in the coupling functions is always decreased. This is in agreement with the filtering idea described above.

The inset of Fig. 6 shows the influence of fluctuations in the site energies. This influence is small, because assuming that B is chosen to maximize the average fidelity, the fluctuations of B will influence only one term in the fidelity as a multiplicative factor that is approximately equal $\cos(\delta B)$, see [1].

Finally, to check that all the effects described above are not the consequence of our special choice of coupling functions (2), we also did the calculation for another type of dynamical coupling/decoupling:

$$J_{xy1} = \begin{cases} 0 & t < 0, \\ J_{xy}(t/\tau)^a & t \in [0,\tau] \\ J_{xy} & t > \tau \end{cases} \tag{5}$$

$$J_{xyN} = \begin{cases} J_{xy} & t < t_f \\ J_{xy}((t_f - t)/\tau + 1)^a & t \in [t_f, t_f + \tau] \\ 0 & t > t_f + \tau. \end{cases} \tag{6}$$

These functions vary from 0 to J_{xy} (and vice versa), and we have chosen $t_i=0$. The parameters a and τ describe the shape and time scale of the coupling/decoupling function. The first maxima of the fidelity for different $a \in [0.1;1]$ are shown in Fig. 7. Here, as in Fig. 6, τ and t_f are chosen to maximize the fidelity. One can see that this type of dynamical coupling also allows us to have better state transfer than for the chain with constant couplings (where the height of the first maximum is F_0). In general, wave packets with bigger width have lower dispersion. Therefore we expect that every smooth monotonic coupling/decoupling function with equal profiles will allow us to improve the fidelity of state transfer.

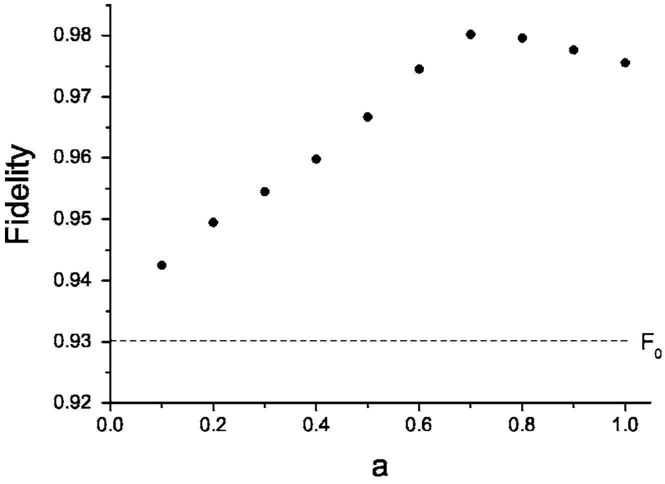

FIG. 7. Fidelity maxima in the case of coupling functions parameterized as $J_{xy}((t)/\tau)^a$, $J_{xy}((t_f-t)/\tau+1)^a$.

In the past, a number of quantum transmission line systems was proposed to achieve a perfect or almost perfect state transfer. A common disadvantage of most of these proposals is the very short time interval, for which the fidelity of the state transfer is high. Manipulating the state in such short time intervals is impossible using current experimental technology. In this paper we have proposed a method that allows one to localize the transferred state on the last qubit of the transmission line, by varying the coupling constants between the first and the last pair of qubits. We have also shown that this method increases the fidelity of the state transfer and that this effect is stable to static disorder in the coupling constants and dynamical fluctuations in the coupling/decoupling functions. We would also like to mention, that applying a sequence of coupling/decoupling pulses may lead to an even better fidelity.[15]

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