Noise as a tool for tracing effects of nonclassical correlations in a degenerate nonequilibrium electron gas

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It is shown that nonclassical interelectron correlations modify the current fluctuation spectrum of a degenerate nonequilibrium macroscopic system: quantum correlations of occupancies of single-electron states are created by interelectron collisions under the Pauli constraints, presenting the electronic analog of Hanbury Brown and Twiss effect (correlation of intensities, not of phase). Calculation of the microwave electronic noise in a biased two- or three-dimensional degenerate channel, performed in quasiclassical approximation, demonstrates the modifications introduced by nonclassical correlations enabling one to trace the effect on the macroscopic level.

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In the literature it is taken for granted that the quantum-correlation effects could be traced exclusively in mesoscopics.[1–3] We propose a method for detection of a nonclassical effect of correlation of intensities of microscopic electronic fluxes in a biased macroscopic conductor. The method is based on the measurement of electronic noise power.

We show that the effect of correlation can be extracted from, say, microwave noise characteristics of the conductor under moderate- to high-field conditions. The method does not need any special scheme by which correlations are generated: under the Pauli constraints, the correlations of occupancies of single-electron states in a degenerate nonequilibrium electron gas are created by interelectron collisions.

In a degenerate electron system, each electron-electron (*e*-*e*) collision creates a two-electron state compatible with the Pauli principle, in that sense—correlated. Generated by the collision event, the correlation survives as long as the processes of transfer of energy from the electron system to the lattice allow. Respectively, correlations of this type contribute to the spectral intensity of current fluctuations at frequencies of the order of the inverse energy relaxation time or lower.

As a result, a rather unusual contribution to the noise characteristics of the conductor emerges, provided that the degenerate system of electrons is sufficiently far from equilibrium. In nonequilibrium, the input of the Pauli constraint induced correlations into the observables such as current fluctuations and noise temperature is of the same order of magnitude as the contribution of the well-known classical additional (kinetic) correlation existing in the nondegenerate nonequilibrium gas.[4–6] Far from equilibrium both contributions do not contain any small parameter and thus are readily detectable, the desired instrument being, e.g., X-band waveguide-type pulsed setup, see Ref. 7, Chap. 8.

Our aim is to show that the current (or voltage) noise spectrum of a degenerate macroscopic nonequilibrium electron system contains a part sensitive to a genuine two-particle effect even if the average current is insensitive to it. Fascinating is the possibility for quantum correlation to reveal itself on the macroscopic level, namely, in the current noise spectrum of a degenerate nonequilibrium collisional electron system.

A possibility to observe the existence of Pauli constraint induced correlations in a degenerate nonequilibrium electron gas by measuring the spatially homogeneous electronic noise in a biased macroscopic conductor containing degenerate electron gas, to all appearance, has not been perceived up to now.

The investigation is performed on the basis of the kinetic theory of quasiclassical fluctuations in degenerate electron gas.[8,9] The theory, valid supposing that the fluctuation time scale of interest is large as compared to the time of formation of single-electron quantum states, predicts appearance, at nonequilibrium, of extraordinary terms in the expressions for the sources of correlation of state occupancies. In turn, these terms—let us call them Pauli source terms or Pauli sources—produce inputs into the equal-time correlation. The structure of the Pauli sources displays nonlocality[2] of *e*-*e* interaction, being caused by Pauli principle and revealing itself in fluctuation phenomena (two-particle effects) even in the quasiclassical limit. More precisely, interelectron collisions in nonequilibrium degenerate gas create quantum correlations, these being displayed in a two-particle phenomenon—in a measurable noise. The effect is based on the intensity correlation—on nonlocality, rather than on phase correlation: the equations for two-particle correlators successively derived from the first principles in the quasiclassical approximation contain the effect of nonlocality. The effect in fact is the electronic version[3] of the Hanbury Brown and Twiss effect[10] (HBT).

At first sight it could seem that the delicate quantum aspects of the electron dynamics could not be approachable in the framework of the Boltzmann-type equations for occupation correlators. However, the quasiclassical equations for time-displaced and equal-time correlators of occupancies of single-electron states in the degenerate weakly interacting system of electrons derived by Kagan[8] and Muradov[9] from the first principles of quantum statistics contain the terms we refer to as Pauli sources. In the framework of the consequent quasiclassical approach, side by side with the classical kinetic correlations, the Pauli cross-correlations emerge, being created by interelectron collisions.

The equation for one-particle distribution obtained in quasiclassical approximation is none other than the usual Boltzmann equation with Pauli principle taken into account by the factors $(1-n)$. The latter can easily be interpreted in the framework of the naîve "semiclassical" approach. However, the intuitive desire simply to substitute $n(1-n)$ for n (this would be sufficient for one-particle problems) would lead in the case of two-particle problems to a mistake, namely, to the loss of substantial quasiclassical terms. Two-particle phenomena, contrary to one-particle ones, already in the quasiclassical approximation prove to be sensitive to nonlocality created by Pauli constraints.

Correlation in nonequilibrium. In nonequilibrium, *e-e* collisions create so-called additional correlation of occupancies of single-electron states, modifying the fluctuation phenomena[4,5] (see also Ref. 6, Sec. 20). For Boltzmann statistics, a net simultaneous flux in the quasimomentum space produced by *e-e* collisions, responsible for creation of equal-time correlation of occupancies of single-electron states $\mathbf{p}$ and $\mathbf{p}'$—the correlating flux—is given by the expression:[4,5]

$$-I^{\mathbf{pp}'} \equiv -\sum_{\mathbf{q}} I^{\mathbf{pp}'}_{\mathbf{p-q},\mathbf{p}'+\mathbf{q}} = -\sum_{\mathbf{q}} W^{\mathbf{pp}'}_{\mathbf{p-q},\mathbf{p}'+\mathbf{q}}[n_{\mathbf{p}}n_{\mathbf{p}'} - n_{\mathbf{p-q}}n_{\mathbf{p}'+\mathbf{q}}]. \tag{1}$$

Here $n_{\mathbf{p}}$ is the electron-distribution function, $W^{\mathbf{pp}'}_{\mathbf{p-q},\mathbf{p}'+\mathbf{q}}$ is the probability of the *e-e* collision changing the quasimomenta of two electrons before the collision, $\mathbf{p}$ and $\mathbf{p}'$, into the quasimomenta $\mathbf{p-q}$ and $\mathbf{p}'+\mathbf{q}$ after the collision. The correlating flux is the number of collisions per unit time, each collision resulting in the appearance of electrons with momenta $\mathbf{p}$ and $\mathbf{p}'$, minus the number per unit time of collisions between the electrons with momenta $\mathbf{p}$ and $\mathbf{p}'$. Thus the correlating flux (1) is none other than the net simultaneous flux into the pair of single-electron states due to *e-e* collisions of the type

$$\mathbf{p},\mathbf{p}' \leftrightarrow \mathbf{p-q},\mathbf{p}'+\mathbf{q}. \tag{2}$$

After summing over $\mathbf{p}'$, the correlation-creating flux turns into the ordinary *e-e* collision term with the minus sign,

$$-\sum_{\mathbf{p}'} I^{\mathbf{pp}'} = -\sum_{\mathbf{p}'\mathbf{q}} I^{\mathbf{pp}'}_{\mathbf{p-q},\mathbf{p}'+\mathbf{q}} = -I^{\mathbf{p}}. \tag{3}$$

The quite important property of the correlating flux (1) is that it vanishes in equilibrium. Indeed, it is easy to verify that $I^{\mathbf{pp}'}$ vanishes after insertion of the Boltzmann distribution function. Differing from zero in nonequilibrium, flux (1) creates a nondiagonal part of a two-electron equal-time correlation function, thus generating the above-mentioned correlation of occupancies of single-electron states called kinetic correlation in Ref. 5. Fluctuation phenomena in a macroscopic system are sensitive to this collision-created classical correlation, in principle detectable in noise experiments.[11]

Sources of correlation in degenerate nonequilibrium systems. Degeneracy introduces new features. First of all, Pauli principle modifies the correlation-creating term (1) as follows:

$$I_1^{\mathbf{pp}'} \equiv (1 - n_{\mathbf{p}} - n_{\mathbf{p}'})\sum_{\mathbf{q}} I^{\mathbf{pp}'}_{\mathbf{p-q},\mathbf{p}'+\mathbf{q}}. \tag{4}$$

Here the correlating flux into the pair of states $\mathbf{p}$, $\mathbf{p}'$ is

$$-\sum_{\mathbf{q}} I^{\mathbf{pp}'}_{\mathbf{p-q},\mathbf{p}'+\mathbf{q}} = -\sum_{\mathbf{q}} W^{\mathbf{pp}'}_{\mathbf{p-q},\mathbf{p}'+\mathbf{q}}[n_{\mathbf{p}}n_{\mathbf{p}'}(1-n_{\mathbf{p-q}})(1-n_{\mathbf{p}'+\mathbf{q}}) - n_{\mathbf{p-q}}n_{\mathbf{p}'+\mathbf{q}}(1-n_{\mathbf{p}})(1-n_{\mathbf{p}'})]. \tag{5}$$

But this is not the only change raised by degeneracy. Quite remarkable is an appearance, side by side with term (4), of an extra term

$$I_2^{\mathbf{pp}'} \equiv (n_{\mathbf{p}'} - n_{\mathbf{p}})\sum_{\mathbf{q}} (I^{\mathbf{p}',\mathbf{p-q}}_{\mathbf{p}'-\mathbf{q},\mathbf{p}} + I^{\mathbf{p}',\mathbf{q-p}'}_{\mathbf{p},\mathbf{q-p}}) \tag{6}$$

in the expression for the source of the equal-time correlation of occupancies of single-electron states in a degenerate electron system. As a consequence, the total source (with the minus sign) of equal-time correlation, referred to below as the correlation source (with the minus sign), is given by the expression

$$I^{\mathbf{pp}'}\{n\} = I_1^{\mathbf{pp}'} + I_2^{\mathbf{pp}'}. \tag{7}$$

Note that the concepts "correlation source" and "correlating flux" coincide only in the absence of degeneracy.

The extra source (6)—Pauli source—contains the distribution function $n_{\mathbf{p}}$ in front of the extra fluxes, thus the Pauli source is of a higher order in the electron density than the classical source of correlation, becoming vanishingly small in the nondegenerate case. The higher-order terms appear also in Eq. (4), in such a way that in the nondegenerate case the expression (7) for the correlation source reduces to Eq. (1), as it should be: in a nondegenerate electron gas only the source (1) is responsible for the creation of additional correlation.

A possibility of existence of the additional correlation is a characteristic feature of out-of-equilibrium systems. Indeed, due to conservation of energy in a collision event, the correlation-creating flux (5), and then the correlation source (7) vanish after the insertion of the Fermi distribution function. Disappearance at equilibrium of the correlation source is highly expected: at equilibrium the spectral intensities of current fluctuations are related, via the fluctuation-dissipation theorem, to the transport coefficients, which are single-particle observables.

The correctness of Eq. (6) obtained in Refs. 8 and 9 is beyond doubts, the derivation being given within the framework of the Keldysh formalism, the underlying approximations thoroughly discussed, the presentation adequate and self-contained. The system considered was supposed to be under the action of an external driving electric field able to substantially displace the system from the equilibrium with a thermal bath. The carriers were supposed to interact with phonons, impurities, and with one another (via the Coulomb potential). Quasiclassical fluctuations were considered, the time scale $1/\omega$ of the fluctuations supposed to be large as compared with $\hbar/\varepsilon_{\mathbf{p}}$—the time of formation of the quantum state having momentum $\mathbf{p}$ and energy $\varepsilon_{\mathbf{p}}$.

The Pauli source $I_2^{\mathbf{pp}'}$, though expressible, as well as $I_1^{\mathbf{pp}'}$, in terms of single-electron distribution function and *e-e* scattering probabilities, can hardly be interpreted within the "naive" semi-classical approach. Though contributing to the correlation source which generates a synchronous correlation of occupancies of states $\mathbf{p}$ and $\mathbf{p}'$, the Pauli term $I_2^{\mathbf{pp}'}$ contains the fluxes

$$\mathbf{p}',\mathbf{p}-\mathbf{q} \leftrightarrow \mathbf{p}'-\mathbf{q},\mathbf{p}, \quad \mathbf{p}',\mathbf{q}-\mathbf{p}' \leftrightarrow \mathbf{p},\mathbf{q}-\mathbf{p}, \tag{8}$$

odd looking from the semiclassical point of view. Indeed, from that point of view, these fluxes tie up the occupancy of the state $\mathbf{p}$ after the collision with the occupancy of the state $\mathbf{p}'$ before the collision, not the occupancies of both such states after the collision. However, as mentioned above, fluxes (8) in fact generate a synchronous correlation of occupancies.

The seeming contradiction between the result of quasiclassical approximation and the "semiclassical" picture arises because the Pauli principle—a quantum constraint—while superimposed on the movement of electrons introduces some sort of nonlocality. In the quasiclassical approximation, the nonlocality evidences itself in two-particle phenomena, but not single-particle phenomena. For the latter—for the ordinary transport and response processes—the "semiclassical" approach works, and locality, natural within the "semiclassical" approach, proves to be compatible with the Pauli principle.

The self-consistent quasiclassical two-particle approach, inherent while studying fluctuation and noise phenomena in nonequilibrium systems with pair collisions, unavoidably requires an *ab initio* derivation of the equation for the equal-time, two-particle distribution function, as performed in Refs. 8 and 9. Appearance of the term $I_2^{\mathbf{pp}'}$ demonstrates the sensitivity of fluctuation phenomena to quantum constraints created by the Pauli principle. Fluctuations, being described by pair-correlation functions, inherently present the two-particle phenomenon, which reduces to the one-particle one only in simplest cases (e.g., at equilibrium). Being conditioned by a primordial two-particle nature of fluctuation phenomena and an intrinsically quantum character of the degeneracy phenomenon, the Pauli constraint induced correlations reveal themselves in the electronic noise (the two-electron phenomenon), but not in the response (the single-electron phenomenon).

Noise as a probe of the Pauli constraint induced correlations in macroscopics. We claim that the Pauli correlations can display themselves on the macroscopic level. Indeed, the contribution from the Pauli correlation source appears in the expression for spectral intensity of spatially homogeneous current fluctuations in a biased, spatially homogeneous two-or three-dimensional macroscopic channel. The macroscopic current (or voltage) noise spectrum of a degenerate nonequilibrium electron system contains a nonvanishing part, sensitive to Pauli constraint correlations.

It was demonstrated in Ref. 12 that in the electron-temperature approximation the current fluctuations in a biased degenerate channel are sensitive to an equal-time correlation of single-electron energies. Namely, the source of correlation,

$$\Lambda_{ee} = \sum_{\mathbf{pp}'} \varepsilon_{\mathbf{p}}\varepsilon_{\mathbf{p}'} I^{\mathbf{pp}'}, \tag{9}$$

enters the expression for the spectral intensity of microwave longitudinal (along the steady current) current density fluctuations [see Ref. 12, Eq. (25)]:

$$(\delta j_\parallel^2)_\omega = \frac{2k_B T\sigma}{V_0}\left\{1 + \frac{2\tau_T E^2 \mathrm{d}\sigma/\mathrm{d}T}{c_e(1+\omega^2\tau_T^2)} \times\left[1 + \frac{1}{2}\frac{\mathrm{d}\ln\sigma}{\mathrm{d}\ln T}\left(1 + \frac{\tau_T\Lambda_{ee}}{2c_e T^2}\right)\right]\right\}. \tag{10}$$

Here k_B is the Boltzmann constant, T the electron temperature, V_0 the volume, E the applied electric field, $\sigma = j/E$ the chord conductivity, $n_T(\varepsilon_{\mathbf{p}})$ the Fermi distribution function corresponding to T, $c_e = \Sigma_{\mathbf{p}}\varepsilon_{\mathbf{p}}\mathrm{d}n_T(\varepsilon_{\mathbf{p}})/\mathrm{d}T$ the specific heat of the degenerate electron gas, and τ_T the relaxation time of the electron-temperature [see Ref. 12, Eq. (14)]. The input of the correlation into the noise spectrum is of the Lorentz shape provided that the electron-temperature approximation works, the width of the Lorentzian being defined by the relaxation time of the electron temperature, τ_T.

Equation (10) was derived for sufficiently high electron densities, when *e-e* collisions are frequent enough to shape the electron energy distribution, making it rather close to the Fermi distribution. The deviation from the Fermi distribution, affording existence of a nonvanishing equal-time correlation of energies Λ_{ee}, satisfies a linear integral equation, with kernel expressed in terms of the *e-e* collision probabilities and data resulting from the electron-lattice scattering.[13]

With the exception of Λ_{ee}, all the quantities on the right-hand side of Eq. (10) are the single-particle observables, in principle measurable separately—from usual transport and/or response experiments. It follows that, by using Eq. (10), the energy-correlation source Λ_{ee} can be qualitatively and even quantitatively evaluated from the electronic noise measurements. This opens the way to test the input of Pauli correlations against that of "classical" kinetic correlation. Indeed, in the source Λ_{ee} one can single out the term identical in form with the total source of correlations of energies in the case of Boltzmann statistics (see Ref. 7, Chap. 12),

$$\Lambda'_{ee} \equiv \sum_{\mathbf{pp}'} \varepsilon_{\mathbf{p}}\varepsilon_{\mathbf{p}'} \sum_{\mathbf{q}} I^{\mathbf{pp}'}_{\mathbf{p}-\mathbf{q},\mathbf{p}'+\mathbf{q}} = 2T^2\left[\frac{E^2\sigma}{T-T_0} - \frac{c_e}{\tau_T}\right]. \tag{11}$$

The term Λ'_{ee}, being expressible in terms of nonlinear transport (or response) regardless of the presence or absence of degeneracy, is extractable from the experiments. It follows that the remaining part of Λ_{ee},

$$\Lambda''_{ee} \equiv -2\sum_{\mathbf{p}\mathbf{p}'} \varepsilon_{\mathbf{p}}\varepsilon_{\mathbf{p}'} n_{\mathbf{p}} \sum_{\mathbf{q}} (I^{\mathbf{p}\mathbf{p}'}_{\mathbf{p}-\mathbf{q},\mathbf{p}'+\mathbf{q}} + I^{\mathbf{p}',\mathbf{p}-\mathbf{q}}_{\mathbf{p}'-\mathbf{q},\mathbf{p}} + I^{\mathbf{p}',\mathbf{q}-\mathbf{p}'}_{\mathbf{p},\mathbf{q}-\mathbf{p}}), \tag{12}$$

is extractable from experiments as the difference $\Lambda''_{ee} = \Lambda_{ee} - \Lambda'_{ee}$. The ratio $\Lambda''_{ee}/\Lambda'_{ee}$ will characterize the contribution of the Pauli correlation against that of the "classical" kinetic correlations.

It follows from Eqs. (4)–(6) that the contribution of the Pauli-constraint-induced correlation to the current-fluctuation spectrum does not contain any small parameter: in general, the contribution due to such a correlation is of the same order of magnitude as the thermal ("Johnson") noise created by chaotic motion of hot electrons (provided that the electron system is substantially displaced from the equilibrium with the thermal bath; the contribution of the additional correlation to the electric noise appears as a term of the order of E^4 rather than E^2, see Ref. 12). To avoid other types of noise ($1/f$ noise, generation-recombination noise, etc.), the measurements of noise described by Eq. (10) usually are performed at microwave frequencies.

The Pauli correlation of occupancies of different $\mathbf{p}$ states can be interpreted as correlation of intensities of "partial" electron beams in the real space. The analogy with the HBT effect—the correlation, under the quantum-statistical constraints, of intensities of the light beams—is obvious. The possibility of observation of an electronic HBT effect in mesoscopic conductors was discussed in Ref. 3.

Of course, noise measurements present only the method of an indirect experimental determination of the Pauli correlations. Our aim was to demonstrate the principal possibility to extract these correlations from experiment by means of the formulas (10) and (11). In concrete cases the questions will arise—how precise the independent measurements of the single-particle transport parameters entering the formulas have to be, and whether the conditions necessary for the applicability of these formulas are fulfilled in the given experiment.

To conclude, let us remind the reader once again the conditions necessary for the extraction of Pauli correlations from noise spectrum of a macroscopic system: the system should be in a nonequilibrium degenerate state in the shaping of which interparticle collisions play a role.

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