Effect of bias-induced spin flips on spin torque: Enhancement of current and spin transfer in magnetic tunnel junctions

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Recent data on the bias dependence of the spin transfer effect in magnetic tunnel junctions have shown that torque remains intact at bias voltages for which the tunneling magnetoresistance has been strongly reduced. We determine the effect of magnons emitted by hot electrons on charge and spin transport in these junctions by using Caroli's formalism. The transport is described in terms of the direct, spin-dependent, and spin-flip probabilities for transmission across the tunnel junction. By adopting values for these adjustable parameters that were found by fitting the bias dependence of the conductance and magnetoresistance, we find that the excitations due to hot electrons, while reducing the magnetoresistance, enhance both the charge current and spin transfer in magnetic tunnel junctions in such a manner that the ratio of the torque to the charge current does not significantly change.

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I. INTRODUCTION

A recent set of experiments have confirmed that the transfer of spin angular momentum from a spin-polarized current to the background magnetization, also known as the spin transfer effect, exists in magnetic tunnel junctions (MTJ's).[1–4] To arrive at the critical current necessary for the current-induced magnetization switching (CIMS) of one of the magnetic electrodes (which is nearly the same for MTJ's as for the metallic multilayers in which this phenomenon was first observed[5]), one applies a bias of the order of several hundred meV. For metallic multilayers, in which CIMS was first observed, a correlation exists between its magnetoresistance (MR) and the critical current for magnetization reversal.[6] What is unusual about the bias dependence of MTJ's is that the spin transfer effect is robust to an applied bias, while the magnetoresistance is quenched (rapidly softens and eventually is driven to zero), i.e., the ratio of the torque to the current remains fairly constant as one increases the bias.[7] Here we give an explanation for this result.

The spin transfer effect has been observed when the magnetic elements are noncollinear; depending on whether the transport is ballistic and diffusive, the origin of this effect is different. Here we consider only magnetic tunnel junctions for which Slonczewski[8] showed that based solely on the equilibrium band structure (wave functions) one expects an out-of-equilibrium coupling of the magnetic electrodes of an MTJ, which is proportional to the sine of the angle θ between them (the spin torque), and one proportional to cos θ, which is known as the effective field term; this latter coupling is in addition to the one that exists in equilibrium (later known as the "interlayer exchange coupling").[8] In his seminal paper, Slonczewski showed that the difference between the spin currents leaving and entering an electrode represents a transfer of spin angular momentum per unit time from the current to the magnetic electrode, which creates a spin torque on the latter. This spin transfer manifests itself by the switching, or reversal, of an electrode that occurs when the current exceeds a critical value. In a recent study, Slonczewski amplified his original work on MTJ's by considering the effect of the applied bias on the barrier profile. He showed that when the junction is forward biased, the ratio of the spin torque to the current remains constant while the tunneling MR (TMR) is rapidly quenched;[9] when the biased is reversed, the torques falls nearly as fast as the TMR. While the first agrees with recent data, the latter does not.[7]

As has been suggested by others,[2,4,7] we will consider the role of the current-induced magnetic excitations that occur due to the hot electrons created by a finite potential difference between magnetic electrodes. It was previously shown that the inelastic spin excitations (spin flips) localized at the interfaces between the magnetic electrodes and the tunnel barrier give rise to a rapid reduction of the MR of the MTJ known as the zero bias anomaly.[10] While these bias-driven spin flips are *partially* able to explain the current-voltage characteristics of the MR of MTJ's, for several cases it is necessary to invoke the energy dependence of the density of states (DOS) in order to obtain a *complete* quenching of the MR.[11]

The guiding principle in MTJ's is that processes in the barrier region, which includes interfaces, have a marked effect on the resistance of the junction, while those in the electrodes affect magnetization reversal.[12] When two electrodes α and β with noncollinear magnetizations form a tunnel junction, the direction of the polarization of the spin current $\vec{I}_s$ is the vector sum of the polarizations $\vec{\rho}$ (parallel to the magnetizations) of the density matrices representing the electrodes.[8] In other words, the spin current is parallel to *neither* electrode's magnetization. Under the condition that spin-flip scattering is a small perturbation of the spin current, i.e., we maintain the zero bias eigenstates, we will show that the net result of the exchange of angular momentum does not cancel because the spin flips in the electrode and the spin current are referred to as noncollinear axes; the component of the *vector* sum of the difference that is transverse to the electrode's magnetization is the torque created by exchange of magnons between noncollinear entities. In this manner, bias increases the spin current and concomitantly the torque on the electrodes in the junction. As we show, the torque due to the magnon exchange between the current and the upstream electrode α is directed toward the other electrode β, and vice versa; therefore, the inelastic torques on the electrodes are in opposite directions while the elastic torques are in the same direction.

II. FORMALISM

The current in a tunnel junction can be written as[13]

$$I = \frac{2e}{h}[\bar{T}_{q\leftarrow p}\mu_p - \bar{T}_{p\leftarrow q}\mu_q], \tag{1}$$

where $\bar{T}_{q\leftarrow p}$ represents the transmission probability from electrode p to electrode q, and μ_p is the electrochemical potential of the pth electrode. When spin phenomena play a role, the $\bar{T}_{q\leftarrow p}$ can be found from the following transmission probabilities written in *spinor* form as (where we use α/β instead of p/q):[14]

$$\hat{T}_{\beta\leftarrow\alpha} = \frac{1}{2}(2\pi)^2\hat{\rho}_\alpha(\hat{t}_{\beta\alpha})^\dagger\hat{\rho}_\beta\hat{t}_{\beta\alpha} \tag{2}$$

and

$$\hat{T}_{\alpha\leftarrow\beta} = \frac{1}{2}(2\pi)^2\hat{\rho}_\beta(\hat{t}_{\alpha\beta})^\dagger\hat{\rho}_\alpha\hat{t}_{\alpha\beta}, \tag{3}$$

where the factor $\frac{1}{2}$ is for each spin channel, the 2π for the $\hat{\rho}_{\alpha/\beta}$ represent the density matrices of the magnetic leads far from the electrode/barrier interfaces, the hats over the symbols signify 2×2 matrices in the spin $\frac{1}{2}$ space of an electron, and $\hat{t}_{\beta\alpha}$ is the transmission amplitude from the lead (deep in the electrode) α to β; see, for example, Ref. 15.

The spinor density matrices will be written as

$$\hat{\rho} = \rho_0 1 + \vec{\sigma}\cdot\vec{\rho}, \tag{4}$$

where the first term represents the charge or spin-independent part and $\vec{\rho}$ the spin-dependent portion or polarization. The diagonal elements of the spinors represent the longitudinal components (parallel to the local magnetization) of these quantities, e.g., $\rho_{\uparrow\uparrow/\downarrow\downarrow}\equiv\rho_{\uparrow/\downarrow}\equiv\rho_0\pm\rho_z$ the average, and z component of the density matrices, while the off-diagonal terms represent transverse components in which the spin of the electron flips. For the matrices $\hat{\rho}_{\alpha/\beta}$ these elements do not appear when they are written in the natural axes which diagonalize them; however, for noncollinear electrodes we have to rotate these matrices so that off-diagonal elements appear and are written as $\rho_{\downarrow\uparrow/\uparrow\downarrow}\equiv\rho_\pm(\theta) = \pm i\sin\theta\rho_z$.[16]

The particle or charge current is the scalar part of Eq. (1) and is found by taking the trace of the spinor transmission probabilities. The trace is invariant under a cyclic permutation of the matrices and by using the Hermitian property for the transmission amplitude $(\hat{t}_{\beta\alpha})^\dagger=\hat{t}_{\alpha\beta}$ one finds $\mathrm{Tr}_\sigma\hat{T}_{\alpha\leftarrow\beta} = \mathrm{Tr}_\sigma\hat{T}_{\beta\leftarrow\alpha}$, so that the charge current between two electrodes is

$$I_c = \frac{2e}{h}\mathrm{Tr}_\sigma\hat{T}[\mu_\alpha-\mu_\beta] = \frac{2e^2V}{h}\mathrm{Tr}_\sigma\hat{T}, \tag{5}$$

where we made the identification for the potential drop $eV\equiv\mu_\alpha-\mu_\beta$; as $e<0$, it follows that $V<0$ for a positive difference in chemical potentials, thus the electron flow that we call "the current" is opposite the conventional one in electrical circuits, but conforms to the convention used to discuss spin transfer.[1,8] The index σ on the trace indicates we are summing over spinors; hereafter we drop this index. The conductance and TMR of tunnel junctions are found from the charge current and its dependence on the configuration of the magnetic electrodes. The effect of bias-driven magnons on these has been previously studied.[10] To understand their effect on the spin current and torque is the purpose of this paper.

The polarization or spin current is defined as the vector part of the spinor current and is found by taking the trace of the product of the Pauli spin vector $\vec{\sigma}$ (written as a spinor) and the spinor transmission probabilities; in this case, $\mathrm{Tr}\vec{\sigma}\hat{T}_{\alpha\leftarrow\beta}=\mathrm{Tr}\vec{\sigma}\hat{\rho}_\beta(\hat{t}_{\alpha\beta})^\dagger\hat{\rho}_\alpha\hat{t}_{\alpha\beta}$, which upon using $(\hat{t}_{\beta\alpha})^\dagger=\hat{t}_{\alpha\beta}$ and doing a cyclic permutation yields $\mathrm{Tr}\hat{\rho}_\alpha(\hat{t}_{\beta\alpha})^\dagger\vec{\sigma}\hat{\rho}_\beta\hat{t}_{\beta\alpha} \neq \mathrm{Tr}\vec{\sigma}\hat{T}_{\beta\leftarrow\alpha}=\mathrm{Tr}\vec{\sigma}\hat{\rho}_\alpha(\hat{t}_{\beta\alpha})^\dagger\hat{\rho}_\beta\hat{t}_{\beta\alpha}$. We find the spin current is written as

$$\vec{I}_s = \frac{2e}{h}[\vec{T}_\alpha\mu_\alpha - \vec{T}_\beta\mu_\beta], \tag{6}$$

where $\vec{T}_{\alpha/\beta}\equiv\mathrm{Tr}\vec{\sigma}\hat{T}_{\beta\leftarrow\alpha/\alpha\leftarrow\beta}$. By defining the average and difference between the chemical potentials $\mu_{\alpha/\beta}=\frac{1}{2}[(\mu_\alpha+\mu_\beta)\pm(\mu_\alpha-\mu_\beta)]$, we write the spin current as

$$\vec{I}_s = \frac{2e}{h}\left\{\frac{1}{2}(\mu_\alpha+\mu_\beta)[\vec{T}_\alpha-\vec{T}_\beta] + eV\frac{1}{2}[\vec{T}_\alpha+\vec{T}_\beta]\right\}, \tag{7}$$

where the first term is the spin current that exists *in* equilibrium, while the second is the out-of-equilibrium part. Again note that we made the identification for the potential drop $eV\equiv\mu_\alpha-\mu_\beta$; it follows that it is the electron flow that we call "the current."

Here we limit ourselves to $T=0$ K so that the electrodes are in their ground states. In the presence of a finite bias between the electrodes, it is possible to create magnons in the electrodes if they are emitted by the spin current. This costs energy so that the chemical potential entering the Fermi distribution is reduced by $\hbar\omega_q^{\alpha/\beta}$ (see Ref. 9); in this case, the chemical potentials $\mu_{\alpha/\beta}$ in Eq. (6) are replaced by $\mu_{\alpha/\beta}-\hbar\omega_q^{\alpha/\beta}\Theta(eV-\hbar\omega_q^{\alpha/\beta})$, where the theta function is a reminder that the energy range of the magnons created in the magnetic electrodes is limited by the bias applied across the junction. By including these inelastic processes, the total spin current, Eq. (7), contains one additional term,

$$\vec{I}_s^{\mathrm{magnon}} = -\frac{2e}{h}\sum_q\hbar\omega_q^{\alpha/\beta}\Theta(eV-\hbar\omega_q^{\alpha/\beta})[\vec{T}_\alpha-\vec{T}_\beta], \tag{8}$$

therefore the *total* spin current is

$$\vec{I}_s = \frac{2e}{h}\left\{ \frac{1}{2}(\mu_\alpha + \mu_\beta)[\vec{T}_\alpha - \vec{T}_\beta] + eV\frac{1}{2}[\vec{T}_\alpha + \vec{T}_\beta] - \sum_q \hbar\omega_q^{\alpha/\beta}\Theta(eV - \hbar\omega_q^{\alpha/\beta})[\vec{T}_\alpha - \vec{T}_\beta] \right\}. \quad (9)$$

From here on in we drop the first term, as we are not interested in this paper in the magnetic coupling that exists between electrodes before a bias is applied to the junction.

The sum over the magnons is written as

$$\sum_q \hbar\omega_q^{\alpha/\beta}\Theta(eV - \hbar\omega_q^{\alpha/\beta}) = \int_0^{eV} g^{\alpha/\beta}(\omega)\hbar\omega d\omega, \quad (10)$$

where the density of states for the magnons $g^{\alpha/\beta}(\omega)$ depends on whether they are created inside the electrodes or at the electrode/barrier interfaces. We will use the same approximation as that used in Ref. 9 and replace the magnon dispersion relation by a simple isotropic parabolic one. The magnons created at the electrode/barrier interface are two dimensional and their DOS is $g^{\alpha/\beta}(\omega)=N^i_{\alpha/\beta}/E_m^{\alpha/\beta}$, where $E_m^{\alpha/\beta}=3kT_c^{\alpha/\beta}/(S^{\alpha/\beta}+1)$, and $N^i_{\alpha/\beta}$ is the number of spins per unit area at the interface; those in the bulk of the electrodes are three dimensional so that $g^{\alpha/\beta}(\omega)=\frac{3}{2}N^b_{\alpha/\beta}/E_m^{\alpha/\beta}\sqrt{\hbar\omega/E_m^{\alpha/\beta}}$, where here $N^b_{\alpha/\beta}$ is the number of spins per unit volume. By using these DOS, we find for interface magnons

$$\vec{I}_s^{\text{magnon}} = -\frac{eN^i_{\alpha/\beta}}{h}\left(\frac{eV}{E_m^{\alpha/\beta}}\right)(eV)\hbar[\vec{T}^i_\alpha - \vec{T}^i_\beta], \quad (11)$$

while for bulk magnons

$$\vec{I}_s^{\text{magnon}} = -\frac{6eN^b_{\alpha/\beta}}{5h}\left(\frac{eV}{E_m^{\alpha/\beta}}\right)^{3/2} eV\hbar[\vec{T}^b_\alpha - \vec{T}^b_\beta], \quad (12)$$

where the superscripts i/b on the transmission probabilities denote, as we presently show, the different transmission amplitudes used for the interface and bulk magnon production, $t_m^{i/b}$. The maximum number of spin flips a spin can make is $2S_\alpha$; it is for this reason that for larger bias $eV > E_m^\alpha$ one replaces eV/E_m^α by $(2-E_m^\alpha/eV)$ in Eq. (11) and by $(2^{2/3}-E_m^\alpha/eV)$ in Eq. (12) so that in the limit as $eV\to\infty$ they approach 2.

III. CALCULATION OF SPIN CURRENT

To complete our calculation of the bias dependence of the spin current in a MTJ, we have to specify the spinor transmission amplitudes

$$\hat{t}_{\beta\alpha} = t_d\hat{1} + t_m\vec{\sigma}\cdot\vec{S}^{\alpha/\beta}, \quad (13)$$

where t_d denotes the direct transmission between the magnetic leads that are used to calculate the spin current in Eq. (7), and t_m is spin-dependent transmission amplitudes ($t_m^{i/b}$) entering the magnon-assisted spin currents entering Eqs. (11) and (12).[17] We leave these amplitudes as parameters used to fit the data; as a general rule, we expect $t_d > t_m^i, t_m^b$ inasmuch as the probability (frequency) of scattering at the interface or in the bulk of an electrode is lower than a direct transmission. As written, the spinor gives the erroneous impression that there is a single spin-flip scattering; however, in the 2×2 spin space of an electron, this is the most general form of the spin scattering of an electron by a magnetic background characterized by the spin *operator* $\vec{S}$, i.e.,

$$\hat{t}_{\beta\alpha} = \begin{pmatrix} t_d + t_m S_z^{\alpha/\beta} & t_m S_-^{\alpha/\beta} \\ t_m S_+^{\alpha/\beta} & t_d - t_m S_z^{\alpha/\beta} \end{pmatrix}. \quad (14)$$

Hot electrons appear only when an electron leaves the electrode at the higher potential, which is α in our example. At the interfaces, the relative probability of scattering these electrons is much less than a direct transmission, e.g., from a fit to the zero point anomaly in magnetic transition-metal tunnel junctions $|t_d/t_m^i|^2=17$.[10] In this case, it is reasonable to replace $S_z^{\alpha/\beta}\Rightarrow S^{\alpha/\beta}$, and the only current-induced excitations in the barrier due to hot electrons are the off-diagonal terms in Eq. (14). However, if a hot electron does not give up its energy in the barrier region, it can excite magnons in the bulk of the electrode at the lower potential μ_β. In this case, there can be multiple exchanges of spin and energy between the electron and the magnetic electrode before it exits into the lead represented by $\hat{\rho}_\beta$. Under these conditions, if an electron enters the reservoir (magnetic lead $\hat{\rho}_\beta$) with its spin in the same direction as it left the lead $\hat{\rho}_\alpha$, there is no net spin-flip; nonetheless the multiple scattering contributes to the transmission amplitudes $t_m^b S_z^{\alpha/\beta}$ and now we should write the transmission probability as proportional to $|t_m^b|^2\langle (S_z^\beta)^2\rangle$ as $\Rightarrow |t_m^b|^2\{(S^\beta)^2 - \frac{1}{2}[\langle S_+^\beta S_-^\beta\rangle + \langle S_-^\beta S_+^\beta\rangle]\}$, i.e., even at $T=0$ K the magnons emitted by the current *lower* the longitudinal component of the transmission in a manner reminiscent of there being a heating of the magnetic electrode by the current. Of course if the electron enters $\hat{\rho}_\beta$ with its spin flipped, the multiple scattering contributes to the transmission amplitudes $t_m^b S_\pm^{\alpha/\beta}$, and to the transmission probability as $|t_m^b|^2\langle S_\pm^\beta S_\mp^\beta\rangle$.

By placing the transmission amplitude $\hat{t}_{\beta\alpha}$, Eq. (13), in the expressions (2) and (3), for the spin current transmission probabilities $\vec{T}_{\alpha/\beta}$ [see the definition after Eq. (6)] we have determined the contributions of the direct and magnetic transmission amplitudes to the out-of-equilibrium spin current, Eqs. (7) and (8). To obtain the trace over the spinors entering these expressions, we have to write them in a common representation. As we are dealing with noncollinear magnetic electrodes, this entails rotating some of the spinors as discussed in Ref. 15; we also used the identity

$$(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) = \vec{a}\cdot\vec{b} + i\vec{\sigma}\cdot(\vec{a}\times\vec{b}), \quad (15)$$

where special attention has to be paid to the ordering when the vectors $\vec{a},\vec{b}$ are operators representing the same spin because their components do not commute with one another, e.g., $\vec{S}\times\vec{S}=i\vec{S}$.

IV. SPIN TORQUE

To obtain the torques on the individual electrodes from the spin currents, we adopt the definition given by Slonczewski that it is the *negative* of the difference between the spin current (electron flows) leaving and entering an electrode.[18] By applying this to the electrode at the higher (α) and lower (β) potentials, we can write $\vec{\tau}^{\alpha} \equiv -\hbar(\vec{I}_s - \vec{I}_s^{\alpha})$ and $\vec{\tau}^{\beta} \equiv -\hbar(\vec{I}_s^{\beta} - \vec{I}_s)$, where $\vec{I}_s$ is the spin current in the junction, while $\vec{I}_s^{\alpha/\beta}$ are the currents in the magnetic leads (reservoirs) α/β. One final hurdle to overcome in obtaining the torques is that the spin current $\vec{I}_s$ has to be written in the coordinate axes of the magnetic leads α/β so as to properly obtain the components of the current *transverse* to the magnetization of the electrodes; this requires further rotations.[16]

A. Elastic contributions

When we use the direct (elastic) term in the transmission amplitude Eq. (13) in Eq. (7), we find for the out-of-equilibrium spin current when we include the unit of angular momentum $\hbar$ for the spin current

$$\vec{I}_s = \frac{2e}{h} eV \frac{1}{2} \hbar [\vec{T}_{\alpha}^{d} + \vec{T}_{\beta}^{d}]$$

$$\vec{I}_s(\hat{\alpha}) = \frac{2e}{h} eV \hbar 4\pi^2 |t_d|^2 [\rho_0^{\alpha} \vec{\rho}^{\beta}(\theta) + \rho_0^{\beta} \vec{\rho}^{\alpha}] \tag{16}$$

when it is referred to the α electrode, and

$$\vec{I}_s(\hat{\beta}) = \frac{2e}{h} eV \hbar 4\pi^2 |t_d|^2 [\rho_0^{\alpha} \vec{\rho}^{\beta} + \rho_0^{\beta} \vec{\rho}^{\alpha}(-\theta)] \tag{17}$$

referred to the β electrode; the rotated elements of the density matrices are defined in Ref. 15. As shown by Slonczewski, the direction of the polarization of the spin current $\vec{I}_s$ can be written in an invariant form as the vector sum of the polarizations $\vec{\rho}$ of the density matrices representing the electrodes,[8] i.e.,

$$\vec{I}_s \sim \vec{\rho}^{\alpha} + \vec{\rho}^{\beta}. \tag{18}$$

In other words, the spin current is parallel to *neither* electrode's magnetization.

In our rotations, we have the downstream electrode β at an angle θ in a clockwise direction with respect to α; as this is opposite to that conventionally chosen, it produces torques in an opposite sense to those of Slonczewski,[8] i.e., we will assume the downstream electrode's magnetization is rotated counterclockwise to the upstream's. So as not to have minus signs throughout, we reverse the direction of θ from hereon in. Then with the currents Eqs. (16) and (17), we reproduce Slonczewski's original result for the spin torques acting on the electrodes by taking their transverse components, i.e.,

$$\tau_y^{\alpha} = 4\pi e |t_d|^2 eV \sin\theta \rho_0^{\alpha} \rho_z^{\beta} \tag{19}$$

and

$$\tau_y^{\beta} = 4\pi e |t_d|^2 eV \sin\theta \rho_0^{\beta} \rho_z^{\alpha}. \tag{20}$$

We note that the torques are in the same direction and in the *same* sense as that found by Slonczewski[8] when one recognizes that we have to replace θ by $-\theta$ in Eqs. (16) and (17) to conform to the conventional sense of rotation. It follows that for *elastic* processes

$$\tau_y^{\beta} = \tau_y^{\alpha}(\alpha \leftrightarrows \beta). \tag{21}$$

The axes we have chosen are such that $\hat{z}_{\alpha}$ is along the magnetization of the α electrode, $\hat{y}_{\alpha}$ lies in the plane of the cross section of the electrode, and $\hat{x}$ is perpendicular to this plane; for the β electrode, $\hat{z}_{\beta}$ is along its magnetization, $\hat{y}_{\beta}$ is perpendicular to it, and in the same plane defined by $\hat{z}_{\alpha}$ and $\hat{y}_{\alpha}$, and they have a common $\hat{x}$ axis.

Here we derived the spin torque by using density matrices for pure states, while the original derivation used wave functions; the physical content of the two is the same.

B. Inelastic contributions: Magnons

Next we consider the contribution from magnons created at the electrode barrier interfaces; these were previously used to understand the zero bias anomaly found in the bias dependence of the TMR of magnetic transition-metal tunnel junctions.[10] By using the spin-dependent scattering in the transmission amplitude, i.e., the second term in Eq. (13), and by taking the components of the ensuing spin current that are transverse to the magnetization of the upstream electrode, we find there is an elastic contribution to the torque [see Eq. (16)],

$$\tau_y^{\alpha} = 2\pi e |t_m^i|^2 N_{\beta}^i eV \sin\theta \frac{1}{2} \sum_{\sigma} (-)^{\sigma} \rho_{\sigma}^{\alpha}(-\theta) \rho_{\sigma}^{\beta} \langle (S_z^{\beta})^2 \rangle, \tag{22}$$

and an inelastic spin-flip contribution [see Eq. (8)],

$$\begin{aligned}\tau_y^{\alpha^{\text{magnons}}} \sim [\vec{T}_{\alpha}^{i} - \vec{T}_{\beta}^{i}] = 2\pi^2 |t_m^i|^2 \sin\theta \frac{1}{2} \Big\{ & N_{\alpha}^i \rho_z^{\beta} [\rho_{\uparrow}^{\alpha} \langle S_+^{\alpha} S_-^{\alpha} \rangle \\ & + \rho_{\downarrow}^{\alpha} \langle S_-^{\alpha} S_+^{\alpha} \rangle] + N_{\beta}^i \rho_z^{\alpha} \cos\theta [\rho_{\uparrow}^{\beta} \langle S_+^{\beta} S_-^{\beta} \rangle + \rho_{\downarrow}^{\beta} \langle S_-^{\beta} S_+^{\beta} \rangle] \\ & - \sum_{\sigma} (-)^{\sigma} \rho_{\sigma}^{\alpha}(-\theta) \rho_{\sigma'}^{\beta} [\langle S_+^{\beta} S_-^{\beta} \rangle + \langle S_-^{\beta} S_+^{\beta} \rangle] \Big\},\end{aligned} \tag{23}$$

where $\rho_{\sigma}(\theta) \equiv \rho_{\sigma} \cos^2 \theta/2 + \rho_{\sigma'} \sin^2 \theta/2$, $\sigma = \uparrow/\downarrow$, $\sigma' = \downarrow/\uparrow$, and $(-)^{\sigma} = \pm$ for $\uparrow/\downarrow$. It follows that $\Sigma_{\sigma}(-)^{\sigma} \rho_{\sigma}^{\alpha}(-\theta) \rho_{\sigma}^{\beta} = [\Sigma_{\sigma}(-)^{\sigma} \rho_{\sigma}^{\alpha} \rho_{\sigma}^{\beta}] \cos^2 \theta/2 - [\Sigma_{\sigma}(-)^{\sigma} \rho_{\sigma}^{\alpha} \rho_{\sigma'}^{\beta}] \sin^2 \theta/2$. At $T=0$ K, and when we do not have multiple scatterings at the interface when an electron eventually tunnels, $\langle (S_z^{\beta})^2 \rangle \Rightarrow (S^{\beta})^2 \hbar^2$ so that the elastic term Eq. (22) adds to the direct transmission Eq. (19) in the proportion $|t_m^i/t_d|^2$, which for transition-metal electrodes is small, 1/17.[10] By making the exchange $\alpha \leftrightarrows \beta$ in Eq. (22), we find the contribution to τ_y^{β} with the *same* sign as τ_y^{α}.

The inelastic contributions, Eq. (23), arise from the exchange of spin flips between the electrons comprising the current and those of the magnetic background; they are represented by off-diagonal elements of the scalar spin interaction $\vec{\sigma}\cdot\vec{S}^{\alpha/\beta}$ [see Eqs. (13) and (14)] so that the angular momentum exchanged between them is equal in magnitude and opposite in sign. For an isolated (single) electrode, or when the electrodes in a junction are collinear, the spin current is polarized along the direction of the magnetization(s). The exchange of a pair of spin flips (magnons) between the current and electrode(s) does not create a torque; its effect is to reduce the magnetization(s) of the electrode(s) while *increasing* the polarization of the spin current. Indeed in this case there are no components of the spin current transverse to the magnetization(s). However, when the electrodes are noncollinear, the ensuing rotations of the spinors, which are needed to evaluate the expectation values of the spin operators $\vec{S}^{\alpha/\beta}$ shown in Eq. (23), create the transverse components of the spin current and thus the additional spin torque due to magnons. The component of the *vector* sum of the difference between the spin angular momentum gained by the current and that lost by the background magnetization that is transverse to the electrode's magnetization is the torque created by exchange of magnons between noncollinear entities.

1. Interface

The contribution at $T=0$ K from the interfacial magnons ($\langle S_+S_-\rangle=2S\hbar^2$) to the torque on the α electrode is in the same direction as the elastic contributions Eqs. (19) and (22). From Eqs. (23) and (11), we find

$$\tau_y^{\alpha\text{interface magnons}} = \pi e|t_m^i|^2 \sin\theta(eV)\left\{\left(\frac{eV}{E_m^\alpha}\right)N_\alpha^i S^\alpha\hbar^2\rho_\uparrow^\alpha\rho_z^\beta + \left(\frac{eV}{E_m^\beta}\right)N_\beta^i S^\beta\hbar^2\left[\rho_z^\alpha\rho_\uparrow^\beta\cos\theta - \sum_\sigma(-)^\sigma\rho_\sigma^\alpha(-\theta)\rho_{\sigma'}^\beta\right]\right\}. \tag{24}$$

However, we find that the contribution from the magnons to the torque on the downstream electrode β is the *negative* of the torque on the upstream electrode α, i.e.,

$$\tau_y^{\beta\text{magnons}} = -\tau_y^{\alpha\text{magnons}}(\alpha\leftrightarrows\beta). \tag{25}$$

This is due to the form of the spin current coming from the magnon production Eq. (8), i.e., the difference between $\vec{T}_\alpha$ and $\vec{T}_\beta$. From a closer inspection of the contributions to Eq. (23) we find that the spin-flip scattering $\langle S_+^\alpha S_-^\alpha\rangle$ creates a torque on the α electrode that is just the *opposite* to the torque created by $\langle S_+^\beta S_-^\beta\rangle$ on the β electrode; also, we find $\langle S_+^\alpha S_-^\alpha\rangle$ creates a torque on the β electrode that is just the *opposite* to the torque created by $\langle S_+^\beta S_-^\beta\rangle$ on the α electrode. This somewhat counterintuitive result can be *a posteriori* understood by the following argument. The elastic spin current is polarized by the weighted average of the polarization of the two magnetic leads; see Eq. (18). From their definition, it follows that the torques on the two are in the same direction. From our results, the inelastic contributions reduce the effect of the polarization of the magnetic leads as epitomized by the density matrices $\vec{\rho}^{\alpha/\beta}$ while at the same time adding to the spin current, albeit along another axis (direction). For the electrode at the higher potential, α, we find from Eq. (23) that the diminution due to $\langle S_+^\alpha S_-^\alpha\rangle$ causes the polarization of the spin current to move toward $\vec{\rho}^\beta$, which is effectively a torque in the *same* direction as the elastic contribution; however, for the downstream electrode β this reorientation of the polarization *reduces* the effect of the elastic term. This can be seen by drawing the *vector* sum of the difference between the spin angular momentum gained by the current and that lost by the background magnetization that is transverse to the electrode's magnetization. By using the relation Eq. (25) and from Eq. (23) we find that spin flips $\langle S_+^\beta S_-^\beta\rangle$ in the downstream electrode lead to a torque on that electrode in the sense *opposite* to the elastic current (as well as a torque opposite in a sense to that created by $\langle S_+^\alpha S_-^\alpha\rangle$ on the α electrode). The net result of all these considerations is epitomized by the relation in Eq. (25).

2. Bulk magnons

If the current does not give up its excess energy, which is the potential difference eV, in tunneling from α to β it remains hot in the downstream β electrode, and the additional torque created by possibly multiple spin-flip scattering in β depends on whether the electrons leave the electrode with their spins in the same direction (NSF) or whether they have flipped (SF), i.e., the NSF and SF scattering cross sections are different.[19] The SF processes produce a transverse component to the spin current; therefore, they affect the torque on both electrodes according to Eq. (25). By using Eq. (23) with only $\langle S_+^\beta S_-^\beta\rangle$ at $T=0$ K in Eq. (12), we find that the contribution from bulk magnons to the torque in the upstream electrode is[20]

$$\tau_y^{\alpha\text{bulk transverse magnons}} = \frac{6}{5}\pi e\left(\frac{eV}{E_m^\beta}\right)^{3/2} eV|t_m^b|^2 N_\beta^b S^\beta\hbar^2\sin\theta \times\left[\rho_z^\alpha\rho_\uparrow^\beta\cos\theta - \sum_\sigma(-)^\sigma\rho_\sigma^\alpha(-\theta)\rho_{\sigma'}^\beta\right], \tag{26}$$

where the expression in square brackets can be written as $[\rho_0^\alpha\rho_\uparrow^\beta-\rho_\uparrow^\alpha(\theta)\rho_\downarrow^\beta]$. From Eqs. (24) and (25) we find the torque on the downstream electrode β by replacing the square brackets in Eq. (26) with $-\rho_z^\alpha\rho_\uparrow^\beta$.

The NSF processes only affect the longitudinal component of the spin current and do not contribute to the torque on the β electrode. However, this reduction of the longitudinal part of the current contributes to the torque on the α electrode when the electrodes are noncollinear; this can be seen by writing the components of the spin current transverse to the magnetization of the α electrode ($\hat{\alpha}$) in terms of its components parallel and perpendicular to $I_s^\pm(\hat{\beta})$,

$$\vec{I}_s\big|_{\perp\hat{\alpha}} = \cos^2\theta/2 I_s^\pm(\hat{\beta}) + \sin^2\theta/2 I_s^\mp(\hat{\beta}) \pm i\sin\theta I_s^z(\hat{\beta}), \tag{27}$$

where $I_s^\pm(\hat{\beta})\equiv\frac{1}{2}(I_s^x\pm I_s^y)$ are the components transverse to $\hat{\beta}$. By using the $t_m^b S_z^\beta$ part of the transmission amplitude $\hat{t}_{\beta\alpha}$, Eq.

(13), in Eq. (12), we obtain $I_s^z(\hat{\beta})$ due to bulk magnons; by placing this in the equation above we find the torque on the upstream electrode at $T=0$ K coming from the multiple spin-flips, that leave the current existing from the downstream electrode non-spin-flipped, is

$$\tau_y^{\alpha^{\text{bulk longitudinal magnons}}} = -\frac{6}{5}\pi e\left(\frac{eV}{E_m^\beta}\right)^{3/2} eV|t_m^b|^2 N_\beta^b S^\beta \hbar^2 \sin\theta\cos\theta\rho_z^\alpha\rho_0^\beta, \tag{28}$$

while $\tau_y^{\beta^{\text{bulk longitudinal magnons}}}=0$, and the minus sign comes from replacing $\langle(S_z^\beta)^2\rangle$ with $(S^\beta)^2-\frac{1}{2}[\langle S_+^\beta S_-^\beta\rangle+\langle S_-^\beta S_+^\beta\rangle]$. The term $(S^\beta)^2$ represents processes in the β electrode in which no spin flips occur at all; therefore, they contribute to the spin current through the second term in Eq. (7), rather than Eq. (12), and produce the same term as the spin-dependent scattering at the barrier/electrode β interface as Eq. (22) except that the transmission coefficient is $|t_m^b|^2$, and the contribution to $\tau_y^\beta=0$.

By collecting the various contributions to the torques, we find for the torque on the upstream electrode

$$\begin{aligned}\tau_y^\alpha = 2\pi e(eV)\sin\theta\Bigg\{&2|t_d|^2\rho_0^\alpha\rho_z^\beta + \frac{1}{2}\Bigg[(|t_m^{i\beta}|^2+|t_m^{b\beta}|^2)\\ &\times\sum_\sigma(-)^\sigma\rho_\sigma^\alpha(-\theta)\rho_\sigma^\beta\Bigg] + \frac{1}{2}\Bigg[\left(\frac{eV}{E_m^\alpha}\right)\frac{|t_m^{i\alpha}|^2}{S^\alpha}\rho_\uparrow^\alpha\rho_z^\beta\\ &+\left(\frac{eV}{E_m^\beta}\right)\frac{|t_m^{i\beta}|^2}{S^\beta}\left(\rho_z^\alpha\rho_\uparrow^\beta\cos\theta-\sum_\sigma(-)^\sigma\rho_\sigma^\alpha(-\theta)\rho_{\sigma'}^\beta\right)\Bigg]\\ &+3/5\left(\frac{eV}{E_m^\beta}\right)^{3/2}\frac{|t_m^{b\beta}|^2}{S^\beta}\left[\rho_z^\alpha\rho_z^\beta\cos\theta-\sum_\sigma(-)^\sigma\rho_\sigma^\alpha(-\theta)\rho_{\sigma'}^\beta\right]\Bigg\},\end{aligned} \tag{29}$$

where $|t_m^{i/b,\alpha/\beta}|^2 \equiv |t_m^{i/b}|^2 N_{\alpha/\beta}^{i/b}(S^{\alpha/\beta})^2\hbar^2$. For the downstream electrode, we find

$$\begin{aligned}\tau_y^\beta = 2\pi e(eV)\sin\theta\Bigg\{&2|t_d|^2\rho_z^\alpha\rho_0^\beta + \frac{1}{2}|t_m^{i\beta}|^2\sum_\sigma(-)^\sigma\rho_\sigma^\alpha\rho_\sigma^\beta(-\theta)\\ &-\frac{1}{2}\Bigg[\left(\frac{eV}{E_m^\alpha}\right)\frac{|t_m^{i\alpha}|^2}{S^\alpha}\left(\rho_\uparrow^\alpha\rho_z^\beta\cos\theta+\sum_a(-)^\sigma\rho_\sigma^\alpha\rho_{\sigma'}^\beta(-\theta)\right)\\ &+\left(\frac{eV}{E_m^\beta}\right)\frac{|t_m^{i\beta}|^2}{S^\beta}\rho_z^\alpha\rho_\uparrow^\beta\Bigg] - 3/5\left(\frac{eV}{E_m^\beta}\right)^{3/2}\frac{|t_m^{b\beta}|^2}{S^\beta}\rho_z^\alpha\rho_\uparrow^\beta\cos\theta\Bigg\}.\end{aligned} \tag{30}$$

V. CHARGE CURRENT

To compare with experimental data, we take the ratio of the torque to the charge current. This was calculated for a MTJ for the cases of parallel and antiparallel alignment of the electrodes, and we have extended it to arbitrary angles between them.[21] The controlling elements for the charge current are the transmission amplitudes across the barrier, which are determined by the parameters in Eq. (13). Whereas the scattering in the bulk of the electrodes t_m^b plays a role for the spin transfer, it is not dominant for the tunneling process itself and therefore has not been singled out in the transfer Hamiltonian approach used to determine the charge current in a MTJ,[10] i.e., the scattering in the bulk of the electrodes alters their resistance, which, for the MTJ's we compare our results to, is very small compared to the resistance (rather small transmission) of the barrier itself. Therefore, while the transmission amplitudes for the charge current take into account the spin-dependent scattering potential across the barrier, those for the torque include an additional parameter t_m^b, i.e., the spin interaction of the conduction electrons with the magnetic background of the electrodes. In a manner of speaking, the magnetic leads in the Landauer or Caroli formalism[14] start just behind the interfaces in the calculation of the charge current, but far from the electrode/barrier interfaces for the torque.

In terms of the transmission amplitudes we adopt here, the charge current across the MTJ at $T=0$ K is[21]

$$\begin{aligned}I_0 = \frac{4\pi e^2 V}{\hbar}\Bigg\{&[|t_d|^2+|t_m^{i\alpha}|^2+|t_m^{i\beta}|^2]\sum_\sigma\rho_\sigma^\alpha\rho_\sigma^\beta(\theta)\\ &+\frac{eV}{E_m^\alpha}\frac{|t_m^{i\alpha}|^2}{S^\alpha}\rho_\downarrow^\alpha\rho_\uparrow^\beta(\theta)+\frac{eV}{E_m^\beta}\frac{|t_m^{i\beta}|^2}{S^\beta}\rho_\uparrow^\alpha\rho_\downarrow^\beta(\theta)\Bigg\},\end{aligned} \tag{31}$$

where in Ref. 10 we had T^d instead of t_d, and $S_{L/R}^z T^J$ instead of $t_m^{i,\alpha/\beta}$.

VI. SIMPLIFICATIONS

These expressions simplify further when the density matrices for the magnetic leads and the electrodes are made of the same material; we find the torque on the α electrode reduces to

$$\begin{aligned}\tau_y^\alpha = 2\pi e^2 V\sin\theta\Bigg\{&2|t_d|^2\rho_0\rho_z+\frac{1}{2}\Bigg[(|t_m^i|^2+|t_m^b|^2)\\ &\times\sum_\sigma(-)^\sigma\rho_\sigma(\theta)\rho_\sigma\Bigg]+\frac{1}{2}\left(\frac{eV}{E_m}\right)\frac{|t_m^i|^2}{S}\Bigg[\rho_\uparrow\rho_z(1+\cos\theta)\\ &-\sum_\sigma(-)^\sigma\rho_\sigma(\theta)\rho_{\sigma'}\Bigg]+3/5\left(\frac{eV}{E_m}\right)^{3/2}\frac{|t_m^b|^2}{S}\Bigg[\rho_z^2\cos\theta\\ &-\sum_\sigma(-)^\sigma\rho_\sigma(\theta)\rho_{\sigma'}\Bigg]\Bigg\},\end{aligned} \tag{32}$$

$$\begin{aligned}=2\pi e^2 V\sin\theta\Bigg\{&\frac{1}{2}(\rho_\uparrow^2-\rho_\downarrow^2)[|t_d|^2+(|t_m^i|^2+|t_m^b|^2)\cos^2\theta/2]\\ &+\frac{1}{2}\left(\frac{eV}{E_m}\right)\frac{|t_m^i|^2}{S}[\rho_\uparrow^2-\rho_\downarrow\rho_\uparrow(\theta)]+3/5\left(\frac{eV}{E_m}\right)^{3/2}\frac{|t_m^b|^2}{S}\\ &\times[\rho_z^2\cos\theta+(\rho_\uparrow^2-\rho_\downarrow^2)\sin^2\theta/2]\Bigg\}.\end{aligned} \tag{33}$$

To bring out the bias dependence, we write this as

$$\tau_y^\alpha = 2\pi e^2 V \sin\theta |t_d|^2 \rho_\uparrow^2 \frac{1}{2}(1-p^2)[1+(\phi_i+\phi_b)\cos^2\theta/2]\left\{1+\left(\frac{eV}{E_m'}\right)\left[\frac{\phi_i}{1+(\phi_i+\phi_b)\cos^2\theta/2}\frac{1-p(\cos^2\theta/2+p\sin^2\theta/2)}{1-p^2} + 6/5\sqrt{\frac{eV}{E_m}}\frac{\phi_b}{1+(\phi_i+\phi_b)\cos^2\theta/2}\frac{(1-p)^2\cos\theta+(1-p^2)\sin^2\theta/2)}{1-p^2}\right]\right\}, \tag{34}$$

where we set $p\equiv\rho_\downarrow/\rho_\uparrow, \phi_{i/b}\equiv|t_m^{i/b}|^2/|t_d|^2$, and $E_m'\equiv SE_m = \frac{3S}{S+1}kT_c$. For the downstream electrode, we find the torque is

$$\tau_y^\beta = 2\pi e^2 V \sin\theta\left\{2|t_d|^2\rho_0\rho_z + \frac{1}{2}|t_m^i|^2\sum_\sigma(-)^\sigma\rho_\sigma(\theta)\rho_\sigma - \frac{1}{2}\left(\frac{eV}{E_m}\right)\frac{|t_m^i|^2}{S}\left[\rho_\uparrow\rho_z(1+\cos\theta)-\sum_\sigma(-)^\sigma\rho_\sigma(\theta)\rho_{\sigma'}\right] - 3/5\left(\frac{eV}{E_m}\right)^{3/2}\frac{|t_m^b|^2}{S}\rho_\uparrow\rho_z\cos\theta\right\}, \tag{35}$$

$$= 2\pi e^2 V\sin\theta\left\{\frac{1}{2}(\rho_\uparrow^2-\rho_\downarrow^2)[|t_d|^2+|t_m^i|^2\cos^2\theta/2] - \frac{1}{2}\left(\frac{eV}{E_m}\right)\frac{|t_m^i|^2}{S}[\rho_\uparrow^2-\rho_\downarrow\rho_\uparrow(\theta)] - 3/5\left(\frac{eV}{E_m}\right)^{3/2}\frac{|t_m^b|^2}{S}\rho_\uparrow\rho_z\cos\theta\right\}, \tag{36}$$

$$= 2\pi e^2 V\sin\theta|t_d|^2\rho_\uparrow^2\frac{1}{2}(1-p^2)\times[1+\phi_i\cos^2\theta/2]\left\{1-\left(\frac{eV}{E_m'}\right)\times\left[\frac{\phi_i}{1+\phi_i\cos^2\theta/2}\frac{1-p(\cos^2\theta/2+p\sin^2\theta/2)}{1-p^2} + 6/5\sqrt{\frac{eV}{E_m}}\frac{\phi_b}{1+\phi_i\cos^2\theta/2}\frac{1}{1+p}\cos\theta\right]\right\}. \tag{37}$$

And the current reduces to

$$I_0 = \frac{4\pi e^2 V}{\hbar}\left\{[|t_d|^2+2|t_m^i|^2]\sum_\sigma\rho_\sigma(\theta)\rho_\sigma + \frac{eV}{E_m}\frac{|t_m^i|^2}{S}\sum_\sigma\rho_\sigma(\theta)\rho_{\sigma'}\right\}, \tag{38}$$

$$= \frac{4\pi e^2 V}{\hbar}\left\{[|t_d|^2+2|t_m^i|^2]\{(\rho_\uparrow^2+\rho_\downarrow^2)\cos^2\theta/2 + 2\rho_\uparrow\rho_\downarrow\sin^2\theta/2\} + \frac{eV}{E_m}\frac{|t_m^i|^2}{S}\{2\rho_\uparrow\rho_\downarrow\cos^2\theta/2 + (\rho_\uparrow^2+\rho_\downarrow^2)\sin^2\theta/2\}\right\}, \tag{39}$$

$$= \frac{4\pi e^2 V}{\hbar}|t_d|^2\rho_\uparrow^2(1+2\phi_i)[(1+p^2)\cos^2\theta/2 + 2p\sin^2\theta/2]\times\left\{1+\left(\frac{eV}{E_m'}\right)\times\left[\frac{\phi_i}{1+2\phi_i}\frac{2p\cos^2\theta/2+(1+p^2)\sin^2\theta/2)}{(1+p^2)\cos^2\theta/2+2p\sin^2\theta/2}\right]\right\}. \tag{40}$$

VII. COMPARISONS TO DATA

To compare these results with experimental data, we take the ratio of the torque to the current needed to produce it, i.e., we obtain the torque per unit current rather than Eqs. (34) and (37), which give the torque in terms of the bias. By taking the ratios of Eqs. (34) and (37) to Eq. (40), we find

$$\frac{\tau_y^{\alpha/\beta}}{I_0} = \frac{\hbar}{2}\sin\theta A^{\alpha/\beta}(\theta)\frac{1\pm b^{\alpha/\beta}(V,\theta)}{1+c(V,\theta)}, \tag{41}$$

where the $\pm$ refers to α/β,

$$A^\gamma(\theta) = \frac{1+(\phi_i+\phi_b\delta_{\gamma\alpha})\cos^2\theta/2}{1+2\phi_i}\times\frac{\frac{1}{2}(1-p^2)}{(1+p^2)\cos^2\theta/2+2p\sin^2\theta/2}, \tag{42}$$

$$b^\gamma(V,\theta) = \left(\frac{eV}{E_m'}\right)\left\{\frac{\phi_i}{1+(\phi_i+\phi_b\delta_{\gamma\alpha})\cos^2\theta/2}\times\frac{1-p(\cos^2\theta/2+p\sin^2\theta/2)}{1-p^2} + 6/5\sqrt{\frac{eV}{E_m}}\frac{\phi_b}{1+(\phi_i+\phi_b\delta_{\gamma\alpha})\cos^2\theta/2}\times\left[\frac{(1-p)^2\cos\theta+(1-p^2)\sin^2\theta/2)}{1-p^2}\delta_{\gamma\alpha} + \frac{1}{1+p}\cos\theta\delta_{\gamma\beta}\right]\right\}, \tag{43}$$

and

$$c(V,\theta) = \left(\frac{eV}{E'_m}\right)\left[\frac{\phi_i}{1+2\phi_i}\frac{2p\cos^2\theta/2 + (1+p^2)\sin^2\theta/2)}{(1+p^2)\cos^2\theta/2 + 2p\sin^2\theta/2}\right], \tag{44}$$

where $\gamma = \alpha/\beta$.

Finally, we evaluate these expressions for MTJ's with Co electrodes for which we previously were able to fit the zero-bias anomaly found in the bias dependence of the TMR of Co/Al$_2$O$_3$/CoFe with the following parameters:[10] $|t_d|^2/|t_m^i|^2 = 17$, $\rho_\uparrow/\rho_\downarrow = 2.1-2.2$, $S=3/2$, and $kT_c = 110$ meV. These yield $\phi_i = 0.06$, $p=0.48 \Rightarrow \frac{1}{2}$, and $E_m = 132$ meV ($E'_m = 198$ meV); ϕ_b is an adjustable parameter inasmuch as it was not critical to fitting the TMR. With these parameters, we find for the upstream electrode

$$\frac{\tau_y^\alpha}{I_0}(\theta\sim 0) \cong \frac{\hbar}{2}\sin\theta\cdot 0.3(1+\phi_b) \times \frac{1 + 0.03\frac{eV}{130}\frac{1}{1+\phi_b}\left[1 + 10\phi_b\sqrt{\frac{eV}{130}}\right]}{1+0.03\frac{eV}{130}} \tag{45}$$

and

$$\frac{\tau_y^\alpha}{I_0}(\theta\sim \pi) \cong \frac{\hbar}{2}\sin\theta\cdot 0.36\frac{1 + 0.04\frac{eV}{130}\left[1 + 13\phi_b\sqrt{\frac{eV}{130}}\right]}{1+0.04\frac{eV}{130}}. \tag{46}$$

For the downstream electrode we find

$$\frac{\tau_y^\beta}{I_0}(\theta\sim 0) \cong \frac{\hbar}{2}\sin\theta\cdot 0.3\frac{1 - 0.03\frac{eV}{130}\left[1 + 20\phi_b\sqrt{\frac{eV}{130}}\right]}{1+0.03\frac{eV}{130}}, \tag{47}$$

and

$$\frac{\tau_y^\beta}{I_0}(\theta\sim \pi) \cong \frac{\hbar}{2}\sin\theta\cdot 0.36\frac{1 - 0.04\frac{eV}{130}\left[1 - 13\phi_b\sqrt{\frac{eV}{130}}\right]}{1+0.04\frac{eV}{130}}. \tag{48}$$

The salient feature of these results is that (i) the bias-dependent torques are in the same direction as the elastic for the upstream electrode and opposite for the downstream, and (ii) the torque increases with bias at least as fast as the charge current for the upstream electrode, i.e., even when we set aside the contribution from the bulk scattering ϕ_b, the ratios in Eqs. (45) and (46) are constant. We should stress that this constancy is not an exact result; rather, the coefficients after rounding off are close, e.g., 0.03 in the numerator of Eq. (45) was arrived at from 0.027. It follows that the torque on the upstream electrode is assisted by the applied bias, which is in agreement with the data on CoFeB/Al$_{2/3}$O$_x$/CoFeB and CoFe/MgO/CoFe (see Refs. 1, 2, and 7).

We stress that for the data available, the upstream electrode is the thinner or "free" layer, while the downstream is thicker and fixed.

Reversed polarity

When the polarity is reversed, the β electrode is at the higher potential (upstream) while α now is downstream. In this case, one must replace $\alpha \rightleftarrows \beta$ in Eqs. (41)–(44), and from Eq. (9) (and previous work[22]) we see that the sign of the elastic contribution to the spin current is reversed, while that coming from the magnons does not change sign.[23] The resulting torques for reverse bias are for the upstream electrode (note that I_0 denotes the magnitude and does not change sign),

$$\frac{\tau_y^\beta}{I_0}(\theta\sim 0) \cong -\frac{\hbar}{2}\sin\theta\cdot 0.3(1+\phi_b) \times \frac{1 - 0.03\frac{eV}{130}\frac{1}{1+\phi_b}\left[1 + 10\phi_b\sqrt{\frac{eV}{130}}\right]}{1+0.03\frac{eV}{130}} \tag{49}$$

and

$$\frac{\tau_y^\beta}{I_0}(\theta\sim \pi) \cong -\frac{\hbar}{2}\sin\theta\cdot 0.36\frac{1 - 0.04\frac{eV}{130}\left[1 + 13\phi_b\sqrt{\frac{eV}{130}}\right]}{1+0.04\frac{eV}{130}}. \tag{50}$$

For the downstream electrode, we find

$$\frac{\tau_y^\alpha}{I_0}(\theta\sim 0) \cong -\frac{\hbar}{2}\sin\theta\cdot 0.3\frac{1 + 0.03\frac{eV}{130}\left[1 + 20\phi_b\sqrt{\frac{V}{130}}\right]}{1+0.03\frac{eV}{130}} \tag{51}$$

and

$$\frac{\tau_y^\alpha}{I_0}(\theta\sim \pi) \cong -\frac{\hbar}{2}\sin\theta\cdot 0.36\frac{1 + 0.04\frac{eV}{130}\left[1 - 13\phi_b\sqrt{\frac{eV}{130}}\right]}{1+0.04\frac{eV}{130}}. \tag{52}$$

In this case, it is the torque on the downstream electrode α that is assisted by the magnons. This is in agreement with the data inasmuch as it is the free layer that is upstream for the forward bias $\mu_\alpha > \mu_\beta$ and is downstream in reverse bias for the MTJ's that have been studied.[2,7] It also conforms the reasoning of Waintal *et al.*[22] When bulk magnons contribute, the magnitude of the torque on the free layer, α, will be somewhat different when the polarity is reversed, e.g., ϕ_b appears differently in the coefficients $A^{\gamma}(\theta)$ and $b^{\gamma}(V,\theta)$ in Eq. (45), where $\gamma=\alpha$, and Eq. (51), where $\gamma=\beta$.

VIII. SUMMARY

We conclude that the bias-induced magnon production in the magnetic electrodes is able to explain the data by Fuchs *et al.*,[7] which were summarized by them as "…the spin torque efficiency is indeed essentially constant over the bias ranges studied." This range was ±300 meV, while our results are valid up to 200 meV [as we mentioned after Eq. (12) for $eV > E_m$, it is necessary to replace $\frac{eV}{130} \Rightarrow (2 - E_m^{\alpha}/eV)$]; this is a strong indication that the magnons created by the spin current in MTJ's play an important role in the spin transfer. This contribution to spin transfer lies outside the conventional elastic contribution Eqs. (19) and (20) found by Slonczewski. It should be kept in mind that while magnon production is able to explain data in a limited range of bias, other effects enter as the bias is increased, e.g., the change of the barrier profile.[11] A recent treatment of this effect has shown that while it can explain the extant data for a forward bias, it is not able to when the bias is reversed.[9]

Our treatment is directly applicable to MTJ's with ferromagnetic transition-metal electrodes, but it is not applicable in its present form to semiconducting junctions, e.g., (Ga,Mn)As/GaAs/(Ga,Mn)As.[3,4] Among other aspects, the electrons (holes) are $j=3/2$ rather than $\frac{1}{2}$, their Fermi energy is quite small, and their Curie temperatures are low. For this case, the applied bias and temperature redistribute (repopulate) the electrons (holes) among the available states; in addition to modifying the transmission amplitudes, this can alter the T_c of the electrodes. We have not taken these effects into account at this time.

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[12] The main distinction between the two is that spin-flip excitations that occur at the electrode/barrier interfaces are part of the tunneling process; they alter the DOS entering the expressions for the tunneling (charge) current, and, as we show, they also contribute to the spin current in the junction. On the other hand, the spin transfer that occurs inside the electrode alters the spin current in the MTJ, but it does *not* alter in an appreciable way the charge current in the junction; it does change the resistance of the electrode, but that is minor compared with changing the tunneling current. These observations, which we now formalize, provide the basis for understanding how in the presence of current-induced spin-flip excitations, one continues to have effective spin transfer of angular momentum between the spin current and magnetic electrodes, while reducing the TMR of MTJ's.

[13] This form is a shorthand for integrating the transmission probabilities over a Fermi distribution whose chemical potential is given by μ. As we do not dwell on the energy dependence of the density of states or the transmission amplitudes, we use this abbreviated form here. See Chap. 2, especially Eq. (2.4.2a) in Supriyo Datta, *Electronic Transport in Mesoscopic Systems* (Cambridge University Press, Cambridge, UK, 1995). See also X. Waintal, E. B. Myers, P. W. Brouwer, and D. C. Ralph, Phys. Rev. B **62**, 12317 (2000); A. Brataas, Yu. V. Nazarov, and G. E. W. Bauer, Phys. Rev. Lett. **84**, 2481 (2000).

[14] C. Caroli, R. Combescot, P. Nozieres, and D. Saint-James, J. Phys. C **4**, 916 (1971); T. N. Todorov, G. A. D. Briggs, and A.

P. Sutton, J. Phys.: Condens. Matter **5**, 2389 (1993); T. N. Todorov, Phys. Rev. B **54**, 5801 (1996). For the extension to spin systems, see, for example, Ref. 11.

[15] P. S. Krstić, X.-G. Zhang, and W. H. Butler, Phys. Rev. B **66**, 205319 (2002).

[16] This follows from writing the spinor as $\hat{\rho}=\rho_0\hat{1}+\vec{\sigma}\cdot\vec{\rho}$, and then rotating the basis of the Pauli matrix $\vec{\sigma}$,

$$R^{\dagger}(\theta)\vec{\sigma}R(\theta)\equiv\vec{\sigma}(\hat{\alpha},\theta)=\begin{pmatrix}\hat{z}(\theta) & \hat{\imath}_-(\theta)\\ \hat{\imath}_+(\theta) & -\hat{z}(\theta)\end{pmatrix},$$

where

$$R(\theta)=\begin{pmatrix}\cos\theta/2 & -i\sin\theta/2\\ -i\sin\theta/2 & \cos\theta/2\end{pmatrix},$$

and we define $\hat{\imath}_{\pm}(\theta)\equiv 1/\sqrt{2}[\hat{x}\pm i\hat{y}(\theta)]$, so that $\hat{y}(\theta)=\cos\theta\hat{y}+\sin\theta\hat{z}$ and $\hat{z}(\theta)=\cos\theta\hat{z}-\sin\theta\hat{y}$. As the density matrix is diagonal when referred to the magnetization of the electrode, $\hat{\rho}=\rho_0\hat{1}+\rho_z\sigma_z$, when one rotates the vector $\vec{\rho}=\rho_z\hat{z}$ we find

$$\rho_x(\theta)=0,$$

$$\rho_y(\theta)=\sin\theta\rho_z,$$

$$\rho_z(\theta)=\cos\theta\rho_z,$$

and

$$\rho_{\pm}(\theta)=\pm i\sin\theta\rho_z=\pm i\rho_y(\theta).$$

There is no need to use $(1/\sqrt{2})$ in the definition of $\rho_{\pm}(\theta)$, as we are not interested in maintaining the normalization of $\vec{\rho}$.

[17] For tunnel junctions, the probability for transmission is quite small; therefore, we make the approximation that the various transmission amplitudes are additive. The sums of their square are not 1 and strictly speaking they do not represent probabilities, only relative ones.

[18] As angular momentum is conserved in the transfer between the spin current and the magnetic background, the *loss* of spin angular momentum in the current $\Delta\vec{I}_s$ shows up as a gain in the momentum of the magnetic background, hence the negative sign in the definition of the torque τ.

[19] Shufeng Zhang and Peter M. Levy, Phys. Rev. B **43**, 11048 (1991).

[20] This expression is derived in the lowest-order Born approximation (one spin flip). We account for multiple scattering by renormalizing the transmission amplitude, i.e., by replacing $t_m^i \Rightarrow t_m^b$. See Ref. 19.

[21] P. M. Levy (private communication).

[22] X. Waintal, E. B. Myers, P. W. Brouwer, and D. C. Ralph, Phys. Rev. B **62**, 12317 (2000); see in particular Fig. 2, which shows that the torque on the free layers changes sign when the direction of the electron current is reversed.

[23] In this context, it is important to remember that the spin current is a dyadic that has three components for its polarization in spin space (which we designate with an arrow $\vec{I}_s$), as well as the direction of the current itself. Reversing the polarity changes the direction of the current, but not the polarization $\vec{I}_s$. It is the sense of the reorientation (cw or ccw) of the polarization of the spin current as one moves from the electrode at the higher to lower potential that is reversed when the polarity is reversed. When one reverses the polarity so that $\mu_\beta > \mu_\alpha$, one has to interchange the indices $\alpha \rightleftarrows \beta$ in Eq. (6); this reverses the sign of the elastic contribution to the torque in Eq. (7). However, the torque coming from the magnons is controlled by Eq. (8), from which we see that one must perform the replacement $\alpha \rightleftarrows \beta$.