Gapless bosonic excitation without symmetry breaking: An algebraic spin liquid with soft gravitons

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A quantum ground state of matter is realized in a bosonic model on a three-dimensional fcc lattice with emergent low energy excitations. The phase obtained is a stable gapless boson liquid phase, with algebraic boson density correlations. The stability of this phase is protected against the instanton effect and superfluidity by self-duality and large gauge symmetries on both sides of the duality. The gapless collective excitations of this phase closely resemble the graviton, although they have a soft $\omega \sim k^2$ dispersion relation. There are three branches of gapless excitations in this phase, one of which is gapless scalar trace mode, the other two have the same polarization and gauge symmetries as the gravitons. The dynamics of this phase is described by a set of Maxwell's equations. The defects carrying gauge charges can drive the system into the superfluid order when the defects are condensed; also the topological defects are coupled to the dual gauge field in the same manner as the charge defects couple to the original gauge field, after the condensation of the topological defects, the system is driven into the Mott insulator phase. In the two-dimensional case, the gapless soft graviton as well as the algebraic liquid phase are destroyed by the vertex operators in the dual theory, and the stripe order is most likely to take place close to the two-dimensional quantum critical point at which the vertex operators are tuned to zero.

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I. INDRODUCTION

Ground states of quantum many-body systems can behave qualitatively differently from classical states. Classical ground states like ferromagnetic order and Néel order can survive from quantum fluctuation at zero temperature. The quantum ferromagnetic state and quantum Néel ordered state can be described in a similar way as their classical counterparts. Searching for nonclassical spin states can be traced back to the early proposal of the "Resonate Valence Bond" state of spin system on frustrated lattices,[1,2] which was shown to be of great importance to the high T_c superconductivity in cuperates.

All the classical orders break certain symmetry, either internal symmetry or space symmetry. Quantum spin ground states without classical orders are termed spin liquids. In order to make sure the quantum ground state has no tendency to order, the low energy emergent gauge symmetry is usually applied, since as is well-known, the gauge symmetry cannot be broken without condensation of matter fields.[3] After two decades of work, several types of nonclassical ground states have been identified. For instance, the spin liquid with Z_2 gauge symmetry has been realized in either the quantum dimer model on triangular lattice[4] or spin-1/2 model on Kagomé lattice.[5] The existence of the Z_2 spin liquids in 2+1 dimensional space is based on the fact that the quantum Z_2 gauge theory has a deconfined phase, which manifests itself as a disordered spin state with fractionalized spin excitations. In these models, the ground states have no classical order, i.e., there is no symmetry breaking. The ground state degeneracy depends on the topology of the space manifold. The excitations contain the deconfined gapped Z_2 gauge charges and the Z_2 vortices (which are usually called visons).

Another type of spin liquids contain gapless collective excitations (which are different from the gapless Magnon excitations in spin ordered state), and the spin-spin correlation functions fall off algebraically, i.e., the state is in a stable critical phase. These algebraic spin liquids usually involve $U(1)$ gauge field, or $U(1)$ gauge field interacting with gapless matter fields. The stability of algebraic spin liquids is very tricky because algebraic spin liquids are critical, presumably there are supposed to be relevant perturbations which can drive the system into an ordered phase. Actually in most cases the algebraic liquid phases are fine-tuned, for instance, the Rokhsar-Kivelson (RK) point of quantum dimer model on square lattice.[6,7] Due to the presence and proliferation of monopoles in 2+1 dimensional systems,[8,9] the gapless photon excitations are generally gapped out, and the matter fields are confined. In the 3+1 dimensional space, the compact gauge theory has a deconfined photon phase. Based on this result, several microscopic models with photon liquid phase have been proposed.[10–12] In the 2+1 dimensional systems, the monopoles are almost always proliferating. The algebraic liquid phase is very hard to survive. The monopoles are only irrelevant at a certain critical point, for instance, the RK point of the quantum dimer model,[6] as well as the transition between valence bond solid phase and Néel state for a spin-1/2 antiferromagnetic system.[13,14] Recently it has been argued that at the large N level (N is the number of flavors of matter fields), there is a stable algebraic liquid phase in 2+1 dimensional space.[15]

Gapless bosonic excitation is one property which characterizes criticality. Since here the critical phase is stable, it implies that even without any continuous symmetry breaking there is a gapless bosonic excitation invulnerable to perturbations. This is actually a quite amazing property. As is well-known to all, the Goldstone theorem is one way to protect the gaplessness of bosonic systems.[16] By breaking continuous global symmetry, the coset of the unbroken symmetry subgroup corresponds to the gapless modes, which are called

Goldstone modes. Almost all the stable gapless bosonic excitations are related to certain continuous symmetry breaking, for instance, the phonon in solids is related to the spatial translational symmetry breaking, the magnon excitation in magnet is related to the $SU(2)$ spin symmetry breaking. However, the gapless critical excitations of algebraic spin liquid phase do not rely on any symmetry breaking. Because of the special gauge symmetry of the gapless excitations, which usually resembles the $U(1)$ gauge symmetry of quantum electrodynamics (QED), it has been proposed that the photon, which is one of the most fundamental particles in the universe, might not be so fundamental, it could be collective excitations of lattice spin models.[11]

All the stable algebraic liquid phases which have been found so far involve $U(1)$ gauge field theory. This $U(1)$ gauge symmetry does not exist in the high energy (microscopic) model, it only emerges at low energy Hamiltonian, due to the effective constraint $\vec{\nabla}\cdot\vec{E}=\rho$ imposed by the spin interaction. ρ is the background static charge distribution. Although the microscopic models which have been proposed so far[10–12] are different on the lattice scale, the 3+1 dimensional photon liquid phases are all the same at long scale. In the current work, a type of algebraic spin (boson) liquid phase has been realized. Based on the standard spin-boson mapping [$S^z=n-\bar{n}$, and $S^\dagger\sim\exp(i\theta)$], the model is presented in the bosonic version. The algebraic boson liquid state studied in this paper broadens the family of algebraic liquid phases. The algebraic liquid phase does not involve $U(1)$ gauge theory. Instead, the gauge symmetry of this model is identical to the gauge symmetry of linearized Einstein gravity. There are three branches of gapless collective excitations in this bosonic algebraic liquid phase. One of the collective excitations is a scalar mode, the other two gapless collective excitations have the same gauge symmetry and polarizations as the gravitons. However, the graviton excitations in our theory have a softened quadratic dispersion, $\omega\sim k^2$. The spin-spin (boson density) correlation functions fall off algebraically, with an exponent bigger than that of the 3+1 dimensional photon liquid. Like gravitational theory, the basic variables in this theory are symmetric rank-2 tensor, which in the gravity language are the linearized metric tensor. The dynamics of the graviton phase is described by a set of Maxwell's equations, with the vectors $\vec{E}$ and $\vec{B}$ replaced by rank two symmetric tensors. The charges and topological defects enter the Maxwell's equations in a special form, and the condensations of charges and topological defects will drive the liquid phase into the superfluid phase and the Mott insulator phase, respectively.

This paper is organized as follows. In the second section, a brief review of the 3+1 dimensional photon spin liquid phase is presented. The discussion of the graviton spin liquid will follow the same logic as the photon spin liquid phase. In the third section, the 2+1 dimensional version of the graviton model is discussed. In the 2+1 dimensional case, the graviton theory is dual to a scalar boson model with quadratic dispersion. However, the vertex operators will generally gap out the graviton excitations. In the forth section, the 3+1 dimensional graviton model is described, and it is shown that the graviton phase (Gaussian phase) is self-dual,

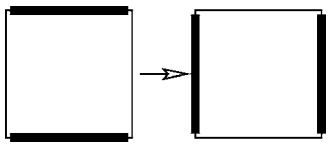

FIG. 1. The flipping term of the dimer model.

and hence stable. In Sec. V, more properties of this model are derived. The boson density correlation function is calculated, and a set of Maxwell's equations are introduced to describe this Gaussian phase. In Sec. VI, we discuss the possible experimental realization of this model, as well as the future work. If the model is written in terms of fermionic operators instead of bosonis operators, nonfermi liquid behavior is expected. It is also possible to develop this theory to be a candidate of quantum gravity theory.

II. REVIEW OF 3+1 DIMENSIONAL PHOTON LIQUID PHASE

Let us first briefly review some basic properties of the 3+1 dimensional photon liquid phase as a warmup. Although many different models have been proposed in the last few years,[10–12] the phases obtained have very similar properties at long scale. Therefore let us take the three-dimensional quantum dimer model on cubic lattice as an example. On the cubic lattice, every site is shared by six links, and only one of those links is occupied by exactly one dimer. On every square face, if two parallel links are occupied by dimers, they can be resonated to dimers perpendicular to the original ones (Fig. 1). Besides this resonating term, another diagonal weight term for each flippable plaquette is also included in the Hamiltonian

$$H=\sum -t(|\,\|\,\rangle\langle=|+\text{H.c.})+V(|\,\|\,\rangle\langle\|\,|+|\,=\,\rangle\langle=|). \qquad (1)$$

The dimer model can be mapped onto a rotor model. A rotor number can be defined on each link to describe the presence or absence of dimers: $n=1$ when the link is occupied by a dimer, and $n=0$ when the link is empty. The Hilbert space of this quantum system is a constrained one, with the constraint $\Sigma_\nu n_{i,\nu}=1$ around each site, the summation is over all six links shared by site i. Notice that $(i,+\hat{a})$ and $(i+\hat{a},-\hat{a})$ denote the same link. Let us define quantity $E_{i,\hat{a}}$ as $E_{i,\hat{a}}=(-1)^{i_x+i_y+i_z}n_{i,\hat{a}}$ and $E_{i,-\hat{a}}=-(-1)^{i_x+i_y+i_z}n_{i,-\hat{a}}$, with $\hat{a}$ equal to $\hat{x}$, $\hat{y}$, and $\hat{z}$. The sign distribution on the XY plane is shown in Fig. 2. Now, the constraint of this system can be rewritten as $\Sigma_{\hat{a}}(E_{i,+\hat{a}}-E_{i-\hat{a},\hat{a}})=\pm 1$, whose compact form is the Gauss's law for electric fields, $\partial_i E_i=\pm 1$. Notice that because the quantity $E_{i,\hat{a}}$ is defined on links, a natural vector notation can be applied: $\vec{E}_i=(E_{i,\hat{x}},E_{i,\hat{y}},E_{i,\hat{z}})$. Here the derivatives are all defined on the lattice $\partial_a E_{i,\hat{b}}=E_{i+\hat{a},\hat{b}}-E_{i,\hat{b}}$, and similar treatments can be found in Refs. 12 and 17.

The background charge distribution ± 1 on the cubic lattice plays a very important role in the solid phase, i.e., the confined phase. The form of the crystalline phase can be determined from Berry's phase induced by the background charge distribution.[12,14] However, as we are focusing on the

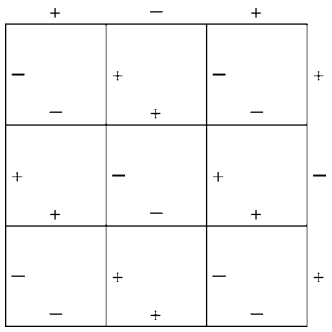

FIG. 2. The sign convention of mapping from rotor number onto the vector electric field $\vec{E}$ in the XY plane.

algebraic liquid phase, the background charge is not very important. Let us instead impose the constraint $\partial_i E_i=0$. Because of this local constraint, the low energy physics will be invariant under the gauge transformation $\vec{A}\rightarrow\vec{A}+\vec{\nabla}f$ [$\vec{A}$ is the conjugate variable of $\vec{E}$, which is related to the phase angle θ associated with the original rotor number n through $A_{i,\hat{a}}=(-1)^{i_x+i_y+i_z}\theta_{i,\hat{a}}$], which is exactly the gauge symmetry of the $U(1)$ gauge theory. Now the effective Hamiltonian of this theory should be invariant under the gauge transformation. The Hamiltonian (1) can be effectively written as

$$H=\sum -\tilde{t}\cos(\vec{\nabla}\times\vec{A})+1/(2\kappa)\vec{E}^2. \tag{2}$$

This Hamiltonian is the three dimensional compact QED, which should have a deconfined photon phase.[8,9] In this phase, the cosine functions in Eq. (2) can be expanded, and the system can be described in the following Gaussian fixed point Lagrangian:

$$L=(\partial_\tau\vec{A})^2-c^2(\vec{\nabla}\times\vec{A})^2. \tag{3}$$

The constraint $\vec{\nabla}\cdot\vec{E}=0$ introduces a Lagrange multiplier A_0 to the Lagrangian $A_0(\nabla_i\cdot E_i)$. After integrating out $\vec{E}$, the gauge invariance can be enlarged to $3+1$ dimensional space time, $A_\mu\rightarrow A_\mu+\partial_\mu f$, with $\mu=0,1,2,3$. The $3+1$ dimensional Lagrangian reads

$$L\sim -F_{\mu\nu}F^{\mu\nu}. \tag{4}$$

The effective Lagrangian for the photon phase has been derived above, and the reason for the existence of the photon phase is actually the remarkable self-duality of this photon phase and the gauge symmetry, as discussed below.

First, because the constraint $\partial_i E_i=0$ is strictly imposed on this system, the matter field, i.e., the defect which violates this constraint, is absent. As is well-known, the $U(1)$ gauge symmetry can be spontaneously broken due to the condensation of the matter fields. However, without matter field, the local gauge symmetry cannot be broken spontaneously.[3] The superfluid phase of the original system implies $\langle\theta\rangle\neq 0$, written in the low energy variables, it reads $\langle\vec{A}\rangle\neq 0$. The nonzero expectation of vector potential $\vec{A}$ breaks the local gauge symmetry. Therefore the dimer superfluid order is ruled out.

The other possible instability of the photon phase is towards the gapped solid phase, which can be analyzed in the dual theory. We can introduce the dual vector $\vec{h}$ and dual momentum vector $\vec{\pi}$ ($\vec{h}$ and $\vec{\pi}$ are both defined on the faces of the cubic lattice) as

$$\vec{E}=\vec{\nabla}\times\vec{h},\quad \vec{\nabla}\times\vec{A}=\vec{\pi}. \tag{5}$$

One can check the commutation relation and see that $\vec{h}$ and $\vec{\pi}$ are a pair of conjugate variables. The Gauss's law constraint on this system is automatically solved by the vector $\vec{h}$. Now, the field theory for the photon phase is self-dual:

$$L=(\partial_\tau\vec{A})^2-c^2(\vec{\nabla}\times\vec{A})^2,\quad \vec{\nabla}\cdot\vec{E}=0,$$

$$L=(\partial_\tau\vec{h})^2-c^2(\vec{\nabla}\times\vec{h})^2,\quad \vec{\nabla}\cdot\vec{\pi}=0. \tag{6}$$

The violation of the dimer constraint (for instance, the hole of the doped dimer model) can be viewed as the gauge charges. The defects couple to the gauge field vector in a gauge invariant manner

$$L_e=-t\cos(\vec{\nabla}\theta^{(e)}-\vec{A})+\cdots. \tag{7}$$

$\theta^{(e)}$ is the phase angle of the defect creation operator, which plays the role of the electric charge in the QED language. There are usually two flavors of matter fields, since besides the $U(1)$ gauge symmetry, there is an extra global $U(1)$ symmetry, which corresponds to the global conservation of total holon number. When the defects condense, the gauge bosons are gapped out by the Higgs mechanism, the system enters the superfluid order, and the global $U(1)$ symmetry is spontaneously broken and becomes the gapless Goldstone phason mode in the superfluid phase.[18,19]

In principle, a vertex operator $\cos(2\pi N\vec{h})$ is supposed to exist in the dual Hamiltonian, due to the fact that $\vec{E}$ only takes on integer values. N is an integer depending on Berry's phase of the vertex operator. When this vertex operator is relevant, it will gap out the photon excitation and drive the system into a crystalline phase, according to Berry's phase. However, the vertex operator is irrelevant in the photon phase. Notice that, because the theory is self-dual, the dual theory has the same gauge invariance $\vec{h}\rightarrow\vec{h}+\vec{\nabla}f$ as the original theory. Also, vector $\vec{\pi}$ is subject to the same constraint as $\vec{E}$, $\partial_i\pi_i=0$ [Eq. (6)]. However, the vertex operator $\cos(2\pi N\vec{h})$ breaks the gauge symmetry and thus the correlation function between two vertex operators is zero at the Gaussian fixed point, i.e., the vertex operator is irrelevant in this Gaussian theory.

Because the theory is self-dual at the Gaussian phase, the magnetic monopoles (the dual charges) should couple to the dual vector potential $\vec{h}$ in the same manner as the coupling between electric charge and original vector potential $\vec{A}$ [Eq. (7)],

$$L_m=-t'\cos(2\pi N\vec{\nabla}\theta^{(m)}-2\pi N\vec{h}). \tag{8}$$

N is again introduced by Berry's phase, corresponding to the multimonopole event. Unless the dual charges (the monopole) condense and break the dual $U(1)$ gauge symmetry, the photon phase is always stable. The gaplessness of this phase

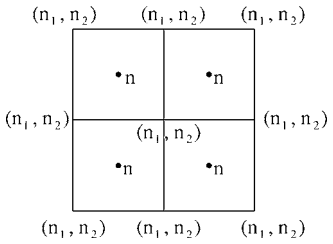

FIG. 3. The structure of the 2D lattice. On each site there are two orbital levels, the occupation number is (n_1,n_2). On each face center there is one orbital level, with occupation number n.

is protected by both the self-duality and the gauge symmetry.

The photon phase is an algebraic liquid phase, the dimer density-density correlation function falls off algebraically. At the point $t=V$ in Eq. (1), the lattice model can be solved exactly, and the equal-time dimer density correlation falls off as[10]

$$\langle [n(0)-\bar{n}][n(r)-\bar{n}]\rangle \sim 1/r^3. \tag{9}$$

III. GRAVITON MODEL IN TWO DIMENSIONAL SPACE

Let us begin with the two-dimensional example. This two-dimensional (2D) model is built on a square lattice with quantities defined on both sites and centers of plaquettes. Let us assume there is one orbital level on the center of each plaquette, and two orbital levels on each site (see Fig. 3). The Hamiltonian of the system contains three terms, $H=H_0+H_1+H_2$. H_1 is merely the nearest neighbor hopping term between the sites and the centers of plaquettes

$$H_1 = \sum_{\langle i,\bar{j}\rangle} \sum_{a=1}^{2} -t(b_{a,i}^{\dagger} b_{\bar{j}} + \text{H.c.}). \tag{10}$$

Here i denotes the site of the lattice, and $\bar{j}$ denotes the center of plaquette, the summation is over all the hoppings between each site and its four nearest neighbor plaquettes. H_2 is an on-site interaction $H_2=\Sigma_{i,\bar{i}}U(n_{\bar{i}}-\bar{n})^2+\Sigma_a U(n_{a,i}-\bar{n})^2$, which fixes the average filling per orbital state. $\bar{n}$ is the average particle number per orbital level, for simplicity it is taken to be 1.

The most important term in this Hamiltonian is H_0. It is a two body interaction between particle numbers with a special form. Each link of the square lattice is shared by two plaquettes and two sites. We denote the link between sites i and $i+\hat{x}$ as $(i,\hat{x})$, and denote the two plaquettes shared by this link as $\bar{i}=i+1/2\hat{x}+1/2\hat{y}$ and $\bar{i}-\hat{y}=i-1/2\hat{y}+1/2\hat{x}$. The term in H_0 which involves this link reads

$$V(n_{i+1/2\hat{x}+1/2\hat{y}} + n_{i-1/2\hat{y}+1/2\hat{x}} + 2n_{1,i} + 2n_{1,i+\hat{x}} - 6\bar{n})^2, \tag{11}$$

and the interaction term in H_0 involving the link $(i,\hat{y})$ is

$$V(n_{i+1/2\hat{x}+1/2\hat{y}} + n_{i-1/2\hat{x}+1/2\hat{y}} + 2n_{2,i} + 2n_{2,i+\hat{y}} - 6\bar{n})^2. \tag{12}$$

Notice that, for links in the $\hat{x}$ direction only n_1 is in this interaction term, and for links in the $\hat{y}$ direction only n_2 is involved. If V is much bigger than t, in the low energy subspace of the Hilbert space the summation in the brackets in both Eqs. (11) and (12) should be zero, this becomes a constraint on the low energy Hilbert space if V is large.

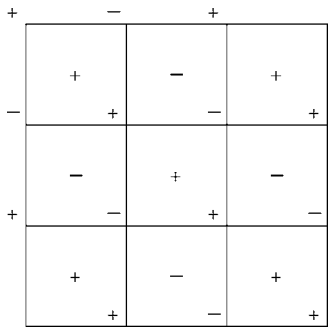

FIG. 4. The sign convention in the definition of symmetric tensor E_{ij}. After introducing the signs on the lattice, the constraint imposed by H_0 can be written compactly as Eq. (13).

Because the square lattice is a bipartite lattice, we can stagger the sign for each sublattice. Let us define variables $E_{xx}(i)=\eta_i(n_{1,i}-1)$, $E_{yy}(i)=\eta_i(n_{2,i}-1)$, and $E_{xy}(\bar{i})=\eta_{\bar{i}}(n_{\bar{i}}-1)$. η's are signs defined on sites and centers of plaquettes, $\eta_i(\eta_{\bar{i}})=\pm 1$, with $+$ for sublattice A $(\bar{A})$ and $-$ for sublattice B $(\bar{B})$. The constraint approximately imposed by H_0 can now be rewritten as

$$2\partial_x E_{xx} + \partial_y E_{xy} = 0,$$

$$\partial_x E_{xy} + 2\partial_y E_{yy} = 0. \tag{13}$$

Again all the derivatives are defined on the lattice. The convention of the staggered signs is depicted in Fig. 4.

The nearest neighbor hopping t term will generate a certain "ring exchange" term by perturbation theory, which is allowed by constraints (11) and (12) at low energy. However, without doing the perturbation literally, one can guess the form of the ring exchange term from the form of the constraint (13). Just like the constraint $\partial_i E_i=0$ generates gauge transformation $A_i \to A_i+\partial_i\varphi$, the current constraint on E_{ij} [Eq. (13)] will generate gauge transformation for its conjugate variable A_{ij}

$$A_{ij} \to A_{ij} + \partial_i f_j + \partial_j f_i, \tag{14}$$

and the low energy ring exchange term generated from perturbation theory should be invariant under this gauge transformation. A_{ij} is related to the phase angles of the original boson creation and annihilation operators by introducing staggered sign distribution $\eta_i: A_{xx}(i)=\eta_i\theta_{1,i}$, $A_{yy}(i)=\eta_i\theta_{2,i}$, and $A_{xy}(\bar{i})=\eta_{\bar{i}}\theta_{\bar{i}}$.

One may have already noticed that the gauge transformation (14) is exactly the gauge transformation for the graviton if we view rank two tensor A_{ij} as the linearized metric tensor on two-dimensional space. Then the only gauge invariant ring exchange is the curvature tensor, which is $R_{\alpha\mu\beta\nu}$. Written in terms of linearized metric tensor, the curvature tensor is[20]

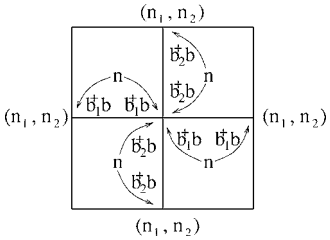

FIG. 5. The ring exchange of the two-dimensional lattice. Elementary hoppings are between each site and its nearest neighbor face center. The gauge invariant ring exchange term can be represented as $\cos(R_{xyxy})$, it involves eight single hoppings of bosons.

$$R_{\alpha\mu\beta\nu} = 1/2(A_{\alpha\nu,\mu\beta} + A_{\mu\beta,\alpha\nu} - A_{\mu\nu,\alpha\beta} - A_{\alpha\beta,\mu\nu}). \quad (15)$$

Here the notation in general relativity has been used, $A_{\alpha\nu,\mu\beta} = \partial_\mu \partial_\beta A_{\alpha\nu}$.

In two-dimensional space, although the curvature tensor has four indices, there is only one nonzero independent component, which is

$$R_{xyxy} = \frac{1}{2}(A_{xx,yy} + A_{yy,xx} - 2A_{xy,xy}). \quad (16)$$

At the eighth order perturbation of t, a ring exchange term $\cos(R_{xyxy})$ is generated. Now the low energy effective Hamiltonian reads

$$H_{eff} = -\tilde{t}\cos(R_{xyxy}) + \frac{1}{2\kappa}(E_{xx}^2 + E_{yy}^2 + aE_{xy}^2). \quad (17)$$

The cosine term in this Hamiltonian is ring exchange, $\tilde{t} \sim t^8/V^7$. If the original boson language is taken, one of the ring exchanges is shown in Fig. 5, one can clearly see that the ring exchange term involves eight independent nearest neighbor hoppings, this term only takes place at the eighth order perturbation of t.

The first conclusion drawn from the gauge symmetry (14) is that this system cannot be in the superfluid phase; because nonzero expectation value of the boson creation operator implies nonzero expectation of A_{ij}. Any linear combination of A_{ij} will break the local gauge symmetry, which is not allowed without matter field.[3] Another way to view this point is through the Hamiltonian. The hopping term H_1 in Eq. (10) has global $U(1)$ symmetry, which corresponds to the global conservation of total boson number. One might wonder whether this global $U(1)$ symmetry can be spontaneously broken, i.e., the system becomes superfluid. However, H_0 opens a very big energy gap to excitations that do not conserve boson number. The big charge gap precludes the possibility of superfluidity. Because of the graviton gauge symmetry, the polarization of the collective excitation should automatically be the same as the gravitons. However, one crucial difference from the photon liquid phase is that here the curvature tensor is the second order derivative of A_{ij}, if there is a phase in which we can expand the cosine functions in Eq. (17), i.e., there is a Gaussian phase or Gaussian fixed point, the dispersion relation of the gapless collective modes should be $\omega \sim k^2$. Unfortunately, just like the monopole proliferation in 2+1 dimensional QED, the topological defects in the current case also generally proliferate and will gap out the graviton excitations.

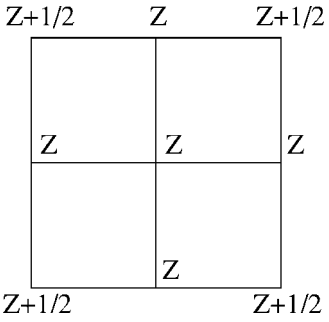

FIG. 6. The distribution of the dual variable h. h cannot all be integers over the whole two-dimensional plane, instead, one-quarter of h have to be half-integers.

The effect of the topological defects can be described in the dual formalism. Dual variables π and h can be defined as follows:

$$E_{xx} = \partial_y^2 h, \quad E_{yy} = \partial_x^2 h,$$

$$E_{xy} = -2\partial_x \partial_y h, \quad 2R_{xyxy} = \pi. \quad (18)$$

h and π are quantities defined on lattice i. One can check that the constraint (13) is solved automatically by the dual variables, and h and π are a pair of conjugate variables, i.e., $[h_i, \pi_j] = i\delta_{ij}$. Because the definition of the dual variable h only involves the second derivatives, the dual Lagrangian should be invariant under the following transformation:

$$h \to h + Ax + By + C, \quad (19)$$

A, B, C are arbitrary constant integers.

The dual Lagrangian now reads

$$L_{dual} = (\partial_\tau h)^2 - \rho_4 h(\partial_x^4 + \partial_y^4 + 4a\partial_x^2\partial_y^2)h + \cdots. \quad (20)$$

The ellipses include vertex operators in this dual theory. Notice that, although E_{ij} are all integers, the representation of E_{xy} in Eq. (18) contains factor 2 on the definition of the dual variables, thus h on some lattice sites have to be half integers, and the Berry's phase will cause oscillation of the signs of the vertex operators on the lattice space. The distribution of h is shown in Fig. 6. The leading unoscillating vertex operators are

$$L_{vertex} = -\alpha\cos(4\pi h) - \gamma[\cos(4\pi\partial_x h) + \cos(4\pi\partial_y h)]. \quad (21)$$

This Lagrangian (20) looks exactly like the Lagrangian at the Rokhsar-Kivelson point for a two-dimensional dimer model on square lattice[21,22] except for the vertex operators. The leading kinetic term of this Lagrangian is proportional to k^4. The k^2 term is ruled out by the symmetry (19). Since the k^2 term does not exist in the dual Lagrangian (20), as long as γ is tuned to zero and ρ_4 is smaller than a critical value, the vertex operator is irrelevant, and the system is a liquid phase without any order.[22] However, the vertex operator proportional to γ is relevant generically (and hence the α term is

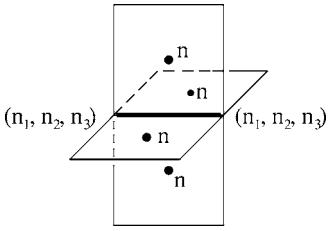

FIG. 7. The distribution of boson numbers on fcc lattice. The link (in bold font) is shared by four plaquettes and two sites. Every site is occupied by three orbital levels, and on every plaquette there is one orbital level.

dangerously irrelevant). The relevant vertex operator will drive the system into crystalline phase. The crystalline pattern close to the critical point with $\gamma=0$ can be predicted from the field theory. The representations of density operators in terms of the low energy field variable are

$$n_1,n_2-1\sim(-1)^x\sin(2\pi\partial_x h)+(-1)^y\sin(2\pi\partial_y h),$$

$$n-1\sim-(-1)^x\sin(2\pi\partial_x h)-(-1)^y\sin(2\pi\partial_y h),$$

$$\cos(R_{xyxy})\sim(-1)^y\cos(2\pi\partial_x h)+(-1)^x\cos(2\pi\partial_y h). \tag{22}$$

When the vertex operators in Eq. (21) are relevant, the order parameters above will take nonzero expectation values. Thus the γ operator tends to drive the system into the $(\pi,0)+(0,\pi)$ state for either particle density or plaquette density, depending on the sign of γ. Each case has four degenerate ground states.

IV. GRAVITON MODEL IN THREE-DIMENSIONAL SPACE

The model in three-dimensional space is defined on the fcc lattice, since physical quantities are defined on both sites and the centers of each plaquette. Let us assume there are three orbital levels on each site, and one orbital level on each face center. The particle number on the face center is denoted as n, and the particle numbers on sites are denoted as (n_1,n_2,n_3) (Fig. 7).

The Hamiltonian for this system still contains three parts, $H=H_0+H_1+H_2$. H_1 is the nearest neighbor hopping between sites and their nearest face centers, and also between adjacent face centers (notice that the adjacent face centers have the same distance as the site and its nearest face center).

$$H_1=\sum_{\langle i,\bar{j}\rangle}\sum_{a=1}^{3}-tb_{a,i}^{\dagger}b_{\bar{j}}-\sum_{\langle\bar{i},\bar{j}\rangle}tb_{\bar{i}}^{\dagger}b_{\bar{j}}+\text{H.c.}, \tag{23}$$

H_2 is the on-site interaction $H_2=\Sigma_{i,\bar{i}}U(n_{\bar{i}}-\bar{n})^2+\Sigma_a U(n_{a,i}-\bar{n})^2$, which fixes the average filling of the fcc lattice. H_0 is the interaction term involving links in all three directions. For example, for the link in $(i,\hat{x})$, the interaction term reads

$$H_0=V(n_{i+1/2\hat{x}+1/2\hat{z}}+n_{i+1/2\hat{x}-1/2\hat{z}}+n_{i+1/2\hat{x}+1/2\hat{y}}+n_{i+1/2\hat{x}-1/2\hat{y}}+2n_{1,i}+2n_{1,i+\hat{x}}-8)^2. \tag{24}$$

The links in $\hat{y}$ and $\hat{z}$ directions are treated similarly. If the bracket in Eq. (24) is expanded, it becomes the usual two body repulsion term.

When H_0 becomes the dominant term in the Hamiltonian, it effectively imposes a constraint on the system. Again the best way to view this constraint is by introducing a staggered sign and defining new variables, similar to the electric field in the dimer model discussed before. Let us define a rank-2 tensor E_{ab}. The off-diagonal terms are defined on face centers as $E_{ab}(\bar{i})=\eta_{\bar{i}}(n_{\bar{i}}-1)$. n is located at one of the $\hat{a}\hat{b}$ face centers; the diagonal term is defined on sites as $E_a(i)=\eta_i(n_{a,i}-1)$ with $a=x,y,z$. $\eta_r=\pm1$ and the distribution of sign η_r is shown in Fig. 4.

After introducing the sign η, the constraint effectively imposed by Eq. (24) can be compactly written as

$$2\partial_x E_{xx}+\partial_y E_{xy}+\partial_z E_{xz}=0,$$

$$\partial_x E_{xy}+2\partial_y E_{yy}+\partial_z E_{yz}=0,$$

$$\partial_x E_{xz}+\partial_y E_{yz}+2\partial_z E_{zz}=0. \tag{25}$$

A violation of this constraint can be interpreted as a charged defect excitation and can drive the system into an ordered boson superfluid state after condensation. We will discuss the transition in Sec. V. In the current section we only focus on the case when the constraint is strictly imposed.

The constraint (25) requires the low energy Hamiltonian to be invariant under the gauge transformation $A_{ij}\rightarrow A_{ij}+\partial_i f_j+\partial_j f_i$. This is precisely the gauge symmetry of the graviton in three-dimensional space. Thus the low energy physics can only involve the linearized curvature tensor. In the three-dimensional space, the curvature tensor has six nonzero components, according to the symmetry of the curvature tensor. Thus now the effective low energy Hamiltonian reads

$$H_{ring}=\sum_{ij,i\neq j}-\tilde{t}_1\cos(R_{ijij})-\sum_{ijk,i\neq j,j\neq k,i\neq k}\tilde{t}_2\cos(R_{ijik})+\frac{1}{2\kappa_1}\left(\sum_{i=1}^{3}E_{ii}^2\right)+\frac{1}{2\kappa_2}\left(\sum_{ij,i\neq j}E_{ij}^2\right). \tag{26}$$

The cosine terms involving the curvature tensor are ring exchange terms generated by the nearest neighbor hopping. Ring exchanges in Eq. (26) are generated at the eighth order perturbation of the nearest neighbor hopping, $\tilde{t}_1$, $\tilde{t}_2\sim t^8/V^7$. All the lower order perturbations only generate terms which do not comply with the constraint (25). The ring exchange term $\cos(R_{xyxy})$ is the same as its two-dimensional counterpart, as shown in Fig. 5.

In order to derive the correct Lagrangian, one needs to introduce a Lagrange multiplier A_{i0} for the constraint (25). The full Lagrangian reads

$$L=\sum_{i,j}(\partial_\tau A_{ij}-\partial_i A_{j0}-\partial_j A_{i0})^2+\sum_{ij,i\neq j}\tilde{t}_1\cos(R_{ijij})+\sum_{ijk,i\neq j,j\neq k,i\neq k}\tilde{t}_2\cos(R_{ijik}). \tag{27}$$

The gauge symmetry can now be enlarged to quantities defined in the 3+1 dimensional space-time: $A_{\mu\nu}\to A_{\mu\nu}+\partial_\mu f_\nu+\partial_\nu f_\mu$ and $A_{00}=0$, $f_0=0$. In this system, the boson superfluid order is again ruled out by the gauge symmetry. Without crystalline order (proven later), the system is in a liquid phase with excitations which have the same gauge symmetry as the graviton. In the linearized Einstein gravity, after taking the traceless-transverse gauge, the spin-2 graviton has only two polarizations.[20] In our theory tracelessness was not imposed to A_{ij}. Therefore there are three gapless collective excitations in the graviton phase, one of which is the scalar trace mode, the other two are described by traceless matrices, if the transverse gauge is taken, the two traceless modes exactly correspond to the two polarizations of the gravitons.

Unlike the quantum dimer model, the curvature tensor is the second spatial derivative of A_{ij}. If there is a Gaussian phase in which we can expand the cosines in Eq. (27), the gapless graviton modes in this Gaussian phase have a soft dispersion $\omega\sim k^2$.

Whether the graviton excitations survive (or equivalently whether crystal order develops) can be studied in the dual theory. If we define the symmetric tensor $\mathcal{E}_{ij}$ as $\mathcal{E}_{ii}=2E_{ii}$, $\mathcal{E}_{ij}=E_{ij}, i\neq j$, the constraint (25) can be solved by defining the dual tensor h_{ij} as

$$\mathcal{E}_{ij}=\epsilon_{iab}\epsilon_{jcd}\partial_a\partial_c h_{bd}. \tag{28}$$

This is a double curl of the symmetric tensor h_{ij}. h_{ij} also lives on the sites and faces of this fcc lattice.

If checking carefully, one can notice that the curvature tensor can also be written in the double curl form

$$\begin{aligned}
&2R_{xyxy}=\epsilon_{zab}\epsilon_{zcd}\partial_a\partial_c A_{bd}, \quad 2R_{xzxz}=\epsilon_{yab}\epsilon_{ycd}\partial_a\partial_c A_{bd},\\
&2R_{yzyz}=\epsilon_{xab}\epsilon_{xcd}\partial_a\partial_c A_{bd}, \quad 2R_{xyxz}=\epsilon_{yab}\epsilon_{zcd}\partial_a\partial_c A_{bd},\\
&2R_{yxyz}=\epsilon_{xab}\epsilon_{zcd}\partial_a\partial_c A_{bd}, \quad 2R_{zxzy}=\epsilon_{xab}\epsilon_{ycd}\partial_a\partial_c A_{bd}.
\end{aligned} \tag{29}$$

Therefore this model is self-dual, as long as we define the dual variables h_{ij} in terms of $\mathcal{E}_{ij}=\epsilon_{iab}\epsilon_{jcd}\partial_a\partial_c h_{bd}$ and its conjugate π_{ij} as follows:

$$\begin{aligned}
&R_{xyxy}=\pi_{zz}, \quad R_{yzyz}=\pi_{xx},\\
&R_{xzxz}=\pi_{yy}, \quad 2R_{xzyz}=\pi_{xy},\\
&2R_{xyxz}=\pi_{yz}, \quad 2R_{xyzy}=\pi_{xz}.
\end{aligned} \tag{30}$$

According to the definition, π_{ij} is subject to the same constraint as E_{ij} [Eq. (25)].

After introducing the dual variables h_{ij} and π_{ij}, the dual Lagrangian reads

$$\begin{aligned}
L_{dual}=&\sum_{ij}(\partial_t h_{ij}-\partial_i h_{j0}-\partial_j h_{i0})^2-\sum_{ij,i\neq j}\rho_1\tilde{R}^2_{ijij}\\
&-\sum_{ijk,i\neq j,j\neq k,i\neq k}\rho_2\tilde{R}^2_{ijik}+\cdots,
\end{aligned} \tag{31}$$

$\tilde{R}_{ijkl}$ is the curvature tensor of h_{ij}. h_{i0} is a Lagrange multiplier, which is introduced for the constraint on π_{ij}. The ellipses include possible vertex operators. Without the vertex operators, this theory is at a Gaussian fixed point and hence in an algebraic liquid phase with soft graviton excitations. If the vertex operators are relevant, they will destabilize the liquid phase and gap out the graviton excitation, and form crystalline order according to Berry's phase. The dual Lagrangian (31) is also invariant under the gauge transformation $h_{\mu\nu}\to h_{\mu\nu}+\partial_\mu f_\nu+\partial_\nu f_\mu$, with $h_{00}=0$ and $f_0=0$. Therefore any kind of vertex operator [for example, $\cos(2N\pi h_{ij})$] is not a gauge invariant operator. The correlation function between two vertex operators at the Gaussian fixed point is zero or correlated at very short range, and hence irrelevant at the Gaussian fixed point. Thus the Gaussian fixed point (also the algebraic spin liquid) is stable against weak perturbations of the vertex operators. Thus this graviton phase is stable due to the same reason as the photon phase, as discussed in the second section of this paper.

V. PROPERTIES OF THE ALGEBRAIC LIQUID PHASE

The most important feature of the algebraic spin liquid phase is the power law correlation between spin operators. In the boson language used in this paper, it is the boson density operators which correlate algebraically. As discussed before, in the algebraic liquid phase, the boson density fluctuation around the average filling $\bar{n}$ can be written as (for instance) $n_{1,i}-\bar{n}\sim\eta_i\epsilon_{1ab}\epsilon_{1cd}\partial_a\partial_c h_{bd}$. The calculation of the equal time boson density correlation can be derived from the correlation functions between h_{ij},

$$\begin{aligned}
\langle(n_{1,i}-\bar{n})(n_{1,j}-\bar{n})\rangle &\sim \eta_i\eta_j\epsilon\epsilon\partial_i\partial_i\epsilon\epsilon\partial_j\partial_j\langle h(i)h(j)\rangle\\
&\sim \eta_i\eta_j\partial^4\left(\frac{1}{r}\right)\sim\eta_i\eta_j\frac{1}{r^5}.
\end{aligned} \tag{32}$$

$r=|i-j|$. Therefore the correlation functions between boson density operators fall off algebraically, with an exponent higher than the exponent of the photon liquid phase.

Because of the gauge symmetry, the operators have to be gauge invariant to have nonzero correlation functions. Therefore the correlation function between original boson creation operators b_i are zero (or shortly correlated with a correlation length which roughly equals the inverse of the charge gap V). The curvature tensors have nonzero correlations, and the correlators also fall off algebraically with the same exponent as that of the density correlator.

The dynamics of the gapless liquid phase can be described by a new set of Maxwell's equations. Define the rank-2 tensor $\mathcal{B}_{ij}=\epsilon_{iab}\epsilon_{jcd}\partial_a\partial_c A_{bd}$, the dynamical equations that describe this liquid phase can be derived directly from the Lagrangian (27),

$$\begin{aligned}
&\partial_i\mathcal{E}_{ij}=0,\\
&\partial_i\mathcal{B}_{ij}=0,\\
&\partial_t\mathcal{E}_{ij}-\kappa\epsilon_{iab}\epsilon_{jcd}\partial_a\partial_c\mathcal{B}_{bd}=0,\\
&\partial_t\mathcal{B}_{ij}+\kappa\epsilon_{iab}\epsilon_{jcd}\partial_a\partial_c\mathcal{E}_{bd}=0.
\end{aligned} \tag{33}$$

Charged excitations and topological defects are absent in these equations, thus the equations correspond to the Max-

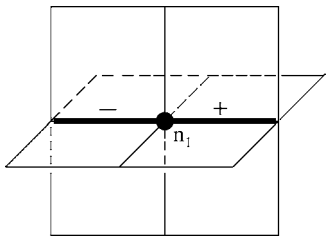

FIG. 8. One of the defects carrying gauge charges. If n_1 on one site is increased by one, it is equivalent to creating a pair of opposite gauge charges on two $\hat{x}$ links sharing this site.

well equations in vacuum. If the constraint (25) is softened, charge density and charge current have to be incorporated in Eq. (33).

The gapless excitations at zero temperature govern the thermodynamical physics at low temperature. For instance, at low temperature, the phonon excitations make a contribution to the specific heat $C_p \sim T^3$. The specific heat from photon excitations in the photon spin liquid also scales as T^3.[12] In the current work, since the graviton excitations have soft dispersion, the low energy specific heat would be $T^{3/2}$, which is larger than the contribution from the phonon.

The violation of constraint (25) corresponds to the defects, which carry gauge charges of gauge fields A_{ij}. Because the gauge field is a rank-2 tensor, the gauge charge should be a vector field. The static vector charge field couples to the gauge field in a similar way with Gauss's law: $\partial_i \mathcal{E}_{ij} = -\rho_j$. For instance, if on one site, we increase n_1 by one, it is equivalent to excite a pair of opposite gauge charges on the two $\hat{x}$ links sharing this site (Fig. 8); if on one plaquette center, $n_{\bar{r}}$ is increased by one, gauge charges are created for four links sharing this plaquette (Fig. 9). Notice that since the constraint imposed by H_0 in the original Hamiltonian (24) is an interaction between particles around each link, now the gauge charge field is defined on links.

The defects should couple to the gauge field in a gauge invariant manner, in the rotor language, the coupling can be written as

$$H_e = -\sum_{i,j} t \cos(\partial_i \theta_j^{(e)} + \partial_j \theta_i^{(e)} - A_{ij}) + \cdots , \qquad (34)$$

$\theta_i^{(e)}$ are the phase angles of gauge charge field creation operators. This coupling is gauge invariant since if $\theta_i^{(e)}$ is added

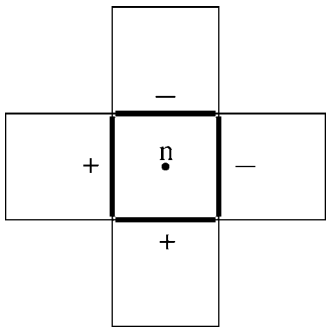

FIG. 9. One of the defects carrying gauge charges. If on one plaquette center, $n_{\bar{r}}$ is increased by one, gauge charges are created for four links sharing this plaquette.

by any arbitrary function f_i, one can always eliminate this extra phase angle by gauge transformation $A_{ij} \rightarrow A_{ij} + \partial_i f_j + \partial_j f_i$. In the Gaussian phase, the defects are gapped, and the phase angles $\theta_i^{(e)}$ are in the disordered phase. If the defects condense, i.e., the phase angles $\theta_i^{(e)}$ take nonzero expectation values, the A_{ij} will be gapped out through the Higgs mechanism. However, the matter field phason modes will not be completely gapped out, this is because of the extra global total particle number conservation besides the gauge charge conservation. After the condensation of all the matter fields, the gauge charge conservation will be broken, and the gauge field is gapped out through the Higgs mechanism. Meanwhile, the spontaneous breaking of the global particle number conservation guarantees the existence of the gapless excitation in the condensate, which is exactly the Goldstone mode. Notice that, although there are in total four flavors of matter fields (n_1, n_2, n_3, and n), different flavors of matter fields are mixed in the nearest hopping term H_1 [Eq. (23)], therefore only the total boson number is conserved. There should be only one Goldstone mode in the condensate. A similar situation takes place in the doped quantum dimer model,[19] where the holes carry both the gauge charge and the global $U(1)$ charge, hence the condensate of holes contains one gapless Goldstone mode.

In the dual formalism, the topological defects can be introduced in the vertex operators of the dual formalism. The topological defects can be introduced in the dual vertex operators

$$H_m = -\sum_{i,j} t \cos[2\pi N(\partial_i \theta_j^{(m)} + \partial_j \theta_i^{(m)}) - 2\pi N h_{ij}], \qquad (35)$$

$\theta_i^{(m)}$ are the phase angles of the creation operators of the topological defects. One can see the topological defects couple to the dual gauge potential h_{ij} in the same manner as the gauge charges couple to the original gauge potential A_{ij}, i.e., the charge and topological charge are dual to each other, which is the same case as the duality between the electric charge and the magnetic monopole. After the condensation of the topological defects $\theta_i^{(m)}$, the gapless graviton excitation h_{ij} is gapped out by the Higgs mechanism. However, in this situation, there is no extra global conservation of the topological defects, therefore in the condensate there will not be any gapless Goldstone mode. The system is in the gapped Mott insulator phase, with crystalline order determined by Berry's phase. Since our emphasis of this paper is about the stable liquid phase, we will not get into the detailed analysis of the crystalline patterns.

After introducing the matter field and the topological defect, the semiclassical Maxwell's equations with charges are

$$\partial_i \mathcal{E}_{ij} = -\rho_j^{(e)},$$

$$\partial_i \mathcal{B}_{ij} = -\rho_j^{(m)},$$

$$\partial_t \mathcal{E}_{ij} - \kappa \epsilon_{iab} \epsilon_{jcd} \partial_a \partial_c \mathcal{B}_{bd} = J_j^{(e)i} + J_i^{(e)j},$$

$$\partial_t \mathcal{B}_{ij} + \kappa \epsilon_{iab} \epsilon_{jcd} \partial_a \partial_c \mathcal{E}_{bd} = J_j^{(m)i} + J_i^{(m)j}. \qquad (36)$$

The current $J_j^{(e)i}$ represents the current with the ith component of matter field flowing in the $\hat{j}$ direction. The equation of conservation law of the gauge charges, as well as the topological defects, reads

$$\partial_t \rho_j^{(e)} = -\partial_i J_i^{(e)j} - \partial_i J_j^{(e)i},$$

$$\partial_t \rho_j^{(m)} = -\partial_i J_i^{(m)j} - \partial_i J_j^{(m)i}. \quad (37)$$

One can clearly see that, because of the flavor mixing in H_1, the vector charge density is not individually conserved. The derivative $\partial_i J_j^{(e)i}$ plays the role of torque density of the vector charge density. This is similar to the spin current,[23] which is also a tensor current, and the conservation equation of the spin current involves torque density[24] due to the fact that spin is not conserved in a system with spin-orbit coupling.

VI. DISCUSSIONS, EXPERIMENTAL REALIZATIONS, AND EXTENSIONS

In this work we constructed models which give rise to collective excitations analogous to the gravitons. In a 2+1 dimensional system, the graviton excitations are unstable against vertex operators in the dual formalism; in 3+1 dimensional space, there is a stable algebraic boson liquid phase which contains gapless graviton excitations with soft dispersion. The graviton liquid phase is self-dual, and the stability of the algebraic phase is guaranteed by the large gauge symmetry on both sides of the duality. The dynamics of the phase is described by a set of Maxwell equations.

We proved the algebraic phase is stable, i.e., the realization of this phase does not require any fine-tuning. However, the original bosonic model is of a special form, its precise realization in experimental systems requires more efforts. Here we only consider the possibility of its realization in the cold atom system trapped in an optical lattice. The original boson model contains standard nearest neighbor hopping between particles, and also contains two body interactions, both on-site and off-site. The on-site repulsion can be obtained from the s-wave scattering between bosons on the same site, and the off-site repulsion has been shown recently to exist in chromium atom condensate due to the long range dipole interaction.[25,26]

The interaction between the three orbital levels on each site and the bosons in the plaquette centers is very anisotropic [Eq. (24)], this could be due to the anisotropy of the orbital level spatial wave functions. In the transition metal oxides materials, the anisotropy of the t_{2g} level electron wave functions gives rise to the Khaliullin model[27,28] which was first introduced to explain the orbital liquid phenomenon, in which the orbital moment is quenched by quantum fluctuation.[29] Recently it has been proposed that, although the ground state of each site of the optical lattice is s-wave, if the particles are pumped to the first excited threefold degenerate p-wave states, the particles will maintain in the excited states for a considerable time, long enough for the particles to equilibrate,[30] therefore the p-wave particles will first form a metastable equilibrium state before they drop to the s-wave ground states. Each of the threefold degenerate p-level wave functions only extends in one direction of the three-dimensional space. Since the wave function overlap is anisotropic in space, the particle on each orbital level interacts more strongly in one certain direction, as long as the optical trap on each site is not too deep, i.e., the wave function is not too localized on each site. The anisotropic interaction is required in our problem. The remaining problems of the experimental realization is how to realize the fcc lattice with laser beams, and how to pump particles on all the sites of the fcc lattice to the excited p-wave states.

So far the models we have considered are purely bosonic models. After the search for unconventional bosonic phases, one might wonder if nonfermi liquid can be obtained in a similar way for fermionic systems. There has been a great deal of study on treating the constraint of no double occupancy of the Hubbard model by introducing $U(1)$ gauge field, and the bosonic holons and fermionic spinons are interacting with this $U(1)$ gauge field. The interacting system has been proposed to explain the nonfermi liquid behavior of the normal state of high T_c cuperates (for instance, Ref. 31). In this kind of theory, the emergent $U(1)$ gauge field plays the most crucial role. For instance, due to the scattering from the gauge fields, the scaling of the resistivity significantly deviates from the T^2 law of normal fermi liquid. The emergence of this gauge field is exactly due to the local constraint $\Sigma_\sigma f_{i,\sigma}^\dagger f_{i,\sigma} + b_i^\dagger b_i = 1$.

In our current work, if all the particles in the original model are fermions, the constraint imposed by H_0 can also be solved by introducing rank-2 tensor gauge field A_{ij}, and when the gauge field is in its deconfined phase, the system can be viewed as fermions interacting with gapless soft graviton mode A_{ij}. The behavior of the fermions is expected to be nonfermi liquid.

In the whole paper, our goal has been carefully limited to algebraic spin liquid. However, one can consider the issue of "quantum gravity" as an extension of this work. Indeed, if in a theory the gauge symmetry of the graviton can emerge at low energy physics, the theory might be a candidate of a new possibility of quantum gravity. Several other systems in condensed matter physics have been proposed to be related to gravity, for instance, spin-2 particles are supposed to exist at the edge states of a four-dimensional quantum Hall model.[32] However, in these systems the gauge symmetry (which is very crucial for gravitons) was not an emergent property. In our work, the collective excitations automatically have the gauge symmetry of the graviton. However, the dispersion relation is quadratic, this is due to the fact that the gauge invariant operator, the curvature tensor, is the second derivative of A_{ij}. The ring exchange term generated from the nearest neighbor hoppings is $\cos(R)$, after the expansion, the leading term is R^2, which is proportional to k^4.

We can make the graviton dispersion linear by introducing by hand a quasigauge invariant term to the low energy Lagrangian:

$$L_{cs} = A_{ij} \epsilon_{iab} \epsilon_{jcd} \partial_a \partial_c A_{bd}. \quad (38)$$

This term is not completely gauge invariant, instead, it is gauge invariant up to a boundary term. This is very

analogous to the Chern-Simons field theory of the quantum Hall effect. However, this term cannot be generated from the microscopic boson model with only nearest neighbor hopping. It is expected that, by coupling to the matter fields, and the matter fields form a certain special state, the k^2 term (38) can be generated from integrating over the matter fields in the partition function, just like how the Chern-Simons field theory is obtained in the quantum Hall state of electrons. Recently, some other authors have proposed a bosonic model which is similar to ours, and the k^2 term (38) is claimed to exist in the long scale physics, therefore a graviton with linear dispersion is supposed to exist.[33]

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