Theory of domain-wall superconductivity in superconductor/ferromagnet bilayers

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We analyze the enhancement of the superconducting critical temperature of superconducting/ferromagnetic bilayers due to the appearance of localized superconducting states in the vicinity of magnetic domain walls in the ferromagnet. We consider the case when the main mechanism of the superconductivity destruction via the proximity effect is the exchange field. We demonstrate that the influence of the domain walls on the superconducting properties of the bilayer may be quite strong if the domain-wall thickness is of the order of the superconducting coherence length.

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I. INTRODUCTION

The coexistence of singlet superconductivity and ferromagnetism is very improbable in bulk compounds but may be easily achieved in artificially fabricated hybrid superconductor (S)–ferromagnet (F) structures.

There are two basic mechanisms responsible for the interaction of the superconducting order parameter with magnetic moments in the ferromagnet: the electromagnetic mechanism (interaction of Cooper pairs with the magnetic field induced by magnetic moments) and the exchange interaction of magnetic moments with electrons in Cooper pairs. The second mechanism enters into play due to the proximity effect, when the Cooper pairs penetrate into the F layer and induce superconductivity there. In S/F bilayers it is possible to study the interplay between superconductivity and magnetism in a controlled manner, since we can change the relative strength of the two competing orderings by varying the layer thicknesses and magnetic content of the F layers. Naturally, to observe the influence of the ferromagnetism on the superconductivity, the thickness of the S layer must be small. This influence is most pronounced if the S layer thickness is smaller than the superconducting coherence length ξ_s. Recently, the observation of many interesting effects in S/F systems became possible due to the great progress in the preparation of high-quality hybrid F/S systems—see Refs. 1–3 for reviews.

In practice, the domains appear in ferromagnets and, near the domain walls, a special situation occurs for the proximity effect. For the purely orbital (electromagnetic) mechanism of superconductivity destruction, the nucleation of the superconductivity in the presence of the domain structure has been theoretically studied in Refs. 4 and 5 for the case of magnetic film with perpendicular anisotropy. The conditions for appearance of superconductivity are more favorable near the domain walls due to the partial compensation of the magnetic induction. Recently, the manifestation of such domain-wall superconductivity (DWS) was revealed in experiment[6] where a Nb film was deposited on top of single-crystal ferromagnetic $BaFe_{12}O_{19}$ covered with a thin Si buffer layer.

As the typical value of the exchange field in ferromagnets $h\sim(100-1000)K$ exceeds by many times the superconducting critical temperature T_{c0}, the exchange mechanism prevails over the orbital one in superconductivity destruction when the electrical contact between S and F layers is good. For the proximity effect mediated by the exchange interaction, the Cooper pairs experience the exchange field averaged over the superconducting coherence length. Naturally, it will be smaller near the domain walls and we may expect that superconductivity would be more robust near them. The local increase of the critical temperature in the presence of magnetic domains was observed experimentally in $Ni_{0.80}Fe_{0.20}$/Nb bilayers (with Nb thickness around 20 nm),[7] and it was attributed to DWS formation.

In the present paper we study theoretically the conditions for appearance of localized superconductivity near the domain wall, taking into account the exchange mechanism of the proximity effect. In Sec. II, we demonstrate that, in the case of a thin F layer and small domain wall thickness, the problem is somewhat similar to that of the domain wall superconductivity in ferromagnetic superconductors.[8,9] For the case when the superconducting coherence length ξ_s exceeds the DW thickness w, we expect a very strong local increase of T_c (see Sec. III). In Sec. IV, we obtain the analytical expression for the critical temperature of the DWS for the case when the DW thickness w exceeds the superconducting coherence length ξ_s. The Appendix presents an extension of this result to arbitrary thickness of the F layers and transparency of the S/F interface. We discuss our results in Sec. V. In particular, we predict the realization of the situation when superconductivity appears only near the DW.

II. EQUATION FOR THE CRITICAL TEMPERATURE IN THIN BILAYERS

We introduce the Usadel equations[10] which are very convenient when dealing with S/F systems in the dirty limit, with critical temperatures T_c and exchange fields h such that $T_c\tau\ll 1$ and $h\tau\ll 1$, where τ is the elastic scattering time.

Near the second-order transition into the superconducting state, the Usadel equations can be linearized with respect to the amplitude of the superconducting gap. In the S region, the linearized Usadel equation is:

$$-D_s\boldsymbol{\nabla}^2\hat{f}_s+2|\omega|\hat{f}_s=2\Delta_s(\boldsymbol{r})\hat{\sigma}_z, \tag{1}$$

where D_s is the diffusion constant in the superconductor and $\omega=(2n+1)\pi T$ is a Matsubara frequency at temperature T.

(The notations are similar to the ones used in Ref. 2, except for the factor i in front of Δ_s.) Equation (1) relates the anomalous Green's function $\hat{f}_s$, which is a matrix in spin space, to the superconducting gap $\Delta_s(x)$; $\hat{\sigma}_{(x,y,z)}$ are Pauli matrices in spin space. In the absence of a supercurrent, the gap can be taken as real.

In the F region, an exchange field $\boldsymbol{h}_f=(h_{f,x},h_{f,y},h_{f,z})$ is acting on the spins of conduction electrons and the linearized Usadel equation for anomalous function $\hat{f}_f$ is

$$-D_f\nabla^2\hat{f}_f+2|\omega|\hat{f}_f+is_\omega(h_{f,x}[\hat{\sigma}_x,\hat{f}_f\hat{f}_f]+h_{f,y}[\hat{\sigma}_y,\hat{f}_f] + h_{f,z}\{\hat{\sigma}_z,\hat{f}_f\})=0. \quad (2)$$

Here, D_f is the diffusion constant in the ferromagnet and we used the abbreviation $s_\omega \equiv \mathrm{sgn}(\omega)$.

In addition, the boundary conditions at the interfaces with vacuum yield $\partial_z\hat{f}_s(z=d_s)=0$ and $\partial_z\hat{f}_f(z=-d_f)=0$, where d_s and d_f are the thicknesses of the S and F layers, respectively, and $z=0$ defines the plane of the interface between the two layers. We also consider that the S and F layers are separated by a thin insulating tunnel barrier. Therefore, the boundary conditions at the interface $z=0$ are[11]

$$\sigma_s\partial_z\hat{f}_s=\sigma_f\partial_z\hat{f}_f, \quad \hat{f}_s=\hat{f}_f+\gamma_B\xi_s\,\partial_z\hat{f}_f|_{z=0}, \quad (3)$$

where σ_s and σ_f are the conductivities in the layers, and γ_B is related to the boundary resistance per unit area R_b through $\gamma_B\xi_s=R_b\sigma_f$, where ξ_s is the superconducting coherence length.

The critical temperature $T=T_c$ at the second-order transition is now obtained from the self-consistency equation for the gap:

$$\Delta(\boldsymbol{r})\ln\frac{T}{T_{c0}}+\pi T\sum_\omega\left(\frac{\Delta(\boldsymbol{r})}{|\omega|}-f_s^{11}(\boldsymbol{r},\omega)\right)=0 \quad (4)$$

(f^{11} is a matrix element of $\hat{f}$), where T_{c0} is the bare transition temperature of the S layer.

In the ferromagnet, the magnitude h_f of the exchange field $\boldsymbol{h}_f$ is fixed. However, its orientation can rotate in the presence of a magnetic domain wall structure. In the following, we assume a one-dimensional domain wall structure, along the x axis. In order to find the critical temperature of the bilayer in the presence of a domain wall, we must find the x and z dependence of the gap and the anomalous functions $\hat{f}_s$ and $\hat{f}_f$ which solve Eqs. (1)–(4). The proximity effect can significantly affect the transition temperature only when the thickness of the S layer is comparable with the superconducting coherence length $\xi_s=\sqrt{D_s/2\pi T_{c0}}$. In order to get tractable expressions, we will consider only the case $d_s \ll \xi_s$. This regime is also well achievable experimentally. Then, $\hat{f}_s$ and Δ_s are almost constant along the z axis. Therefore, we can average Eq. (1) over the thickness of the S layer, integrate the term $\partial_z^2\hat{f}$, and make use of the boundary condition at the interface with vacuum. Finally, we get the following equation at $z=0$:

$$-D_s\partial_x^2\hat{f}_s+\frac{D_s}{d_s}\partial_z\hat{f}_s+2|\omega|\hat{f}_s=2\Delta_s\hat{\sigma}_z. \quad (5)$$

The characteristic scale for the proximity effect in the F layer is rather set by the coherence length $\xi_f=\sqrt{D_f/h_f}$, where h_f is the typical amplitude of the exchange field in the ferromagnet. In this section, we will address the case of a very thin F layer: $d_f \ll \xi_f$. Then, from Eq. (2) we can derive similarly the Usadel equation averaged over the thickness d_f, at $z=0$:

$$-D_f\partial_x^2\hat{f}_f-\frac{D_f}{d_f}\partial_z\hat{f}_f+2|\omega|\hat{f}_f+is_\omega(h_{f,x}[\hat{\sigma}_x,\hat{f}_f]+h_{f,y}[\hat{\sigma}_y,\hat{f}_f] + h_{f,z}\{\hat{\sigma}_z,\hat{f}_f\})=0. \quad (6)$$

Let us note right now that the assumption of a very thin F layer is quite hard to achieve experimentally, as we will discuss at the end of Sec. V. In the Appendix, we will consider the case of arbitrary thickness for the F layer.

For simplicity, we also consider the case of low interface resistance ($\gamma_B \to 0$) where the proximity effect is maximal. In this regime, $\hat{f}_f(x)\approx\hat{f}_s(x)\equiv\hat{f}(x)$. By proper linear combination of Eqs. (3), (5), and (6), we can form a single equation in $\hat{f}(x)$:

$$-D\partial_x^2\hat{f}+2|\omega|\hat{f}+is_\omega(h_x[\hat{\sigma}_x,\hat{f}_f]+h_y[\hat{\sigma}_y,\hat{f}_f] + h_z\{\hat{\sigma}_z,\hat{f}_f\})=2\Delta\hat{\sigma}_z, \quad \Delta=\frac{\eta_s}{\eta_s+\eta_f}\Delta_s, \quad (7)$$

where $\eta_s=\sigma_sd_s/D_s$ and $\eta_f=\sigma_fd_f/D_f$. Therefore, the thin bilayer is described by the same equations as for a magnetic superconductor,[9] with the effective diffusion constant, exchange field, and BCS coupling constant

$$D=\frac{D_s\eta_s+D_f\eta_f}{\eta_s+\eta_f}, \quad \boldsymbol{h}=\frac{\eta_f}{\eta_s+\eta_f}\boldsymbol{h}_f, \quad \tilde{\lambda}=\frac{\eta_s}{\eta_s+\eta_f}\lambda, \quad (8)$$

respectively. An equation similar to Eq. (7) was derived for a thin normal-metal/superconductor bilayer, in the absence of exchange field ($\boldsymbol{h}=\boldsymbol{0}$) in Ref. 12. There, it was shown that the reduction of the coupling constant $\tilde{\lambda}$ leads to a rapid decrease of the bilayer critical temperature. In the following, we do not consider these effects. Rather, we dwell on the case when $\tilde{\lambda}\approx\lambda$ and the reduction of T_c is mainly due to the effective exchange field $\boldsymbol{h}$. This situation occurs at $\eta_s \gg \eta_f$. [When the S and F layers have comparable diffusion constant and conductivity, the renormalization factors in Eq. (8) have a simple interpretation in terms of volume ratios. In particular, the condition $\eta_s \gg \eta_f$ results in $d_s \gg d_f$.] Thus, $D\approx D_s$ and $\boldsymbol{h}\approx(\eta_f/\eta_s)\boldsymbol{h}_f$. Let us note that the amplitude of $\boldsymbol{h}$ is strongly reduced compared to $\boldsymbol{h}_f$; eventually it is of the order of Δ_s, and thus it leads to the possible coexistence of magnetism and superconductivity in the bilayer.

The phase diagram of magnetic superconductors with constant exchange field was studied long ago.[13] The second-order transition line from the normal to the superconducting state at the critical temperature $T=T_c(h)$ is given by the equation

$$\ln\frac{T}{T_{c0}}+2\pi T\,\mathrm{Re}\sum_{\omega>0}\left\{\frac{1}{\omega}-\frac{1}{\omega+ih}\right\}=0, \tag{9}$$

where h is the amplitude of $\boldsymbol{h}$. At zero temperature, the critical field is $h_c^{(2)}=\Delta_0/2$, where $\Delta_0\simeq 1.76T_{c0}$ is the superconducting gap. However, at $T<T^*\approx 0.56T_{c0}$, the transition into the superconducting state is of the first order and the critical field at zero temperature is rather $h_c=\Delta_0/\sqrt{2}$.

In the presence of a domain structure in the ferromagnet, the average exchange field experienced by the electrons near domain walls is smaller than in the domains. This may lead to the enhancement of the superconducting critical temperature. On the basis of the Usadel equation (7) with the self-consistency equation (4), we consider now this problem in the case of narrow domain walls in Sec. III and large domain walls in Sec. IV.

III. NARROW DOMAIN WALL

In this section, we consider the case of thin domain walls characterized by the domain wall thickness $w\ll\xi_s$. In Ref. 8, the zero-temperature critical field was obtained in the context of magnetic superconductors. Here, we revise the result and obtain the phase diagram at finite temperature.

We model the exchange field $\boldsymbol{h}$ acting on the electrons with a step function: $h_z(x)=h\,\mathrm{sgn}(x)$, $h_y=h_z=0$. The structure of the Usadel equation (7) in spin space simplifies greatly and we have

$$-\frac{D}{2}\partial_x^2 f^{11}+[|\omega|+is_\omega h_z(x)]f^{11}=\Delta, \tag{10}$$

while $f^{12}=f^{21}=0$ and $f^{22}_{\omega}=-f^{11}_{-\omega}$. Its solution for a given Δ is

$$f^{11}(x)=\int dy\,\mathcal{G}(x,y)\Delta(y), \tag{11}$$

where $\mathcal{G}$ is the Green's function associated with the homogeneous differential equation (10); $\mathcal{G}$ is defined by

$$\mathcal{G}(x,y)=\begin{cases}\dfrac{e^{-\kappa x}}{D\kappa}\left(e^{\kappa y}+\dfrac{\kappa-\kappa^*}{\kappa+\kappa^*}e^{-\kappa y}\right) & \text{for } x>y>0,\\[2ex] \dfrac{2}{D(\kappa+\kappa^*)}e^{-\kappa x}e^{\kappa^* y} & \text{for } x>0>y,\end{cases} \tag{12}$$

where $\kappa=\sqrt{2(|\omega|+is_\omega h)/D}$, while $\mathcal{G}(x,y)=\mathcal{G}(y,x)$ and $\mathcal{G}(x,y)=\mathcal{G}(-x,-y)^*$.

We look for a symmetric solution $\Delta(-x)=\Delta(x)$. Writing the self-consistency equation (11) in Fourier space, we get the equation defining the critical temperature $T=T_{cw}$ for DWS formation:

$$\left(\ln\frac{T}{T_{c0}}+2\pi T\,\mathrm{Re}\sum_{\omega>0}\frac{1}{\omega}-\frac{1}{\omega+ih+Dp^2/2}\right)\Delta_p$$
$$=2T\sum_{\omega>0}\int dk\frac{h\sqrt{D}[(\omega^2+h^2)(\sqrt{\omega^2+h^2}-\omega)]^{1/2}}{[h^2+(\omega+Dp^2/2)^2][h^2+(\omega+Dk^2/2)^2]}\Delta_k. \tag{13}$$

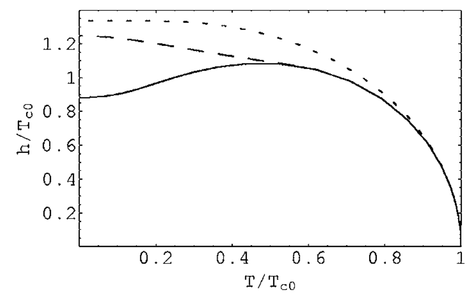

FIG. 1. Phase diagram. The straight line is the critical line $h_c^{(2)}(T)$ of the second-order transition into a uniform superconducting state. At $T<T^*=0.56T_{c0}$, the transition into a uniform state at $h_c(T)$ is of the first order (dashed line). The critical line $h_{cw}(T)$ corresponding to domain wall superconductivity is of the second order (dotted line).

Close to T_{c0}, at $h\ll T_{c0}$, the critical temperature for the transition into a uniform superconducting state can be obtained analytically from Eq. (9):

$$\frac{T_{c0}-T_c(h)}{T_{c0}}=\frac{7\zeta(3)}{4\pi^2}\frac{h^2}{T_{c0}^2}-\frac{31\zeta(5)}{16\pi^4}\frac{h^4}{T_{c0}^4}+\cdots . \tag{14}$$

On the other hand, Eq. (13) for the DWS can be simplified:

$$\left(\frac{T_{cw}(h)-T_c(h)}{T_{c0}}+\frac{\pi}{8}\frac{Dp^2}{T_{c0}}\right)\Delta_p=A\frac{h^2}{T_{c0}^2}\sqrt{\frac{D}{\pi T_{c0}}}\int dk\,\Delta_k, \tag{15}$$

where $A=(8\sqrt{2}-1)\zeta(\frac{7}{2})/(8\pi^3)$. This equation is solved straightforwardly and we get the increase of critical temperature near T_{c0} due to the DWS:

$$\frac{T_{cw}(h)-T_c(h)}{T_{c0}}\approx 8A^2\frac{h^4}{T_{c0}^4}. \tag{16}$$

The corresponding shape of the order parameter near the transition is given by

$$\Delta(x)\sim\exp\left(-B\frac{|x|}{\xi_s}\frac{T_{c0}-T_{cw}(h)}{T_{c0}}\right), \tag{17}$$

where $B=16\pi\sqrt{2}A/[7\zeta(3)]$. Thus, near T_{c0}, the localized superconductivity is characterized by exponential decay without oscillation of the superconducting order parameter, with its maximum at the domain wall position.

Away from T_{c0}, Eq. (13) does not contain any small parameter. Its structure is that of a linear integral equation whose kernel is a superposition of separable terms. This form of the kernel is known to be convenient for numerical calculation. As a result, we obtained the second-order critical line at any temperature (see Fig. 1). The critical line is significantly increased compared to the critical line for the transition into a uniform superconducting state. In particular, at zero temperature, the critical field for localized superconductivity at $T=0$ is $h_{cw}\simeq 1.33T_{c0}\sim 0.76\Delta_0$ and lies above the critical field for the first-order transition into a uniform state, $h_c\simeq 0.71\Delta_0$. It is of interest to note that, if the effective exchange field in the bilayer is between h_c and h_{cw}, then the special situation occurs when only DWS can be realized in

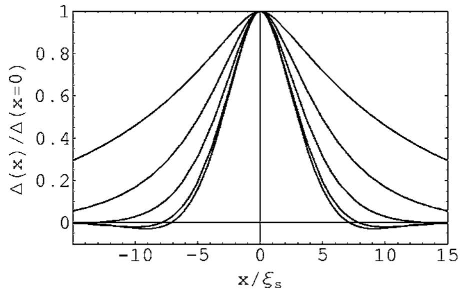

FIG. 2. Self-consistent gap just below the second-order transition line $h_{cw}(T)$ into a localized domain wall superconducting state at different temperatures $T=(0,0.2,0.4,0.6,0.8)T_{c0}$ (from narrowest to widest).

the system, but no superconductivity far away from the domain walls.

We also plot the self-consistent order parameter at different temperatures in Fig. 2. In addition to the decay, it shows small oscillations along the direction perpendicular to the domain wall at low temperatures.

As it was pointed out in Sec. II, the transition into a uniform superconducting state happens to be of the first order at $T<T^*$ in the bilayer. Therefore, one should also worry about the possibility for a change of the transition order along the critical line corresponding to DWS. We considered this possibility by solving the nonlinear Usadel equations perturbatively up to the third-order terms in the gap Δ. We found that this equation corresponds to a saddle point of the free energy density (per unit thickness of the bilayer and per unit length of the magnetic domain wall) functional:

$$\mathcal{F}=\mathcal{F}_2+\mathcal{F}_4+\cdots,$$

$$\mathcal{F}_2=\nu\int dx\left\{|\Delta|^2\ln\frac{T}{T_{c0}}+2\pi T\,\mathrm{Re}\sum_{\omega>0}\frac{|\Delta|^2}{\omega}-\Delta^* f_\omega^{11}\right\},$$

$$\mathcal{F}_4=\frac{\pi T\nu}{2}\mathrm{Re}\sum_{\omega>0}\int dx\left(\frac{D}{2}(\partial_x f_\omega^{11})^2+\Delta f_\omega^{11}\right)(\bar{f}_\omega^{11})^2, \quad (18)$$

where ν is the density of states at the Fermi level in the normal state, and $\bar{f}_\omega^{11}=(f_{-\omega}^{11})^*$ where f_ω^{11} is given by Eq. (11). At the second-order transition, the term $\mathcal{F}_2$, which is quadratic in Δ, vanishes when Eq. (13) is satisfied. The phase transition is stable provided that the term $\mathcal{F}_4$, which is quartic in Δ, remains positive along the transition line. We checked numerically that this was indeed the case. In particular, at $T=0$ and h close to h_{cw}, we found

$$\mathcal{F}_2=\nu\ln\frac{h}{h_{cw}}\int dx|\Delta(x)|^2$$

$$\simeq 2.87\nu\sqrt{\frac{D}{2h_{cw}}}\ln\frac{h}{h_{cw}}\Delta(x=0)^2,$$

$$\mathcal{F}_4=0.26\nu\sqrt{\frac{D}{2h_{cw}}}\frac{\Delta(x=0)^4}{h_{cw}^2}. \quad (19)$$

Thus we found that the transition into DWS remains of the second order at all temperatures.

IV. LARGE DOMAIN WALL

In this section, we consider the case of a large domain wall with thickness $w\gg\xi_s$. The domain wall can be described with an exchange field $\boldsymbol{h}=h(\cos\phi,\sin\phi,0)$; the rotation angle $\phi(x)$ varies monotonically between $\phi(-\infty)=-\pi/2$ and $\phi(\infty)=\pi/2$. We find that the critical temperature at DWS nucleation is given by a Schrödinger equation for a particle in the presence of a potential well whose profile is proportional to $-(\partial_x\phi)^2$. This result is not specific to the thin bilayer considered in this section. In the Appendix, we extend it to bilayers with arbitrary thickness of the F layer and arbitrary transparency of the S/F interface.

At the transition, the linearized Usadel equation (7) must be solved, that is,

$$-\frac{D}{2}\partial_x^2 f^{11}+|\omega|f^{11}-i\frac{hs_\omega}{2}(e^{-i\phi}f^{21}-e^{-i\phi}f^{12})=\Delta,$$

$$-\frac{D}{2}\partial_x^2 f^{12}+|\omega|f^{12}+ihs_\omega e^{-i\phi}f^{11}=0,$$

$$-\frac{D}{2}\partial_x^2 f^{21}+|\omega|f^{21}-ihs_\omega e^{i\phi}f^{11}=0, \quad (20)$$

while $f^{22}=-f^{11}$.

If the spatial dependence of ϕ is neglected, the gap Δ is uniform along the layer and the solutions are readily found:

$$f_0^{11}=\frac{\Delta|\omega|}{\omega^2+h^2},\quad f_0^{12}=\frac{i\Delta hs_\omega e^{-i\phi}}{\omega^2+h^2}=-e^{-2i\phi}f_0^{21}, \quad (21)$$

As a result, the critical temperature T_c into a uniform superconducting state naturally does not depend on ϕ and is again given by Eq. (9).

Now, let us assume that ϕ varies slowly. We solve the Usadel equation (20) perturbatively by looking for a solution $\hat{f}\approx\hat{f}_0+\hat{f}_1$. In first approximation, $\hat{f}_0$ is still given by Eq. (21), where ϕ, and, possibly, Δ, now slowly depend on x. The correction $\hat{f}_1$ induced by their spatial dependence is determined by the set of equations

$$|\omega|f_1^{11}-i\frac{hs_\omega}{2}(e^{-i\phi}f_1^{21}-e^{-i\phi}f_1^{12})=\frac{D}{2}\partial_x^2 f_0^{11},$$

$$|\omega|f_1^{12}+ihs_\omega e^{-i\phi}f_1^{11}=\frac{D}{2}\partial_x^2 f_0^{12},$$

$$|\omega|f_1^{21}-ihs_\omega e^{i\phi}f_1^{11}=\frac{D}{2}\partial_x^2 f_0^{21}, \quad (22)$$

By appropriate linear combination of these equations, one finds that

$$f^{11} \approx \frac{|\omega|}{\omega^2+h^2}\Delta + \frac{Dh^2}{2(\omega^2+h^2)^2}(\phi')^2\Delta + \frac{D}{2}\frac{\omega^2-h^2}{(\omega^2+h^2)^2}\Delta''. \tag{23}$$

Here, the primes stand for derivatives along x. Inserting this solution in the self-consistency equation (4) we obtain the equation for the gap,

$$-\frac{1}{2m}\Delta''(x) + U(x)\Delta(x) = E\Delta(x), \tag{24}$$

where

$$E = -\ln\frac{T}{T_c}, \quad \frac{1}{2m} = D\pi T\sum_{\omega>0}\frac{(\omega^2-h^2)}{(\omega^2+h^2)^2},$$

$$U(x) = -D\pi T(\phi')^2\sum_{\omega>0}\frac{h^2}{(\omega^2+h^2)^2}. \tag{25}$$

Equation (24) is a linearized Ginzburg-Landau equation for a magnetic superconductor in the presence of a domain wall. It can be easily checked that the effective mass m is always positive and $U(x)$ is negative. Therefore, Eq. (24) looks like a one-dimensional Schrödinger equation for a particle in a potential well $U(x)$. It is well known that a bound state with $E<0$ always forms in such a potential. As a result, the second-order transition into a localized superconducting state always appears more favorable than that into a uniform state.

Let us estimate now the magnitude of the critical temperature increase.

Close to T_{c0}, the spatial variation for Δ is set by the temperature-dependent coherence length $\xi(T) = \xi_s\sqrt{T_{c0}/(T_{c0}-T)}$ which diverges at the transition. Therefore, $\xi_s \ll w \ll \xi(T)$ and the potential well can be approximated by a δ potential: $U(x)\approx -(\pi D h^2/48T_{c0}^3 w)\delta(x)$, while $1/2m \sim (\pi D/8T_{c0})$. Therefore we get the estimate

$$\frac{T_{cw}-T_c}{T_c} \simeq \frac{\pi^6}{36}\left(\frac{h}{2\pi T_{c0}}\right)^4\frac{\xi_s^2}{w^2} \propto \frac{\xi_s^2}{w^2}\frac{(T_{c0}-T_c)^2}{T_{c0}^2}. \tag{26}$$

This increase is small in the ratio $(\xi_s/w)^2$, and another reduction factor comes from the smallness of the critical exchange field h near T_{c0} [see Eq. (14)].

At $T\to 0$, the second-order transition into a uniform state occurs at the exchange field $h_c^{(2)}=\Delta_0/2$. On the other hand, the effective mass in Eq. (24) diverges as $1/2m \sim (\pi D/T)e^{-h/T}$. The fact that m remains positive at finite temperatures is related to the absence of instability toward a modulated superconducting (Fulde-Ferrell-Larkin-Ovchinnikov) state in magnetic superconductors in the presence of strong disorder, $\tau T_{c0}\ll 1$.[14] Due to its large inertia, the particle now resides in the minimum of the potential well (zero-point fluctuations can be neglected): $U_{\min} = -(\pi D/16hw^2)$. The corresponding increase in critical exchange field at $T=0$ is $(h_{cw}-h_c^{(2)}) = \pi D/16w^2$. This increase is of the order of magnitude $\sim(\xi_s/w)^2 h_c^{(2)} \ll h_c^{(2)}$. Actually, at low temperature the transition into the superconducting state in the uniform exchange field is first order. We may expect that the domain wall superconductivity in this situation also appears by a first-order transition.

At intermediate temperature, we may obtain the critical temperature $T=T_{cw}$ with a specific choice of the spatial dependence of the rotation angle ϕ. Assuming that $\phi(x) = 2\arctan[\tanh(\xi/2w)]$, we get[15]

$$\ln\frac{T}{T_c} = \frac{T_{c0}}{8T_c}\frac{\xi_s^2}{w^2}F\left(\frac{h}{2\pi T_c}\right), \tag{27}$$

where

$$F(u) = \text{Re}\,\psi_1(Z)\left(-1+\sqrt{1-\frac{2\,\text{Re}\,[i\psi(Z)+u\psi_1(Z)]}{u\,\text{Re}\,\psi_1(Z)}}\right)^2, \tag{28}$$

where ψ and ψ_1 are digamma functions, and $Z=1/2+iu$.

We should note, however, that the transition into a uniform superconducting state becomes of the first order at $T<T^*$ and results in significant increase of the critical line $h_c(T)$. Most probably, such a change of the transition order should also be considered for localized superconductivity.

V. DISCUSSION

A qualitative picture of the effect of domains walls on the superconducting properties now emerges from our calculations.

First, the electrons of the Cooper pairs that travel across the S/F interface experience the exchange field h_f in the F layer. This proximity effect results in an effective pair breaking that weakens the superconductivity in the S layer. As a result, the critical temperature T_c of the bilayer gets suppressed in comparison with the critical temperature T_{c0} of the bare S film: $T_{c0}-T_c\sim\tau_s^{-1}$. There, the pair-breaking time τ_s can be estimated from Eq. (14): at $h\ll T_{c0}$, $\tau_s^{-1}\sim h^2/T_{c0}$, where h is the effective exchange field that would act directly in the bare S layer to yield the same pair-breaking effect due to h_f in the bilayer. In particular, when the S/F interface is transparent and F layer is thin, we found that $h \sim h_f d_f/(d_s+d_f)$, where d_f and d_s are the thicknesses of F and S layers, respectively. On the other hand, when $h\gg T_{c0}$, there is no superconducting transition in the bilayer.

Second, in the vicinity of magnetic domain walls in the F layer, this pair-breaking mechanism becomes less effective. Therefore, localized superconductivity may appear with critical temperature $T_{cw}>T_c$. When the domain wall width w is large in comparison with the superconducting coherence length ξ_s, the exchange field rotates by the angle $\theta\sim\xi_s/w$ on the scale of proximity effect. Therefore, the decrease of the average exchange field close to the wall is estimated as $h-h_{av}\sim\theta^2 h$. Correspondingly, the pair-breaking time is increased by $\Delta\tau_s/\tau_s\sim\theta^2$. In analogy with the theory of superconductivity at twinning-plane boundaries,[16] one can estimate the increase of T_c:

$$\Delta T \equiv T_{cw}-T_c \sim \frac{\theta^2}{\tau_s}\frac{w}{\xi(T)}. \tag{29}$$

Here, the temperature-dependent correlation length $\xi(T) \sim \xi_s\sqrt{T_c/\Delta T}$ is the spatial extension of the superconducting gap and it diverges close to the superconducting transition; the pair breaking is only reduced on the small portion of the gap corresponding to the width w of the domain wall. On the end, the formula yields the estimate $\Delta T/T_c \sim (\xi_s/w)^2/(\tau_s T_c)^2$. Combining this result with the estimates for τ_s given in the preceding paragraph, we finally retrieve Eq. (26) qualitatively. This result holds when the width w is much larger than ξ_s. At smaller domain wall width, weakening of pair breaking effect works on the characteristic scale ξ_s of the proximity effect and w should be replaced by ξ_s in the estimate. Therefore, the large enhancement of T_{cw}, of the order of T_c), can be expected when $w \lesssim \xi_s$ and $h \sim T_{c0}$.

In the present work, we confirmed quantitatively this estimate in a number of situations. In particular, we obtained that, in the case of strong enhancement of T_{cw} (at $w \lesssim \xi_s$), for a thin F layer, the transition into the DWS state remains of the second order, with critical line above the transition into a uniform superconducting state of the second order at $T > T^* = 0.56 T_{c0}$, and of the first order at $T < T^*$. We predicted that only DWS could appear in bilayers with appropriate parameters.

The above physical picture does not depend on the sample's cleanness. However, the Usadel equations that we used to make quantitative estimates hold only for very dirty ferromagnets with $h\tau \ll 1$. For cleaner systems, one should solve the more difficult Eilenberger equations. We believe the results will be qualitatively similar. On the other hand, for ferromagnets with larger exchange field, the finite spin polarization is not longer negligible and leads to the suppression of proximity effect. Therefore, we do not expect the DWS formation by this mechanism.

Let us now come back to the assumption of a very thin F layer that was taken in the calculations. Usually, the exchange field in a ferromagnet is much larger than the gap in a superconductor. Therefore, the coherence length ξ_f is much smaller than ξ_s (1–5 nm for the former, compared to 10–50 nm for the latter). Therefore, the regime $d_f \ll \xi_f$ is quite hard to achieve with real samples of diffusive ferromagnets. Extending the calculation for a thin F layer to arbitrary thickness of the layer and arbitrary transparency (characterized by γ_B) of the interface is quite straightforward when domain walls are large, as we show in the Appendix. As is well known, the behavior of the critical temperature of the transition into a uniform superconducting state is quite rich in this case and may even oscillate as a function of the parameters such as d_f or γ_B. However, the physics of DWS appears to be quite similar to the one derived for thin bilayer. In particular, in the case of thick F layer ($d_f \gg \xi_f = \sqrt{D_f/h_f}$), the effective exchange field that would enter the above qualitative estimates would be $h \sim (\xi_s/d_s)\sqrt{h_f T_{c0}}$. Clearly, at $\xi_s \gg d_s$, as we assumed from the beginning, this field is much larger than T_{c0} and leads therefore to superconductivity suppression. However, h is strongly reduced if the S/F interface is opaque, which may lead to the F/S coexistence and to DWS appearance, as studied in this work. Nevertheless, the enhancement of T_c still is small by the factor $\xi_s/w \ll 1$ in the situation described in the Appendix.

These considerations suggest two possible directions to extend the range of existence of DWS. In the present work, DWS was analyzed for S layers with thickness d_s much smaller than coherence length ξ_s. On the other hand, DWS should not appear when the superconductor is hardly affected by proximity effect, at $d_s \gg \xi_s$. It would be of interest to consider the intermediate case when d_s and ξ_s are of the same order. This was studied for instance in the absence of magnetic domains in Ref. 17. Maybe a more important point would be to address the case of DWS in S/F bilayers with narrow domain walls and large thickness of the F layer. However, both problems require considerably more numerical work, which goes beyond the scope of this paper.

A lot of attention has been devoted to the study of long-range triplet proximity effect which develops in S/F structures when the direction of exchange field in the ferromagnet varies spatially.[2,18,19] In our calculation, such long range triplet component is also present, as it is clear from the Appendix: in Eq. (A6), the term $\gamma_0 \mathrm{ch} q_0 \tilde{z}$ in the direction transverse to the bilayer is generated only because of the presence of the domain wall, and it decays with typical length $\xi_T \sim \sqrt{D_f/T} \gg \xi_f$. However, γ_0 does not enter f^{11} and, therefore, is not important for the determination of the critical temperature of transition into DWS. In all the calculations we presented, there is no long-range triplet component in the direction transverse to the wall either: the typical length for the superconducting gap is determined by the conventional short-range proximity effect with decay length $\sim \xi_s$. We would like to emphasize also that the T_c enhancement due to the appearance of DWS is maximized for narrow domain walls (see Sec. III), when the matrix elements of the anomalous Green's function that would give rise to a long-range triplet component are exactly zero. Therefore, the physics of DWS discussed here is not directly related to such phenomena.

This observation is in agreement with Ref. 18 where a calculation of the critical temperature of S/F bilayers in the presence of spiral magnetic order in the F layer was presented. There also the long-range triplet component was found not to contribute to the result. On the other hand, the long-range triplet component may be important for other properties such as density of states in the ferromagnetic layer with domain structure deposited on top of a bulk superconducting substrate.[19]

We would like also to emphasize that our calculations differ from the study of S/F bilayers in the presence of spiral magnetic order in the F layer.[18] These works can be interpreted as considerations on magnetic domain structure only in as much as the width of the domains L and the width of the walls w are identical. A consequence is that the superconducting gap is spatially uniform along the bilayer in Ref. 18. In contrast, our theory really shows that, in the more realistic case when $w \ll L$, truly localized superconducting states can appear. In addition, consideration of the effect of spiral magnetic order corresponding to $\phi(x) = Qx$ in Eqs. (24) and (25), where Q is the wave vector of the spiral, can be immediately calculated from the Schrödinger-like equation (24), at least when $Q\xi_s \ll 1$. Critical temperature enhancement due to spiral magnetic order and corresponding to a uniform gap $\Delta(x)$ follows straightforwardly from the observation that $\phi'(x) = Q$, which enters Eq. (25), is constant.

The influence of magnetic domain walls on the superconducting properties of S/F bilayers was considered in other contexts. The enhancement of the Andreev subgap current at low temperature was predicted in Refs. 22 and 23. The critical magnetization for spontaneous vortex nucleation was determined in Ref. 24.

In conclusion, we analyzed in this work the enhancement of the superconducting critical temperature of superconducting/ferromagnetic bilayers due to the appearance of localized superconducting states in the vicinity of magnetic domain walls in the ferromagnet. We considered the case when the main mechanism of the superconductivity destruction via the proximity effect is the exchange field. We demonstrated that the influence of the domain walls on the superconducting properties of S layer may be quite strong if the domain wall thickness is of the order of the superconducting coherence length.

We interpreted qualitatively and quantitatively the amplitude of this effect, and we pointed out the special case when parameters of the bilayer are such that only localized superconductivity may form in these systems.

For a magnetic film with perpendicular anisotropy, the orbital effect provides an additional mechanism for domain wall superconductivity[4] and it may be easily taken into account. On the other hand for a film with easy plane magnetic anisotropy the domain wall will be a source of magnetic field in the adjacent S layer and locally weakens the superconductivity.[20] This mechanism will also depend on the precise structure of the magnetic domain wall (Bloch wall in thick ferromagnets, Néel wall in thin ferromagnets). The role of the orbital mechanism in the domain wall superconductivity may be important only if the magnetic induction is comparable with the upper critical field of the superconducting film. As we emphasized in the Introduction, this is usually not the case when the contact between S and F layers is good. We completely disregarded the orbital mechanism in the present work.

The domain wall superconductivity in S/F bilayers opens an interesting way to manipulate the superconducting properties through the domain structure. In particular the motion of the domain wall in the F layer may be accompanied by the displacement of the narrow superconducting region in the S layer.

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APPENDIX: THICK F LAYER

As mentioned previously, h_f is usually large in ferromagnets. Therefore the condition of thin F layer, $d_f \ll \xi_f$, is hardly reached. We would like to extend the results of the previous section to the more realistic situation of a finite size F layer. The difficulty is that the set of differential equations (1)–(4) to be solved are now two dimensional. We managed to solve it for the case of a large domain wall only. In the F layer, we parametrize the exchange field rotation with a slowly varying angle $\phi(x)$ such that $\boldsymbol{h}_f = h_f(\cos\phi, \sin\phi, 0)$. The linearized Usadel equations (2) in the F layer are

$$-\frac{D_f}{2}\triangle f^{11} + |\omega| f^{11} - i\frac{h_f s_\omega}{2}(e^{-i\phi}f^{21} - e^{-i\phi}f^{12}) = 0,$$

$$-\frac{D_f}{2}\triangle f^{12} + |\omega| f^{12} + i h_f s_\omega e^{-i\phi} f^{11} = 0,$$

$$-\frac{D_f}{2}\triangle f^{21} + |\omega| f^{21} - i h_f s_\omega e^{i\phi} f^{11} = 0. \quad \text{(A1)}$$

where $\Delta = \partial_x^2 + \partial_z^2$, while $f^{22} = -f^{11}$. When the spatial dependence of ϕ is neglected, the general form of the solutions which satisfy the boundary condition at the F/vacuum interface is

$$f_0^{11} = F_+ \cosh q_+\tilde{z} + F_- \cosh q_-\tilde{z},$$

$$f_0^{12} = s_\omega e^{-i\phi}\{F_0 \cosh q_0\tilde{z} + F_+ \cosh q_+\tilde{z} - F_- \cosh q_-\tilde{z}\},$$

$$f_0^{21} = -s_\omega e^{i\phi}\{-F_0 \cosh q_0\tilde{z} + F_+ \cosh q_+\tilde{z} - F_- \cosh q_-\tilde{z}\}, \quad \text{(A2)}$$

where $\tilde{z} = z + d_f$, $q_0 = \sqrt{2|\omega|/D_f}$, and $q_\pm = \sqrt{2(\pm i h_f + |\omega|)/D_f}$.

We determine now the amplitudes of the eigenmodes F_0 and $F_\pm$. For this, we first determine f^{11} and $\partial_z f^{11}$ at $z=0$ in the F layer. Making use of the boundary conditions (3), we can now insert them in the Usadel equation (5) in the S layer. We proceed similarly for f^{12} and f^{21}. In the end, we get from (5) three equations which determine the amplitudes we are looking for. We find $F_0 = 0$, while

$$F_\pm = \frac{\Delta}{2\Omega_\pm}, \quad \Omega_\pm = |\omega| C_\pm + \alpha q_\pm \sinh q_\pm d_f, \quad \text{(A3)}$$

where $C_\pm = \cosh q_\pm d_f + \gamma_B \xi_s q_\pm \sinh q_\pm d_f$ and $\alpha = D_s\sigma_f / 2d_s\sigma_s$.

Inserting Eqs. (A2) and (A3) into (3) to determine $\hat{f}_s$, and then inserting f_s^{11} in the self-consistency equation (4), we get the equation defining the critical temperature $T = T_c(h)$ for a uniform superconducting state:

$$0 = \ln\frac{T}{T_{c0}} + 2\pi T \,\mathrm{Re}\sum_{\omega>0}\left\{\frac{1}{|\omega|} - \frac{1}{|\omega| + \Gamma_+}\right\},$$

$$\Gamma_+ = \frac{\alpha q_+}{\coth q_+ d_f + \gamma_B \xi_s q_+}. \quad \text{(A4)}$$

This result is the same as Eq. (47) of Ref. 1. Whether the transition is of the second order [as described by Eq. (A4)] or of the first order was considered in Ref. 21.

Let us now consider the effect of a domain wall. As in Sec. IV, we will look for a solution $\hat{f} \approx \hat{f}_0 + \hat{f}_1$. In leading order, the spatial dependence of ϕ and the gap Δ is ignored, and $\hat{f}_0$ is still given by Eqs. (A2) and (A3). The correction $\hat{f}_1$ accounts for the slow variations of ϕ and Δ along the x axis; it is determined by the set of equations:

$$-\frac{D_f}{2}\partial_z^2 f_1^{11} + |\omega| f_1^{11} - i\frac{h_f s_\omega}{2}(e^{-i\phi}f_1^{21} - e^{-i\phi}f_1^{12}) = \frac{D_f}{2}\partial_x^2 f_0^{11},$$

$$-\frac{D_f}{2}\partial_z^2 f_1^{12}+|\omega| f_1^{12}+ih_f s_\omega e^{-i\phi} f_1^{11}=\frac{D_f}{2}\partial_x^2 f_0^{12},$$

$$-\frac{D_f}{2}\partial_z^2 f_1^{21}+|\omega| f_1^{21}-ih_f s_\omega e^{i\phi} f_1^{11}=\frac{D_f}{2}\partial_x^2 f_0^{21}. \qquad (A5)$$

Ignoring the x dependence of the right-hand side of the above differential equations, we can find the exact z-dependent function $\hat{f}_1$ which solves them. We obtain

$$f^{11}=\sum_{a=\pm}\left[\cosh q_a\tilde{z}\left(\frac{\Delta}{2\Omega_a}+a\frac{iD_f\phi'^2\Delta}{16h_f\Omega_a}+\gamma_a\right)-\tilde{z}\sinh q_a\tilde{z}\frac{\Delta''-\frac{1}{2}\phi'^2\Delta}{4\Omega_a q_a}\right],$$

$$f^{12,21}=\pm s_\omega e^{\mp i\phi}\left\{\pm\gamma_0\cosh q_0\tilde{z}+\sum_{a=\pm}\left[\left(\frac{a\Delta}{2\Omega_a}+a\gamma_a-\frac{iD_f\phi'^2\Delta}{16h_f\Omega_a}\pm\frac{D_f(2\phi'\Delta'+\phi''\Delta)}{4h_f\Omega_a}\right)\cosh q_a\tilde{z}-a\tilde{z}\sinh q_a\tilde{z}\frac{\Delta''-\frac{1}{2}\phi'^2\Delta}{4\Omega_a q_a}\right]\right\}, \qquad (A6)$$

where the integration constants γ_0 and $\gamma_\pm$ still remain to be determined. For this purpose, we insert Eq. (A6) into Eq. (3) in order to get $\hat{f}_s$ and $\partial_z\hat{f}_s$. Inserting them in Eq. (5), we thus obtain a set of three equations which allow us to determine them. In particular, we find

$$\gamma_\pm=\pm\frac{iD_f\phi'^2\Delta}{16h_f\Omega_\pm}+\frac{\Xi_\pm}{4q_\pm\Omega_\pm^2}\left(\Delta''-\frac{1}{2}\phi'^2\Delta\right)+\frac{D_s}{4}\left(\frac{C_\pm}{\Omega_\pm^2}\Delta''\mp\frac{i\phi'^2\Delta}{\Omega_\pm}\,\mathrm{Im}\,\frac{C_\pm}{\Omega_\pm}\right) \qquad (A7)$$

where $\Xi_\pm=|\omega|S_\pm+\alpha(\sinh q_\pm d_f+d_f q_\pm\cosh q_\pm d_f)$ and $S_\pm=d_f\sinh q_\pm d_f+\gamma_B\xi_s(\sinh q_\pm d_f+d_f q_\pm\cosh q_\pm d_f)$.

Finally, we can insert Eqs. (A7) into (A6), and then into (3), in order to obtain f^{11} in the S layer. Then, we insert it in the self-consistency equation (4). At the end, we find that the superconducting gap at the transition into the DWS state is still determined by the Schrödinger equation (24), with effective coefficients

$$E=-\ln\frac{T}{T_c},$$

$$\frac{1}{2m}=\pi T\,\mathrm{Re}\sum_{\omega>0}\left(\frac{D_s}{\omega_+^2}+\frac{\alpha(\sinh 2q_+d_f+2q_+d_f)}{2q_+\Omega_+^2}\right)$$

$$U(x)=-\pi T(\phi')^2\sum_{\omega>0}\left[D_s\left(\mathrm{Im}\,\frac{1}{\omega_+}\right)^2-\mathrm{Re}\,\frac{\alpha(\sinh 2q_+d_f+2q_+d_f)}{4q_+\Omega_+^2}-\frac{D_f}{2h_f}\,\mathrm{Im}\,\frac{1}{\omega_+}\right]. \qquad (A8)$$

where $\omega_+=|\omega|+\Gamma_+$.

In the limit of a thin F layer ($q_+d_f\to 0$) and large transparency of the S/F interface ($\gamma_B\to 0$), we note that $\Gamma_+\simeq\alpha q_+^2 d_f\simeq(\eta_f/\eta_s)h_f$, and it is easily checked that the above formulas for $1/2m$ and $U(x)$ reduce to Eq. (25) from Sec. II, when $\eta_f\ll\eta_s$.

For large F films and transparent S/F interface, we find that the critical temperature is given by Eq. (A4), where $\Gamma_+=(1+i)h$ and $h=\alpha\sqrt{h_f/D_f}$. Γ_+ can be interpreted as a combination of both exchange field and spin-flip terms with equal weight. In this case, it is known that the transition into a uniform state is of the second order.[21] The coefficients of Eq. (A8) also simplify to the form

$$E=-\ln\frac{T}{T_c},\quad \frac{1}{2m}=\pi TD_s\,\mathrm{Re}\sum_{\omega>0}\frac{1}{[\omega+(1+i)h]^2},$$

$$U(x)=-\pi TD_s(\phi')^2\sum_{\omega>0}\left(\mathrm{Im}\,\frac{1}{\omega+(1+i)h}\right)^2. \qquad (A9)$$

Let us note that the effective exchange field and spin-flip parameter scale is $h\sim(\xi_s/d_s)\sqrt{h_f T_{c0}}$ if the S and F layers have comparable diffusion constants and conductivities. When $d_s\ll\xi_s$, as we assumed from the beginning, this leads to $h\gg T_{c0}$, and therefore to complete superconductivity suppression. However, we expect that our results hold qualitatively in the more general case $\xi_s\lesssim d_s$ when superconductivity is not completely suppressed.

[1] A. I. Buzdin, Rev. Mod. Phys. 77, 935 (2005).
[2] F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Rev. Mod. Phys. 77, 1321 (2005).
[3] I. F. Lyuksyutov and V. L. Pokrovsky, Adv. Phys. 54, 67 (2005).
[4] A. I. Buzdin and A. S. Mel'nikov, Phys. Rev. B 67, 020503(R) (2003).
[5] A. Yu. Aladyshikin, A. I. Buzdin, A. A. Fraerman, A. S. Mel'nikov, D. A. Ryzhov, and A. V. Sokolov, Phys. Rev. B 68, 184508 (2003).
[6] Z. Yang, M. Lange, A. Volodin, R. Szymczak, and V. Moshchalkov, Nat. Mater. 3, 793 (2004).
[7] A. Yu. Rusanov, M. Hesselberth, J. Aarts, and A. I. Buzdin, Phys. Rev. Lett. 93, 057002 (2004).
[8] A. I. Buzdin, L. N. Bulaevskii, and S. V. Panyukov, Zh. Eksp.

Teor. Fiz. **87**, 299 (1984) [Sov. Phys. JETP **60**, 174 (1984)].

[9] A. I. Buzdin, L. N. Bulaevskii, M. L. Kulić, and S. V. Panyukov, Adv. Phys. **34**, 176 (1985).

[10] L. Usadel, Phys. Rev. Lett. **95**, 507 (1970).

[11] M. Yu. Kuprianov and V. F. Lukichev, Zh. Eksp. Teor. Fiz. **94**, 139 (1988) [Sov. Phys. JETP **67**, 1163 (1988)].

[12] Ya. V. Fominov and M. V. Feigel'man, Phys. Rev. B **63**, 094518 (2001).

[13] D. Saint-James, D. Sarma, and E. J. Thomas, *Type II Superconductivity* (Pergamon, New York, 1969).

[14] L. G. Aslamazov, Sov. Phys. JETP **28**, 773 (1969).

[15] L. D. Landau and E. M. Lifschitz, *Quantum Mechanics, Non-Relativistic Theory* (Pergamon Press, Oxford, 1965).

[16] I. N. Khlyustikov and A. I. Buzdin, Adv. Phys. **36**, 271 (1987).

[17] Ya. V. Fominov, N. M. Chtchelkatchev, and A. A. Golubov, Phys. Rev. B **66**, 014507 (2002).

[18] T. Champel and M. Eschrig, Phys. Rev. B **72**, 054523 (2005); Phys. Rev. B **71**, 220506(R) (2005).

[19] A. F. Volkov, Ya. V. Fominov, and K. B. Efetov, Phys. Rev. B **72**, 184504 (2005).

[20] E. Sonin, Pis'ma Zh. Tekh. Fiz. **14**, 1640 (1988) [Sov. Tech. Phys. Lett. **14**, 714 (1988)].

[21] S. Tollis, Phys. Rev. B **69**, 104532 (2004).

[22] N. M. Chtchelkatchev and I. S. Burmistrov, Phys. Rev. B **68**, 140501(R) (2003).

[23] R. Mélin and S. Peysson, Phys. Rev. B **68**, 174515 (2003).

[24] I. S. Burmistrov and N. M. Chtchelkatchev, Phys. Rev. B **72**, 144520 (2005).