Underscreened Kondo impurities in a Luttinger liquid

P. Durganandini[1] and Pascal Simon[2]

[1]*Department of Physics, University of Pune, Pune 411 007, India*

[2]*Laboratoire de Physique et Modélisation des Milieux Condensés, CNRS et Université Joseph Fourier, 38042 Grenoble, France*

We study the problem of underscreened Kondo physics in an interacting electronic system modeled by a Luttiger liquid (LL). We find that the leading temperature dependence of thermodynamical quantities such as the specific heat, spin susceptibility are Fermi liquidlike in nature. However, anomalous power law exponents are seen in the subleading terms. We also discuss possible realizations through single and double quantum dot configurations coupled to LL leads and its consequences for electronic transport. The leading low temperature transport behavior is seen to exhibit in general, non-Fermi liquid LL behavior unlike the thermodynamical quantities.

I. INTRODUCTION

There has been a resurgence of interest in the study of underscreened Kondo models in recent years due to their possible role in the observed breakdown of Fermi liquid behavior in the neighborhood of a quantum critical point in many heavy fermion materials[1–5] as well as the possibility that such underscreened models may be realized for quantum dot configurations.[6–8] Although the thermodynamics of these models are well known, the dynamical properties have been studied only recently.[2] In particular, it has been emphasized that at zero temperature, the presence of free spins in the underscreened models gives rise to singular scattering leading to what has been termed as "singular" Fermi liquid behavior.[5] Electronic transport through quantum dots which has parameter regimes with underscreening have also been studied.[6–8] Correlations between the electrons can modify impurity effects quite dramatically. An example is provided by interacting electrons in one dimension (1D). Such systems have the property that any arbitrary Coulomb repulsion between the electrons generically drives the system away from Fermi liquid (FL) to a Tomonaga-Luttinger liquid (LL)[9–11] behavior. In the low energy limit, the charge and spin degrees of freedom are separated and described by collective charge and spin density excitations, each moving with a characteristic Fermi velocity. As a result, electron correlation functions show spin charge separation as well as anomalous power law dependences. Such one dimensional Luttinger liquids can be realized as very narrow quantum wires[12] or edge states in fractional quantum Hall liquids[13] or single walled carbon nanotubes,[14] etc. The effects of scalar impurities in a LL have been well studied and shown to lead to effects like "breaking" or "healing" of the 1D chain.[15] The problem of a spin 1/2 magnetic impurity in a LL has also been largely studied.[16–24] It has been shown that while the ground state is a singlet state just like for the ordinary Kondo problem, the LL properties of the conduction electrons show up in the anomalous power law scaling for the Kondo temperature as well as the thermodynamics. An interesting question to ask is how underscreened Kondo physics manifests itself in a LL. In this paper, we study the problem of underscreened Kondo physics in a LL using boundary conformal field theory methods to analyze the renormalization group flows and to obtain the thermodynamical properties. We find that the leading temperature dependence of thermodynamical quantities such as the specific heat, spin susceptibility is FL-like in nature. However, the anomalous LL power law exponents are seen in the subleading terms. We also discuss possible realizations through single and double quantum dot configurations coupled to LL leads and the consequences for electronic transport. The low temperature transport behavior is seen to exhibit non-Fermi liquid behavior unlike the thermodynamical quantities.

The plan of our paper is as follows. In Sec. II, we analyze the renormalization group flows of the effective low energy model using boundary conformal field theory methods and obtain the thermodynamical properties. In the next section (Sec. III), we discuss possible realizations through single and double dot configurations coupled to LL. We then discuss electronic transport through such systems. We conclude by summarizing our results.

II. FIXED POINT ANALYSIS: A BOUNDARY CONFORMAL FIELD THEORY APPROACH

In the low energy, long wavelength limit, interacting electrons moving in a finite size 1D space extending from $-L$ to L can be described by the linearized continuum Hamiltonian with a four Fermi interaction:

$$H_0 = \int_{-L}^{L} dx [iv_F \psi_\sigma^\dagger(x)\partial_x \psi_\sigma(x) + U(\psi_\sigma^\dagger(x)\psi_\sigma(x))^2], \quad (1)$$

where v_F is the Fermi velocity and U denotes the strength of the repulsive density-density interaction. The one dimensional fermion field $\psi_\sigma(x)(\sigma=\uparrow,\downarrow)$ can be expanded about the Fermi points $\pm k_F$ in terms of the left moving and right moving fields as follows:

$$\psi_\sigma = e^{-ik_Fx}\psi_{L\sigma}(x) + e^{ik_Fx}\psi_{R\sigma}(x). \quad (2)$$

The left and right moving fermions may be bosonized as (Ref. 25) follows:

$$\psi_{L\uparrow/\downarrow} \sim \exp - i\sqrt{\frac{\pi}{2}}\left(\sqrt{g_c}\phi_c - \frac{1}{\sqrt{g_c}}\tilde{\phi}_c \pm \sqrt{g_s}\phi_s \pm \frac{1}{\sqrt{g_s}}\tilde{\phi}_s\right),$$

(3)

$$\psi_{R\uparrow/\downarrow} \sim \exp i\sqrt{\frac{\pi}{2}}\left(\sqrt{g_c}\phi_c - \frac{1}{\sqrt{g_c}}\tilde{\phi}_c \pm \sqrt{g_s}\phi_s \mp \frac{1}{\sqrt{g_s}}\tilde{\phi}_s\right).$$

(4)

Here $\phi_{c,s}, \tilde{\phi}_{c,s}$ are linear combinations of the bosons $\phi_{L,\uparrow\downarrow}, \phi_{R,\uparrow\downarrow}$ introduced to represent the fermion fields $\psi_{L,\uparrow\downarrow}, \psi_{R,\uparrow\downarrow}$:

$$\phi_c \sim \frac{1}{\sqrt{2}}(\phi_{L,\uparrow} + \phi_{R,\uparrow} + \phi_{L,\downarrow} + \phi_{R,\downarrow}),$$

(5)

$$\tilde{\phi}_c \sim \frac{1}{\sqrt{2}}(\phi_{L,\uparrow} - \phi_{R,\uparrow} + \phi_{L,\downarrow} - \phi_{R,\downarrow}),$$

(6)

$$\phi_s \sim \frac{1}{\sqrt{2}}(\phi_{L,\uparrow} + \phi_{R,\uparrow} - \phi_{L,\downarrow} - \phi_{R,\downarrow}),$$

(7)

$$\tilde{\phi}_s \sim \frac{1}{\sqrt{2}}(\phi_{L,\uparrow} - \phi_{R,\uparrow} - \phi_{L,\downarrow} + \phi_{R,\downarrow}).$$

(8)

In the absence of an external magnetic field, the parameter $g_s = 1$. The parameter g_c takes the value one for free fermions and has a U dependent value less than one for repulsive interaction. The low energy effective bulk Hamiltonian for the interacting fermions can be written then in terms of a free theory of charge and spin bosons with the interactions parametrized by g_c and g_s and moving with Fermi velocities v_c and v_s, respectively as, as follows:

$$H_0 = \frac{1}{2}\sum_{\alpha=c,s} v_\alpha \int_{-L}^{L} dx \partial_\mu \phi_\alpha \partial^\mu \phi_\alpha.$$

(9)

Let us now consider the effect of a magnetic impurity of magnitude $S > 1/2$ placed at the origin. We can describe the interaction of the impurity spin $\vec{S}$ with the conduction electrons at the site 0 through the spin exchange interaction:

$$H_K = J_K \psi^\dagger(0)\frac{\vec{\sigma}}{2}\psi(0) \cdot \vec{S}$$

$$= J_K\left[\psi_L^\dagger(0)\frac{\vec{\sigma}}{2}\psi_L(0) + \psi_R^\dagger(0)\frac{\vec{\sigma}}{2}\psi_R(0)\right.$$

$$\left. + \psi_L^\dagger(0)\frac{\vec{\sigma}}{2}\psi_R(0) + \psi_R^\dagger(0)\frac{\vec{\sigma}}{2}\psi_L(0)\right] \cdot \vec{S},$$

(10)

where the two terms in the second line of Eq. (11) describe forward scattering and the terms in the third line of Eq. (11) describe backward scattering. Finally, J_K is the Kondo coupling.

A. CFT analysis for free fermions

In the following we briefly recall some results for the corresponding problem with noninteracting fermions.[25] For free fermions, it is convenient to impose the boundary conditions $\psi(-L) = \psi(L)$ and define a parity definite even-odd basis: $\psi_{e(o),L/R}(x) = \psi_{L/R}(x) \pm \psi_{R/L}(-x)$, $x > 0$. The fermion fields satisfy the boundary conditions $\psi_{e(o),L}(0) = \pm \psi_{e(o),R}(0)$. In this basis, the Hamiltonian can be written

$$H = H_0 + H_K$$

$$= \int_0^L dx i v_F[\psi_{e,L,\sigma}^\dagger(x)\partial_x \psi_{e,L,\sigma}(x) + \psi_{o,L,\sigma}^\dagger(x)\partial_x \psi_{o,L,\sigma}(x)]$$

$$+ J_K \psi_{eL}^\dagger(0)\frac{\vec{\sigma}}{2}\psi_{eL}(0) \cdot \vec{S}.$$

(11)

Thus, the odd channel electrons decouple from the interaction and the theory can be described entirely in terms of the left moving even channel electrons on the 1D space 0 to L with the Kondo interaction at the origin. The problem reduces therefore to that of the usual single channel Kondo problem interacting with a spin S impurity. In the absence of the impurity, the free fermion theory can be described by the $SU(2)_{c,k=1} \times SU(2)_{s,k=1}$WZW model with certain specified "gluing" conditions for the charge and spin degrees of freedom. The Kondo interaction is a local interaction involving only the spin degrees of freedom. The renormalization group equations tell us that the Kondo interaction is marginally relevant for antiferromagnetic (AFM) coupling while it is marginally irrelevant for ferromagnetic (FM) coupling. The weak coupling fixed point is therefore stable for FM coupling but unstable for AFM coupling. For AFM Kondo coupling, the theory flows to the strong coupling (SC) fixed point $J_K = \infty$ with the Kondo scale set by $T_k = D \exp-(1/J_K\rho)$ (assuming a constant density of states ρ for the conduction electrons). In the $J_K = \infty$ limit, the ground state can be understood in terms of the Nozières-Blandin[26] picture of quenching of part of the impurity spin by the conduction electrons which leads to a $\pi/2$ phase shift for the conduction electrons. The $\pi/2$ phase shift corresponds to a change in the boundary conditions for the even channel fermions $\psi_{e,L}(0) = -\psi_{e,R}(0)$. Therefore, the strong coupling FP theory corresponds to that of a decoupled impurity spin of magnitude $s = S - 1/2$ and a free fermion theory with renormalized boundary conditions. The renormalization of the boundary conditions in the strong coupling limit leads to a modification of the gluing conditions for the charge and spin degrees of freedom which correspond here simply to "fusion" with the spin 1/2 WZW primary field in the spin sector.[25] Such a renormalization of the effects of a local interaction into boundary conditions lies at the heart of the boundary critical phenomena. If the boundary condition renormalizes to a fixed point (FP), then the effective theory may be described by the appropriate boundary conformal field theory (BCFT). The operator content of the BCFT can be obtained by imposing modular invariance on the theory. The stability as well as the physics around the FP can be

determined by analyzing all possible perturbations near the FP with the boundary operators.[25]

The high temperature or the weak coupling limit physics is governed by the marginally relevant Kondo interaction. Standard perturbative methods can be used to obtain the behavior of various physical quantities like the entropy, specific heat, spin susceptibility, etc.,[25,27] which show as expected, a logarithmic divergence at temperature $T=T_K$ for AFM coupling. In the low temperature or strong coupling limit for AFM coupling, the leading perturbation around the strong coupling FP is that of a ferromagnetic spin exchange coupling between the leftover spin $s=S-1/2$ impurity and the phase shifted conduction electrons via virtual nearest neighbor hoppings.[25] Since the residual ferromagnetic Kondo coupling is marginally irrelevant, the strong coupling fixed point is stable. Leading corrections to the zero temperature entropy can be obtained by a perturbative calculation in the marginally irrelevant residual FM coupling.[25,27] This gives the low temperature entropy as follows:

$$S_{imp}(T \ll T_K) = \ln(2s + 1)$$
$$- \frac{\pi^2}{3} s(s+1)(2\lambda\rho)^3 [1 - (2\lambda\rho)\ln(T/T_K)$$
$$+ 6(2\lambda\rho)^2 \ln^2(T/T_K) + \cdots], \qquad (12)$$

where the first term denotes the degeneracy of the residual impurity spin and λ denotes the strength of the FM coupling between the leftover impurity spin and the conduction electrons. Usual scaling arguments show that $\lambda\rho$ scales as $\lambda\rho \sim \frac{1}{\ln(T/T_K)}$. The specific heat then has the leading temperature dependence

$$C_{imp}(T \ll T_K) = \pi^2 s(s+1) \frac{1}{[\ln(T/T_K)]^4} + \cdots. \qquad (13)$$

In the presence of a weak magnetic field, the impurity spin susceptibility can be computed as

$$\chi_{imp}(T \ll T_K) = \frac{(g\mu_B)^2 s(s+1)}{3T} \left[1 - \frac{1}{[\ln(T/T_K)]} + \cdots \right]. \qquad (14)$$

Thus the marginal exchange coupling between the residual free impurity spin and the conduction electrons leads to the singular Fermi liquid behavior.[5]

If a magnetic field H is added, at low temperature $T \ll H \ll T_K$, the residual impurity spin becomes polarized and the ground state degeneracy is lifted. Since there are no impurity spin fluctuations, there is no FM coupling between the residual impurity spin and the conduction electrons. The leading boundary perturbation is now the spin two object with dimension two just as in the ordinary Kondo problem (Ref. [25]):

$$\lambda_2 (\psi_{e,L\uparrow}^\dagger \psi_{e,L\downarrow})^2, \qquad (15)$$

which leads to the usual regular FL behavior for the various physical quantities.

B. CFT analysis for interacting electrons

It is not possible in general to describe the boundary conditions for the interacting electron problem in a simple way as for the free fermion theory, however, the possible conformally invariant boundary conditions for the interacting electron theory with a magnetic impurity [see Eq. (11)] turn out to be particularly simple within the bosonic language—the only conformally invariant boundary conditions being either the Dirichlet or Neumann boundary conditions. The bulk theory (in the absence of the Kondo interaction) can be identified with both the charge and spin bosons satisfying the Neumann boundary conditions.[19,28] The operator content around this fixed point can be identified, it turns out the backscattering component of the electron spin operator in Eq. (11) is the lowest dimensional parity invariant operator which can couple to the impurity spin. This operator has dimension $(1+g_c)/2$ (Ref. [19]) which is less than 1 for $g_c < 1$. Hence this term is relevant for either sign of the Kondo coupling. The weak coupling fixed point is therefore unstable for both ferromagnetic and antiferromagnetic perturbations and flows to the strong coupling fixed points $J_K = +\infty$ for AFM coupling and $J_K = -\infty$ for FM coupling. The corresponding Kondo scale is given by $T_K \propto |J_{K\rho}|^{2/(1-g_c)}$.[16] At the AFM SC FP, one can argue using the usual Nozières and Blandin picture that the impurity spin gets locked with the electron at site 0 forming an effective spin of magnitude $s=S-1/2$ which gets decoupled from the rest of the chain. The effective theory therefore becomes that of an open chain with one site removed and a decoupled impurity spin of magnitude $s=S-1/2$. At the FM SC FP, the impurity spin is ferromagnetically coupled to the electron at site 0 to form an effective spin $S+1/2$ which in turn couples with the electrons at the sites -1 and $+1$ to form an effective spin $s=S-1/2$. The effective theory is that of an open chain with three sites removed and a decoupled impurity spin of magnitude $s=S-1/2$. Thus, in the $L \to \infty$ limit, both the AFM and FM SC FP are described by an effective theory of two decoupled semi-infinite LL and a decoupled spin of magnitude $s=S-1/2$. The two decoupled semi-infinite LL can be described by a BCFT with Dirichlet boundary conditions on the charge and spin bosons.[19,28] We now determine the stability of the SCFP by analyzing all possible perturbations around the fixed point. The two decoupled channels can interact with each other and with the remaining spin $S-1/2$ impurity via the boundary operators. From the boundary operator content,[28] one can see that the lowest dimensional boundary interactions which can occur are: (i) the spin exchange coupling between the boundary spin current operator in each decoupled chain and the leftover free impurity spin of size $s=S-1/2$:

$$\lambda_1 \left[\psi_{L,1}^\dagger \frac{\vec{\sigma}}{2} \psi_{L,1} + \psi_{L,2}^\dagger \frac{\vec{\sigma}}{2} \psi_{L,2} \right] \cdot \vec{s}, \qquad (16)$$

with dimension 1. Since this coupling is generated as in the usual FL case by virtual hopping's between the nearest neighbor site electrons (next nearest neighbor site electrons for FM Kondo coupling) and the decoupled leftover impurity

spin, it is ferromagnetic in nature and hence marginally irrelevant;

(ii) the hopping of fermions between the two channels via spin flip scattering with the leftover impurity spin:

$$\lambda_2 \left[\psi_{L,1}^\dagger \frac{\vec{\sigma}}{2} \psi_{L,2} + \psi_{L,2}^\dagger \frac{\vec{\sigma}}{2} \psi_{L,1} \right] \cdot \vec{s}, \tag{17}$$

which has dimension $(1+g_c)/2g_c$ and is irrelevant for $g_c < 1$;

(iii) the hopping of a fermion between the two channels:

$$\lambda_3 (\psi_{L\uparrow,1}^\dagger \psi_{L\uparrow,2} + \psi_{L\downarrow,1}^\dagger \psi_{L\downarrow,2}) + \text{H.c.}, \tag{18}$$

with dimension $(1+g_c)/2g_c$;

(iv) the hopping of a charge two spin singlet between the two channels:

$$\lambda_4 ((\psi_{L\uparrow}^\dagger \psi_{L\downarrow}^\dagger)_1 (\psi_{L\uparrow} \psi_{L\downarrow})_2) + \text{H.c.}, \tag{19}$$

with dimension $2/g_c$;

(v) the hopping of a charge neutral spin 2 object between the two channels:

$$\lambda_5 ((\psi_{L\uparrow}^\dagger \psi_{L\downarrow})_1 (\psi_{L\uparrow}^\dagger \psi_{L\downarrow})_2) + \text{H.c.}, \tag{20}$$

which has dimension 2;

(vi) a potential scattering term:

$$\lambda_6 ((\psi_{L\uparrow}^\dagger \psi_{L\downarrow})_1 + (\psi_{L\uparrow}^\dagger \psi_{L\downarrow})_2), \tag{21}$$

which has dimension 1;

(vii) and a spin two object:

$$\lambda_7 [((\psi_{L\uparrow}^\dagger \psi_{L\downarrow})_1)^2 + ((\psi_{L\uparrow}^\dagger \psi_{L\downarrow})_2)^2], \tag{22}$$

which has dimension 2.

The potential scattering term is an exactly marginal operator and can only lead to a shift of the ground state energy. Since all these operators are irrelevant for $g_c < 1$, the fixed point is stable to these perturbations.

We next discuss the physics around the weak and strong coupling FP. The Kondo backscattering term governs the physics near the weak coupling FP. For $T \gg T_K$, the leading temperature dependence of the entropy, specific heat and the impurity spin susceptibility can be obtained as:

$$S_{imp}(T \gg T_K) = \ln(2S+1) + A S(S+1)(T_K/T)^{(1-g_c)} + \cdots, \tag{23}$$

$$C_{imp}(T \gg T_K) = A(g_c - 1)S(S+1)(T_K/T)^{(1-g_c)} + \cdots, \tag{24}$$

$$\chi_{imp}(T \gg T_K) = \frac{(g\mu_B)^2 S(S+1)}{3T}[1 - B(T_K/T)^{(1-g_c)} + \cdots], \tag{25}$$

where A and B are nonuniversal dimensionless constants depending on g_c and the electron density of states and the dots denote subleading terms.

The lowest dimension boundary perturbation near the strong coupling FP is the marginally irrelevant exchange coupling between the boundary spin current operator in each channel and the residual impurity spin. The next lowest dimensional boundary perturbation is the electron hopping term between the two channels via spin flip scattering with the residual impurity spin. The leading corrections to the low temperature entropy can be expressed in terms of the irrelevant coupling parameters λ_i with $i = 1, \ldots 7$ (redefined in terms of dimensionless quantities) as follows:

$$S_{imp}(T \ll T_K) = \ln(2s+1) - \frac{\pi^2}{3}s(s+1)[\lambda_1^3 + c_2\lambda_2^2 + \cdots]$$
$$+ c_3\lambda_3^2 + \cdots. \tag{26}$$

The first term in the above equation denotes the degeneracy of the left-over impurity spin, the next two terms are due to the two lowest dimension boundary operators interacting via the residual impurity spin and the dots inside the bracket indicate subleading terms due to interactions with the residual impurity. The next term indicates the boundary contribution from electron tunneling between the two channels without interaction with the residual impurity spin and the final dots indicate higher order contributions from residual spin impurity independent boundary operators. It is easy to see from the scaling dimensions of the boundary operators that λ_1 scales as $\lambda_1 \sim \frac{1}{\ln(T/T_K)}$ while λ_2 and λ_3 scales as $(T/T_K)^{(1-g_c)/2g_c}$. The temperature behavior of the specific heat can be obtained as follows:

$$C_{imp}(T \ll T_K) = \pi^2 s(s+1) \left[\frac{1}{[\ln(T/T_K)]^4} + c_2(T/T_K)^{(1-g_c)/g_c} \right.$$
$$\left. + \cdots \right] + c_3(T/T_K)^{(1-g_c)/g_c} + \cdots. \tag{27}$$

Similarly, the zero field impurity spin susceptibility is given as follows:

$$\chi_{imp}(T \ll T_K) = \frac{(g\mu_B)^2 s(s+1)}{3T} \left[1 - \frac{1}{\ln(T/T_K)} \right.$$
$$\left. + c_2(T/T_K)^{(1-g_c)/g_c} + \cdots \right] + c_0 + \cdots, \tag{28}$$

c_0, c_2, c_3 in the above equations are nonuniversal constants depending on g_c and the electron density of states. We see therefore that while the lowest dimension boundary perturbation leads to the same singular low temperature thermodynamic properties as for the underscreened Kondo problem in a FL, the temperature dependence of the subleading terms which come from the electron tunneling term between the two channels with spin-flip scattering, are governed by the anomalous LL exponents and reflect the non-Fermi liquid nature of the system.

If we add a magnetic field H, the residual spin impurity fluctuations are suppressed at low temperature $T \ll H$, $H \ll T_K$. Therefore the leading boundary perturbation is now the same as that in the fully screened case, namely the elec-

tron tunneling operator between the two channels with no spin-flip scattering [the term (iii) in the list of boundary operators] with dimension $(1+g_c)/2g_c$. The leading behavior of the thermodynamical quantities like the specific heat will be the same as in the fully screened Kondo case.[17,19] In particular, the low temperature specific heat has a leading anomalous LL power law behavior instead of the logarithmic dependence while the impurity spin susceptibility shows to leading order the expected paramagnetic behavior.

C. Zero backward Kondo scattering

More generally, we can distinguish between forward and backward scattering strengths and express the interaction Eq. (11)

$$H_K = J_{K,f}\left[\psi_L^\dagger(0)\frac{\vec{\sigma}}{2}\psi_L(0) + \psi_R^\dagger(0)\frac{\vec{\sigma}}{2}\psi_R(0)\right]\cdot\vec{S}$$
$$+ J_{K,b}\left[\psi_L^\dagger(0)\frac{\vec{\sigma}}{2}\psi_R(0) + \psi_R^\dagger(0)\frac{\vec{\sigma}}{2}\psi_L(0)\right]\cdot\vec{S}. \quad (29)$$

For any generic values of $J_{K,f}, J_{K,b}$, the couplings flow to the strong coupling FP described above. But for $J_{K,b}=0$, the theory reduces to a two channel Kondo problem (with the left and right moving electrons corresponding to the two channels) interacting with a spin S magnetic impurity. The charge sector decouples from the theory and it is sufficient to consider only the spin sector to study the stability of the weak coupling fixed point. As is well known, the weak coupling fixed point is stable for ferromagnetic (FM) coupling $(J_{K,f}>0)$ while it is unstable for antiferromagnetic (AFM) Kondo coupling $(J_{K,f}<0)$. The Kondo temperature has the usual exponential coupling dependence. We can distinguish between three different cases for the low temperature physics. While for $S=1/2$, the low temperature physics corresponds to the two channel overscreened Kondo physics, for $S=1$, the conduction electrons form a singlet with the impurity spin leading to fully screened Kondo physics, for $S>1$, the conduction electrons form a singlet with part of the impurity spin and the rest is left over as a decoupled spin of size $S-1$. For $S=1$, the low temperature physics exhibits the usual regular Fermi liquid behavior while for $S>1$, the low temperature physics is governed by the marginally irrelevant ferromagnetic coupling between the residual spin of size $S-1$ and the conduction electrons. The latter again leads in the low temperature limit to the singular Fermi liquid behavior [Eqs. (12)–(14)] described earlier.

III. QUANTUM DOT REALIZATIONS

We now discuss possible scenarios where we might observe such physics. One possible realization would be to couple a single quantum dot with spin S to an interacting semiconducting wire (a LL wire) in the geometries shown in Figs. 1(a) and 1(b). Another possibility is to couple two quantum dots with spin S to a LL wire as shown in Fig. 1(c).
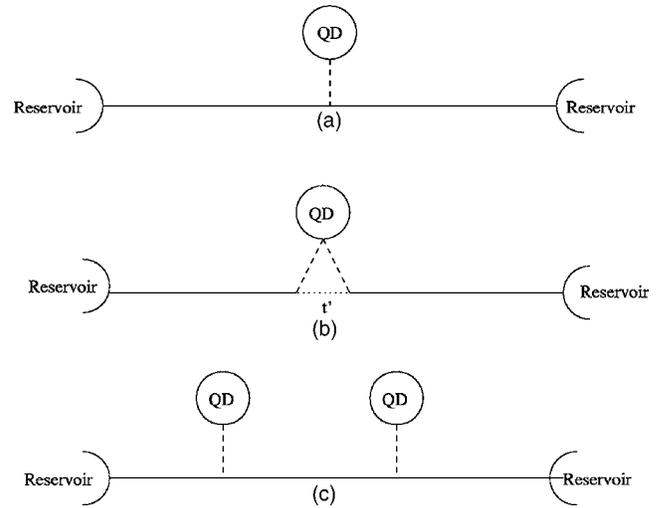
FIG. 1. (a) A quantum dot coupled to one site of a LL lead. (b) Quantum dot coupled to two sites of a LL lead and (c) two quantum dots attached to a LL lead.

A. Quantum dot with spin S side coupled to one site of a LL wire

Let us first consider transport through the spin impurity realized as a QD coupled to LL lead in the geometry shown in Fig. 1(a). In the side coupled geometry, the Kondo effect appears as an anomalously strong reflection or backscattering rather than as transmission. At high temperatures, the weak coupling FP dictates the temperature dependence of the conductance and is essentially governed by the behavior of the backscattering Kondo scattering process. Therefore the high temperature linear conductance has the leading temperature dependence

$$G(T) - G_0 \sim -G_0 S(S+1)(T/T_K)^{(g_c-1)}, \quad (30)$$

where $G_0=2e^2/h$ is the unitary conductance predicted in the absence of the coupling to the QD. This is in contrast to the FL lead case where the conductance has the temperature dependence

$$G(T) - G_0 \sim -G_0\frac{\pi^2 S(S+1)}{4(\ln(T/T_K))^2}, \quad (31)$$

due to the marginal nature of the Kondo exchange interaction. The leading temperature dependence of the conductance in the low temperature limit is governed by the hopping of an electron between the two semi-infinite LL leads via spin flip scattering and the electron tunneling operator with no spin flip scattering. The low temperature conductance is therefore of the form:

$$G(T) \sim G_0(a1s(s+1) + a2)(T/T_K)^{1-g_c/g_c}, \quad (32)$$

where a_1 and a_2 are some nonuniversal constants. This is in contrast to the FL lead case which shows a logarithmic temperature behavior

$$G(T) \sim G_0 \frac{\pi^2 s(s+1)}{4(\ln(T_K/T))^2}. \tag{33}$$

Thus we find that the leading low temperature transport behavior is governed by the subleading electron tunneling terms between the two channels without and with residual impurity spin-flip scattering and therefore shows non-Fermi liquid behavior with an anomalous power law behavior with the power law exponent being dictated by the LL interaction strength.

In a finite magnetic field, $T \ll H \ll T_K$, the leading temperature dependence of the conductance is the same as that for the fully screened case. However, since the dimension of the boundary operator governing the transport process in the two cases is the same, the conductance has the same temperature dependence as for the underscreened case [see Eq. (32)]. The main difference between the two cases being the absence of the spin-dependent term.

B. Quantum dot with spin S side coupled to two sites of a LL wire

We next consider electronic transport through a dot configuration where the dot is coupled to two different sites of the LL chain as shown in Fig. 1(b). When the electron density is at half-filling and for $1/2 < g < 1$, it is well known that the charge sector in the LL model becomes massive but the spin sector still remains massless. It can be then shown[17,18] that this model is a realization of the case $J_{K,b}=0$ discussed at the end of the previous section. As discussed in the previous section, the problem becomes then essentially that of a two channel Kondo problem with a spin S impurity. Such a model has been previously studied.[21] For $S=1/2$, the low temperature transport exhibits overscreened Kondo behavior:

$$G(T) \sim G_0(T/T_K)^{1/2}. \tag{34}$$

For $S=1$, the transport shows FL behavior

$$G(T) \sim G_0(T/T_K)^2, \tag{35}$$

while for $S>1$, the conductance shows the underscreened behavior given in Eq. (33) with $s=S-1$.

For electron densities away from half-filling, the problem can be thought of that of a quantum dot in an embedded geometry[29] with potential scattering. A similar analysis as in the $S=1/2$ case[29,30] shows that for $0 < g_c < 1/2$, the strong coupling fixed point is the same as that for the two channel Kondo FP with a spin S impurity, while for $1/2 < g_c < 1$, one obtains the strong coupling FP of the single channel LL with a spin S impurity. We note that this implies that in contrast to noninteracting electrons, one will not get a Kondo resonance in general for interacting electrons (for $1/2 < g_c < 1$) except at some particular value of the gate voltage where the backscattering term vanishes. Off resonance, the low T conductance has the same behavior as in 32. For gate voltages very close to the resonance voltage, $G(T) - G_0 \propto -G_0(a_3 + a_{-4}(s(s+1)T^{(1-g_c)})$ where a_3 and a_4 are nonuniversal constants. We note that such a scaling behavior was observed in

earlier studies of the gate voltage dependence of the linear conductance through a Kondo spin $1/2$ quantum dot coupled to LL leads.[24]

C. Two spin S quantum dots side-coupled to a LL wire

Another possibility is to couple two quantum dots with spin S to a LL wire as shown in Fig. 1(c). The latter problem is equivalent to that of two magnetic impurities in a LL.[31–33] When there is more than one magnetic impurity, there are two competing effects: the Kondo spin exchange interaction between each impurity spin and the conduction electron spin and the induced Ruderman-Kittel-Kasuya-Yosida (RKKY) spin exchange interaction between the impurities (the RKKY interaction is modified in the presence of electron interaction[31]). The ground state of the system depends on which of these interactions dominate. If the Kondo interaction strength is greater than the RKKY interaction, then one expects single impurity physics. The impurity spin then forms a singlet with the conduction electrons and gets decoupled. The electrons then see only an effective potential scatterer at each impurity site. So effectively, for two impurities, the chain behaves as if there are two barriers. Generically, one should expect the zero temperature conductance to be zero. However, there is the interesting possibility of resonant tunneling in the Kondo limit (for not very large distances between the two impurities and if the resonant tunneling conditions are satisfied) just like for symmetric double barriers.[15] On the other hand, if the RKKY interaction dominates, there can be different kinds of physics depending on whether there is FM or AFM interaction between the two impurities. For AFM coupling between the spins, one expects the two impurities to lock into an effective singlet state which is essentially like a nonmagnetic impurity. In the side coupled configuration, one expects the nonmagnetic impurity to have no effect on the conduction electrons and therefore lead to the unitary value for the zero temperature conductance. Thus, while the Kondo limit and AFM exchange limit both show a singlet phase, they exhibit different physics in that in the Kondo limit, one expects breaking of the chain except under some circumstances where resonant tunneling can occur while in the RKKY AFM limit, one expects healing of the chain. For strong FM RKKY interaction, the problem effectively becomes that of a spin $2S$ impurity interacting with a LL, the problem therefore becomes effectively the underscreened Kondo problem discussed in the previous sections. The low temperature conductance then has the temperature dependence given in Eq. (32) reflecting LL behavior. We also mention that recent experiments[34,35] on quantum dots with a nonlocal RKKY interaction have motivated studies of transport in such coupled quantum dot systems.[36–39] However, these studies do not consider the effect of electron-electron interactions.

IV. CONCLUSION

To summarize, we have analyzed the problem of underscreened Kondo physics in a LL. We find that the leading temperature dependence of thermodynamical quantities like

the specific heat, spin susceptibility are FL-like in nature. However, the anomalous LL power law exponents are seen in the subleading terms. We have also discussed possible realizations through single and double quantum dot configurations coupled to LL leads and the consequences for electronic transport. The leading low temperature transport behavior is seen to exhibit in general, non-Fermi liquid LL behavior unlike the thermodynamical quantities.

[1] Q. Si, J. L. Smith, and K. Ingersent, Int. J. Mod. Phys. B **13**, 2331 (1999).

[2] P. Coleman, C. Pepin, Q. Si, and R. Ramazashvili, J. Phys.: Condens. Matter **13**, R723 (2001).

[3] Q. Si, S. Rabello, K. Ingersent, and J. L. Smith, Nature (London) **412**, 804 (2001).

[4] P. Coleman and C. Pepin, Phys. Rev. B **68**, 220405(R) (2003).

[5] P. Mehta, N. Andrei, P. Coleman, L. Borda, and G. Zarand, Phys. Rev. B **72**, 014430 (2005).

[6] A. Posazhennikova and P. Coleman, Phys. Rev. Lett. **94**, 036802 (2005).

[7] M. Pustilnik and L. Glazman, J. Phys.: Condens. Matter **16**, R513 (2004).

[8] W. Hofstetter and H. Schoeller, Phys. Rev. Lett. **88**, 016803 (2002).

[9] S. Tomonaga, Prog. Theor. Phys. **5**, 544 (1950).

[10] J. M. Luttinger, J. Math. Phys. **4**, 1154 (1963).

[11] F. D. Haldane, J. Phys. C **14**, 2585 (1981).

[12] O. M. Auslaender, A. Yacoby, R. de Picciotto, K. W. Baldwin, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **84**, 1764 (2000).

[13] A. M. Chang, Rev. Mod. Phys. **75**, 1449 (2003).

[14] C. Dekker, Phys. Today **52**, 22 (1999).

[15] C. L. Kane and M. P. A. Fisher, Phys. Rev. B **46**, 15233 (1992).

[16] D. H. Lee and J. Toner, Phys. Rev. Lett. **69**, 3378 (1992).

[17] A. Furusaki and N. Nagaosa, Phys. Rev. Lett. **72**, 892 (1994).

[18] P. Frojdh and H. Johannesson, Phys. Rev. Lett. **75**, 300 (1995).

[19] P. Durganandini, Phys. Rev. B **53**, R8832 (1996).

[20] K. Le Hur, Phys. Rev. B **59**, R11637 (1999); **61**, 1853 (2000).

[21] S. Eggert, D. P. Gustafsson, and S. Rommer, Phys. Rev. Lett. **86**, 516 (2001).

[22] A. Furusaki, J. Phys. Soc. Jpn. **74**, 73 (2005).

[23] V. Ravi Chandra, S. Rao, and D. Sen, Europhys. Lett. **75**, 797 (2006).

[24] S. Andergassen, T. Enss, and V. Meden, Phys. Rev. B **73**, 153308 (2006).

[25] I. Affleck, Nucl. Phys. B **336**, 517 (1990); I. Affleck and A. W. W. Ludwig, *ibid.* **360**, 641 (1991); Phys. Rev. B **48**, 7297 (1993).

[26] P. Nozieres and A. Blandin, J. Phys. (France) **41**, 193 (1980).

[27] A. C. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge University Press, Cambridge, 1993).

[28] E. Wong and I. Affleck, Nucl. Phys. B **417**, 403 (1994).

[29] P. Simon and I. Affleck, Phys. Rev. B **64**, 085308 (2001).

[30] M. Fabrizio and A. O. Gogolin, Phys. Rev. B **51**, 17827 (1995); A. Gogolin and N. V. Prokof'ev, *ibid.* **50**, 4921 (1994).

[31] R. Egger and H. Schoeller, Phys. Rev. B **54**, 16337 (1996).

[32] K. Hallberg and R. Egger, Phys. Rev. B **55**, R8646 (1997).

[33] J. Igarashi, T. Tonegawa, M. Kaburagi, and P. Fulde, Phys. Rev. B **51**, 5814 (1995).

[34] N. J. Craig, J. M. Taylor, E. A. Lester, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Science **304**, 565 (2004).

[35] S. Sasaki, S. Kang, K. Kitagawa, N. Yamaguchi, S. Miyashita, T. Maruyama, H. Tamura, T. Akazaki, Y. Hirayama, and H. Takayanagi, Phys. Rev. B **73**, 161303(R) (2006).

[36] P. Simon, R. Lopez, and Y. Oreg, Phys. Rev. Lett. **94**, 086602 (2005); L. Glazman and M. Vavilov, *ibid.* **94**, 086805 (2005).

[37] P. Simon, Phys. Rev. B **71**, 155319 (2005).

[38] H. Tamura and L. Glazman, Phys. Rev. B **72**, 121308(R) (2005).

[39] P. Durganandini (unpublished).