Coherent charge transport through an interface between a superconductor and hopping insulator: Role of barrier properties

M. Kirkengen,[1] J. Bergli,[1] and Y. M. Galperin[1,2,3]

[1]*Department of Physics, University of Oslo, P.O. Box 1048 Blindern, 0316 Oslo, Norway*
[2]*Center for Advanced Materials and Nanotechnology at the University of Oslo, Argonne National Laboratory,*
9700 South Cass Avenue, Argonne, Illinois 60439, USA
[3]*A. F. Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia*

At low temperatures and voltages tunneling transport through an interface between a superconductor and hopping insulator is dominated by coherent two-electron tunneling between the Cooper-pair condensate and pairs of localized states [Kozub *et al.*, Phys. Rev. Lett. **96**, 107004 (2006)]. By detailed analysis of such transport we show that the interface resistance is extremely sensitive to the properties of the tunneling barriers, as well as to the asymptotic behavior of the localized states. In particular, a dramatic cancellation takes place for hydrogenlike impurities and ideal barriers. However, some disorder can lift the cancellations, restoring the interface transport. We also study the non-Ohmic behavior of the interface resistor and show that it is sensitive to the Coulomb correlation of the occupation probabilities of the involved localized states. It is expected that the non-Ohmic contribution to the *I-V* curve will experience pronounced mesoscopic (fingerprint) fluctuations.

PACS number(s): 72.20.Ee, 74.45.+c, 74.50.+r, 73.50.−h

I. INTRODUCTION

Tunneling charge transport through a superconductor-semiconductor interface has been extensively studied in connection with hybrid devices. Since the seminal paper by Blonder, Tinkham, and Klapwijk[1] it is well known that two mechanisms of charge transfer can be important. The fist mechanism is single-electron tunneling while the second one is due to Andreev reflection from the interface. Though the Andreev transport is proportional to the tunneling transparency squared and seems to be less important than the contribution of single-electron tunneling, at low temperatures it can dominate. The reason is that the energy cost for transfer of a quasiparticle is $\gtrsim \Delta$, and at low temperatures single-electron transport is suppressed $\propto e^{-\Delta/T}$.[2] Here Δ is the energy gap in the superconductor while T is the temperature in energy units. Since quasiparticle transport is proportional to the first power of the tunneling transparency, Andreev transport will dominate when $e^{-\Delta/T}$ is much less than the tunneling transparency.

Extensive theoretical[1,3–5] and experimental[6–14] effort has been focused on relatively low-resistive structures where the semiconductor part of the device has *metallic* conductance. This is natural having in mind coherent hybrid devices showing the Josephson effect.

Contrary to these activities, in this paper we address coherent charge transfer through the interface between a superconductor (SC) and a *hopping insulator* (HI)—i.e., a system where transport occurs via hops between localized (*non-propagating*) single-particle states. This is the case, in particular, in many experiments aimed at studies of hopping conductance where superconducting electrodes are conventionally used; see, e.g., Ref. 15. Investigations of hopping are usually conducted at low temperatures, much less than the transition temperature of the leads, and in situations when the tunneling barrier at the interface is not too strong. Consequently, one can expect that single-particle transport is sig-

nificantly suppressed compared to Andreev-like charge transfer.

The problem of charge transfer between a SC and a HI was first addressed in Ref. 16, where it was shown that at low temperatures transport is governed by *time-reversal reflection*, where electrons tunnel into the superconductor from localized states in the hopping insulator located near the interface. This process is similar to the so-called *crossed* Andreev charge transfer discussed previously in connection with various mesoscopic systems.[17,18] Electron hopping from the superconductor to impurities near the surface of the insulator and back again has been proposed as a source of qubit decoherence for some systems.[19] According to Ref. 16, the interface resistance can compete with the bulk resistance of the hopping insulator, and in this way can be important for an interpretation of the experimental results on hopping conductance. Since the time-reversal contribution disappears when the leads are driven into the normal state, the interface resistance of Andreev type can be removed by applying a relatively weak magnetic field, causing a superconductor–to–normal-metal transition in the leads.[16] This anomalous magnetoresistance provides a natural way to separate the interface contribution and compare it with the bulk resistance of the hopping insulator.

In this paper we extend the analysis of Ref. 16 in two directions. *First*, we consider the influence of the properties of the tunneling barrier on the charge transfer, which turns out to be surprisingly sensitive to the barrier roughness. Namely, we found that interference effects in tunneling can lead to a significant *increase* in the interface resistance due to fine cancellations of the contributions to the two-particle tunneling probability. The roughness of the barrier suppresses these effects, and in this way influences the interface resistance. *Second*, we consider *non-Ohmic* transport through the interface. We will show that the interface contribution to the resistance can be strongly nonlinear and that the nonlinear behavior is essentially related to the Coulomb correlation of

the occupation numbers of the localized states adjacent to the interface. One can expect a rich pattern of reproducible (fingerprint) fluctuations in the I-V curve due to pronounced non-Ohmic contributions of individual pairs. Thus, combined studies of the linear interface resistance and average nonlinear I-V curve and its fluctuations may tell a lot about barrier details and about the formation of the depletion zone near the barrier.

The paper is organized as follows. In Sec. II we set the stage presenting the model of Ref. 16 for coherent charge transport. Detailed calculations for the Ohmic case are presented in Sec. III where we show how interference-based cancellations occur. In Sec. IV we discuss several generalizations of the model, including the effects of modification of the tails of localized wave functions and barrier details. In Sec. V, starting from calculation of the non-Ohmic contributions of the individual pairs, we demonstrate how an account of the Coulomb correlations leads to non-Ohmic behavior of the interface conductance.

II. MODEL

As shown in Ref. 16, contact resistance can be governed either by the interface tunneling barrier or by deformation of the hopping cluster in the HI in the vicinity of the interface. Here, for brevity, we will focus on the situation where the interface resistance is dominated by the barrier.

We start with the case of linear conductance where it is natural to use the Kubo linear response theory.[20] According to this theory, the conductance $\mathcal{G}$ is expressed through the susceptibility,

$$\chi(\omega) = i \int_0^\infty \langle [\hat{I}^\dagger(t), \hat{I}(0)] \rangle e^{i\omega t} dt, \qquad (1)$$

as $\mathcal{G} = \lim_{\omega \to 0} \mathrm{Im}\, \chi(\omega)/\omega$. Here $\hat{I}(t)$ is the current operator and we will use units where $\hbar = 1$.

Let a superconductor and a hopping insulator occupy adjacent three-dimensional (3D) semispaces separated by a tunneling barrier (TB). The presence of the barrier simplifies calculations which will be made in the lowest nonvanishing approximation in the tunneling amplitude T_0. This models the Schottky barrier usually present at a semiconductor-metal interface. Then the current operator is defined as[21]

$$\hat{I}(t) = ie \int d^2 r\, d^2 r'\, T(\mathbf{r}, \mathbf{r}')[a^\dagger(\mathbf{r}, t) b(\mathbf{r}', t) - \text{H.c.}],$$

where $\mathbf{r}$ is the coordinate on the superconductor side of the interface plane, $\mathbf{r}'$ is the coordinate on the semiconductor side, $a^\dagger(\mathbf{r}, t)$ and $b(\mathbf{r}, t)$ are creation and annihilation operators in the semiconductor and superconductor, respectively, and d is the electron localization length under the barrier.

The Matsubara formalism (see, e.g., Ref. 22) allows calculation of the susceptibility by analytical continuation of the so-called *Matsubara susceptibility* defined as

$$\chi_M(\Omega) = \int_0^\beta \langle T_\tau \hat{I}(\tau) \hat{I}(0) \rangle e^{i\Omega\tau} d\tau.$$

Here T_τ means ordering in "imaginary time" τ, $\beta \equiv 1/T$, and the temperature T is measured in units of energy. Integration over the imaginary time actually means the average over a grand canonical ensemble with temperature T and chemical potential μ.

The operators $\hat{I}^\dagger$ and $\hat{I}$ are time-dependent interaction picture operators. Changing to Schrödinger-type operators we write

$$\chi_M(\Omega) = \int_0^\beta \langle T_\tau I(\tau) I(0) e^{\int H_T d\tau} \rangle e^{i\Omega\tau} d\tau,$$

where H_T is the tunneling Hamiltonian given by the expression

$$H_T(\tau) = \int d^2 r\, d^2 r'\, T(\mathbf{r}, \mathbf{r}')[a^\dagger(\mathbf{r}, \tau) b(\mathbf{r}', \tau) + \text{H.c.}], \quad (2)$$

where integration is performed along the interface. Here $T(\mathbf{r}, \mathbf{r}')$ is the tunneling amplitude which in general is dependent on the coordinates for both entry to and exit from the barrier.

Let us first assume that

$$T(\mathbf{r}, \mathbf{r}') = T_0 \delta(\mathbf{r} - \mathbf{r}'), \qquad (3)$$

as was done in Ref. 16. Then

$$H_T(\tau) = T_0 \int d^2 r\, [a^\dagger(\mathbf{r}, \tau) b(\mathbf{r}, \tau) + \text{H.c.}]. \qquad (4)$$

Because single-electron transitions are forbidden by the energy gap, we have to expand the expression for χ to second order in H_T. Of the many possible contractions, we are only interested in two-electron transitions in both directions. We get a total of 12 different contractions. Of these, half will be only the Hermitian conjugate of the other half, and a symmetry consideration reduces the number of relevant contractions to 3. They are

$$(A) \quad [a_\tau^\dagger b_\tau - a_\tau b_\tau^\dagger][a_0^\dagger b_0 - a_0 b_0^\dagger][a_1^\dagger b_1 + a_1 b_1^\dagger][a_2^\dagger b_2 + a_2 b_2^\dagger],$$

$$(B1) \quad [a_\tau^\dagger b_\tau - a_\tau b_\tau^\dagger][a_0^\dagger b_0 - a_0 b_0^\dagger][a_1^\dagger b_1 + a_1 b_1^\dagger][a_2^\dagger b_2 + a_2 b_2^\dagger],$$

$$(B2) \quad [a_\tau^\dagger b_\tau - a_\tau b_\tau^\dagger][a_0^\dagger b_0 - a_0 b_0^\dagger][a_1^\dagger b_1 + a_1 b_1^\dagger][a_2^\dagger b_2 + a_2 b_2^\dagger].$$

One can show that the first one (A) is small at $eV \ll \Delta$—i.e., when the single-electron transport is suppressed—while the two others $B1$ and $B2$ give equal contributions. We will therefore follow only $B1$ through the further analysis. To perform calculations we introduce Green's functions in the usual way:[22]

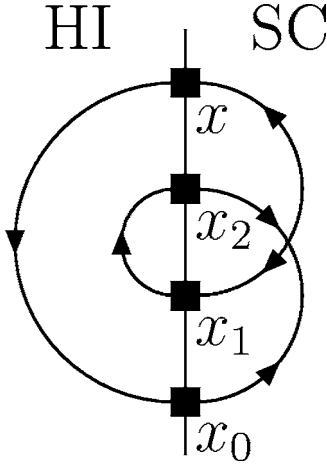$$\langle T_\tau b(\mathbf{r}, \tau) b(\mathbf{r}', \tau') \rangle = F(x - x'),$$

FIG. 1. Diagram relevant for the Matsubara susceptibility.

$$\langle T_\tau b^\dagger(\mathbf{r},\tau)b^\dagger(\mathbf{r}',\tau')\rangle = F^\dagger(x-x'),$$

$$\langle T_\tau a(\mathbf{r},\tau)a^\dagger(\mathbf{r}',\tau')\rangle = G(x,x').$$

Here $x_i \equiv (\mathbf{r}_i, \tau_i)$, $F(x-x')$ is the anomalous Green's function in the superconductor, and $G(x,x')$ is the Green's function of the insulator. We get

$$\langle T_\tau \hat{I}(\tau)\hat{I}(0)\rangle = e^2 T_0^4 \int d^2r d^2r_0 dx_1 dx_2$$
$$\times F(x-x_1)F^\dagger(x_2-x_0)G(x_0,x)G(x_2,x_1). \quad (5)$$

Here $dx \equiv d^2r d\tau$. We then take the discrete Fourier transforms of the Green's functions to write them as functions of Ω and ω_i rather than τ and τ_i. In this case we will have to require that Ω and ω take discrete values of $\Omega_m = 2\pi mT$ and $\omega_n = (2n+1)\pi T$. Performing the integrations over τ, τ_1, and τ_2, we get δ functions for the relations between the different discrete frequencies, giving

$$\chi(\Omega_m) = 2Te^2 T_0^4 \sum_{\omega_n} \int \prod d^2r_i F(\mathbf{r}-\mathbf{r}_1,\omega_n)$$
$$\times F^\dagger(\mathbf{r}_0-\mathbf{r}_2,-\omega_n)$$
$$\times G(\mathbf{r}_0,\mathbf{r},\omega_n-\Omega_m)G(\mathbf{r}_2,\mathbf{r}_1,-\omega_n). \quad (6)$$

The Feynman diagram corresponding to this expression is shown in Fig. 1.

We assume the localized states to have hydrogenlike wave functions, centered on impurities in positions $\mathbf{r}_s$, with energies ϵ_s and localization length a,

$$\Psi_s(\mathbf{r}) = (\pi a^3)^{-1/2} e^{-|\mathbf{r}-\mathbf{r}_s|/a}, \quad (7)$$

and the insulator Green's function is

$$G(\mathbf{r},\mathbf{r}',\omega_n) = \sum_s \frac{\Psi_s^*(\mathbf{r})\Psi_s(\mathbf{r}')}{i\omega_n - \epsilon_s}. \quad (8)$$
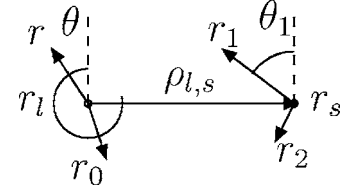
For the anomalous Green's function we use

FIG. 2. Coordinates for the spatial integration.

$$F(R,\omega_n) = \int \frac{d^3p}{(2\pi\hbar)^3} \frac{\Delta}{\Delta^2 + \xi_p^2 + \omega_n^2} e^{-i\mathbf{p}\cdot\mathbf{R}/\hbar}$$
$$= \frac{\pi g_m \Delta}{2\sqrt{\Delta^2+\omega_n^2}} \frac{\sin(Rk_F)}{Rk_F} e^{-(R\sqrt{\Delta^2+\omega_n^2})/\pi\xi\Delta}. \quad (9)$$

Here $\xi_\mathbf{p} = (p^2 - p_F^2)/2m$, ξ is the coherence length in a superconductor and $g_m = mp_F/\pi^2$ is the density of states in a metal.

III. CALCULATIONS

So far we just followed Ref. 16, but we will now demonstrate how the oscillations of the anomalous Green's functions, $\propto \sin Rk_F$, lead to a significant decrease of the result compared to the simple estimates presented there. It turns out that these oscillations give rise to pronounced cancellations in the susceptibility for the case of a hydrogenlike impurity state.

We now define $\mathbf{r}_s$ and $\mathbf{r}_l$ as coordinates of the impurities contributing to $G(\mathbf{r}_2,\mathbf{r}_1)$ and $G(\mathbf{r}_0,\mathbf{r})$, respectively. Then the Matsubara susceptibility can be expressed as

$$\chi_M(\Omega_m) = \frac{Te^2|T_0|^4 g_m^2}{2a^6} \sum_{sln} \frac{\Delta^2}{\Delta^2 + \omega_n^2}$$
$$\times \frac{I_{sl}^2}{(-i\omega_n - \epsilon_s)(i\omega_n - i\Omega_m - \epsilon_l)}, \quad (10)$$

$$I_{sl} = \int d^2r d^2r_1 \frac{\sin k_F|\mathbf{r}-\mathbf{r}_1|}{k_F|\mathbf{r}-\mathbf{r}_1|} \exp\left(-\frac{|\mathbf{r}_1-\mathbf{r}_s|+|\mathbf{r}-\mathbf{r}_l|}{a}\right.$$
$$\left. -\frac{(|\mathbf{r}-\mathbf{r}_1|)\sqrt{\Delta^2+\omega_n^2}}{\pi\xi\Delta}\right). \quad (11)$$

It is safe to assume that $|\mathbf{r}-\mathbf{r}_1|$ and $|\mathbf{r}_0-\mathbf{r}_2|$ are of the order of the distance ρ_{sl} between the impurities projected onto the interface, but with variations of the order of a, where $k_F a \gg 1$. The localization length a can be estimated as the Bohr radius $a_0 = 4\pi\kappa\hbar^2/m^*e^2$. Assuming $m_* \approx 0.1 m_e$, where m_e is the mass of a free electron, and $\kappa \approx 10$, we get $k_F a \approx 100$. Since the superconductor localization length is much greater, $\xi \gg a$, we can safely replace $|\mathbf{r}-\mathbf{r}_1| \rightarrow \rho_{sl}$ in Eq. (11) in all places *except* $\sin k_F|\mathbf{r}-\mathbf{r}_1|$, which oscillates rapidly. Choosing the coordinates as shown in Fig. 2 we obtain

$$I_{sl} = \frac{1}{k_F\rho_{sl}} e^{-(\rho_{sl}/\pi\xi)(\sqrt{\Delta^2+\omega_n^2})/\Delta}$$
$$\times \int d^2r d^2r_1 \sin k_F|\mathbf{r}-\mathbf{r}_1| e^{-(|\mathbf{r}_1-\mathbf{r}_s|+|\mathbf{r}-\mathbf{r}_l|)/a}.$$

As $\mathbf{r}$ and $\mathbf{r}_1$ are located in the interface plane, it is natural to choose the origins for $\mathbf{r}$ and $\mathbf{r}_1$ to be at the projections of $\mathbf{r}_s$ and $\mathbf{r}_l$, respectively, into the interface plane of the. Then $d^2 r_i = r_i dr_i d\theta_i$,

$$|\mathbf{r} - \mathbf{r}_l| = \sqrt{r_1^2 + z_l^2}, \quad |\mathbf{r}_1 - \mathbf{r}_s| = \sqrt{r_1^2 + z_s^2},$$

$$|\mathbf{r}_1 - \mathbf{r}_2| = \sqrt{(\rho_{sl} + r\sin\theta - r_1\sin\theta_1)^2 + (r\cos\theta - r_1\cos\theta_1)^2},$$

where z_s and z_l are the distances from the impurities to the interface.

The minimal distance between the impurities taking part in the coherent transport, $\rho_{sl}^{\min}$, is limited by Coulomb correlation. If $a/\rho_{s,l}^{\min} \ll (k_F a)^{-1}$, one can neglect the items containing $\cos\theta_i$ in the above expression. Then $|\mathbf{r} - \mathbf{r}_1| \approx \rho_{sl} + r\sin\theta - r_1\sin\theta_1$ and the integrals over $\mathbf{r}$ and $\mathbf{r}_1$ can be calculated exactly using the formula

$$\int_{-\pi}^{\pi} d\theta \sin(k_F r\sin\theta + \phi) = \mathrm{Im} \int_{-\pi}^{\pi} d\theta e^{i k_F r \sin\theta + i\phi}$$

$$= \mathrm{Im}\, e^{i\phi} \int_{-\pi}^{\pi} d\theta e^{i k_F r \sin\theta}$$

$$= 2\pi \sin(\phi) J_0(kr).$$

After integrating over both angles we get

$$I_{sl} = \frac{(2\pi)^2 \sin k_F \rho_{sl}}{k_F \rho_{sl}} e^{(-\rho_{sl}/\pi\xi)(\sqrt{\Delta^2 + \omega_n^2})/\Delta} I_r(z_s) I_r(z_l),$$

$$I_r(z) = \int_0^\infty r dr J_0(k_F r) e^{-\sqrt{r^2 + z^2}/a}. \tag{12}$$

The integral I_r will be referred to extensively in later sections, as most of the modifications we will discuss change this integral only, leaving all other calculations unaltered. We can then use the following identity:[23]

$$\int_0^\infty x dx e^{-p\sqrt{x^2 + z^2}} J_0(cx) = p(p^2 + c^2)^{-3/2}(1 + z\sqrt{p^2 + c^2})$$

$$e^{-z\sqrt{p^2 + c^2}} [\mathrm{Re}(p) > |\mathrm{Im}(c)|,$$

$$\mathrm{Re}(z) > 0.$$

Combining all integrals, this gives

$$I_{sl} = \frac{(2\pi)^2 a^4 \sin k_F \rho_{sl}}{k_F \rho_{sl}} \frac{\mathcal{F}(z_l)\mathcal{F}(z_s)}{(1 + k_F^2 a^2)^3} e^{(-\rho_{sl}/\pi\xi)(\sqrt{\Delta^2 + \omega_n^2})/\Delta},$$

$$\mathcal{F}(z) = \left(1 + \frac{z}{a}\sqrt{1 + k_F^2 a^2}\right) e^{(-z/a)\sqrt{1 + k_F^2 a^2}}.$$

Using the assumption that $k_F a \gg 1$ the function $\mathcal{F}(z)$ simplifies to

$$\mathcal{F}(z) \approx (1 + k_F z) e^{-z k_F}.$$

The essential observation here is that the expression for I_{sl} contains a factor $e^{-(z_l + z_s)k_F}$, which again should be squared for

the final result. This conclusion contradicts Ref. 16, where the factor $e^{-(z_l + z_s)/a}$ was predicted. The strong decay of I_{sl} as a function of z_l and z_s means that only pairs very close to the interface can contribute to the Cooper-pair transfer. This means that the theory as presented above and in Ref. 16 proves its own inadequacy, since the assumption that the wave functions of the localized states are bulk hydrogenlike ones requires z_l and z_s to be at least of the order of a. For closer impurities, the wave function is modified by the vicinity of the surface, and the result becomes strongly dependent on unknown properties of the surface states. In this paper we will not consider these close impurities, but discuss how details of the barrier may change the above result back to the $e^{-(z_l + z_s)/a}$ of Ref. 16 and thus allow the main contribution to come from pairs farther from the barrier.

Note also the extreme accuracy to which the positive and negative contributions to $I_r(z)$ cancel. Between two adjacent zeros of $J_0(k_F r)$ the integral is of order 1 for small r (for $r > a$ it gets damped by the exponential), yet the final integral is of order $e^{-k_F z} \approx e^{-k_F a} \approx 10^{-44}$ if $k_F a = 100$.

A closer analysis of the integral over r shows that it accumulates a negative value of the order of $e^{-z/a}$ for small r, which is almost canceled by an equivalent positive contribution for very large r. The cancellation is found to be strongly dependent on the exact shape of the wave function. Consequently, one may conclude that the cancellation can be lifted by specific properties of the tunneling amplitude, which could introduce an effective cutoff of the integration over r. In the following Sec. IV several models will be discussed, where the importance of hitherto ignored details in the tunneling barrier will be made clear and the importance of the assumed wave function for the localized states will be discussed.

To complete the calculation of the conductance, we must now perform a summation over the Matsubara frequencies in the standard way, replacing

$$T \sum_{\omega_n} f(i\omega_n) = \oint \frac{d\epsilon}{4\pi i} f(\epsilon) \tanh\frac{\epsilon}{2T}.$$

Under the assumptions we have made, this integration is independent of the details of the spatial integration and will not be affected by the modifications introduced in later calculations. Integrating over the contour shown in Fig. 3, we get an expression for conductance depending on the discrete variable Ω_m. Making an analytical continuation of this function and taking the limit as Ω_m goes to zero, we finally get an expression for the conductance:

$$G = \frac{\pi(2\pi)^4 a^2 e^2 |T_0|^4 g_m^2}{2(k_F a)^{12} T} \sum_{s \neq l} \frac{\sin^2(k_F \rho_{sl})}{(k_F \rho_{sl})^2} e^{-2\rho_{sl}/\pi\xi}$$

$$\times \mathcal{F}(z_s)\mathcal{F}(z_l) n(\epsilon_l) n(\epsilon_s) \delta(\epsilon_s + \epsilon_l). \tag{13}$$

Here $n(\epsilon) = (e^{\epsilon/T} + 1)^{-1}$ is the Fermi distribution.

As shown in Ref. 16, it is important to include the effect of Coulomb interaction between the occupied sites which results in additional energy $U_C = e^2 a^2 \kappa \rho_{sl}^3$. However, accounting for this interaction does not change the strong cancellation. We will come back to the role of the Coulomb interac-
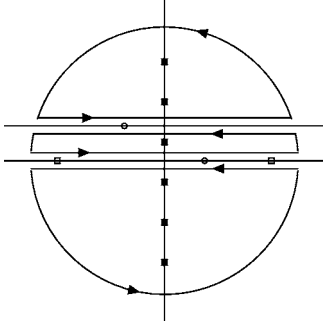
FIG. 3. Integration contour. The values $\epsilon = \epsilon_s$ and $\epsilon_l - i\Omega_m$ are shown by circles ($\bigcirc$) and the values $\epsilon = \pm\Delta$ are shown by open squares ($\square$), while the poles of $\tanh(\epsilon/2T)$ are shown by solid squares.

tion in Sec. V where we discuss non-Ohmic conductance. In the next section we discuss how robust the cancellation is and how it is influenced by the properties of the tunneling barrier.

IV. OHMIC CONDUCTANCE

To understand how robust the cancellation shown in the previous section is, we will analyze several aspects of Ohmic transport through the interface.

A. Importance of the impurity wave function

The hydrogenlike wave function is a typical approximation for shallow centers in semiconductors. However, crossed Andreev transport can also take place in mesoscopic devices—e.g., between a bulk superconductor and a pair of quantum dots. This is, in particular, the case for the previously suggested spin entangler.[18]

If the above cancellation is correct also for that case, then crossed Andreev transport would be hardly feasible since the dots would have to be located virtually at the interface. However, the tails of the wave functions of the electrons localized at quantum dots are far from being hydrogen like. In general, they are dependent on the design of the quantum dots. In particular, for the lateral quantum dots designed by a properly engineered gate potential one can expect parabolic confinement. In this case the wave function tail is Gaussian rather than exponential.

To check whether the above cancellation exists in this case we have repeated the calculations of Sec. III, but replacing $\Psi_s(\mathbf{r})$ with $\Psi_G(\mathbf{r}) = (2\pi a^2)^{-1/2} e^{-|\mathbf{r} - \mathbf{r}_s|^2/2a^2}$. As a result, the contribution of a given pair decays as $e^{-(z_s + z_l)/a}$ and one returns to the estimates of Ref. 16. Thus the design of quantum dots chosen for the spin entangler can be crucial for its potential success.

B. Importance of barrier properties

In Sec. III we assumed that the tunneling amplitude is *local* and *coordinate independent*, $T(\mathbf{r}, \mathbf{r}') = T_0 \delta(\mathbf{r} - \mathbf{r}')$. It means that during tunneling an electron can transfer its momentum to some disorder-induced scatterers and the tunnel-

ing amplitude is independent of the incident angle. To study the role of this simplification, we will proceed as follows. First, we consider the case of an *ideal* barrier for which the tunneling amplitude depends only on the incident angle. We will show that such dependence does not remove the cancellation and the decay $\propto e^{-(z_l + z_s)k_F}$ persists. Then we will consider the case of a barrier with fluctuating strength. We will find that fluctuations of a scale $\lesssim a$ can strongly facilitate transport restoring the $e^{-(z_l + z_s)/a}$ dependence.

To make these consideration more specific let us assume that the effective barrier thickness d fluctuates along the interface. Then the tunneling amplitude is nonlocal and the tunneling Hamiltonian acquires the general form of Eq. (2). Consequently, the coordinates of the Green's functions for the HI and SC sides are different and the proper diagram has the form of Fig. 4 rather than that of Fig. 1.

For the following it is convenient to normalize the tunneling amplitude to T_0,

$$T(\mathbf{r}, \mathbf{r}') \equiv T_0 f(\mathbf{r}, \mathbf{r}').$$

Then the spatial integral can be written as [cf. Eq. (5)]

$$\int \prod d^2 r_i d^2 r_i' f(\mathbf{r}, \mathbf{r}') f(\mathbf{r}_0, \mathbf{r}_0') f(\mathbf{r}_1, \mathbf{r}_1') f(\mathbf{r}_2, \mathbf{r}_2')$$

$$\times \sum_{\omega_n} F(\mathbf{r} - \mathbf{r}_1, \omega_n) F^\dagger(\mathbf{r}_0 - \mathbf{r}_2, \omega_n) G(\mathbf{r}', \mathbf{r}_0', \omega_n - \Omega_m)$$

$$\times G(\mathbf{r}_2', \mathbf{r}_1', \omega_n). \tag{14}$$

Following the previous calculations, this can be split into separate, identical integrations for each impurity. To separate the roles of barrier thickness fluctuations and dependence of the incident angle let us express the tunneling amplitude as

$$f(\mathbf{r}, \mathbf{r}') = g(\mathbf{r}) h(\mathbf{r}' - \mathbf{r}), \tag{15}$$

where $g(\mathbf{r})$ describes spatial variations in the barrier, while $h(\mathbf{r}' - \mathbf{r})$ accounts for the dependence on the incident angle. Here both vectors $\mathbf{r}$ and $\mathbf{r}'$ belong to the interface plane. The function $h(\mathbf{r}' - \mathbf{r})$ can be assumed to depend only on $|\mathbf{r}' - \mathbf{r}|$.

a. Smooth Barrier. Let us start with the case when $g(\mathbf{r}) = g_0 = $ constant. The basic spatial integration is

$$I(r, r_0, \omega_n) \equiv \int d^2 r'' G(|\mathbf{r} + \mathbf{r}''|, r_0', \omega_n) h(r''). \tag{16}$$

Here we have taken into account that the Green's function $G(\mathbf{r}, \mathbf{r}_1, \omega_n)$ depends only on $\sqrt{z^2 + r^2}$ and $\sqrt{z^2 + r_1^2}$. Let us now assume that $h(r'')$ decays much more rapidly than G. That allows us to expand the integrand in powers of x'' and y'', keeping only the second order (the first-order term vanishes on integration),

$$G(|\mathbf{r} + \mathbf{r}''|, r_0', \omega_n) \approx G + \frac{x''^2}{2} \frac{\partial^2 G}{\partial x^2} + \frac{y''^2}{2} \frac{\partial^2 G}{\partial y^2},$$

with $G \equiv G(r, r_0', \omega_n)$. Now let us consider the simplest case of a rectangular barrier for which the function $h(r)$ can be modeled as $h(r) = d^{-2} e^{-B(r/d)^2}$ where $B \gtrsim 1$ is some dimensionless constant. This model follows from an assumption that the barrier is uniform along the surface and rectangular.
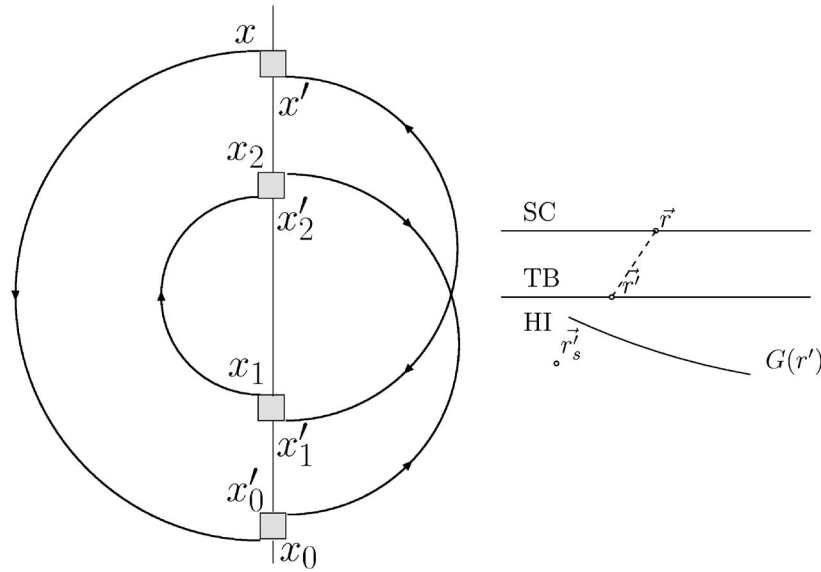
FIG. 4. Left panel: the diagram of Fig. 1 for the case of nonlocal tunneling transparency. Right panel: a more detailed sketch of the coordinates.

Then the tunneling exponent can be written as

$$-d\sqrt{2m[U - E(1 - k_{\parallel}^2/k^2)]} \approx -\lambda_0 - B(r/d)^2,$$

where $\lambda_0 \equiv d\sqrt{2m(U-E)}$, $B \equiv \lambda_0 E/2(U-E)$, and U is the barrier height, while E is the electron energy. Using this model and explicit expression (8) for the Green's function we obtain

$$I(r, r_0, \omega_n) = G(r, r_0', \omega_n)c(r),$$

$$c(r) \equiv \frac{\pi g_0}{B}\left[1 + \frac{d^2}{4Ba^2}\left(\frac{r^2}{r^2 + z^2} - \frac{2a}{\sqrt{r^2 + z^2}} + \frac{ar^2}{\sqrt{r^2 + z^{23}}}\right)\right].$$

The integral I_r, Eq. (12), now changes to

$$I_r(z) = \int_0^{\infty} r\,dr\,J_0(k_F r)e^{-\sqrt{r^2 + z^2}/a}c(r).$$

At $d \ll a$ the correction induced by the angular dependences of the transmission is small, but in principle, it could be sufficient to lift the cancellations that give the $e^{-k_F z}$ dependence. However, a combination of analytical and numerical analyses shows that the additional terms also lead to the $e^{-k_F z}$ dependence. We therefore conclude that the dependence of the tunneling transparency of a uniform barrier on the incident angle still leads to $e^{-k_F z}$ decay of the crossed Andreev transport.

b. Inhomogeneous barrier. We now turn to the situation were we have fluctuations in the barrier strength that facilitate tunneling through the places where the barrier is thin. Let us assume that the typical size of these regions, λ, is much less than the localization length a, but larger than k_F^{-1},

$$k_F^{-1} \ll \lambda \ll a.$$

Now we cannot assume $g(\mathbf{r})$ to be constant over the region spread by the impurity potential. For simplicity, we will in this case write $h(\mathbf{r}'') = \delta(\mathbf{r}'')$, as we have previously shown

that these corrections do not change the principal behavior of the transport. The shape of $g(\mathbf{r})$ is dependent on the roughness and on the relative positions of the barrier minimum and the impurity center. For a barrier with a parabolic minimum one can show that $g(r)$ has a Gaussian shape,

$$g(r) = e^{-r^2/a^2\sigma}.$$

The general analysis of this situation is complicated, as the impurity center may not coincide with the center of the barrier minimum. Two simplified cases are still sufficient to shed light on the situation.

Let us for simplicity start with the case when the minimum in the barrier strength coincides with the projection of the impurity center on the interface. In this case the integral for $I_r(z)$ similar to Eq. (12) can be analyzed in detail. It turns out that with increasing z it crosses over from $e^{-k_F z}$ to $e^{-z/a}$ at some z^* which depends on σ. The quantity z^* decreases with decrease of σ, $z^* = a$ at $1/\sigma = 0.15$. Thus the barrier inhomogeneity (modeled by small σ) facilitates transport by eliminating the cancellation (see Fig. 5).

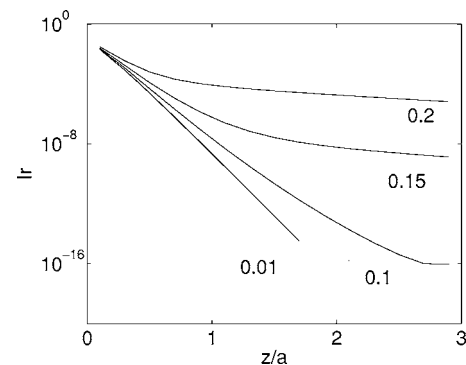

FIG. 5. The dependence $I_r(z)$ for $k_F a = 20$ and $1/\sigma = 0.01, 0.1, 0.15, 0.2$.

Now we can relax the previous assumption that the barrier strength minimum occurs exactly at the projection point of an impurity center and consider the situation where the minimum is off center, but varies so fast that $G(r)$ can be considered constant in comparison. In this case we can replace $G(r)=G(r_{min})$, r_{min} being the barrier minimum coordinate, and move the origin of the integration to r_{min}. The integral will then again give a simple $e^{-z/a}$ dependence. Thus we see that the cutoff introduced by a clear minimum in the barrier is sufficient to change the z dependence of the conductance per impurity.

If there are several minima within one single impurity, one could imagine that these could give new interference effects. However, integration over several minima corresponds to simply summing up the contribution from the separate integrals. Each minimum will be coupled to a minimum on another impurity, giving a prefactor of $\sin^2 \rho_{sl}$. When several sines are added at each impurity, we get

$$(\sin \rho_{sl} + \sin \rho_{s'l'})^2 = \sin^2 \rho_{sl} + \sin^2 \rho_{s'l'} + 2 \sin \rho_{sl} \sin \rho_{s'l'}.$$

When averaging over several pairs, the $\sin^2$ terms will survive, while the cross terms will average to zero. We therefore assume several minima within the range of each impurity to be equivalent to several separate pairs in the total averaging. Considering the probability of a pair accepting a Cooper pair, the effect on one minimum of the pair already being occupied due to another minimum should be negligible.

The characteristic localization length of the electron under the barrier we call α. In order for these considerations to be relevant, the barrier thickness d must vary with several α over a length scale much shorter than a. Changes in barrier thickness that do not meet this condition are better analyzed in terms of the following model.

c. Barrier with a blocklike disorder. The model discussed above relies on a change in barrier thickness that is of the order of α. Because of the very fine cancellations in the integral I_r, Eq. (12), much smaller changes in barrier height can be important, provided they are on a length scale of the order of a. This can be demonstrated using a simple model based on an analysis of the integral

$$I_r(R,z) \equiv \int_0^R r dr J_0(k_F r)e^{-\sqrt{r^2+z^2}/a}$$

as a function of the cutoff R. Obviously, $I_r(z) = \lim_{R \to \infty} I_r(R,z)$. By splitting the integration over x into intervals divided by the zeros of the Bessel function, we get successive contributions of alternating signs and close absolute values. The result approaches the $e^{-k_F z}$ behavior seen before when X goes to infinity. The absolute value of the sum $S(n,z)$ of an even number of the intervals defined as

$$S(n,z) = \sum_{m=0}^{2n} \int_{R_m}^{R_{m+2}} r dr J_0(k_F r)e^{-\sqrt{r^2+z^2}/a}$$

$$= \int_0^{R_{2n+2}} r dr J_0(k_F r)e^{-\sqrt{r^2+z^2}/a},$$

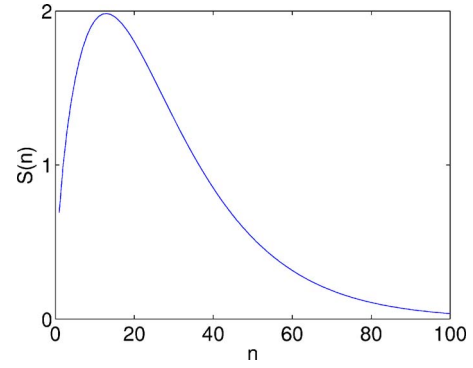where R_m/a is the mth zero of the Bessel function, $R_0=0$,

FIG. 6. (Color online)$S(n)$, the integral cutoff after $2n$ zeros of the Bessel function, $k_F a=100$, $z/a=1$.

will therefore have a maximum for some n, as shown in Fig. 6.

This maximal value of $S(n,z)$ varies with z as $e^{-z/a}$, and the corresponding cutoff radius $R^* \equiv R_{n_{max}}$ is a slowly varying function of the ration z/a, corresponding to 20–100 zeros of the Bessel function.

Based on this property we construct a simple model of a barrier with a blocklike disorder assuming

$$g(r) = 1 + \eta \Theta(R^* - r), \tag{17}$$

where $\Theta(r)$ is the Heaviside step function. Since this contribution only slowly decays with z, even a small barrier variation η can give significant contributions. Since the first term in Eq. (17) leads to a decay $\propto e^{-k_F z}$ while the second contribution decays $\propto e^{-z/a}$, we only need $\eta > e^{-(k_F - 1/a)z}$.

Figure 7 shows the optimal cutoff radius R^* as a function of z, as well as $I_r(R^*,z)$, compared to a graph showing $e^{-z/a}$.

Obviously, small variations in inhomogeneity size R will give large variations in the result; this can be remedied by smearing the cutoff over a period or two of the Bessel function. If we assume the center of the barrier reduction to be slightly displaced from the impurity center, the integration over angles can probably be found to be some such smeared step function. At some point, the smearing will be such that the cancellation is no longer lifted and the barrier minimum gives insignificant contribution.
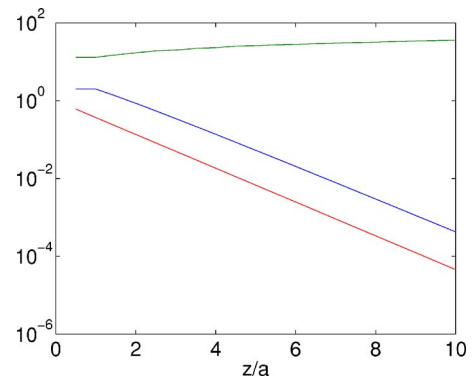

FIG. 7. (Color online)The functions $n_{max}(z)$ (top line) and $I_r(R^*,z)$ (middle line) for $k_F a=100$. The bottom line shows the function $e^{-z/a}$.

$$eV > 0 \quad eV = 0 \quad eV < 0 \qquad eV > 0 \quad eV = 0 \quad eV < 0$$
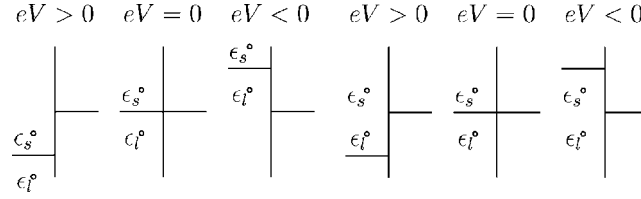

FIG. 8. Electrochemical potential in HI (left) and SC (right) outside (left panel) and inside (right panel) the depletion zone.

Thus we see that a small variation in the barrier thickness, as long as it is at the proper length scale and centered on the impurity center, can drastically reduce the interface resistance of the barrier.

V. NON-OHMIC CONDUCTANCE

Having established that there exists a range of applicability of the underlying model we now address the non-Ohmic behavior of the interface conductance. To figure out the nonlinear properties one has to compare eV with other relevant energy scales: temperature T, intersite Coulomb repulsion energy U_C, and energy splitting $\epsilon_s - \epsilon_l$ of a pair. In addition, we have to consider the width of the δ function in energy that selects which pairs may contribute. This width can be estimated as a typical inverse lifetime of an electron at a localized site forming the pair. We always assume $eV \ll \Delta$, excluding in this way the possibility of single-electron transport.

The net current over the barrier can be seen as the difference between current from the superconductor to the insulator, $I_{S \rightarrow I}$ and current from the insulator to the superconductor $I_{I \rightarrow S}$. These currents are in turn determined by the transition probability and the occupation probabilities for the involved states. Following Ref. 24 we will assume all matrix elements to be energy independent. Thus the only variations in the transition rate are due to the occupation probabilities. We must also remember the eV dependence of the δ function, describing the conservation of energy, which regulates which pairs contribute to the transport.

Since the superconducting condensate has a macroscopic number of states at the level of the chemical potential, the current $I_{S \rightarrow I}$ is only dependent on the probability of finding an empty pair in the insulator, while $I_{I \rightarrow S}$ requires an occupied pair. In both cases the relevant pair will have to satisfy energy conservation.

To keep track of realistic situations we assume that the entire voltage drop occurs at the barrier, but allow for a small portion of the insulator near the interface to be filled up or emptied by electrons due to the voltage drop over the barrier. This region models the depletion zone of a semiconductor heterojunction. For impurities outside the depletion zone, the Fermi level is assumed to be fixed relative to the impurity energy levels. In this case the δ function in energy must be chosen as $\delta(\epsilon_s + \epsilon_l + U_C - 2eV)$ and the occupancy numbers are given as before, as $n(\epsilon)$. Very close to the barrier, inside the depletion layer, we instead use a picture where we keep the impurity energy levels constant relative to the superconductor condensate, but adjust the Fermi level to get $n(\epsilon, eV) = n(\epsilon + eV)$ (see Fig. 8).

In both cases we have to consider the Coulomb energy U_C, and the simplest way of accommodating it is by describing each pair as a four-level system corresponding to the four following configurations: (i) both sites are empty, (ii), (iii) one site is occupied, and (iv) both sites are occupied. The configurations are shown in Fig. 9.

For simplicity we disregard Coulomb interaction with charges outside the pair. We can then write a partition function for the four-level system and use this to find the probabilities of a pair being empty, allowing a Cooper pair to fill it, or if filled, allowing one Cooper pair to be created. Using the energies defined in the figure, the partition function can be written as

$$Z = 1 + e^{-(\epsilon_s - \mu)/T} + e^{-(\epsilon_l - \mu)/T} + e^{-(\epsilon_s + \epsilon_l - 2\mu + U_C)/T}$$

and the probabilities of the different configurations in similar notation:

$$P_{00} = Z^{-1}, \quad P_{10} = Z^{-1} e^{-(\epsilon_s - \mu)/T},$$

$$P_{01} = Z^{-1} e^{-(\epsilon_l - \mu)/T}, \quad P_{11} = Z^{-1} e^{-(\epsilon_s + \epsilon_l - 2\mu + U_C)/T}.$$

Here μ is the chemical potential in the superconductor.

The current through the interface is proportional to the difference

$$P_{00} - P_{11} = Z^{-1}(e^{-(\epsilon_s + \epsilon_l - 2\mu + U_C)/T} - 1). \tag{18}$$

With the inclusion of the Coulomb energy this can no longer be factorized into separate occupation probabilities of the two impurities of the pair, but if we set $U_C = 0$, it can be seen that the difference simplifies to the former results.

If we consider the pairs inside the depletion zone, we use $\mu = -eV$. Taking into account the energy conservation law requiring $\epsilon_s + \epsilon_l + U_C = 0$ one can express the difference $P_{00} - P_{11}$ as

$$\frac{1 - e^{2eV/k_B T}}{1 + e^{(-\epsilon_s + eV)/k_B T} + e^{(\epsilon_s + U_C + eV)/k_B T} + e^{2eV/k_B T}}.$$

In this case, the energy conservation law is independent of eV, so the transitions are suppressed until the voltage reaches $|\epsilon_s| + U_C$; then, it rapidly rises, before $P_{00} - P_{11}$ saturates at unity. For the pairs outside the depletion zone, we can write $\mu = 0$ and the expression for $P_{00} - P_{11}$ is
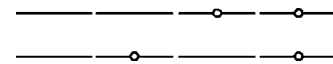

FIG. 9. Energy levels for four configurations of the pair: $\epsilon_1 = 0$, $\epsilon_{10} = \epsilon_s$, $\epsilon_{01} = \epsilon_l$, and $\epsilon_{11} = \epsilon_s + \epsilon_l + U_C$.
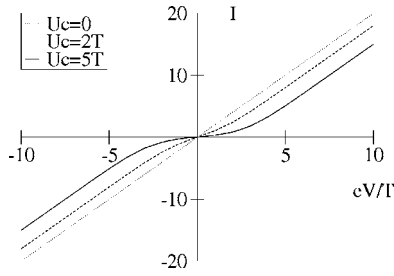
FIG. 10. Current-voltage curve for U_C/T=0, 1, and 2.

$$\frac{e^{2eV/T} - 1}{1 + e^{-\epsilon_s/T} + e^{(\epsilon_s + 2eV + U_C)/T} + e^{2eV/T}}.$$

While these expressions are somewhat similar, the main difference lies in the eV dependence of the conservation law, which means that as the voltage changes, the choice of pairs satisfying energy conservation will change, so a single pair will pass into and out of the allowed range instead of saturating.

In the following we assume the Fermi level in a semiconductor to be located inside the impurity band, which is sufficiently larger than $\Delta \gg eV$. Therefore, when performing the summation over all pairs, we choose a uniform distribution of ϵ_s. In this case, both expressions will give exactly the same results, although the physics behind them are slightly different. Neglecting the Coulomb interaction between the components of the pair, we simply get a linear relation in both cases. Thus Ohmic behavior would persist even though eV can exceed the temperature T. For pairs inside the depletion zone, this is due to inclusion and saturation of more pairs as eV becomes larger than the energy splitting of the pairs. If a given pair has started contributing, one channel has been opened and it will not terminate with increasing eV. Outside the depletion zone, pairs will only contribute for the width of the δ function, so as voltage rises, other pairs will take over the transport, but the number of pairs that can contribute will rise linearly with the voltage.

A typical *I-V* curve is shown in Fig. 10. This curve is calculated under a simplifying assumption that the Coulomb correlation energy U_C is kept constant of the same order as the temperature T since the contribution of the *sl* pair is $\propto \rho_{sl}^{-2} e^{-\rho_{sl}/\xi}$ and cut off at small ρ by the requirement $U_C \sim T$.[16] For these pairs the Coulomb interaction is essentially screened and does not block two-electron tunneling. This assumption significantly simplified the calculation compared with averaging over all correlation energies, but does not change the conclusion. As we see, the transport is suppressed at $eV \lesssim U_C$, while for $eV > U_C$ transport will return to a Ohmic behavior, as shown in Fig. 10.

Thus we predict a rather unusual situation when the conductance evolves for a low-field Ohmic to a high-field Ohmic regime through an intermediate non-Ohmic one. Such behavior is a hallmark of the Coulomb correlation between the occupation numbers of the pairs responsible for crossed Andreev reflections.

VI. CONCLUSIONS

In conclusion, we have shown that the pair tunneling through a barrier at the interface between a superconductor

and hopping insulator is extremely sensitive to the properties of the tunneling barrier. This sensitivity is due to rapid oscillations (at scale $\sim k_F^{-1}$) of the electron wave functions in a superconductor compared to the characteristic scale a of variation of the localized wave function in a hopping insulator. These oscillations cause dramatic cancellations in the tunneling probability if the barrier is uniform. However, this cancellation of the interference origin can be suppressed if the barrier is inhomogeneous, as was demonstrated for different models of a barrier. In particular, the fluctuations in the barrier strength of the scale λ falling within the window $1/k_F \ll \lambda < a$ suppress the cancellations and restore the transport even if their relative amplitude η is very small. For a barrier with a blocklike disorder with the scale $\sim a$ we obtained an estimate for suppression of the oscillations $\eta \gtrsim e^{-k_F a}$ for the pairs located at the distance $\sim a$ from the interface. This happens if the beneficial barrier fluctuations must coincide with particularly positioned impurities with the right energies and means that the number of impurities contributing to conductance will be relatively small and the relative importance of "successful" pairs will increase. Consequently, one can expect pronounced mesoscopic—sample-specific and reproducible—fluctuations in both Ohmic and non-Ohmic conductance. Such fluctuations will be especially pronounced when the barrier contains large transparency fluctuations (punctures). One can expect that the fluctuations will have different behavior depending on the location of the relevant pairs with respect to the position of the depletion zone near the interface. We plan to study mesoscopic fluctuations of the Andreev transport between a superconductor and a hopping insulator in more detail as a separate project.

Another specific feature of the two-electron charge transfer is sensitivity of non-Ohmic transport to Coulomb correlation in the occupation numbers of the relevant pairs. This correlation leads to non-Ohmic behavior at *low* voltages, $eV \lesssim U_C, T$, while at higher voltages the transport turns out to be Ohmic. This reentrant behavior is a hallmark of the Coulomb correlation.

The experimental relevance of these calculations is mainly for the study of hopping systems connected by superconducting leads. In such experiments the temperature is often less than 1 K and quasiparticle conduction is exponentially suppressed. So while we get a result proportional to $|T_0|^4$, this may still easily be greater than the quasiparticle contribution $\propto |T_0|^2 e^{-\Delta/T}$. If we also include cancellations from a perfect barrier, we get an additional factor of the order of $e^{-k_F a}$. Under such conditions quasiparticle conduction may dominate over Andreev transport down to temperatures $T \lesssim \Delta/(\ln T_0^{-1} + k_F a)$, ignoring preexponential factors. The roughness of the barrier makes Andreev transport more favorable and decreases the interface resistance.

[1] G. E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **25**, 4515 (1982).

[2] I. Giaever, Phys. Rev. Lett. **5**, 147 (1960).

[3] B. J. van Wees, P. de Vries, P. Magnee, and T. M. Klapwijk, Phys. Rev. Lett. **69**, 510 (1992).

[4] C. W. J. Beenakker, Phys. Rev. B **46**, 12841 (1992); see Rev. Mod. Phys. **69**, 731 (1997) for a review.

[5] A. F. Volkov, Phys. Lett. A **174**, 144 (1993).

[6] A. Kastalsky, A. W. Kleinsasser, L. H. Greene, R. Bhat, F. P. Milliken, and J. P. Harbison, Phys. Rev. Lett. **67**, 3026 (1991).

[7] H. Takayanagi and T. Kawakami, Phys. Rev. Lett. **54**, 2449 (1985).

[8] Nishino, M. Miyake, Y. Harada, and U. Kawabe, IEEE Electron Device Lett. **EDL-6**, 297 (1985).

[9] Z. Ivanov and T. Claeson, Jpn. J. Appl. Phys., Part 1 **26**, Suppl. 26-3, 1617 (1987).

[10] A. W. Kleinsasser, T. N. Jackson, D. McInturff, F. Rammo, G. D. Pettit, and J. M. Woodall, Appl. Phys. Lett. **55**, 1909 (1989).

[11] J. Nitta, T. Akazaki, H. Takayanagi, and K. Arai, Phys. Rev. B **46**, 14286 (1992).

[12] J. Nitta, T. Akazaki, H. Takayanagi, and K. Arai, Phys. Rev. B **46**, 14286 (1992).

[13] J. Nitta, T. Akazaki, and H. Takayanagi, Phys. Rev. B **49**, 3659 (1994).

[14] T. Akazaki, H. Takayanagi, and J. Nitta, Appl. Phys. Lett. **68**, 418 (1996).

[15] N. V. Agrinskaya, V. I. Kozub, and R. Rentzsch, JETP **84**, 814 (1997).

[16] V. I. Kozub, A. A. Zyuzin, Y. M. Galperin, and V. Vinokur, Phys. Rev. Lett. **96**, 107004 (2006).

[17] G. Deutsher and D. Feinberg, Appl. Phys. Lett. **76**, 487 (2000); G. Falci, D. Feinberg, and F. W. J. Hekking, Europhys. Lett. **54**, 255 (2001); D. Feinberg, Eur. Phys. J. B **36**, 419 (2003); S. Russo, M. Kroug, T. M. Klapwijk, and A. F. Morpurgo, Phys. Rev. Lett. **95**, 027002 (2005).

[18] P. Recher, E. V. Sukhorukov, and D. Loss, Phys. Rev. B **63**, 165314 (2001); O. Sauret, T. Martin, and D. Feinberg, *ibid.* **72**, 024544 (2005).

[19] L. Faoro, J. Bergli, B. L. Altshuler, and Y. M. Galperin, Phys. Rev. Lett. **95**, 046805 (2005); A. Grishin, I. V. Yurkevich, and I. V. Lerner, Phys. Rev. B **72**, 060509(R) (2005).

[20] R. Kubo, J. Phys. Soc. Jpn. **12**, 570 (1957).

[21] J. Bardeen, Phys. Rev. Lett. **6**, 57 (1961).

[22] A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinskii, *Quantum Field Theoretical Methods in Statistical Physics* (Pergamon Press, New York, 1965).

[23] A. P. Prudnikov, Yu. A. Brychkov, and O. I. Marichev, *Integrals and Series* (Gordon and Breach, New York, 1986) (Eq. 2.12.10.7).

[24] Y. M. Galperin, V. L. Gurevich, and D. A. Parshin, Zh. Eksp. Teor. Fiz. **85**, 1757 (1983) [Sov. Phys. JETP **58**, 1023 (1983)].