Josephson plasmon versus amplitude modes in a superconducting tunnel junction

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The mutual interaction between Josephson plasmon and energy-gap amplitude collective oscillations in a SIS junction is considered where S denotes a superconducting electrode and I is a dielectric barrier. Using a simple tunneling model we find that both modes can be stable for a range of magnitudes of the tunneling interaction and Cooper coupling strength.

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Renewed interest in collective oscillations in Josephson junctions has been generated by recent progress in quantum information technology.[1–6] Quantization of the Josephson plasmon in SIS junctions (where S is a superconductor and I is a dielectric barrier) has been successfully used in single and double quantum logic gates.[1–6] Several types of collective oscillations have been studied to date. The Josephson plasmon (JP) has attracted the most attention due to its utilization in various superconducting devices. The JP involves oscillations of the Josephson supercurrent about its equilibrium value. A simple classical analogy is a pendulum with a mass $m=C(\hbar/2e)^2$ and a rigidity $k=\hbar J_c/2e$; here C is the junction capacitance and J_c is the critical current. In such a pendulum, the phase difference φ across the junction serves as a coordinate, while the velocity $\dot{\varphi}$ is related to the charge $Q=C\hbar\dot{\varphi}/2e$ accumulated on the junction electrodes; here we used the Josephson relationship $V=\hbar\dot{\varphi}/2e$ relating the voltage V and φ. The JP pendulum frequency is then $\omega_{\mathrm{JP}}=\sqrt{2eJ_c/\hbar C}$.

Other types of collective oscillations are associated with the phase and amplitude of the superconducting energy gap $\Delta=|\Delta|e^{i\Phi}$. In a bulk BCS superconductor, two modes have been discussed:[7] one involves a small oscillation $\varphi(t)$ of the phase, $\Phi(t)=\Phi_0+\varphi(t)$, while the other is an oscillation $\delta(t)$ of the gap amplitude, $|\Delta(t)|=\Delta_0+\delta(t)$; here Φ_0 and Δ_0 denote the equilibrium values and t is the time. The first of these modes is referred to as the Anderson-Bogoliubov (AB) mode,[8,9] and would occur at the electron plasma frequency $\omega_{\mathrm{p}}=\sqrt{4\pi ne^2/(\varepsilon_0 m^*)}$ (where n is the electron concentration, m^* is the electron effective mass; typically $\omega_{\mathrm{p}}/\varepsilon_{\mathrm{F}}\approx 10$ and $\Delta/\varepsilon_{\mathrm{F}}\approx 10^{-3}-10^{-4}$ for a Fermi energy $\varepsilon_{\mathrm{F}}\approx 1$ eV); this property results from the fact that an oscillating phase necessarily produces an oscillating current which in turn creates an oscillating charge density.[10] The second mode, which was examined by Littlewood and Varma[11] (LV), is found to lie at or above the absorption threshold $2\Delta/\hbar$, and hence is strongly damped. In previous work,[8–14] the AB, LV, and JP modes were discussed independently of each other, and their interrelationship was largely ignored.

In the SIS Josephson junction sketched in Fig. 1 one would expect both symmetric (+) and antisymmetric (−) AB and LV modes, $\omega^{\pm}_{\mathrm{AB}}$ and $\omega^{\pm}_{\mathrm{LV}}$. The branches ω^{+}_{AB} and $\omega^{\pm}_{\mathrm{LV}}$ lie near ω_{p} and 2Δ, respectively. In low-transparency junctions, the $\omega^{\pm}_{\mathrm{LV}}$ modes are strongly damped,[14] while the ω^{-}_{AB} mode is typically stable (since $\omega^{-}_{\mathrm{AB}}<2\Delta$) and actually coincides with the aforementioned JP mode, i.e., $\omega^{-}_{\mathrm{AB}}=\omega_{\mathrm{JP}}=\sqrt{2eJ_c/\hbar C}$. In general, all the relevant modes (i.e., $\omega^{\pm}_{\mathrm{LV}}$ and ω^{-}_{AB}) interact with each other, which means that the Josephson supercurrent oscillations are hybridized with oscillations involving elementary Cooper pair processes.

In this paper we study how the above AB, LV, and JP modes are related to each other. Using a model approach we derive analytical equations that describe the interrelationship between the Josephson plasmon, AB, and LV modes. Our earlier calculations[15] addressed collective modes using a different quasiclassical approach considering more complex SISIS junction. Here we consider a simpler SIS junction using a many-body method. It provides illustrative analytical relationships between different branches of the collective modes in the SIS junction. Finally we will discuss the possibility of experimentally observing the collective oscillations, e.g., in a tunneling experiment.

Our approach is based on the well-known tunneling model[14–18] from which we obtain the collective oscillations (CO) and their mutual interaction. Along with terms considered in Ref. 14 we include additional contributions from the Coulomb charging $\hat{W}_{\mathrm{Q}}$ and Josephson $\hat{W}_{\mathrm{J}}$ energies of the junction.[19] We will see that the terms $\hat{W}_{\mathrm{Q}}$ and $\hat{W}_{\mathrm{J}}$ have a different symmetry structure as compared to the formerly studied case.[14] The charging energy originates from a finite capacitance $C\neq 0$ of the Josephson junction, which was not accounted for in the former work. This allows a more consistent study of both the phase and amplitude oscillations of

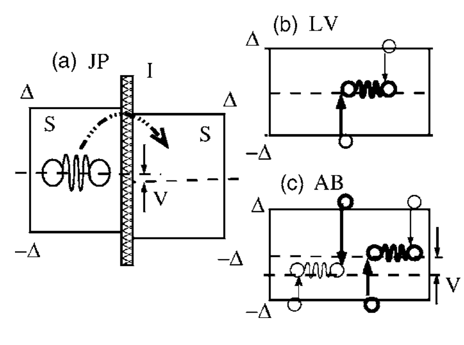

FIG. 1. Schematic of collective oscillations in a SIS junction. (a) The Josephson plasmon (JP) results from oscillations of the supercurrent across the SIS junction, which are accompanied by small voltage oscillations $V=(\hbar/2e)\dot{\varphi}$. (b) The LV amplitude mode in a bulk superconductor. (c) The AB phase mode in a bulk superconductor. Out-of-phase processes and particles are shown by thinner lines.

the energy gap Δ. Although the tunneling model is applicable in the low-transparency limit, it allows a concise and systematic delineation of the CO branches.

We start by writing the total effective Hamiltonian in the form $\tilde{H}=\hat{H}-\mu\hat{N}$, where

$$\hat{H}=\hat{H}_0+\hat{H}_{\text{int}}. \tag{1}$$

The first term is given by

$$\hat{H}_0=\sum_{\mathbf{p},\sigma,i}\varepsilon_{\mathbf{p}}\hat{a}^{\dagger}_{i,\mathbf{p},\sigma}\hat{a}_{i,\mathbf{p},\sigma}+\sum_{\mathbf{p},\sigma,i\neq j}\bar{t}_{\mathbf{p}}\hat{a}^{\dagger}_{i,\mathbf{p},\sigma}\hat{a}_{j,\mathbf{p},\sigma}, \tag{2}$$

where $\varepsilon_{\mathbf{p}}$ is the kinetic energy of an electron with momentum $\mathbf{p}$ and $\bar{t}_{\mathbf{p}}$ is the tunneling matrix element between the two superconducting electrodes denoted by i and j. The interaction term involves three contributions that we write as

$$\hat{H}_{\text{int}}=\hat{H}_{\text{C}}+\hat{H}_{\text{T}}+\hat{W}; \tag{3}$$

the first two terms represent the Cooper coupling and tunneling interaction, while the third term, which we write as

$$\hat{W}=\hat{W}_{\text{Q}}+\hat{W}_{\text{J}}, \tag{4}$$

accounts for the charge on, and the current flowing between, the junction electrodes. $\hat{H}_{\text{C}}$ and $\hat{H}_{\text{T}}$ can be written in the form

$$\hat{H}_{\text{C}}=-g\sum_{\mathbf{q},i,\sigma}(\langle\hat{\Phi}^{\dagger}_{i,\mathbf{q},\sigma}\rangle\hat{\Phi}_{i,\mathbf{q},\sigma}+\langle\hat{A}^{\dagger}_{i,\mathbf{q},\sigma}\rangle\hat{A}_{i,\mathbf{q},\sigma}), \tag{5}$$

$$\hat{H}_{\text{T}}=T_{\text{J}}\sum_{\mathbf{q},i\neq j,\sigma}(\langle\hat{\Phi}^{\dagger}_{i,\mathbf{q},\sigma}\rangle\hat{\Phi}_{j,\mathbf{q},\sigma}+\langle\hat{A}^{\dagger}_{i,\mathbf{q},\sigma}\rangle\hat{A}_{j,\mathbf{q},\sigma}), \tag{6}$$

where $i,j=\overline{1,2}$, $g=|\alpha_{\mathbf{q}}|^2$, $\alpha_{\mathbf{q}}$ is the electron-phonon coupling matrix element, $T_{\text{J}}=|\bar{t}_{\mathbf{p}}|^2$, and we defined the operators

$$\hat{\Phi}_{i,\mathbf{q},\sigma}=\sum_{\mathbf{p}}(\hat{a}^{\dagger}_{i,\mathbf{p},\sigma}\hat{a}^{\dagger}_{i,-\mathbf{p}-\mathbf{q},-\sigma}-\hat{a}_{i,-\mathbf{p}+\mathbf{q},-\sigma}\hat{a}_{i,\mathbf{p},\sigma})/\sqrt{2},$$

$$\hat{A}_{i,\mathbf{q},\sigma}=\sum_{\mathbf{p}}(\hat{a}^{\dagger}_{i,\mathbf{p},\sigma}\hat{a}^{\dagger}_{i,-\mathbf{p}-\mathbf{q},-\sigma}+\hat{a}_{i,-\mathbf{p}+\mathbf{q},-\sigma}\hat{a}_{i,\mathbf{p},\sigma})/\sqrt{2}, \tag{7}$$

which are associated with the phase and amplitude of the order parameter, respectively. The Coulomb and Josephson energies $\hat{W}_{\text{Q}}$ and $\hat{W}_{\text{J}}$ of the SIS junction in the pendulum approximation[19] are expressed through the operators (7)

$$\hat{W}_{\text{Q}}=\frac{m}{2}\sum_{\mathbf{q},\sigma}\langle\dot{\hat{\phi}}_{\mathbf{q},\sigma}\rangle\dot{\hat{\phi}}_{\mathbf{q},\sigma},\quad \hat{W}_{\text{J}}=\frac{k}{2}\sum_{\mathbf{q},\sigma}\langle\hat{\phi}_{\mathbf{q},\sigma}\rangle\hat{\phi}_{\mathbf{q},\sigma}, \tag{8}$$

where $\hat{\phi}_{\mathbf{q},\sigma}=\hat{\Phi}_{i,\mathbf{q},\sigma}-\hat{\Phi}_{j,\mathbf{q},\sigma}$ and $\hat{\alpha}_{\mathbf{q},\sigma}=\hat{A}_{i,\mathbf{q},\sigma}-\hat{A}_{j,\mathbf{q},\sigma}$ are operators of the phase and gap amplitude difference across the SIS junction, $m=C(\hbar/2e)^2$ is the mass, and $k=\hbar J_c/2e$ is the rigidity. The quantum mechanical operator $\dot{\hat{\phi}}_{\mathbf{q},\sigma}$ is related to the electric charge operator $\hat{Q}$ through the Josephson relationship $\dot{\hat{\phi}}_{\mathbf{q},\sigma}=(2e/\hbar)\hat{Q}/C$. The charge operator $\hat{Q}$ generally does not commute with the phase difference operator $\hat{\phi}_{\mathbf{q},\sigma}$, i.e., $[\hat{Q},\hat{\phi}_{\mathbf{q},\sigma}]=2ei$. However, we are interested in the Josephson plasmon with the lowest excitation energy, when one can use $\dot{\hat{\phi}}_{\mathbf{q},\sigma}\to i\omega\hat{\phi}_{\mathbf{q},\sigma}$. Although all the interlayer coupling terms (6) and (8) originate from the tunneling interaction [second term in Eq. (2)], they apparently have a different meaning and structure; namely, Eq. (6) describes the coupling between two linear phase oscillations in adjacent layers while Eq. (8) is a nonlinear part of energy of the Josephson SIS junction, which depends on the interlayer phase difference. In this way Eq. (8) describes a pendulum with a kinetic energy $\hat{W}_Q$ and potential energy $\hat{W}_J$.

We calculate the collective mode spectrum by locating the poles of the susceptibility $\hat{\chi}^{(ij)}(\mathbf{q},i\Omega_n)$ characterizing the linear response of the system.[14,20] This susceptibility is given by the expression

$$\hat{\chi}^{(ij)}(\mathbf{q},i\Omega_n)=-\int_0^{\beta}d\tau\,e^{i\Omega_n\tau}\langle\hat{T}_{\tau}\hat{O}^{(i)}_{\mathbf{q}}(\tau)\hat{O}^{\dagger(j)}_{\mathbf{q}}(0)\rangle, \tag{9}$$

where $\hat{O}_{\mathbf{q}}$ is the four-component vector

$$\hat{O}_{\mathbf{q}}=(\hat{\Phi}_{i,\mathbf{q},\sigma},\hat{\Phi}_{j,\mathbf{q},\sigma},\hat{A}_{i,\mathbf{q},\sigma},\hat{A}_{j,\mathbf{q},\sigma})$$

and $\hat{T}_{\tau}$ is the time ordering operator. In the random phase approximation this susceptibility is given by

$$\hat{\chi}=\hat{\chi}^{(0)}-\hat{\chi}^{(0)}\hat{\mathcal{V}}\hat{\chi}=(\hat{I}+\hat{\chi}^{(0)}\hat{\mathcal{V}})^{-1}\hat{\chi}_0. \tag{10}$$

The seqular equation for the eigenfrequencies $\omega_{\mathbf{q}}$ takes the form

$$\det(\hat{I}+\hat{\chi}^{(0)}\hat{\mathcal{V}})=0; \tag{11}$$

here $\hat{\chi}^{(0)}=-\hat{1}B^{(+)}\oplus\hat{1}B^{(-)}$ is the susceptibility of a noninteracting system, $\hat{1}$ is a unit 2×2 matrix, $\oplus$ means the direct matrix sum,[22] and the auxiliary function $B^{\pm}_{\mathbf{q},n}$ is defined as

$$B^{\pm}_{\mathbf{q},n}=-\int\frac{d\mathbf{p}}{(2\pi)^2}\frac{E+E'}{2EE'}\frac{(EE'+\varepsilon\varepsilon'\pm\Delta^2)}{(i\Omega_n)^2-(E+E')^2}, \tag{12}$$

where Ω_n is the Bose Matsubara frequency, $E=E_{\mathbf{p}}$, $E'=E_{\mathbf{p}+\mathbf{q}}$, and $\varepsilon\equiv\varepsilon_{\mathbf{p}}$, $\varepsilon'\equiv\varepsilon_{\mathbf{p}+\mathbf{q}}$. The interaction $\hat{\mathcal{V}}$ has the form

$$\hat{\mathcal{V}}=[(g+\omega^2m/2-k/2)\hat{1}+(k/2-\omega^2m/2-T_{\text{J}})\hat{\tau}_1]$$
$$\oplus(g\hat{1}-T_{\text{J}}\hat{\tau}_1).$$

The function $B^{+}_{\mathbf{q},\omega}$ in Eq. (12) can be rewritten as

$$B^{+}_{\mathbf{q},\omega}=f-Q(\mathbf{q},\omega), \tag{13}$$

where

$$f=\sum_{\mathbf{p}}^{\omega_c}\frac{1}{E_{\mathbf{p}}}\tanh\frac{E_{\mathbf{p}}}{2T}, \tag{14}$$

which in equilibrium satisfies $1-gf=0$,

$$Q_{\mathbf{q},\omega}=\int\frac{d\mathbf{p}}{(2\pi)^2}\frac{E+E'}{4EE'}\frac{\omega^2-(\varepsilon'-\varepsilon)^2}{\omega^2-(E+E')^2}, \tag{15}$$

and ω_c is the cutoff frequency. Equation (11) for the eigenfrequencies yields the following algebraic equations for the collective modes:

$$D'=0=(1-B_{-}g)^2-B_{-}^2T_{\text{J}}^2 \quad \text{(LV mode)},$$

$$D'' = 0 = (1 - B_+ g)^2 - B_+^2 T_J^2 + B_+(1 - B_+ g + B_+ T_J)(k - m\omega^2) \quad \text{(phase mode)},$$

which in the limit $q \to 0$ yields

$$Q(q,\omega) = N(0)\mathcal{I}(\omega)(\omega^2 - v_F^2 q^2/2)/(2\Delta)^2; \tag{16}$$

here $\mathcal{I}(\omega) = (4\tilde{\omega}\tilde{\Omega})^{-1}\ln[(\tilde{\Omega}+\tilde{\omega})/(\tilde{\Omega}-\tilde{\omega})]$, $\tilde{\omega} = \omega/2\Delta$, and $\tilde{\Omega} = \sqrt{\tilde{\omega}^2 - 1}$. The auxiliary function $\mathcal{I}(\omega)$ has a threshold at $\tilde{\omega} = 1$ ($\omega = 2\Delta$) related to pair breaking.

The function $B_-(\mathbf{q},\omega)$ is rewritten as

$$B_-(\mathbf{q},\omega) = f - P(\mathbf{q},\omega), \tag{17}$$

where

$$P(\mathbf{q},\omega) = \int \frac{d\mathbf{p}}{(2\pi)^2} \frac{E+E'}{4EE'} \frac{\omega^2 - (\varepsilon' - \varepsilon)^2 + (2\Delta)^2}{\omega^2 - (E+E')^2} \simeq N(0)\mathcal{I}(\omega)[\omega^2 - v_F^2 q^2/2 - (2\Delta)^2]/(2\Delta)^2. \tag{18}$$

If $T_J = 0$, one obtains $D'' = 0 = 1 - B_- g$, which gives the dispersion relation for the LV mode as $\omega^2 - v_F^2 q^2/2 - (2\Delta)^2 = 0$ where v_F is the Fermi velocity. For $T_J \neq 0$, approximating $\mathcal{I}(\omega) = -\theta(2\Delta - \omega)$ one gets for the $LV_\pm$ modes

$$\omega_{LV\pm}^2 = (2\Delta)^2\left(1 - \frac{1}{\lambda}\frac{D/\lambda}{D/\lambda \mp 1}\right) + \frac{1}{2}v_F^2 q^2, \tag{19}$$

where D is the junction transparency, and $\lambda = gN(0)$ with $N(0)$ the density of states. In the same approximation the equation for the AB_- mode (JP) is obtained as

$$\frac{m}{g}(\omega_{JP}^2 - \omega^2) = \frac{\lambda}{(2\Delta)^2}\left(\omega^2 - \frac{1}{2}v_F^2 q^2\right)\left(\frac{m}{g}(\omega_{JP}^2 - \omega^2) - 1\right).$$

Solving this equation one obtains[21]

$$\omega_{1,2} = \pm\sqrt{\Omega^2 \pm \sqrt{\Omega^4 - B_-}}, \tag{20}$$

where $\Omega^2 = q^2 v_F^2/4 + 2\Delta^2/\lambda + \omega_{JP}^2/2 - g/2m$ and $B_- = \omega_{JP}^2(4\Delta^2/\lambda + q^2 v_F^2/2) - gq^2 v_F^2/(2m)$. From Eq. (20) one can see that in the limit $C \to \infty$ (i.e., for large-area junctions) and when $q \to 0$ only the LV_- mode remains [which depends only on g and $\lambda = N(0)g$] while the JP mode vanishes since $\omega_{JP} = 0$.

In Fig. 2(a) we plot $\omega(q)$ versus the coupling strength g. Here we used the following parameters: $v_F = 1$ (in units of $\pi\Delta\xi_0/\hbar$, ξ_0 being the BCS coherence length), $\Delta = 1 + i0.05$ (where the imaginary part models an inelastic scattering rate and is introduced to smooth the singularities), $q = 0.2$, $k = 0.7$, $m = 2$, $T_J = 0.4$, and $\lambda = 0.3g$. One can see that the amplitude modes are strongly affected by the pairing interaction and change significantly as g increases. The phase mode ω_{AB}^+ is shown for the hypothetical case of a neutral superconductor; it moves to very high frequencies ($\sim$10 eV in metals) when intralayer Coulomb repulsion is taken into account.

In Fig. 2(b) we show the dispersion law for $\omega_{AB}^-(q)$ (which coincides with the Josephson plasmon) for the same parameters as used in Fig. 2(a) for $g = 1$ and 2. One can see that the wave vector dependence of the Josephson plasmon, $\omega_{AB}^-(q)$, is very weak.[21]

In Fig. 2(c) we plot the JP and LV_- branches for

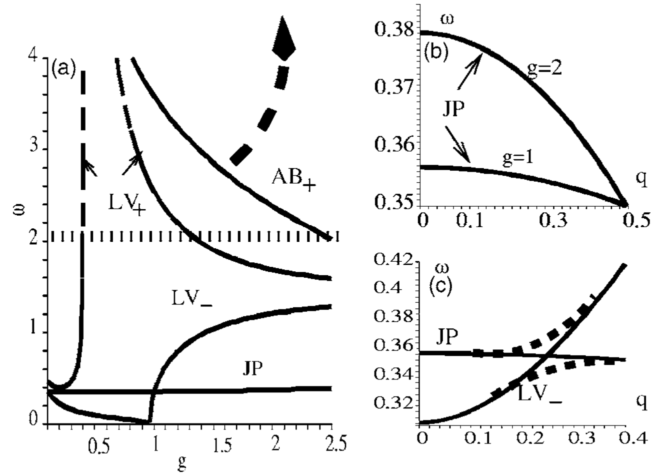

FIG. 2. Mode frequencies and their dispersion for the phase and amplitude modes. (a) The mode frequencies $\omega_{AB}^\pm$ and $\omega_{LV}^\pm$ versus the electron-phonon interaction strength g. The AB_- mode corresponds to the JP. The dotted line denotes the absorption threshold by quasiparticles at $\omega \geq 2\Delta/\hbar$. (b) The Josephson plasmon frequency versus the wave vector. (c) Schematic representation of the coupling between the phase and amplitude modes.

$\hbar\omega_{JP}/\Delta = 0.3$ with the remaining parameters the same as for the $g = 1$ curve in Fig. 2(b). The two branches JP and LV_- cross each other, but are expected to split [as depicted by the dashed curves in Fig. 2(c)] in the vicinity of the mode crossing if a bias dc supercurrent $j \neq 0$ is applied across the SIS junction. The splitting gap is evaluated as $\Delta_{spl} \simeq \iota(\iota^2 + 1)^{-1}(1 - \iota^2)^{-1/2}\omega_{JP}$ (where $\iota = j/j_c$, and j_c is the critical supercurrent).

In Fig. 3 we plot the $\chi_{11}(\omega)$ (which corresponds to the phase oscillations $\omega_{AB}^\mp$) and $\chi_{44}(\omega)$ (which is related to amplitude oscillations $\omega_{LV}^\pm$) components of the dynamic susceptibility for $q = 0.3$. Note that curve 1 [where $g = 1$ and other parameters the same as for the LV_+ curve in Fig. 2(a)] exhibits a sharp Josephson plasmon peak at $\tilde{\omega} = 0.37$ and a wider peak at $\tilde{\omega} = 1.42$ corresponding to phase oscillations. Curve 2 was plotted for a stronger electron-phonon interaction strength, $g = 2$. The upper peak is shifted to a higher frequency $\tilde{\omega} = 1.8$ and broadens. We emphasize that the upper peaks are for a neutral superconductor only, and, as noted above, are shifted to very high frequencies as the Coulomb repulsion is turned on. Curves 3 and 4 show $\chi_{44}(\omega)$ for $g = 1$ and 2 and are related to amplitude oscillations. One can see that a strong pairing interaction destroys the amplitude mode ω_{LV}^-.

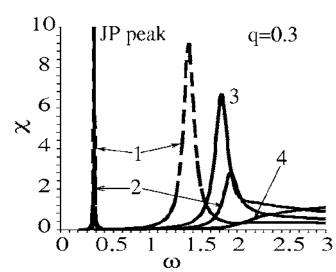

FIG. 3. The susceptibility function for the phase and amplitude oscillations.

The collective modes discussed in this paper can be detected by measuring the real and imaginary parts of the complex ac impedance $Y_{\mathrm{ac}}(\omega)$ versus frequency ω in SIS tunnel junctions. The functions $\mathrm{Re}\{Y_{\mathrm{ac}}(\omega)\}$ and $\mathrm{Im}\{Y_{\mathrm{ac}}(\omega)\}$ are out of phase with each other and are determined by the ac Josephson and quasiparticle currents. The LV and AB modes can be further distinguished by applying a dc magnetic field parallel to the junction. The interplay between different types of collective modes can be studied in SIS junctions having a relatively high transparency ($D \simeq 10^{-3}-10^{-2}$, for which the tunneling approximation still works) at low temperatures. For the case of Nb-I-Nb junctions with $\lambda = 0.82$ and $v_{\mathrm{F}} = 1.37 \times 10^{8}$ cm/s, one finds a 1% decrease of $\omega_{\mathrm{LV}\pm}$ below the absorption threshold $2\Delta/\hbar$ already at transparency $D \sim 10^{-2}$. The effect is even stronger in Sn-I-Sn junctions where λ is smaller. A different tendency takes place for the AB$_-$ mode ω_{JP}, which is shifted by $\sim \sqrt{2\Delta\omega_{\mathrm{JP}}/(\hbar\sqrt{\lambda})}$. Since the energy gap Δ is essentially smaller in Sn than in Nb, the shift of ω_{JP} in Sn-I-Sn junctions is also less significant. We conclude that the collective oscillations of the superconducting gap amplitude and phase in "clean" Josephson junctions may be long lived for appropriate values D and λ.

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[1] A. J. Berkley, H. Xu, R. C. Ramos, M. A. Gubrud, F. W. Strauch, P. R. Johnson, J. R. Anderson, A. J. Dragt, C. J. Lobb, and F. C. Wellstood, Science **300**, 1548 (2003).
[2] Yu. A. Pashkin *et al.*, Nature (London) **421**, 823 (2003).
[3] Y. Nakamura, Yu. A. Pashkin, and J. S. Tsai, Nature (London) **398**, 786 (1999).
[4] D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. H. Devoret, Science **296**, 286 (2002).
[5] J. R. Friedman *et al.*, Nature (London) **406**, 43 (2000).
[6] C. H. van der Wal *et al.*, Science **290**, 773 (2000).
[7] J. B. Ketterson and S. N. Song, *Superconductivity* (Cambridge University Press, Cambridge, UK, 1998).
[8] P. W. Anderson, Phys. Rev. **112**, 1900 (1958).
[9] N. N. Bogoliubov, V. V. Tolmachev, and D. V. Shirkov, *A New Method in the Theory of Superconductivity* (Consultants Bureau, New York, 1959).
[10] A. Bardasis and J. R. Schrieffer, Phys. Rev. **121**, 1050 (1961).
[11] P. B. Littlewood and C. M. Varma, Phys. Rev. B **26**, 4883 (1982).
[12] A. J. Leggett, Prog. Theor. Phys. **36**, 161 (1966).
[13] L. N. Bulaevskii, Sov. Phys. JETP **37**, 1133 (1973); L. N. Bulaevskii, M. P. Maley, and M. Tachiki, Phys. Rev. Lett. **74**, 801 (1995); L. N. Bulaevskii, V. L. Pokrovsky, and M. P. Maley, *ibid.* **76**, 1719 (1996).
[14] W. C. Wu and A. Griffin, Phys. Rev. Lett. **74**, 158 (1995); Phys. Rev. B **51**, 15317 (1995); **52**, 7742 (1995).
[15] S. E. Shafranjuk and J. B. Ketterson, Phys. Rev. B **72**, 212506 (2005).
[16] S. E. Shafranjuk, M. Tachiki, and T. Yamashita, Phys. Rev. B **53**, 15136 (1996); **55**, 8425 (1997); **57**, 582 (1998).
[17] M. H. Cohen, L. M. Falicov, and J. C. Philips, Phys. Rev. Lett. **8**, 316 (1962).
[18] W. L. McMillan, Phys. Rev. **138**, A442 (1965).
[19] K. K. Likharev, *Dynamics of Josephson Junctions and Circuits* (Gordon & Breach, New York, 1986).
[20] L. P. Kadanoff and I. I. Falko, Phys. Rev. **136**, A1170 (1964).
[21] Here we ignore the q dependence associated with coupling to the electromagnetic degrees of freedom, which leads to propagation at the Swihard velocity at short wavelengths.
[22] G. A. Korn and T. M. Korn, *Mathematical Handbook for Scientists and Engineers* (McGraw-Hill, New York, 1968).