Control of spin relaxation in semiconductor double quantum dots

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We propose a scheme to manipulate the spin relaxation in vertically coupled semiconductor double quantum dots. Up to 12 orders of magnitude variation of the spin relaxation time can be achieved by a small gate voltage applied vertically on the double dot. Effects of different parameters such as the dot size, barrier height, interdot distance, and magnetic field on the spin relaxation are investigated in detail. The conditions to achieve a large variation is addressed.

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I. INTRODUCTION

Spin-related phenomena in semiconductor nanostructures have attracted much interest recently due to the fast growing field of spintronics.¹ Among different structures, quantum dots (QDs) have attracted a lot of attention as they provide a versatile system to manipulate the spin and/or the electronic states.² Many proposals for spin qubits, spin filters, spin pumps, and spin quantum gates have been proposed and/or demonstrated based on different kinds of QDs.²⁻¹¹ Manipulation and understanding of the spin coherence in QDs are of great importance in the design and the operation of these spin devices. There are many theoretical and experimental investigations on the spin relaxation in single QDs,12-18 double QDs,19,20 and quasi-one-dimensional coupled QDs,^{21,22} due to the Dresselhaus²³ or Rashba²⁴ spin-orbit couplings. In this paper, we propose a feasible and convenient way to manipulate the spin relaxation in double QDs by a small gate voltage. We show that up to 12 orders of magnitude variation of the longitudinal spin relaxation time (SRT) can be tuned in such a system.

II. MODEL AND FORMULISM

We consider a single electron spin in two vertically coupled QDs. Each QD is confined by a parabolic potential $V_c(\mathbf{r}) = \frac{1}{2}m^*\omega_0^2\mathbf{r}^2$ (therefore the effective dot diameter $d_0 = \sqrt{\hbar \pi/m^*\omega_0}$) along the *x*-*y* plane in a quantum well of width *d* with its growth direction along the *z* axis. A gate voltage V_d together with a magnetic field *B* are applied along the growth direction. A schematic of the potential of the coupled quantum wells is plotted in the inset of Fig. 1(a) and the potential is given by²⁵

$$V_{z}(z) = \begin{cases} eEz + \frac{1}{2}eV_{d}, & \frac{1}{2}a < |z| < \frac{1}{2}a + d, \\ eEz + \frac{1}{2}eV_{d} + V_{0}, & |z| \leq \frac{1}{2}a, \\ \infty & \text{otherwise,} \end{cases}$$
(1)

in which V_0 represents the barrier height between the two coupled QDs, *a* is the barrier width, and $E=V_d/(a+2d)$ denotes the electric field due to the gate voltage. The origin

of the z axis is chosen to be the center of the barrier between the two QDs. By solving the Schrödinger equations along the z axis $d^2\psi_z/d\xi_i^2 - \xi_i\psi_z = 0$ with $\xi_1 = 2^{1/3}(m^*/\hbar^2e^2E^2)^{1/3}(eEz - \varepsilon + eV_d/2)$ for $\frac{1}{2}a < |z| < \frac{1}{2}a + d$ and $\xi_2 = 2^{1/3}(m^*/\hbar^2e^2E^2)^{1/3}(eEz - \varepsilon + eV_d/2 + V_0)$ for $|z| \leq \frac{1}{2}a$, one obtains the wave function



FIG. 1. (Color online) (a) SRT vs the electric field. Solid curve, perturbation result; dotted curve, exact diagonalization result. Inset: Schematic of the potential along the vertical (z) direction. (b) Upper panel: Weighted scattering rates $\overline{\Gamma}_{i \rightarrow j}$ between different energy levels (from "spin up" to "spin down") vs the electric field. $\overline{\Gamma}_{total}$ is the total weighted scattering rate from the spin-up to the spin-down states. Lower panel: Energy level ε_{λ} of the z direction of the double QD vs the electric field.

$$\psi_{z}(z) = \begin{cases} A_{1}\operatorname{Ai}(\xi_{1}) + A_{2}\operatorname{Bi}(\xi_{1}), & -(a/2+d) < z < -a/2, \\ B_{1}\operatorname{Ai}(\xi_{2}) + B_{2}\operatorname{Bi}(\xi_{2}), & |z| \leq \frac{1}{2}a, \\ C_{1}\operatorname{Ai}(\xi_{1}) + C_{2}\operatorname{Bi}(\xi_{1}), & a/2 < z < a/2+d, \end{cases}$$
(2)

in which $Ai(\xi)$ and $Bi(\xi)$ are the Airy functions. The coefficients together with the eigenenergy ε_{λ} can be obtained from the boundary conditions $\psi_{z\lambda}(z=\pm(a/2+d))=0$, the continuity conditions at $z = \pm \frac{1}{2}a$, and the condition of normalization $\int \psi_{z\lambda}^*(z)\psi_{z\lambda}(z)dz=1$. The electron Hamiltonian in the x-y plane is $H_e = H_0 + H_{so}$, where $H_0 = (P_x^2 + P_y^2)/(2m^*) + V_c(\mathbf{r})$ $+H_B$ is the electron Hamiltonian without the spin-orbit interaction, in which $\mathbf{P} = (P_x, P_y) = -i\hbar \nabla + (e/c)\mathbf{A}$ with \mathbf{A} =(B/2)(-y,x) is the electron momentum operator. m^* is the electron effective mass. $H_B = \frac{1}{2}g\mu_B B\sigma_z$ is the Zeeman energy with σ_z denoting the Pauli matrix. $H_{so} = (\gamma/\hbar^3) \Sigma_{\lambda} \langle P_z^2 \rangle_{\lambda}$ $\times (-P_x \sigma_x + P_y \sigma_y)$ is the Dresselhaus spin-orbit coupling²³ with $\langle P_z^2 \rangle_{\lambda} \equiv -\hbar^2 \int \psi_{z\lambda}^*(z) \partial^2 / \partial z^2 \psi_{z\lambda}(z) dz$ and $\gamma = 27.5 \text{ Å}^3 \text{ eV}.^{26}$ For a small applied gate voltage, the Rashba spin-orbit coupling²⁴ is unimportant in this study.²⁷ The eigenenergy of H_0 is $E_{nl\sigma} = \hbar \Omega (2n + |l| + 1) - \hbar l \omega_B + \sigma E_B$, in which Ω = $\sqrt{\omega_0^2 + \omega_B^2}$, $\omega_B = eB/(2m^*)$ and $E_B = \frac{1}{2}g\mu_B B$. The eigenfunction $\langle \mathbf{r} | n l \sigma \rangle = N_{n,l} (\alpha r)^{|l|} e^{-(\alpha r)^2} L_n^{|l|} ((\alpha r)^2) e^{il\theta} \chi_{\sigma}$ with $N_{n,l} = [\alpha^2 n! / \pi (n+|l|)!]^{1/2}$ and $\alpha = \sqrt{m^* \Omega / \hbar}$. $L_n^{|l|}$ is the generalized Laguerre polynomial. χ_{σ} represents the eigenfunction of σ_z . In these equations $n=0,1,2,\ldots,l=0,\pm 1,\pm 2,\ldots$, and σ $=\pm 1$ are quantum numbers. From the eigenfunction of H_0 , one can construct the wave function $|\Psi_{\ell}\rangle$ of H_{e} by either perturbation calculations^{12,14} modified by the right energy corrections pointed out by Cheng et al. 15 or the exact diagonalization approach.¹⁵

The SRT τ is calculated from $\tau^{-1} = \sum_{if} f_i \Gamma_{i \to f}$ in which $f_i = C \exp[-E_i/(k_B T)]$ denotes the Maxwell distribution of the *i*th level with C standing for the normalization parameter and

$$\Gamma_{i \to f} = \frac{2\pi}{\hbar} \sum_{\mathbf{q}\lambda_1} |M_{\mathbf{q}\lambda_1}|^2 |\langle f| e^{i\mathbf{q}\cdot\mathbf{r}} |i\rangle|^2 [\bar{n}_{\mathbf{q}\lambda_1} \delta(E_f - E_i - \hbar \omega_{\mathbf{q}\lambda_1}) + (\bar{n}_{\mathbf{q}\lambda_1} + 1) \delta(E_f - E_i + \hbar \omega_{\mathbf{q}\lambda_1})]$$
(3)

is the transition rate from the *i*th level to the *f*th one due to the electron-phonon scattering due to the deformation potential with $|M_{\mathbf{q}sl}|^2 = \hbar \Xi^2 q/2Dv_{sl}$ and the piezoelectric coupling for the longitudinal phonon mode with $|M_{qpl}|^2$ $=(32\hbar \pi^2 e^2 e_{14}^2 / \kappa^2 D v_{sl})[(3q_x q_y q_z)^2 / q^7]$ and for the two trans- $\sum_{j=1,2} |M_{\mathbf{q}pt_j}|^2$ verse phonon modes with $= (32\hbar\pi^2 e^2 e_{14}^2 / \kappa^2 D v_{st} q^5) [q_x^2 q_y^2 + q_y^2 q_z^2 + q_z^2 q_x^2 - (3q_x q_y q_z)^2 / q^2].$ $\bar{n}_{a\lambda}$, represents the Bose distribution function of phonon with mode λ_1 and momentum **q** at the temperature T. Here Ξ =7 eV stands for the acoustic deformation potential; D $=5.3 \times 10^3$ kg/m³ is the GaAs volume density; $e_{14}=1.41$ $\times 10^9$ V/m is the piezoelectric constant; and $\kappa = 12.9$ denotes the static dielectric constant. The acoustic phonon spectra are given by $\omega_{\mathbf{q}ql} = v_{sl}q$ for the longitudinal mode and $\omega_{\mathbf{q}pt}$ $=v_{st}q$ for the transverse mode with $v_{sl}=5.29\times10^3$ m/s and v_{st} =2.48×10³ m/s being the corresponding sound velocities.

The states *i* and *f* in Eq. (3) are the eigenstates of the Hamiltonian H_e . In order to demonstrate the physics clearly, we first use the corrected perturbation method by Cheng *et al.* ¹⁵ to study the SRT. For the double-dot system, we need to include the lowest two energy levels of the *z* direction which we label as $|1_z\rangle$ and $|2_z\rangle$ [Eq. (2)]. In the *x*-*y* plane, the lowest six energy levels of H_0 for each QD are considered, i.e., $|00+\rangle$, $|00-\rangle$, $|01+\rangle$, $|01-\rangle$, $|0-1+\rangle$, and $|0-1-\rangle$. The wave functions of the lowest four states of $H_e + P_z^2/2m^* + V_z$ constructed from these levels are therefore given by

$$|\Psi_1\rangle = |00+\rangle|1_z\rangle - \mathcal{B}_1|0-1-\rangle|1_z\rangle, \qquad (4)$$

$$|\Psi_2\rangle = |00-\rangle|1_z\rangle - \mathcal{A}_1|01+\rangle|1_z\rangle, \tag{5}$$

$$|\Psi_3\rangle = |00+\rangle |2_z\rangle - \mathcal{B}_2 |0-1-\rangle |2_z\rangle, \tag{6}$$

$$|\Psi_4\rangle = |00 - \rangle |2_z\rangle - \mathcal{A}_2 |01 + \rangle |2_z\rangle, \tag{7}$$

with the corresponding energies being:

$$E_1 = E_{00+,1} - |\mathcal{B}_1|^2 (E_{0-1-,1} - E_{00+,1}), \tag{8}$$

$$E_2 = E_{00-,1} - |\mathcal{A}_1|^2 (E_{01+,1} - E_{00-,1}), \tag{9}$$

$$E_3 = E_{00+,2} - |\mathcal{B}_2|^2 (E_{0-1-,2} - E_{00+,2}), \tag{10}$$

$$E_4 = E_{00-,2} - |\mathcal{A}_2|^2 (E_{01+,2} - E_{0-,2}). \tag{11}$$

In these equations $E_{nl\sigma,\lambda} = E_{nl\sigma} + \varepsilon_{\lambda}$, $\mathcal{B}_{\lambda} = i\alpha \gamma_{\lambda}^{*} [1 - eB/(2\hbar \alpha^{2})]/(E_{0-1-,\lambda} - E_{00+,\lambda})$, and $\mathcal{A}_{\lambda} = i\alpha \gamma_{\lambda}^{*} [1 + eB/(2\hbar \alpha^{2})]/(E_{01+,\lambda} - E_{00-,\lambda})$ with $\gamma_{\lambda}^{*} = \gamma \langle P_{z}^{2} \rangle_{\lambda} / \hbar^{2}$. $\lambda \times (=1,2)$ is the quantum number on the *z* axis. Now we calculate the spin-flip rates from the spin-up states $|\Psi_{2m-1}\rangle$ to the spin-down ones $|\Psi_{2m}\rangle$ (m=1,2) due to the electron-phonon scattering. There are nine spin-flip scattering rates. The scatting rate from the spin-up state *i* to the spin-down one *f* reads

$$\Gamma_{i \to f} = |\mathcal{A}_f - \mathcal{B}_i|^2 \{n_q + [1 + \operatorname{sgn}(i - f)]/2\} q^3 \int_0^{\pi/2} d\theta$$
$$\times [C_{LD}q^2 \sin^3\theta + C_{LP}q^2 \sin^7\theta \cos^2\theta + C_{TP} \sin^5\theta$$
$$\times (\sin^4\theta + 8\cos^4\theta)] e^{-q^2 \sin^2\theta/2} |I_{if}(q \cos\theta)|^2, \quad (12)$$

in which $I_{if}(q_z) = \langle \psi_{zi} | e^{iq_z z} | \psi_{zf} \rangle$ and $q = |E_i - E_f| / (\hbar v_\lambda \alpha)$. $C_{LD} = \Xi^2 \alpha^3 / (8 \pi \hbar v_{sl}^2 D)$, $C_{LP} = 9e^2 e_{14}^2 \alpha \pi / (\hbar \kappa^2 D v_{sl}^2)$, and $C_{TP} = \pi e^2 e_{14}^2 \alpha / (\hbar \kappa^2 D v_{sl}^2)$ in Eq. (12) are the coefficients of the electron-phonon scattering due to the deformation potential and due to the piezoelectric coupling for the longitudinal phonon mode and two transverse phonon modes, respectively.

III. NUMERICAL RESULTS

In Fig. 1 we plot the SRT of a typical double dot with $d_0=20 \text{ nm}, a=10 \text{ nm}, d=5 \text{ nm}, V_0=0.4 \text{ eV}, \text{ and } B=0.1 \text{ T}$ at

T=4 K as a function of electric field E. The solid curve in Fig. 1(a) is the result from the perturbation approach. It is interesting to see that the SRT is increased about seven orders of magnitude when the electric field is tuned from 0.1 to 1.3 kV/cm. The physics of such a gate-voltage-induced dramatic change can be understood as follows. When the gate voltage is small, due to the large well height V_0 and/or the large interdot distance a, the electron wave function (along the z axis) of the lowest subband of each well is mostly localized in that well due to the high barrier between them, and hence the difference of the lowest two energy levels is very small (about 10^{-4} eV). When the gate voltage is high enough, electrons can tunnel through the barrier and the wave functions in the two wells get a larger overlap. Therefore the separation between the lowest two levels ε_1 and ε_2 increases. This can be seen from Fig. 1(b) where the energies of the lowest two levels along the z axis ε_1 and ε_2 are plotted against the electric field E. From Eqs. (8)–(11) one can see that the first two levels $(E_1 \text{ and } E_2)$ and the next two levels $(E_3 \text{ and } E_4)$ are mainly separated by the energy along the z axis, i.e., ε_1 and ε_2 . Such an increase makes the electronphonon scattering more efficient when the energy difference $\varepsilon_2 - \varepsilon_1$ is not too big. Therefore, by applying the gate voltage, one finds that the SRT first decreases. Nevertheless, with further increase of the gate voltage, half of the lowest four levels are quickly removed from the spin relaxation channel and the SRT is enhanced. As a result, there is a minimum of the SRT with the gate voltage. This can be seen from the same figure where the weighted scattering rates $(\Gamma_{i\to f})$ $=f_i\Gamma_{i\to f}$ between different levels are plotted versus the electric field. The leading contribution to the total scattering rate comes from $\Gamma_{3\rightarrow 2}$ at small field regime. When the electric field increases from 0.5 to 1.3 kV/cm, $\overline{\Gamma}_{3\rightarrow 2}$ decreases rapidly due to the separation of ε_{λ} with the electric field but $\overline{\Gamma}_{1\rightarrow 2}$ stays almost unchanged as both levels E_1 and E_2 correspond to the same lowest level ε_1 along the *z* axis. Finally, for large field, $\bar{\Gamma}_{1\rightarrow2}$ defines the total scattering rate. It is further noted that, although we performed the average of the initial and the sum of the final states in calculating the SRT, the leading contribution comes from the scattering from E_3 to E_2 at low electric field and the scattering from E_1 to E_2 at large one.

The large variation of $\overline{\Gamma}_{3\to2}$ around 1 kV/cm can be estimated as follows. As the electron-phonon scattering due to the piezoelectric coupling of the two transverse phonon modes is at least one order of magnitude larger than the other modes, we only consider the third term in Eq. (12). From our calculation, $\varepsilon_1 = [3.25 \times 10^{-4}E \text{ (kV/cm)} + 0.15129] \text{ eV}$ and $\varepsilon_2 = [1.68 \times 10^{-3}E \text{ (kV/cm)} + 0.1513] \text{ eV}$. The energy splitting between E_2 and E_3 can be approximated by $\varepsilon_2 - \varepsilon_1$. Therefore $\Delta E_{23} = [1.36 \times 10^{-3}E \text{ (kV/cm)} + 5 \times 10^{-5}] \text{ eV}$ approximately and $q = \Delta E_{23}/(\hbar v_{st}\alpha)$. As the variation of $|I_{12}(q \cos \theta)|$ in Eq. (12) is within one order of magnitude, we approximately bring it out of the integral. Then the remaining integral $\int_0^{\pi/2} d\theta \sin^5 \theta(\sin^4 \theta + 8 \cos^4 \theta) e^{-q^2 \sin^2 \theta/2}$ can be carried out analytically and reads $\frac{1}{2}B(\frac{1}{2};5)\Phi(5;\frac{11}{2};-q^2/2) + 4B(\frac{5}{2},3)\Phi(3;\frac{11}{2};-q^2/2)$ with $B(\mu;\nu)$ and $\Phi(\alpha;\gamma;z)$ being the Beta function and the degenerate hypergeometric func-



FIG. 2. SRT calculated from the exact diagonalization approach vs the electric field at different (a) magnetic fields with $d_0=20$ nm and (b) QD diameters with B=0.1 T. In the calculation a=10 nm, d=5 nm, $d_0=20$ nm, $V_0=0.4$ eV, and T=4 K.

tion, respectively. When E=0.1 kV/cm, the value of the integral is 10^{-1} and when E=1.3 kV/cm, it becomes 10^{-6} . Meanwhile, with the change of the electric field from 0.1 to 1.3 kV/cm, although $q^3 |\mathcal{A}_f - \mathcal{B}_i|^2$ is increased by one order of magnitude, $|I_{23}|^2$ is decreased by one order of magnitude and the distribution function f_3 is decreased by another two orders of magnitude. Therefore, $\overline{\Gamma}_{3\to 2}$ decreases by about seven orders of magnitude when E is tuned from 0.1 to 1.3 kV/cm.

As pointed out by Cheng *et al.* ¹⁵ and confirmed later by Destefani and Ulloa,²⁸ due to the strong spin-orbit coupling, the perturbation approach is inadequate in describing the SRT even when the second-order energy corrections are included. Therefore, in Fig. 1(a) we further plot the SRT calculated from the exact diagonalization method as a dotted curve. Similar results are obtained, although again the SRT from the exact diagonalization approach differs from the perturbation one.

Now we investigate the magnetic field and dot size dependence of the SRT in Fig. 2(a) and 2(b) by the exact diagonalization approach. Again one observes a dramatic increase of the SRT by tuning the electric field up to a certain value and then the SRT is insensitive to the electric field. For small dot size ($d_0 = 10$ nm), one even observes a 12 orders of magnitude change of the SRT by tuning the gate electric field to 2.6 kV/cm. The dramatic variation of the SRT has been explained above. Now we discuss why the SRT decreases with increasing magnetic field and dot size as observed in Fig. 2 in the electric-field-insensitive part. From Fig. 1(b) one finds that $\Gamma_{1\rightarrow 2}$ is the leading contribution to the total scattering rate in this part. The energy splitting between the first and second levels $\Delta E_{12} \propto B$. As ΔE_{12} is about 10^{-5} eV, n_q $\approx k_B T / \Delta E_{12}$ and $n_q q^3 \propto (\Delta E_{12})^2$. Moreover $|\mathcal{A}_1 - \mathcal{B}_1|^2$ = $(\alpha \gamma_1^* 4 E_B \omega_B)^2 / (\hbar^2 \Omega \omega_0^2)^2 \propto B^4$ approximately. As a result, the coefficient before the integral of the electron-transverse phonon scattering due to the piezoelectric coupling is proportional to B^6 . Although the integral has a marginal decrease with increasing *B*, $\overline{\Gamma}_{1\rightarrow 2}$ still increases with *B*. Similarly, one can explain the change of the SRT with the dot diameter d_0 .

It is noted that in order to obtain the large variation of the SRT by a gate voltage, it is important that the barrier between the QDs should be large enough so that without a gate voltage, the two dots are decoupled (and there is no energy



FIG. 3. SRT calculated from the exact diagonalization approach vs the electric field at different (a) barrier heights V_0 with the barrier width a=10 nm and (b) barrier widths a with $V_0=0.4$ nm. In the calculation, d=5 nm, $d_0=20$ nm, and B=0.1 T. T=4 K.

splitting along the *z* axis). This can be clearly seen from Fig. 3: With the decrease of the barrier height V_0 or the interdot distance *a*, the tunability of the SRT by the gate voltage decreases.

The double-dot system proposed in our scheme can be easily realized with the current technology.^{29,30} Nevertheless, it is not essential to use such a high-barrier-height system to obtain the large spin manipulation. For the ordinary barrier heights widely used in experiments (about one order of magnitude lower than V_0 used above), one can still achieve a similar manipulation by increasing the distance *a* between the two QDs as shown in Fig. 4 where the barrier height $V_0=0.05$ eV. One finds that for small V_0 , if the barrier width *d* is large enough, one can still get a large change of SRT. In particular, in the case of a=30 nm, 11 orders of magnitude change of SRT is obtained by a small gate field.

IV. CONCLUSIONS

In conclusion, we have proposed a feasible scheme to manipulate the spin relaxation in GaAs vertical double QDs by a small gate voltage. The SRT calculated can be tuned by up to 12 orders of magnitude by an electric field from the gate voltage less than 3 kV/cm. This provides a unique way to control the spin relaxation and to make spin-based logical gates. The conditions needed to realize such a large tunability are addressed. The double-dot system proposed in our scheme can be easily realized in experiments. Finally, the



FIG. 4. SRT calculated from the exact diagonalization approach vs the electric field at different interdot distances *a* with low barrier height $V_0=0.05$ eV. In the calculation, d=5 nm, $d_0=20$ nm, B=0.1 T, and T=4 K.

proposed large orders of magnitude change due to the gate voltage will not be reduced by the hyperfine interaction with nuclear spins^{31,32} as the SRT due to this mechanism in our case is around 10³ s at 0.1 T. Finally we point out that, differing from the earlier reports^{21,33} where a strong variation of the SRT is obtained from the anticrossing of the energy levels induced by the Rashba spin-orbit coupling by increasing the magnetic field³³ or the interdot distance,²¹ there is no anticrossing or crossing of the energy levels in our scheme. Moreover, the tunability of the scheme proposed in the present paper is more applicable as one only needs to tune a very small gate voltage (to tune the electric field from 0.1 to 1.2 kV/cm) to obtain a surge of the SRT by up to 12 orders of magnitude, in contrast to the large magnetic field of several tesla needed to obtain a variation up to seven orders of magnitude.33,34

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