Temperature dependence of the spin susceptibility of a clean Fermi gas with repulsion

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Spin susceptibility of a clean Fermi gas with repulsion in any dimension is considered using a supersymmetric low energy theory of interacting spin excitations and renormalization scheme recently proposed by Aleiner and Efetov. We generalize this method to include the coupling to the magnetic field. As a result, we obtain for the correction $\delta\chi$ to the Pauli susceptibility a nonanalytic temperature dependence of the form $T^{d-1}\gamma_b^2(T)$ in dimensions $d=2$, 3, where $\gamma_b(T)$ is an effective d-dependent logarithmically renormalized backscattering amplitude. In one dimension, $\delta\chi$ is proportional to $\gamma_b(T)$, and we reproduce a well known result obtained long ago by a direct calculation.

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I. INTRODUCTION

According to Landau's Fermi liquid theory[2] low energy properties of interacting fermion systems are similar to those of an ideal Fermi gas. For many purposes interaction effects can be incorporated into renormalized parameters like the effective mass. Using the Landau theory one might expect that thermodynamic quantities like $C(T)/T$ and $\chi(T)$, where $C(T)$ is the specific heat and $\chi(T)$ is the spin susceptibility, have an expansion in powers of T^2, as known from the standard Sommerfeld expansion for the ideal Fermi gas.

This would mean extending the analogy too far, however. In fact, it is known that interaction between fermions can induce so-called nonanalytic corrections, that are absent for the ideal Fermi gas. The form of these corrections strongly depends on the dimensionality of the system. As a result of theoretical studies the leading temperature dependence of $C(T)/T$ was found to be $T^2\ln T$ in $d=3$ and T in $d=2$ at low temperatures (see Refs. 3–10 and Refs. 11–15, respectively). For the spin susceptibility a linear in T dependence in $d=2$ was obtained in Refs. 13–21, while for $d=3$ a nonanalytic $Q^2\ln Q$ dependence of the wave-vector dependent spin susceptibility[15–18,22] was found not to be paralleled by a similar temperature dependence.

The model of a weakly interacting Fermi gas allows for a controlled perturbative expansion in the strength of the interaction potential. In such an approach the corrections cited above appear in the second order of perturbation theory. However, calculations in higher orders are difficult and a full analysis of high order corrections has not been performed previously.

Recently, a new low energy supersymmetric field theory for weakly interacting electrons was introduced in Ref. 1. This is some kind of a bosonization that includes not only charge but also spin degrees of freedom. The scattering processes responsible for the nonanalytic corrections are quasi-one-dimensional in character, which leads to logarithmic contributions originating from interactions between spin excitations. As concerns the charge excitations in $d>1$ studied previously within other bosonization schemes,[9,10,23–31] their contribution is less singular and does not lead to the effects discovered in Ref. 1. The low energy theory of Ref. 1 resembles the supersymmetry approach,[32,33] well known in the theory of disordered systems and random matrix theory.

In the model of interacting spin excitations, logarithmic corrections to interaction amplitudes were found in any dimension, motivating a renormalization group study. The origin of the logarithms can be easily understood. Consider the interaction of two spin modes with propagators $(v_F\mathbf{np}-i\omega)^{-1}$, but opposite directions $\mathbf{n}$, $-\mathbf{n}$ on the Fermi surface. Then one finds for perturbative corrections to the interaction amplitudes integrals of the type (compare Fig. 4)

$$T\sum_{\omega}\int d\mathbf{p}\frac{1}{(v_F\mathbf{np})^2+\omega^2}\propto\ln(\varepsilon_\infty/T), \tag{1.1}$$

ε_∞ is the largest energy in the model.

The integral over momenta transverse to $\mathbf{n}$ has to be regularized. Of course, in dimensions $d>1$, the region of phase space for which the excitations move in almost antiparallel directions $\mathbf{n}_1\sim-\mathbf{n}_2$ is rather restricted and in general the logarithm is cut by $\max[T/\varepsilon_\infty,|\mathbf{n}_1+\mathbf{n}_2|]$. The question arises how the logarithms found at this level affect the temperature dependence of observable quantities, since eventually one needs to perform a (weighted) average over all directions. To this end the specific heat was studied in Ref. 1. It was found that $C(T)/T$ generally depends on an effective amplitude of backward scattering, that displays a complicated dependence on $\ln T$, in such a way that the results reduced to well known $\delta C_{d=2}\propto T^2$ and $\delta C_{d=3}\propto T^3\ln T$ when replacing the effective backscattering constants by the bare ones.

The evident question arises whether similar corrections exist for the spin susceptibility. We will show in this paper that this is really so. To this end we generalize the formalism introduced in Ref. 1 to include the external magnetic field. A convenient diagrammatic representation makes apparent the relation to conventional diagrammatic approaches. It turns out that, unlike for the thermodynamic potential, the contributions to the spin susceptibility in $d>1$ are determined by all the renormalized interaction amplitudes of the model in various combinations. In $d=2$ we find nonanalytic contributions $\delta\chi(T)\propto T\gamma_b^2(T)$ and $\gamma_b(T)$ is an effective backscattering

constant that includes logarithmic corrections. In $d=3$ we confirm the absence of $T^2 \ln T$ terms in the second order in the interaction. This does, however, not mean the absence of nonanalytic corrections in three dimensions. Here we sum leading logarithmic corrections to the T^2 behavior and come to the result $\delta\chi(T) \propto T^2 \gamma_b^2(T)$. Within the same formalism the one-dimensional case can be also considered and we reproduce the temperature dependence first obtained by Dzyaloshinskii and Larkin.[38] Although the one-dimensional case is not the main focus of the approach, we consider the result obtained as an important check of the overall consistency.

The paper is organized as follows. In Sec. II we introduce the model that serves as a starting point for our subsequent analysis. In Sec. III we decouple the interaction part via a Hubbard-Stratonovich transformation in both the charge and spin channel and reformulate the partition function in terms of charge and spin excitations. The charge and the spin excitations decouple from each other and calculating the spin susceptibility we can concentrate on the spin sector. A representation for the partition function in this sector is derived in Sec. IV using the supersymmetry technique. A low energy effective action is obtained and rules of the perturbation theory are introduced as well as a convenient diagrammatic representation. In Sec. V we analyze corrections to the magnetic field vertices in a renormalization group scheme. In Sec. VI we study in our formalism temperature dependence of the spin susceptibility in one spatial dimension. Then we turn to the calculation of nonanalytic corrections to the temperature dependent spin susceptibility in two and three dimensions in Sec. VII before concluding with a discussion of our results in Sec. VIII.

II. MODEL

We introduce our model by specifying the partition function in the imaginary time formalism (Refs. 34 and 35)

$$\mathcal{Z} = \int \mathcal{D}(\chi^*, \chi) \exp(-\mathcal{S}). \tag{2.1}$$

The fermionic fields χ, χ^* depend on coordinates $\mathbf{r}$ and imaginary time τ and carry a spin label σ. They obey the antiperiodic boundary conditions $\chi_\sigma(\mathbf{r}, \tau) = -\chi_\sigma(\mathbf{r}, \tau+\beta)$, where $\beta = 1/T$ and T is the temperature.

We write the action $\mathcal{S}$ as the sum of three parts

$$\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_b + \mathcal{S}_{int}. \tag{2.2}$$

$\mathcal{S}_0$ describes free motion, $\mathcal{S}_b$ stands for the coupling of the spin to an external field $\mathbf{b}$, and $\mathcal{S}_{int}$ is the interaction of fermions,

$$\mathcal{S}_0 = \int dx \chi_\sigma^*(x) \left[-\partial_\tau - \hat{H}_0 \right] \chi_\sigma(x), \tag{2.3}$$

$$\mathcal{S}_b = \int dx \chi_\sigma^*(x) \mathbf{b} \boldsymbol{\sigma}_{\sigma\sigma'} \chi_{\sigma'}(x), \tag{2.4}$$

$$\mathcal{S}_{int} = \frac{1}{2} \int dx dx' \chi_\sigma^*(x) \chi_{\sigma'}^*(x') v(x-x') \chi_{\sigma'}(x') \chi_\sigma(x). \tag{2.5}$$

Here and in the following summation over repeated spin indices is implied and $v(x-x') = V(\mathbf{r}-\mathbf{r}')\delta(\tau-\tau')$, where $V(\mathbf{r}-\mathbf{r}')$ is the interaction potential. We use the notation

$$x = (\mathbf{r}, \tau) \qquad \int dx = \int_0^\beta d\tau \int d\mathbf{r} \tag{2.6}$$

and $\hat{H}_0 = \frac{\hat{\mathbf{p}}^2}{2m} - \mu$, where $\hat{\mathbf{p}}$ is the momentum operator and μ is the chemical potential. The standard relation between the partition function $\mathcal{Z}$ and the thermodynamic potential Ω is

$$\Omega = -T \ln \mathcal{Z}. \tag{2.7}$$

The spin susceptibility is then obtained as the second derivative with respect to b,

$$\chi(x-x') = -\left. \frac{\delta^2 \Omega[b]}{\delta b(x) \delta b(x')} \right|_{b=0}. \tag{2.8}$$

We will mostly be interested in the static spin susceptibility χ_s for a spatially homogeneous external field, in which case after the Fourier transform the limit of vanishing frequencies should be taken before taking the external wave vector to zero.

Let us also introduce the following convention for the integration over momenta:

$$\int d\mathbf{q} = \int \frac{d^d q}{(2\pi)^d} \tag{2.9}$$

in dimension d. For the sake of notational convenience we sometimes write $\int_{\mathbf{r}}, \int_{\mathbf{q}}, \int_\tau$, symbolizing $\int d\mathbf{q}$, $\int d\mathbf{r}$, and $\int_0^\beta d\tau$, respectively. $\int d\hat{\mathbf{n}}$ stands for the integration over the solid angle normalized to unity.

III. DERIVATION OF THE BOSONIZED ACTION

In this section we present the derivation of the model that will be used for the further analysis of the spin susceptibility. It describes low lying charge and spin excitations in the system. A derivation in the absence of external sources has been presented in Ref. 1. Here we include the coupling to the magnetic field, so that we will mainly focus on the changes introduced by the magnetic field.

A. Decoupling into slow pairs

For the interaction part we perform decoupling in two different channels by singling out slow pairs in the following way:

$$\mathcal{S}_{int} \to \tilde{\mathcal{S}}_{int} = \tilde{S}_{int,1} + \tilde{S}_{int,2}, \tag{3.1}$$

$$\tilde{S}_{int,1} = \int dp_1 dp_2 dq V(\mathbf{q}) \chi_\sigma^*(p_1) \chi_\sigma(p_1 - q) \chi_{\sigma'}^*(p_2) \chi_{\sigma'}(p_2 + q), \tag{3.2}$$

$$\tilde{S}_{int,2} = -\int dp_1 dp_2 dq V(\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{q}) \times \chi^*_\sigma(p_1)\chi_{\sigma'}(p_1 - q)\chi^*_{\sigma'}(p_2)\chi_\sigma(p_2 + q). \quad (3.3)$$

Here we denoted

$$\int dp_i = T\sum_{\varepsilon_{n_i}} \int d\mathbf{p}_i, \quad (3.4)$$

where the sum goes over fermionic frequencies $\varepsilon_{n_i} = \pi T(2n_i + 1)$ and

$$\int dq = T\sum_{\Omega_n} \int d\mathbf{q} f(\mathbf{q}), \quad (3.5)$$

where the sum goes over bosonic frequencies $\Omega_n = 2\pi nT$, further $p_i = (\mathbf{p}_i, \varepsilon_{n_i})$ and $q = (\mathbf{q}, \Omega_n)$.

The cutoff function f, introduced in Eq. (3.5), is defined as follows:

$$f(\mathbf{p}) = f_0(pr_0), \quad p = |\mathbf{p}|, \quad (3.6)$$

where $f_0(t)$ has the properties $f_0(0) = 1$ and $f(t) \to 0$ smoothly for $t \to \infty$. The function f has been introduced to avoid double counting when singling out regions of small momentum transfer in Eqs. (3.1)–(3.3), since without the cutoff both $\tilde{S}_{int,1}$ and $\tilde{S}_{int,2}$ would identically reproduce the original S_{int}. Accordingly, $k_c = r_0^{-1}$ is a momentum cutoff that is much smaller than the Fermi momentum $k_c \ll p_F$, but much larger than typical momenta for the excitations of the low-energy theory that we wish to construct. We denote also the cutoff energy $\varepsilon_\infty = r_0^{-1} v_F$. Additional decoupling in the Cooper channel is not included, since this would amount to overcounting of relevant scattering processes (compare the related discussion in Ref. 1, Sec. II A).

For a short range potential we can further simplify our considerations by setting $V_2 = V(\mathbf{q} \ll p_F)$. Since important momenta are close to the Fermi surface we can write $V_1(\theta_{12}) = V(\mathbf{p}_1 - \mathbf{p}_2) = V\left[2p_0 \sin\left(\frac{\theta_{12}}{2}\right)\right]$, where θ_{12} is the angle between momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, $\theta_{12} = \widehat{\mathbf{p}_1 \mathbf{p}_2}$.

For the further development of the theory it will be crucial to separate explicitly interactions in the triplet and singlet channel.

$$V_s(\theta_{12}) = V_2 - \frac{1}{2}V_1(\theta_{12}), \quad V_t(\theta_{12}) = \frac{1}{2}V_1(\theta_{12}). \quad (3.7)$$

The action separates into a charge and a spin sector,

$$\tilde{S}_{int} = S_{int,s} + S_{int,t},$$

$$S_{int,s} = \frac{1}{2}\int dp_1 dp_2 dq \rho(p_1, -q) V_s(\theta_{12}) \rho(p_2, q),$$

$$S_{int,t} = -\frac{1}{2}\int dp_1 dp_2 dq \mathbf{S}(p_1, -q) V_t(\theta_{12}) \mathbf{S}(p_2, q), \quad (3.8)$$

where the charge $\rho(p,q)$ and spin densities $\mathbf{S}(p,q)$ are

$$\rho(p,q) = \chi^\dagger\left(p - \frac{q}{2}\right)\chi\left(p + \frac{q}{2}\right), \quad (3.9)$$

$$\mathbf{S}(p,q) = \chi^\dagger\left(p - \frac{q}{2}\right)\boldsymbol{\sigma}\chi\left(p + \frac{q}{2}\right), \quad (3.10)$$

and we turned to a spinor notation $\chi = (\chi_\uparrow, \chi_\downarrow)$.

Finally, one may decouple the interaction term $\tilde{S}_{int}$ using a Hubbard-Stratonovich transformation with a field $\phi_\mathbf{n}(x) \equiv i\varphi_\mathbf{n}(x) + \boldsymbol{\sigma}\mathbf{h}_\mathbf{n}(x)$. Here $\varphi_\mathbf{n}(x)$ and $\mathbf{h}_\mathbf{n}(x)$ are real bosonic fields, so that $\phi_\mathbf{n}(\mathbf{r}, \tau) = \phi_\mathbf{n}(\mathbf{r}, \tau + \beta)$ and $\mathbf{n}$ is the direction of momentum $\mathbf{p}$ on the Fermi surface, $\mathbf{n} = \mathbf{p}/|\mathbf{p}|$. The result is the following representation of the partition function:

$$\mathcal{Z} = \mathcal{N}\int \mathcal{D}\phi W_s[\varphi] W_t[\mathbf{h}] \mathcal{Z}[\mathbf{b}, \mathbf{h}, \varphi]. \quad (3.11)$$

The weight functions W_s, W_t are shown below in Eqs. (3.18) and (3.19), and $\mathcal{N}$ is a simple normalization constant that will not be displayed from now on. The partition function $\mathcal{Z}[\mathbf{b}, \mathbf{h}, \varphi]$ describes the fermion motion for fixed configuration of fields $\mathbf{b}, \mathbf{h}, \varphi$,

$$\mathcal{Z}[\mathbf{b}, \mathbf{h}, \varphi] = \int D(\chi^*, \chi) \exp(-S_{eff}[\mathbf{b}, \mathbf{h}, \varphi]), \quad (3.12)$$

where the effective action $\mathcal{S}_{eff}$ has the form

$$S_{eff}[\mathbf{b}, \mathbf{h}, \varphi] = \mathcal{S}_0 + \mathcal{S}_b[\mathbf{b}] + \int d\mathbf{p} d\mathbf{r}_1 d\mathbf{r}_2 \chi^\dagger(\mathbf{r}_1, \tau) \times \phi_\mathbf{n}\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}\right)\chi(\mathbf{r}_2, \tau) e^{i\mathbf{p}(\mathbf{r}_1 - \mathbf{r}_2)}. \quad (3.13)$$

By comparing Eq. (2.4) with Eq. (3.13) we observe the following simple relation:

$$\mathcal{S}_{eff}[\mathbf{b}, \mathbf{h}, \varphi] = \mathcal{S}_{eff}[0, \mathbf{h} + \mathbf{b}, \varphi], \quad (3.14)$$

which enables us to remove the field $\mathbf{b}$ from $\mathcal{S}_{eff}$ by a shift in $\mathbf{h}$ at the expense of changing the weight W_t accordingly.

Now we can write down a representation of the partition function in the presence of the magnetic field as a weighted integral over field configurations

$$\mathcal{Z} = \int \mathcal{D}\phi W_s[\varphi] W_t[\mathbf{h} - \mathbf{b}] \mathcal{Z}[\phi], \quad (3.15)$$

where

$$\mathcal{Z}[\phi] = \int D(\chi^*, \chi) \exp(-\mathcal{S}_{eff}[\phi]), \quad (3.16)$$

$$\mathcal{S}_{eff}[\phi] = \mathcal{S}_0 + \int d\mathbf{p} d\mathbf{r}_1 d\mathbf{r}_2 e^{i\mathbf{p}(\mathbf{r}_1 - \mathbf{r}_2)} \times \chi^\dagger(\mathbf{r}_1, \tau)\phi_\mathbf{n}\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}\right)\chi(\mathbf{r}_2, \tau). \quad (3.17)$$

The weights $W_s[\varphi]$ and $W_t[\mathbf{h}]$ are

$$W_s[\varphi] = \exp\left[-\frac{1}{2} \int d\hat{\mathbf{n}}_1 d\hat{\mathbf{n}}_2 d\mathbf{q} d\tau \varphi^*_{\mathbf{n}_1}(\mathbf{q},\tau) \times V_s^{-1}(\theta_{12},\mathbf{q}) \varphi_{\mathbf{n}_2}(\mathbf{q},\tau) \right] \tag{3.18}$$

$$W_t[\mathbf{h}] = \exp\left[-\frac{1}{2} \int d\hat{\mathbf{n}}_1 d\hat{\mathbf{n}}_2 d\mathbf{q} d\tau \mathbf{h}^\dagger_{\mathbf{n}_1}(\mathbf{q},\tau) \times V_t^{-1}(\theta_{12},\mathbf{q}) \mathbf{h}_{\mathbf{n}_2}(\mathbf{q},\tau) \right] \tag{3.19}$$

and $V_{s,t}(\theta_{12},\mathbf{q}) = V_{s,t}(\theta_{12}) f(\mathbf{q})$.

Equations (3.15)–(3.19) represent the final result of this subsection.

The model is still quite general and in order to make progress further approximations have to be introduced, where the focus will be on calculation of $\mathcal{Z}[\phi]$. Since it was possible to remove the field $\mathbf{b}$ from this term, the further derivation of the theory parallels that of Ref. 1 up to the point, where the weighted integral over the field configurations of ϕ is performed. In the following we will outline the main steps here in order to introduce our notations and prepare the subsequent discussion of the model.

B. Bosonized action

In Sec. III B 1 the derivation of the representation for $\mathcal{Z}[\phi]$ in the so-called quasiclassical approximation is outlined following Ref. 1. Section III B 2 deals with some additional contribution from the high energy sector and the resulting change of weights.

1. Charge and spin modes

Integration over the fields χ, χ^* in the expression, Eq. (3.16), for $\mathcal{Z}[\phi]$ results in

$$\mathcal{Z}[\phi] = \exp(\mathrm{Tr}\ln(-\partial_\tau - \mathcal{H}_0 + \hat{\Phi})), \tag{3.20}$$

where the symbol Tr includes integration over coordinates as well as trace tr_σ in spin space. The operator $\hat{\Phi}$ acts in accordance with Eq. (3.17). In the next step, we use a standard trick introducing an auxiliary integration over parameter u, which enables one to formally avoid expanding the logarithm while keeping track of appropriate combinatorial factors. Instead, the Green's function for fixed field configuration $G(x,x'|u\phi)$ comes into play.

$$\frac{\mathcal{Z}[\phi]}{\mathcal{Z}[0]} = \exp\left[\mathrm{Tr} \int_0^1 du \partial_u \ln\left(-\partial_\tau - \hat{H}_0 + u\hat{\Phi}\right) \right] \tag{3.21}$$

$$= \exp\left(-i\mathrm{Tr} \int_0^1 du \left(\hat{\Phi} G\right)(x,x|u\phi) \right). \tag{3.22}$$

The Green's function,

$$G(x,x'|\phi) = \frac{i}{\mathcal{Z}[\phi]} \int D(\chi,\chi^*)\chi(x)\chi^\dagger(x') e^{-\mathcal{S}_{eff}[\phi]} \tag{3.23}$$

enters at coinciding points and this is why it is advantageous to Fourier transform with respect to the difference of coordinates

$$G(x,x'|\phi) = \int (d\mathbf{p}) e^{i\mathbf{p}(\mathbf{r}-\mathbf{r}')} \bar{G}_{\mathbf{p}}\left(\frac{\mathbf{r}+\mathbf{r}'}{2}, \tau, \tau' | \phi \right). \tag{3.24}$$

Using the fact that $\bar{G}_{\mathbf{p}}$ is sharply peaked at the Fermi surface one splits $\int d\mathbf{p} \sim \nu \int d\hat{\mathbf{n}} \int d\xi_{\mathbf{p}}$, where $\xi_{\mathbf{p}} = \mathbf{p}^2/2m - \mu$ and ν is the single particle density of states at the Fermi surface per spin direction. After integration over $\xi_{\mathbf{p}}$ the quasiclassical Green's function

$$g_{\mathbf{n}}(\mathbf{r},\tau,\tau'|\phi) \equiv \frac{1}{\pi} \int d\xi_{\mathbf{p}} \bar{G}_{\mathbf{p}}(\mathbf{r},\tau,\tau'|\phi) \tag{3.25}$$

enters the expression

$$\mathcal{Z}[\phi]/\mathcal{Z}[0] = \exp\left(-i\pi\nu \int_{\mathbf{r}\tau} \int_0^1 du\ \mathrm{tr}[\phi_{\mathbf{n}}(x) g_{\mathbf{n}}(\mathbf{r},\tau,\tau|u\phi)] \right). \tag{3.26}$$

One of the main results of Ref. 1, which we only cite here, is a set of *decoupled* differential equations for the charge ρ and spin $\mathbf{S}$ components of $g_{\mathbf{n}}(x,\tau,\tau)$ in the decomposition

$$i\pi g_{\mathbf{n}}(x,\tau,\tau) = i\rho_{\mathbf{n}}(x) + \mathbf{S}_{\mathbf{n}}(x)\boldsymbol{\sigma}, \tag{3.27}$$

namely

$$\hat{L}_{\mathbf{n},u} \mathbf{S}_{\mathbf{n}}(x,u) = -u\partial_\tau \mathbf{h}_{\mathbf{n}}(x), \tag{3.28}$$

$$\hat{L}_{\mathbf{n},0} \rho_{\mathbf{n}}(x,u) = -u\partial_\tau \varphi(x), \tag{3.29}$$

where

$$\hat{L}_{\mathbf{n},u} = -\partial_\tau + iv_F \mathbf{n}\nabla + 2iu\hat{h}_{\mathbf{n}}. \tag{3.30}$$

$\hat{h}_{\mathbf{n}}(x)$ is a matrix in the spin space with components $\hat{h}_{ij} = -\varepsilon_{ijk}\mathbf{h}_k$, so that $\hat{h}\mathbf{S} = \mathbf{h} \times \mathbf{S}$.

This result was obtained with the help of a generalized Schwinger ansatz[36] for $g_{\mathbf{n}}$

$$g_{\mathbf{n}}(\mathbf{r},\tau,\tau'|\phi) = \mathcal{T}_{\mathbf{n}}(\mathbf{r},\tau) g_0(\tau-\tau') \mathcal{T}^{-1}_{\mathbf{n}}(\mathbf{r},\tau'), \tag{3.31}$$

where g_0 is the Green's function for free fermions ($\phi=0$). It was further assumed that ϕ varied smoothly on the scale of the Fermi wavelength $\lambda_F = p_F^{-1}$.

Finally, substituting the decomposition Eq. (3.27) into Eq. (3.26), one finds

$$\mathcal{Z}[\phi] = \mathcal{Z}[0]\mathcal{Z}_\rho[\varphi]\mathcal{Z}_s[\mathbf{h}], \tag{3.32}$$

where

$$\mathcal{Z}_\rho[\varphi] = \exp\left(2\nu \int dudxd\hat{\mathbf{n}} \varphi_{\mathbf{n}}(x)\rho_{\mathbf{n}}(x,u) \right), \tag{3.33}$$

$$\mathcal{Z}_s[\mathbf{h}] = \exp\left(-2\nu \int dudxd\hat{\mathbf{n}}\mathbf{h}_{\mathbf{n}}(x)\mathbf{S}_{\mathbf{n}}(x,u)\right), \quad (3.34)$$

and $\mathbf{S}$ and ρ fulfill the differential equations Eqs. (3.28) and (3.29). The equation for ρ is readily solved using a Fourier transform but we concentrate in the following on the spin sector here.

Generally, one finds

$$\mathcal{Z} = \mathcal{Z}[0]\mathcal{Z}_\rho\mathcal{Z}_s, \quad (3.35)$$

where

$$\mathcal{Z}_\rho = \int \mathcal{D}\varphi W_s[\varphi]\mathcal{Z}_\rho[\varphi], \quad (3.36)$$

$$\mathcal{Z}_s = \int \mathcal{D}\mathbf{h} W_t[\mathbf{h}-\mathbf{b}]\mathcal{Z}_s[\mathbf{h}]. \quad (3.37)$$

Our main task is to calculate the partition function $\mathcal{Z}_s$.

2. Weight functions

As is well known, the quasiclassical approximation does not capture contributions that originate from frequencies of the order of the Fermi energy ε_F. In the limit of weak interactions considered here we may incorporate such contributions into the model by replacing weights W_s, W_t of Eqs. (3.18) and (3.19) by weights $\mathcal{W}_s$, $\mathcal{W}_t$

$$\mathcal{W}_s[\varphi] = \exp\left[-\frac{\nu}{2}\int dxd\hat{\mathbf{n}}\varphi_{\mathbf{n}}(x)\left[\hat{\Gamma}_s^{-1}\varphi\right](x,\mathbf{n})\right], \quad (3.38)$$

$$\mathcal{W}_t[\mathbf{h}] = \exp\left[-\frac{\nu}{2}\int dxd\hat{\mathbf{n}}\mathbf{h}_{\mathbf{n}}(x)\left[\hat{\Gamma}_t^{-1}\mathbf{h}\right](x,\mathbf{n})\right], \quad (3.39)$$

where

$$\hat{\Gamma}_s = \hat{f}\frac{\nu\hat{V}_s}{1+2\nu\hat{V}_s}, \quad \hat{\Gamma}_t = \hat{f}\frac{\nu\hat{V}_t}{1-2\nu\hat{V}_t} \quad (3.40)$$

and we adopted the convention

$$[\hat{f}p](x,\mathbf{n}) = \int d\mathbf{r}_1\bar{f}(\mathbf{r}-\mathbf{r}_1)p_{\mathbf{n}}(\mathbf{r}_1,\tau), \quad (3.41)$$

$$[\hat{V}_{t,s}b](x,\mathbf{n}_1) = \int d\hat{\mathbf{n}}_2 V_{t,s}(\theta_{12})b_{\mathbf{n}_2}(x). \quad (3.42)$$

The above argument is valid, however, only for the quadratic in the $\mathbf{h}$ part of the weight function W_t entering Eq. (3.37). It does not hold for the part containing $\mathbf{b}$, since the Green's zfunction, for which the quasiclassical approximation was used, depends only on the field $\mathbf{h}$. Therefore, writing Eq. (3.37) we should use the following form for the weight $W_t[\mathbf{h}-\mathbf{b}]$:

$$W_t[\mathbf{h}-\mathbf{b}] \to \mathcal{W}_t[\mathbf{h},\mathbf{b}] = \mathcal{W}_t[\mathbf{h}]\exp\left[-\frac{1}{2}\int dx\mathbf{b}(x)\left[\hat{V}_t^{-1}\mathbf{b}\right](x)\right] \times\exp\left[\int dxd\hat{\mathbf{n}}\mathbf{b}(x)\left[\hat{V}_t^{-1}\mathbf{h}\right](x,\mathbf{n})\right]. \quad (3.43)$$

Here and in what follows we omit the cutoff function f, whenever the momentum is determined by the external field $\mathbf{b}$. This cannot change results since we are interested only in small external momenta, $|q| \ll r_0^{-1}$, which allows us to put $f(\mathbf{q})=1$.

It is clear from Eq. (3.37) that the magnetic field couples only to the spin degrees of freedom and the charge sector does not play any role for the spin susceptibility. Therefore, we can concentrate on the spin sector, described by Eq. (3.37) with the weight $W_t[\mathbf{h}-\mathbf{b}]$ determined by Eq. (3.43).

IV. SUPERSYMMETRIC REPRESENTATION

In Sec. IV A we derive a representation for $\mathcal{Z}_s$, Eq. (3.37), in terms of a functional integral over superfields. A detailed derivation has been presented in Ref. 1. Therefore we only highlight the main ideas here and relegate more technical details of the construction of the model to Appendix A. In Sec. IV B we collect the relevant definitions of supervectors and supermatrices that enter the final model. This model is then presented in Sec. IV C, rules of the perturbation theory are formulated in Sec. IV D, and a convenient diagrammatic representation is introduced in Sec. IV E.

A. $\mathcal{Z}_s$ as an integral over supervectors

Using Eq. (3.34) and Eqs. (3.28) and (3.30) one arrives at the following form for the partition function $\mathcal{Z}_s[\mathbf{h}]$:

$$\mathcal{Z}_s[\mathbf{h}] = \exp\left(2\nu\int dudxd\hat{\mathbf{n}}\mathbf{h}_{\mathbf{n}}(x)\left[u\hat{L}_{\mathbf{n},u}^{-1}\partial_\tau\mathbf{h}_{\mathbf{n}}\right](x)\right). \quad (4.1)$$

If one could find $\hat{L}_{\mathbf{n},u}^{-1}$ exactly for all $u\neq 0$, the problem would be solved. However, since this is hardly possible for an arbitrary u and $\mathbf{h}$, we have to resort to some approximation scheme. For this purpose it is advantageous to reexpress $\hat{L}_{\mathbf{n},u}^{-1}$ in terms of a Gaussian functional integral. Using either bosonic (complex) or fermionic (Grassmann) fields separately one would have to deal with an $\mathbf{h}$-dependent normalization factor of the Gaussian integral, which is inconvenient. This complication can be avoided by introducing an integral that includes both bosonic and fermionic variables on equal footing, as it has been used for a long time in the theory of disordered systems, where the technique is known as the supersymmetry method.[32,33] In the context of the present problem this approach has been introduced in Ref. 1.

When using the Gaussian functional integration one should be careful, however, since the operator $\hat{L}$ is not Hermitian. In particular, the sign of the $\mathbf{h}$-dependent term is not fixed and thus the convergence of the Gaussian integral over

bosonic variables, for which one requires a positive real part of the kernel, is not easily ensured. Fortunately, it is known how to overcome this difficulty.[37] One can construct from the operator $\hat{L}$ Hermitian operators $\hat{L}'=(\hat{L}+\hat{L}^\dagger)/2$ and $\hat{L}''=-i(\hat{L}-\hat{L}^\dagger)/2$ and arrange them into a new *Hermitian* matrix operator

$$\hat{M}=\begin{pmatrix} \hat{L}' & i\hat{L}'' \\ -i\hat{L}'' & -\hat{L}' \end{pmatrix}_H. \tag{4.2}$$

The corresponding vector space will be called "Hermitized" or H space. One can reconstruct $\hat{L}^{-1}$ by summing certain matrix elements of the inverse of $\hat{M}^{-1}$ as was shown in Ref. 37.

The implementation of the ideas presented above leads to the following identity:

$$\mathcal{Z}_s[\mathbf{h}]=\exp\left(\nu\int_{XX'}\bar{\mathcal{F}}_{\mathbf{h}}(X)\mathcal{H}^{-1}_{X,X'}\mathcal{F}_{\mathbf{h}}(X')\right). \tag{4.3}$$

Here we use the collective variables

$$X=(x,z),\quad x=(\mathbf{r},\tau),\quad z=(u,\mathbf{n}) \tag{4.4}$$

and the integration measure is specified as follows:

$$\int dX=\int_X=\int dxdz,\quad \int dz=\int_0^1 du\int d\mathbf{n}, \tag{4.5}$$

where $\int dx$ has been introduced in Eq. (2.6).

To make contact with the previous discussion, we note that $(\Lambda\mathcal{H})^{-1}$ corresponds to $\hat{M}^{-1}$ of Eq. (4.2), where Λ is some constant matrix introduced below. $\bar{\mathcal{F}}_{\mathbf{h}}$ and $\mathcal{F}_{\mathbf{h}}$ are linear in $\mathbf{h}$ and their role is merely to select relevant components of $\mathcal{H}^{-1}$, the sum of which gives $\hat{L}^{-1}_{\mathbf{n}}$. The operator $\mathcal{H}^{-1}_{X,X'}$ can be written in terms of a Gaussian functional integral as follows:

$$\begin{aligned}-\frac{1}{4i\nu}\mathcal{H}^{-1}_{X,X'}&=\langle\boldsymbol{\psi}_X\bar{\boldsymbol{\psi}}_{X'}\rangle\\&=\int\mathcal{D}(\boldsymbol{\psi},\bar{\boldsymbol{\psi}})\boldsymbol{\psi}_X\bar{\boldsymbol{\psi}}_{X'}\\&\quad\times\exp\left(2i\nu\int_X\bar{\boldsymbol{\psi}}_X(\mathcal{H}+i\delta\Lambda)\boldsymbol{\psi}_X\right),\end{aligned} \tag{4.6}$$

where $\boldsymbol{\psi}$ and $\bar{\boldsymbol{\psi}}=\boldsymbol{\psi}^\dagger\Lambda$ are supervectors (see below). They contain both complex and Grassmann variables on equal footing, which leads to the simple normalization of the Gaussian integral

$$1=\int\mathcal{D}(\boldsymbol{\psi},\bar{\boldsymbol{\psi}})\exp\left(2i\nu\int_X\bar{\boldsymbol{\psi}}_X(\mathcal{H}+i\delta\Lambda)\boldsymbol{\psi}_X\right). \tag{4.7}$$

The identities, Eqs. (4.3) and (4.6), are the basic building blocks for the derivation of the model we want to work with. In the next section we will define all quantities involved in more detail, making it possible to verify Eq. (4.3) by explicit computation. *Construction* of the theory that follows Ref. 1 and adopts the notations used in this paper is included in Appendix A for the interested reader.

It may be worth making a comment concerning the angular integration $\int d\mathbf{n}$. It will turn out later that for our purposes the most important scattering processes are forward and backward scattering. By forward scattering we mean scattering processes, in which both the incoming and outgoing fermions have almost parallel momenta. Backward scattering refers to a process, in which both incoming and outgoing momenta for each fermion are almost antiparallel to each other, while incoming and outgoing momenta of different fermions are almost parallel.

It is then convenient to split the angular integration $\int d\hat{\mathbf{n}}$ into two half-spheres, one half-sphere contains "left movers" the other one "right movers." The arbitrariness involved in fixing the boundary in dimension $d>1$ will not become important due to the quasi-one-dimensional character of the relevant scattering processes. When separating sectors of left and right moving particles it is then only necessary to integrate over one half sphere and we denote this angular integration as $\int d\mathbf{n}$ with normalization $\int d\mathbf{n}=1/2$. As an example, the angular integration in Eq. (4.1) is now written

$$\int d\hat{\mathbf{n}}\mathbf{h}_{\mathbf{n}}\hat{L}^{-1}_{\mathbf{n}}\partial_\tau\mathbf{h}_{\mathbf{n}}=\int d\mathbf{n}\mathbf{h}_{\mathbf{n}}\hat{L}^{-1}_{\mathbf{n}}\partial_\tau\mathbf{h}_{\mathbf{n}}+\mathbf{h}_{-\mathbf{n}}\hat{L}^{-1}_{-\mathbf{n}}\partial_\tau\mathbf{h}_{-\mathbf{n}}. \tag{4.8}$$

Let us perform two more manipulations to arrive at a form, where only the averaging with weight $\mathcal{W}_t$ remains to be done. Starting from Eq. (4.7) one may verify by shifting fields $\boldsymbol{\psi}$, $\bar{\boldsymbol{\psi}}$, that

$$\begin{aligned}\mathcal{Z}_s[\mathbf{h}]&=\exp\left(\nu\int_{XX'}\bar{\mathcal{F}}_{\mathbf{h}}(X)\mathcal{H}_{X,X'}\mathcal{F}_{\mathbf{h}}(X')\right)\\&=\int\mathcal{D}(\boldsymbol{\psi},\bar{\boldsymbol{\psi}})\exp\left(2i\nu\int_X\bar{\boldsymbol{\psi}}_X(\mathcal{H}+i\delta\Lambda)\boldsymbol{\psi}_X\right)\\&\quad\times\exp\left(\sqrt{-2i}\nu\int_X[\bar{\mathcal{F}}_{\mathbf{h}}(X)\boldsymbol{\psi}_X+\bar{\boldsymbol{\psi}}_X\mathcal{F}_{\mathbf{h}}(X)]\right).\end{aligned} \tag{4.9}$$

Just as $\hat{L}$, $\mathcal{H}$ contains a part $\mathcal{H}_{\mathbf{h}}$, that is linear in the field $\mathbf{h}$. We split off this part by writing

$$\mathcal{H}=\mathcal{H}_0+\mathcal{H}_{\mathbf{h}}. \tag{4.10}$$

The final form of the model is then obtained by averaging the $\mathbf{h}$-dependent part of $\mathcal{Z}_s[\mathbf{h}]$ with the weight $\mathcal{W}_t[\mathbf{h}-\mathbf{b}]$ [compare with Eq. (3.37)].

$$\mathcal{Z}_s=\int\mathcal{D}(\boldsymbol{\psi},\bar{\boldsymbol{\psi}})\exp\left(2i\nu\int_X\bar{\boldsymbol{\psi}}_X(\mathcal{H}_0+i\delta\Lambda)\boldsymbol{\psi}_X\right)\mathcal{B}[\boldsymbol{\psi},\bar{\boldsymbol{\psi}},\mathbf{b}] \tag{4.11}$$

$$\mathcal{B}[\boldsymbol{\psi},\bar{\boldsymbol{\psi}},\mathbf{b}] = \int \mathcal{D}\mathbf{h}\mathcal{W}_t[\mathbf{h}-\mathbf{b}]\exp\left[2i\nu\int_X \bar{\boldsymbol{\psi}}_X\mathcal{H}_{\mathbf{h}}\boldsymbol{\psi}_X\right] \times\exp\left(\sqrt{-2i\nu}\int_X [\bar{\mathcal{F}}_{\mathbf{h}}(X)\boldsymbol{\psi}_X + \bar{\boldsymbol{\psi}}_X\mathcal{F}_{\mathbf{h}}(X)]\right). \tag{4.12}$$

In the next subsection we will give the precise definition for $\boldsymbol{\psi},\bar{\boldsymbol{\psi}},\mathcal{F},\bar{\mathcal{F}}$, and $\mathcal{H}$. Equations (4.3) and (4.6) are discussed in Appendix A. What remains then is to obtain the final model by explicitly computing $\mathcal{B}$, Eq. (4.12).

B. Relevant supervectors and supermatrices

1. Supervector $\boldsymbol{\psi}$ and its conjugation

Let us first introduce the supervector $\boldsymbol{\psi}$ depending on coordinates $X=(x,z)$ [cf. Eq. (4.4)]. It has components in the sectors of left- and right-moving particles labeled as n, the graded space of bosonic $\mathbf{S}$ and fermionic $\boldsymbol{\chi}$ variables labeled as g, the Hermitized space labeled by H, and the spin space labeled by s. An additional sector is introduced, which simplifies calculations with the model. It has been termed "electron-hole" eh space in Ref. 1 and plays a similar role as the time-reversal sector in the σ-model description of disordered systems (Ref. 33)

$$\boldsymbol{\psi} = \frac{1}{\sqrt{2}}\begin{pmatrix}\boldsymbol{\phi}^*\\ \boldsymbol{\phi}\end{pmatrix}_{eh}, \quad \boldsymbol{\phi}(\mathbf{n}) = \begin{pmatrix}\boldsymbol{\varphi}(\mathbf{n})\\ \boldsymbol{\varphi}(-\mathbf{n})\end{pmatrix}_n, \tag{4.13}$$

where

$$\boldsymbol{\varphi} = \begin{pmatrix}\boldsymbol{\chi}\\ \mathbf{S}\end{pmatrix}_g, \quad \boldsymbol{\chi} = \begin{pmatrix}\boldsymbol{\chi}^1\\ \boldsymbol{\chi}^2\end{pmatrix}_H, \quad \mathbf{S} = \begin{pmatrix}\mathbf{S}^1\\ \mathbf{S}^2\end{pmatrix}_H. \tag{4.14}$$

Both $\mathbf{S}^i$ and $\boldsymbol{\chi}^i$ are vectors in the spin space

$$\mathbf{S}^i = \begin{pmatrix}S^i_x\\ S^i_y\\ S^i_z\end{pmatrix}_s, \quad \boldsymbol{\chi}^i = \begin{pmatrix}\chi^i_x\\ \chi^i_y\\ \chi^i_z\end{pmatrix}_s. \tag{4.15}$$

The components $\chi^i_x,\chi^i_y,\chi^i_z$ are anticommuting (Grassmann) fields.

The conjugate vector $\bar{\boldsymbol{\psi}}$ is defined

$$\bar{\boldsymbol{\psi}} = \boldsymbol{\psi}^\dagger\Lambda, \tag{4.16}$$

where

$$\Lambda = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}_H \tag{4.17}$$

is the third Pauli matrix in the Hermitized space. An important symmetry that arises due to introducing the (eh) sector is

$$\bar{\boldsymbol{\psi}} = (C\boldsymbol{\psi})^T, \tag{4.18}$$

where C is the following matrix:

$$C = \begin{pmatrix}C_0 & 0\\ 0 & -C_0\end{pmatrix}_H, \quad C_0 = \begin{pmatrix}c_1 & 0\\ 0 & c_2\end{pmatrix}_g \tag{4.19}$$

and matrices c_i have structure in the eh sector.

$$c_1 = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}_{eh}, \quad c_2 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}_{eh}. \tag{4.20}$$

For Grassmann variables the convention $(\chi^*)^* = -\chi$ is used. The conjugation of matrices is introduced as follows:

$$\bar{A} = CA^TC^T, \tag{4.21}$$

where the special transposition appropriate for supermatrices[33] should be used. The important property

$$\bar{\psi}A\phi = \bar{\phi}\bar{A}\psi, \tag{4.22}$$

where ψ, ϕ are supervectors, is one of the main motivations for introducing the (eh) sector. When calculating higher cumulants later using Wick's theorem the number of contractions can be reduced considerably with the help of relation Eq. (4.22).

2. The matrix $\mathcal{H}$

The matrix $\mathcal{H}$ is split into an $\mathbf{h}$-dependent and an $\mathbf{h}$-independent part

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\mathbf{h}}$$

$$\mathcal{H}_0 = -iv_0\tau_3\Sigma_3\mathbf{n}\nabla - \Lambda_1\partial_\tau, \quad \mathcal{H}_{\mathbf{h}} = -2i\tau_3\hat{\mathbb{H}}_{\mathbf{n}}. \tag{4.23}$$

Different constant matrices in this expression are

$$\Lambda_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}_H, \quad \Sigma_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}_n, \quad \tau_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}_{eh}, \tag{4.24}$$

and

$$\mathbb{H}_{\mathbf{n}}(x) = \begin{pmatrix}\hat{h}_{\mathbf{n}}(x) & 0\\ 0 & \hat{h}_{-\mathbf{n}}(x)\end{pmatrix}_n. \tag{4.25}$$

3. Vector $\mathcal{F}_{\mathbf{h}}$

Vector $\mathcal{F}$ does not have the full symmetry in supersymmetric g space. Instead, it projects onto the bosonic sector. The role of the fermionic fields in Eq. (4.9) is only to provide the normalization. We present $\mathcal{F}_{\mathbf{h}}$ as a product of an $\mathbf{h}$-dependent and an $\mathbf{h}$-independent part. The latter one is

$$\mathcal{F}_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ 1\end{pmatrix}_g \otimes \begin{pmatrix}1\\ 1\end{pmatrix}_n \otimes \begin{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix}_{eh}\\ \begin{pmatrix}-1\\ 1\end{pmatrix}_{eh}\end{pmatrix}_H. \tag{4.26}$$

The charge conjugated vector is $\bar{\mathcal{F}}_0 = (C\mathcal{F}_0)^T$, where only c_2 is effective when evaluating the right-hand side. $\mathcal{F}_{\mathbf{h}}$ and $\bar{\mathcal{F}}_{\mathbf{h}}$ are then given as follows:

$$\mathcal{F}_{\mathbf{h},\alpha}(X) = \partial_X(\alpha)\mathbb{H}_{\mathbf{n}}(x)\mathcal{F}_0, \tag{4.27}$$

and $\bar{\mathcal{F}}_{\mathbf{h}} = (C\mathcal{F}_{\mathbf{h}})^T$, where

$$\partial_X(\alpha) = \begin{pmatrix} 1 & 0 \\ 0 & u[\alpha\partial_\tau + (1-\alpha)iv_0\mathbf{n}\boldsymbol{\nabla}\Sigma_3] \end{pmatrix}_{eh}. \quad (4.28)$$

Here an additional parameter α has been introduced into the model and we will comment on it in the following subsection.

4. Parameter α and the weight $\mathcal{W}_t$

In view of Eq. (4.1) one would expect only ∂_τ to enter Eq. (4.28), corresponding to $\alpha=1$. Choosing different values of α, however, may be convenient as we will see below when studying the renormalization of the model. We will set $\alpha=1/2$ there, treating temporal and spatial derivatives in a symmetric way. For $\alpha\neq 1$ relation Eq. (4.4) needs to be modified and this modification eventually changes the weight $\mathcal{W}_t$. Any physical quantity calculated with the model is of course independent of the choice of α. As is shown in Appendix A, when introducing parameter $\alpha\neq 1$, relation Eq. (4.4) takes the form

$$\mathcal{Z}_s[\mathbf{h}] = \exp\left(-\nu(1-\alpha)\int_{\hat{\mathbf{n}},x}\mathbf{h}_{\mathbf{n}}^2(x)\right) \times \exp\left(-4i\nu^2\int_{XX'}\overline{\mathcal{F}_{\mathbf{h},\alpha}}(X)\langle\boldsymbol{\psi}_X\bar{\boldsymbol{\psi}}_{X'}\rangle\mathcal{F}_{\mathbf{h},\alpha}(X')\right). \quad (4.29)$$

For $\alpha=1$ it coincides with Eq. (4.4) and one can use the form for $\mathcal{B}$ given in Eq. (4.12). For general α it seems natural to absorb the exponential in the first line of Eq. (4.29), into the weight $\mathcal{W}_t$ and thus change $\mathcal{B}$ to

$$\mathcal{B}[\boldsymbol{\psi},\bar{\boldsymbol{\psi}},\mathbf{b}] = \int \mathcal{D}\mathbf{h}\mathcal{W}_t[\mathbf{h},\mathbf{b},\alpha]\exp\left[2i\nu\int_X \bar{\boldsymbol{\psi}}_X\mathcal{H}_{\mathbf{h}}\boldsymbol{\psi}_X\right] \times \exp\left[\sqrt{-2i}\nu\int_X[\overline{\mathcal{F}_{\mathbf{h},\alpha}(X)}\boldsymbol{\psi}_X + \overline{\boldsymbol{\psi}_X}\mathcal{F}_{\mathbf{h},\alpha}(X)]\right], \quad (4.30)$$

where

$$\mathcal{W}_t[\mathbf{h},\mathbf{b},\alpha] = \mathcal{W}_t[\mathbf{h},\mathbf{b}]e^{-\nu(1-\alpha)\int_{\hat{\mathbf{n}},x}\mathbf{h}_{\mathbf{n}}^2(x)}. \quad (4.31)$$

Clearly, this change in $\mathcal{W}_t$ only affects the quadratic form in $\mathbf{h}$ but not the part containing $\mathbf{b}$. Therefore, to make the change explicit we may write here

$$\mathcal{W}_t[\mathbf{h},\mathbf{b}=0,\alpha] = \exp\left[-\frac{\nu}{2}\int dx(d\mathbf{n})\mathbf{h}_{\mathbf{n}}(x)[\hat{\Gamma}_t^{-1}(\alpha)\mathbf{h}](x,\mathbf{n})\right], \quad (4.32)$$

where

$$2\hat{\Gamma}_t(\alpha) = \hat{f}\frac{2\nu\hat{V}_t}{1-2\nu\alpha\hat{V}_t}. \quad (4.33)$$

The final step in the derivation of the model is the calculation of $\mathcal{B}$ in Eq. (4.30).

C. Effective low energy theory

From Eq. (4.11) together with $\mathcal{B}$ given in Eq. (4.30) we find

$$\mathcal{Z}_s = \int \mathcal{D}(\boldsymbol{\psi},\bar{\boldsymbol{\psi}})\exp\left(-\sum_i \mathcal{S}_i\right). \quad (4.34)$$

Next we specify the different parts $\mathcal{S}_i$ of the effective action.

The interaction-independent part is

$$\mathcal{S}_0 = -2i\nu\int dX\bar{\boldsymbol{\psi}}_X(\mathcal{H}_0 + i\delta\Lambda)\boldsymbol{\psi}_X. \quad (4.35)$$

There are three different interaction vertices present in the theory

$$\mathcal{S}_2 = -2i\nu\sum_{ij}\lambda_{ij}\int dXdX_1 \times(\bar{\psi}_{X,\delta}\tau_3\Pi_j\partial_X\mathcal{F}_0)\Gamma^i_{X,X_1}(\overline{\mathcal{F}_0\partial_{X_1}}\Pi_j\tau_3\psi_{X_1,\delta}), \quad (4.36)$$

$$\mathcal{S}_3 = -4\sqrt{-2i}\nu\sum_{ij}\lambda_{ij}\varepsilon_{\delta\beta\gamma}\int dXdX_1 \times(\bar{\psi}_{X,\delta}\tau_3\Pi_j\partial_X\mathcal{F}_0)\Gamma^i_{X,X_1}(\overline{\mathcal{F}_0\partial_{X_1}}\Pi_j\tau_3\psi_{X_1,\delta}), \quad (4.37)$$

$$\mathcal{S}_4 = -4\nu\sum_{ij}\lambda_{ij}\varepsilon_{\delta\beta\gamma}\varepsilon_{\delta\beta_1\gamma_1}\int dXdX_1 \times(\bar{\psi}_{X,\beta}u\tau_3\Pi_j\psi_{X,\gamma})\Gamma^i_{X,X_1}(\bar{\psi}_{X_1,\beta_1}u_1\tau_3\Pi_j\psi_{X_1,\gamma_1}). \quad (4.38)$$

Summation over spin indices is implied and we use the totally antisymmetric tensor $\varepsilon_{\alpha\beta\gamma}$ with $\varepsilon_{123}=1$. This part of the action would be sufficient for a calculation of the thermodynamic potential in the absence of a magnetic field and it coincides with the action written in Ref. 1. Here we used the notation

$$\Gamma^i_{X,X'} = \gamma_i(\widehat{\mathbf{nn}'})f(\mathbf{r}-\mathbf{r}')\delta(\tau-\tau') \quad (4.39)$$

and

$$\gamma_1(\widehat{\mathbf{nn}_1}) = \left(\frac{\nu\hat{V}_t}{1-2\nu\alpha\hat{V}_t}\right)(\widehat{\mathbf{n},\mathbf{n}_1}) \equiv \gamma_f^0, \quad (4.40)$$

$$\gamma_2(\widehat{\mathbf{nn}_1}) = \left(\frac{\nu\hat{V}_t}{1-2\nu\alpha\hat{V}_t}\right)(\widehat{\mathbf{n},-\mathbf{n}_1}) \equiv \gamma_b^0. \quad (4.41)$$

Matrices Π_i are

$$\Pi_1 = \mathbb{1}, \quad \Pi_2 = \Sigma_3. \quad (4.42)$$

The form of

$$\lambda_{ij} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad (4.43)$$

was determined from the following identities:

$$\Sigma_+ A \Sigma_+ + \Sigma_- A \Sigma_- = \frac{1}{2}(\Pi_1 A \Pi_1 + \Pi_2 A \Pi_2), \tag{4.44}$$

$$\Sigma_+ A \Sigma_- + \Sigma_- A \Sigma_+ = \frac{1}{2}(\Pi_1 A \Pi_1 - \Pi_2 A \Pi_2). \tag{4.45}$$

Taking into account that relevant scattering events are quasi-one-dimensional, as will be seen later, Eqs. (4.44) and (4.45) also explain the labeling in Eqs. (4.40) and (4.41), where γ_1 is classified as forward scattering and γ_2 as backward scattering.

The presence of a magnetic field introduces three more terms, namely

$$\mathcal{S}_{b0} = -\nu\eta \int dx \mathbf{b}^2(x), \tag{4.46}$$

$$\mathcal{S}_{b1} = -2\nu\sqrt{-2i}\eta \int dX b_\delta(x)(\bar{\psi}_{X,\delta}\tau_3\partial_X \mathcal{F}_0), \tag{4.47}$$

$$\mathcal{S}_{b2} = 4\nu\varepsilon_{\delta\beta\gamma}\eta \int dX\, b_\delta(x)(\bar{\psi}_{X,\beta} u \tau_3 \psi_{X,\gamma}). \tag{4.48}$$

In these expressions

$$\eta = \frac{1}{1 - 2\alpha\nu\overline{V_t}}, \tag{4.49}$$

where the bar in $\overline{V_t}$ means averaging over the full solid angle. We remind that η, the interaction amplitudes Γ_i as well as ∂_X, $\bar{\partial}_X$ depend on parameter α introduced in (4.28). Here we suppressed the label for the sake of brevity.

D. Rules of perturbation theory

1. Gaussian averages

A perturbation theory can be set up in a standard way using a cumulant expansion and Wick's theorem. Gaussian averages are taken with respect to $\mathcal{S}_0$ and it is therefore convenient to work with the matrix Green's function

$$\hat{\mathcal{G}}(X_1, X_2) = -4i\nu\langle \boldsymbol{\psi}_{X_1} \otimes \bar{\boldsymbol{\psi}}_{X_2}\rangle_0, \tag{4.50}$$

$$\langle(\cdots)\rangle_0 = \int \mathcal{D}\boldsymbol{\psi}(\cdots)\exp(-\mathcal{S}_0[\boldsymbol{\psi}]). \tag{4.51}$$

Due to supersymmetry no normalization factor arises. $\hat{\mathcal{G}}$ is a matrix in spin space but its spin structure is trivial and we denote

$$\hat{\mathcal{G}}_{\alpha\beta}(X_1, X_2) = \mathcal{G}_{\mathbf{n}_1}(x_1 - x_2)\delta_{\alpha\beta}\delta_{\mathbf{n}_1,\mathbf{n}_2}\delta(u_1 - u_2). \tag{4.52}$$

The Fourier transform of $\mathcal{G}$ is introduced as follows:

$$\mathcal{G}_{\mathbf{n}_1}(x_1 - x_2) = T\sum_{\omega_n} \int d\mathbf{p}\mathcal{G}_{\mathbf{n}_1}(\mathbf{p}, \omega_n) e^{i\mathbf{p}(\mathbf{r}_1 - \mathbf{r}_2) - i\omega_n(\tau_1 - \tau_2)}. \tag{4.53}$$

Here $\omega_n = 2\pi T n$ are bosonic Matsubara frequencies and

$$\mathcal{G}_{\mathbf{n}}(\mathbf{p}, \omega_n) = \frac{1}{v_F \mathbf{n}\mathbf{p}\tau_3\Sigma_3 + i\omega\Lambda_1 - i\delta\Lambda_1}. \tag{4.54}$$

Similar to the specific heat,[1], the temperature dependence of the susceptibility is determined by nonzero Matsubara frequencies only. Therefore, the term containing the infinitesimal δ in the Green's functions will not become important in our calculations and will not be written from now on. The matrix Green's function $\mathcal{G}_{\mathbf{n}}$ is diagonal in spin space.

In addition to averages of the type written in Eq. (4.50) one has to account for nonstandard averages of the type $\langle \psi_{X_1,\alpha}\psi_{X_2,\beta}\rangle_0 \neq 0$ and $\langle \bar{\psi}_{X_1,\alpha}\bar{\psi}_{X_2,\beta}\rangle_0 \neq 0$, which arise due to the *eh* sector. It is, however, sufficient to work with Eq. (4.50), since expressions involving such nonstandard averages can easily be transformed to a more standard form with the help of the relation $\bar{\boldsymbol{\psi}}_{X_1} A \boldsymbol{\psi}_{X_2} = \bar{\boldsymbol{\psi}}_{X_2} \bar{A} \boldsymbol{\psi}_{X_1}$, which is valid for any supermatrix A. The charge conjugation operation $A \to \bar{A}$ has been defined in Eq. (4.21).

It is often convenient to work with a generalization of the trace operation tr, used for conventional matrices, to the so-called supertrace str.[33] It is defined

$$\mathrm{str}\begin{pmatrix} a & \sigma \\ \rho & b \end{pmatrix}_g = \mathrm{tr}a - \mathrm{tr}b. \tag{4.55}$$

Two more useful relations are

$$\mathrm{str}(AB) = \mathrm{str}(BA), \tag{4.56}$$

$$\bar{\psi}_{1,\alpha} A \psi_{2,\beta} = -\mathrm{str}(A(\psi_{2,\alpha}\bar{\psi}_{1,\beta})). \tag{4.57}$$

A, B are arbitrary supermatrices. It follows from the definition of the supertrace that for matrices A_i that have the full symmetry in g space, $\mathrm{str}(A_i) = 0$. The matrix Green's function $\mathcal{G}$ is a particularly important example. One immediately concludes that the following relations (and straightforward generalizations thereof) hold for such matrices

$$\langle(\bar{\boldsymbol{\psi}} A_1 \boldsymbol{\psi})\rangle_0 = \langle(\bar{\boldsymbol{\psi}} A_1 \boldsymbol{\psi})(\bar{\boldsymbol{\psi}} A_2 \boldsymbol{\psi})\rangle_0 = 0. \tag{4.58}$$

These important relations considerably reduce the number of diagrams to be considered in the perturbation theory.

E. Diagrammatic representation

Figure 1 displays the building blocks that we will use for the diagrammatic representation of the perturbation theory. The three interaction vertices reflect the structure of $\mathcal{S}_2$, $\mathcal{S}_3$, and $\mathcal{S}_4$. They are plotted in such a way that *small* momenta flow *along* the interaction line. The dotted lines symbolize the structure $\mathcal{F}_0\bar{\mathcal{F}}_0$. These lines carry neither momentum nor frequency. They are, however, convenient to make contact with conventional diagrams formulated in terms of electron Green's functions as we will discuss now.

To this end it is instructive to make a comparison with the original model. Since the charge channel has been separated, we need to consider fermions interacting in the triplet channel only, i.e., the free action Eq. (2.3) and an interaction part of the form

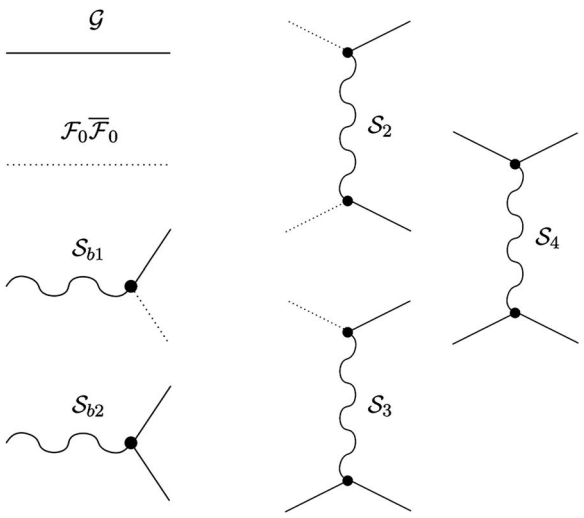

FIG. 1. Basic building blocks of the perturbation theory.

$$\mathcal{S}_{int,t} = -\frac{1}{2}\int_{p_1 p_2 q} \mathbf{S}(p_1,-q)\Gamma_t(\theta_{12})\mathbf{S}(p_2,q), \qquad (4.59)$$

where we remind that $\mathbf{S}(p,q)=\chi^\dagger(p-\frac{q}{2})\boldsymbol{\sigma}\chi(p+\frac{q}{2})$ and we used four-dimensional notation for momenta and energies.

In contrast, the effective low energy model of Eq. (4.34) is formulated in terms of spin modes. The structure of the terms appearing after expansion of $\int \mathbf{h}\hat{L}_{\mathbf{h}}\partial_\tau \mathbf{h}$ in Eq. (4.1) in the field $\mathbf{h}$ is the translation to the spin mode language of a closed Fermion loop with $n \geq 2$ fields $\mathbf{h}$ coupled to it. The expansion of $\hat{L}_{\mathbf{h}}$ in powers of $\mathbf{h}$ is performed here assuming that $\hat{L}_{\mathbf{h}=0} \propto (v_0 \mathbf{np} - i\omega)^{-1}$ describes free propagation of spin modes [compare to $\mathcal{S}_0$ of Eq. (4.35)]. After integration over $\mathbf{h}$, which reduces to contracting pairs of fields $\langle \mathbf{hh} \rangle$, one obtains a theory of *interacting* spin modes. Not all fields $\mathbf{h}$ in $\int \mathbf{h}\hat{L}_{\mathbf{h}}\partial_\tau \mathbf{h}$ enter in an equivalent way, however, and this explains the presence of three different interaction terms in the model. When contracting two fields $\mathbf{h}$ that appear due to an expansion of $\hat{L}_{\mathbf{h}}$ in $\mathbf{h}$, one finds an interaction vertex of the type represented by $\mathcal{S}_4$ (see Fig. 1). If only one such field is involved one comes to $\mathcal{S}_3$, otherwise to $\mathcal{S}_2$.

Let us summarize the discussion with the help of Figs. 2 and 3. Here a closed loop of fermionic Green's functions

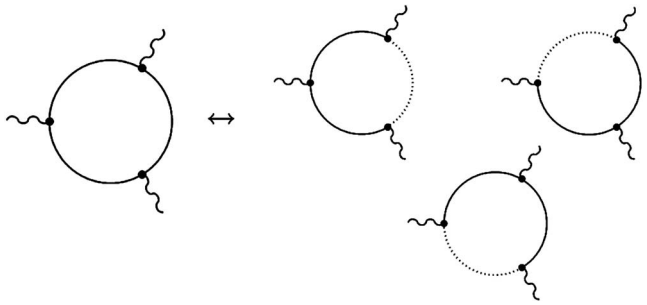

FIG. 2. A closed loop of fermionic Green's function contrasted with the spin mode representation.

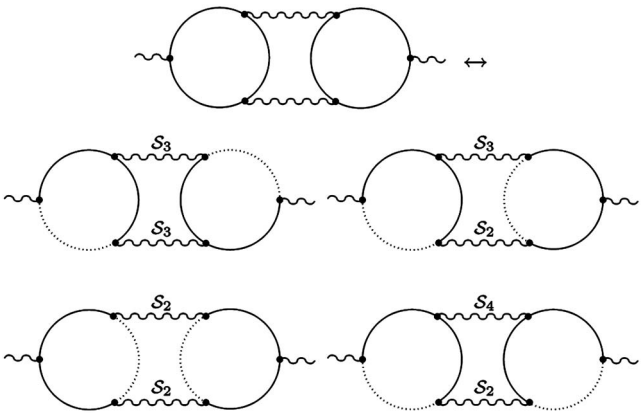

FIG. 3. A particular diagram for the susceptibility formulated in the fermionic and the spin mode representation.

(Fig. 2) as well as a particular diagram for the susceptibility (Fig. 3) are shown formulated first in terms of fermionic Green's functions and then also as a diagram for interacting spin modes in the effective theory. We see that one diagram of the conventional perturbation theory produces several diagrams of the expansion in the spin modes (we marked the corresponding interactions vertices by $\mathcal{S}_i$, $i=2,3,4$).

At first glance it looks as if our effective perturbation theory has become even more complicated than the original one. However, it is not so because it is not sufficient to just write the diagrams. One should calculate them singling out the most interesting low energy contributions. This singling out has already been performed when deriving the effective theory for the spin excitations. A larger number of the diagrams in the theory corresponds to different possibilities of obtaining low energy contributions when integrating in the diagrams of the conventional perturbation theory.

The perturbative expansion obtained from the low energy effective action is equivalent to expressions obtained from Eqs. (2.3) and (4.59) after expanding $\xi_{\mathbf{p}+\mathbf{q}} \sim \xi_{\mathbf{p}} + v_F \mathbf{nq}$ in the vicinity of the Fermi surface in each loop and subsequent integration in $\xi_{\mathbf{p}}$. The present model, however, organizes the terms in a way that is much more convenient for identifying the most important contributions.

Let us remark that the field $\mathbf{b}$ enters the diagrams in the same way as $\mathbf{h}$ and that for each loop there is one fixed angle $\mathbf{n}$, which is as a direct consequence of the integration over $\xi_{\mathbf{p}}$.

F. Bare spin susceptibility

The static spin susceptibility χ at $T=0$,

$$\chi = 2\nu\eta_{\alpha=1}, \qquad (4.60)$$

where η is given by Eq. (4.49), can be obtained straightforwardly from $\mathcal{S}_{b0}$, Eq. (4.46), for $\alpha=1$. For representations with an arbitrary $\alpha \neq 1$, the term $\mathcal{S}_{b0}$ alone does not provide the full answer. For general α one should consider the combination $\mathcal{S}_{b0} - 1/2\langle\langle \mathcal{S}_{b1}^2 \rangle\rangle$ with $\mathcal{S}_{b1}$ from Eq. (4.47). The average in this formula is to be taken with the full quadratic form $\mathcal{S}_0 + \mathcal{S}_2$. This procedure effectively amounts to a ladder summation. In particular, we will later choose $\alpha = 1/2$. In this

case one arrives at Eq. (4.60) with the help of the identity $2\nu\eta_1=2\nu\eta_{1/2}+2\nu\eta_{1/2}^2(1-\nu\bar{V}_t\eta_{1/2})^{-1}$, where the first term is obtained from $\mathcal{S}_{b0}$ and the second one from $-1/2\langle\langle\mathcal{S}_{b1}^2\rangle\rangle$.

V. RENORMALIZATION

As sketched in the Introduction, logarithmic corrections appear in the model and they can be summed in a renormalization scheme as it has been done for the interaction amplitudes in Ref. 1. The appearing logarithms and thus the renormalized amplitudes generally depend on the deviation in angles from the ideal forward or backward scattering case. It is therefore immediately clear that this renormalization scheme cannot include as effective charges physical quantities like the susceptibility that do not depend on such angles. This is why we consider the renormalization of interaction amplitudes and external vertices first and then include the renormalized values into a perturbation theory for the susceptibility.

A. Generalization

During the process of the renormalization additional terms may appear in the action. To consistently take such terms into account, they should be included into the model from the beginning. To this end a generalization of Eqs. (4.36)–(4.38) is introduced in Sec. V A 2. This step is prepared in Sec. V A 1.

1. Decomposition

In Eqs. (4.36)–(4.38) the following decomposition of supermatrix P was used

$$P=\sum_{i=1}^{2}P^i,\quad P^i=\frac{1}{2}\sum_{k=1}^{2}\lambda_{ik}\Pi_kP\Pi_k, \tag{5.1}$$

where $[A^1,\Sigma_3]=0$ and $\{A^2,\Sigma_3\}=0$ for arbitrary matrices A.

For a generalization we consider four supermatrices Π_i with $[\Pi_i,\Pi_j]=0$ and $\Pi_i^2=1$, namely

$$\Pi_1=1,\quad \Pi_2=\Sigma_3,\quad \Pi_3=\Lambda_1\tau_3,\quad \Pi_4=\Lambda_1\Sigma_3\tau_3. \tag{5.2}$$

We decompose a supermatrix P in such a way that $P=\Sigma_{i=1}^4P^i$ and

$$[P^1,\Pi_2]=0,\quad \{P^1,\Pi_3\}=0, \tag{5.3}$$

$$[P^2,\Pi_2]=0,\quad [P^2,\Pi_3]=0, \tag{5.4}$$

$$\{P^3,\Pi_2\}=0,\quad [P^3,\Pi_3]=0, \tag{5.5}$$

$$\{P^4,\Pi_2\}=0,\quad \{P^4,\Pi_3\}=0. \tag{5.6}$$

An explicit formula for P^i can be given

$$P^i=\frac{1}{4}\sum_{k=1}^{4}\lambda_{ik}\Pi_kP\Pi_k,\quad \lambda_{ik}=\begin{pmatrix}1&1&-1&-1\\1&1&1&1\\1&-1&1&-1\\1&-1&-1&1\end{pmatrix}. \tag{5.7}$$

It is easily seen that

$$\mathrm{str}[P_iP_j]=\delta_{ij}\mathrm{str}[P_i^2],\quad \mathrm{str}[PQ]=\sum_{k=1}^{4}\mathrm{str}[P_kQ_k]. \tag{5.8}$$

The following useful relation can be checked by direct computation:

$$\sum_{k_1,k_2=1}^{4}\lambda_{i_1k_1}\lambda_{i_2k_2}[AL_{k_1k_2}BL_{k_1k_2}]=4\delta_{i_1,i_2}\sum_{k=1}^{4}\lambda_{i_1k}[A\Pi_kB\Pi_k], \tag{5.9}$$

where $L_{k_1k_2}=\Pi_{k_1}\Pi_{k_2}$.

2. Generalized action

After this preparation we introduce the generalized model. We start with S_4,

$$\mathcal{S}_4=-2\nu\sum_{ij=1}^{4}\lambda_{ij}\varepsilon_{\delta\beta\gamma}\varepsilon_{\delta\beta_1\gamma_1}\int dXdX_1(\bar{\psi}_{X,\beta}u\tau_3\Pi_j\psi_{X,\gamma})\hat{\Gamma}_i(X,X_1)\times(\bar{\psi}_{X_1,\beta_1}u_1\tau_3\Pi_j\psi_{X_1,\gamma_1}). \tag{5.10}$$

The amplitudes $\hat{\Gamma}_i$ are

$$\hat{\Gamma}_i(X,X_1)=\Gamma_i(\widehat{\mathbf{nn}}_1;u,u_1,(\mathbf{r}-\mathbf{r}_1)^\perp)f(\mathbf{r}-\mathbf{r}_1)\delta(\tau-\tau') \tag{5.11}$$

and by comparison with Eq. (4.37) one finds their bare values

$$\Gamma_i(\theta,u,u_1;\mathbf{r}^\perp)=\begin{cases}\gamma_f^0(\theta)\ i=1,2,\\ \gamma_b^0(\theta)\ i=3,4,\end{cases} \tag{5.12}$$

where $\mathbf{r}^\perp=\mathbf{r}-(\mathbf{nr})$ is the component of $\mathbf{r}$ transverse to $\mathbf{n}$ and one should keep in mind that important initial and final angles $\mathbf{n}$, $\mathbf{n}_1$ are almost parallel or almost antiparallel to each other. The initial values for Γ_i do not depend on u, u_1, and $\mathbf{r}^\perp$ but develop such a dependence under renormalization.

The generalization of Eq. (4.36) reads

$$\mathcal{S}_2=-i\nu\sum_{ij}\sum_{\sigma_1,\sigma_2=\pm}\lambda_{ij}\int dXdX_1(\bar{\psi}_{X,\delta}\tau_3\Pi_j\partial_X\mathcal{F}_{\sigma_1})\hat{\mathbf{\Delta}}_i^{\sigma_1\sigma_2}(X,X_1)\times(\overline{\mathcal{F}_{\sigma_2}\partial_{X_1}}\Pi_j\tau_3\psi_{X_1,\delta}). \tag{5.13}$$

In this formula $\mathcal{F}_\pm=\tau_\pm\mathcal{F}_0$, $\overline{\mathcal{F}_\pm}=\bar{\mathcal{F}}_0\tau_\mp$ and $\tau_\pm=(1\pm\tau_3)/2$ are projection operators that change under charge conjugation as $\bar{\tau}_\pm=\tau_\mp$. In analogy to Eq. (5.11) we defined

$$\hat{\mathbf{\Delta}}_i^{\sigma_1\sigma_2}(X,X_1)=\mathbf{\Delta}_i^{\sigma_1\sigma_2}(\widehat{\mathbf{nn}}_1;u,u_1,(\mathbf{r}-\mathbf{r}_1)^\perp)f(\mathbf{r}-\mathbf{r}_1)\delta(\tau-\tau'). \tag{5.14}$$

Due to the relation $(\mathcal{F}_0\overline{\mathcal{F}_0})^i=0,(i=1,4)$ only (2,3) components enter the action. The bare values of the vertices are equal to

$$\mathbf{\Delta}_i^{\sigma_1\sigma_2}(\theta,u,u_1;\mathbf{r}_\perp)=\begin{cases}\gamma_f^0(\theta)\ i=2,\\ \gamma_b^0(\theta)\ i=3.\end{cases} \tag{5.15}$$

Finally, we write the cubic term in the form

$$\mathcal{S}_3 = -2\sqrt{-2i}\varepsilon_{\delta\beta\gamma}\sum_{ij}\sum_{\sigma=\pm}\lambda_{ij}\int dXdX_1(\bar{\psi}_{X,\delta}u\tau_3\Pi_j\psi_{X,\beta})$$
$$\times\hat{\mathcal{B}}_i^{\sigma}(X,X_1)(\overline{\mathcal{F}_{\sigma}\partial_{X_1}}\tau_3\Pi_j\psi_{X_1,\gamma}) \tag{5.16}$$

with

$$\hat{\mathcal{B}}_i^{\sigma}(X,X_1) = \mathcal{B}_i^{\sigma}(\mathbf{n}\hat{\mathbf{m}}_1;u,u_1,(\mathbf{r}-\mathbf{r}_1)^{\perp})f(\mathbf{r}-\mathbf{r}_1)\delta(\tau-\tau'), \tag{5.17}$$

where the bare values of this vertex are

$$\mathcal{B}_i^{\sigma}(\theta,u,u_1;\mathbf{r}_{\perp}) = \begin{cases} \gamma_f^0(\theta) & i=1,2, \\ \gamma_b^0(\theta) & i=3,4. \end{cases} \tag{5.18}$$

As we will see, in the approximation we consider, a generalization for the terms $\mathcal{S}_0, \mathcal{S}_{b1}, \mathcal{S}_{b2}$ will be necessary only in the $1d$ case.

B. Renormalization scheme

We use a standard momentum shell renormalization group scheme. Separating fast and slow fields in the action we integrate over the fast fields and determine in this way the flow of coupling constants as a function of a running cutoff. In our case this amounts to a resummation of the perturbation theory in the leading logarithmic approximation. A quantity y is expanded in a series of the form

$$y = \sum_n [\gamma \ln(\cdots)]^n a_n(\gamma), \tag{5.19}$$

and one attempts to find a Taylor expansion of $a_n(\gamma)$. We assume during the renormalization that the coupling constants γ are small, $\gamma \ll 1$.

In our case it is convenient to define fast fields $\boldsymbol{\phi}$ and slow fields $\boldsymbol{\Psi}$ with respect to the frequency only. The reason is the anisotropy in momentum. As one can see, relevant momenta $p_{\parallel}$ are of the order of ω/v_F, while momenta $\mathbf{p}_{\perp}$ do not contribute to the logarithm and enter as parameters. Thus we write

$$\boldsymbol{\psi}(X) = \boldsymbol{\Psi}(X) + \boldsymbol{\phi}(X), \tag{5.20}$$

where the fast fields $\boldsymbol{\phi}$ have the frequencies ω in the interval,

$$\kappa\omega_c < |\omega| < \omega_c \tag{5.21}$$

while the slow ones $\boldsymbol{\Psi}$ carry frequencies

$$|\omega| < \kappa\omega_c, \tag{5.22}$$

where ω_c is the running cutoff and $\kappa < 1$. Fast modes are integrated over in the Gaussian approximation using averages of the form

$$\langle\cdots\rangle_0 = \int d\boldsymbol{\phi}(\cdots)\exp(-\mathcal{S}_0[\boldsymbol{\phi}]). \tag{5.23}$$

This results in a change in $\mathcal{S}$

$$\delta\mathcal{S}[\boldsymbol{\Psi}] = -\ln\langle\exp(-\mathcal{S}[\boldsymbol{\Psi}+\boldsymbol{\phi}])\rangle_0 - \mathcal{S}[\boldsymbol{\Psi}], \tag{5.24}$$

that will now be determined explicitly. In diagrams the Green's function of the fast modes will be denoted by a thick solid line in order to discriminate it from the Green's function of slow modes.

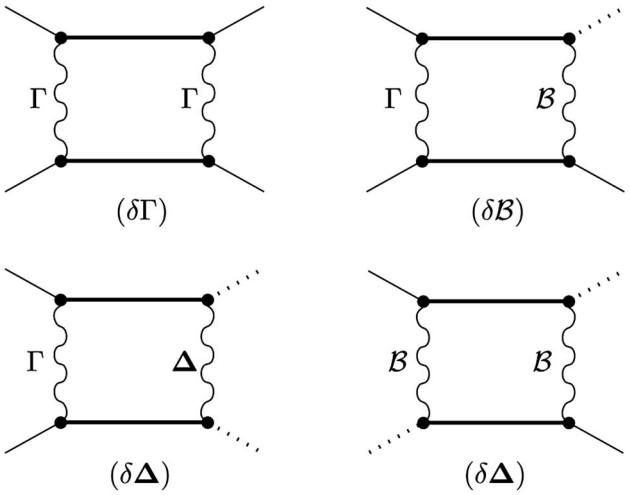

FIG. 4. Relevant diagrams for the renormalization of the interaction amplitudes as found in Ref. 1.

C. Renormalization of interaction amplitudes

The renormalization of the interaction amplitudes was considered in Ref. 1. Here we merely summarize the results, since we will use them later on. Let us note that for the renormalization group the symmetric choice $\alpha = 1/2$ is most convenient. Relevant diagrams are shown in Fig. 4.

The result of the analysis in Ref. 1 was that the model is reproduced under renormalization and the changes in $\Gamma_i, \boldsymbol{\Delta}_i$ and $\mathcal{B}_i$ can conveniently be written in the form

$$\delta\hat{\Gamma}_i = \mathfrak{B}_{\Gamma_i}(\hat{\Gamma}_j;\hat{\mathcal{B}}_j;\hat{\boldsymbol{\Delta}}_j)\delta\xi, \tag{5.25}$$

$$\delta\hat{\mathcal{B}}_i = \mathfrak{B}_{\mathcal{B}_i}(\hat{\Gamma}_j;\hat{\mathcal{B}}_j;\hat{\boldsymbol{\Delta}}_j)\delta\xi, \tag{5.26}$$

$$\delta\hat{\boldsymbol{\Delta}}_i = \mathfrak{B}_{\boldsymbol{\Delta}_i}(\hat{\Gamma}_j;\hat{\mathcal{B}}_j;\hat{\boldsymbol{\Delta}}_j)\delta\xi, \tag{5.27}$$

where

$$\delta\xi = uu_1\mu_d\bar{f}_{\perp}\left(\frac{\mathbf{r}_{\perp}}{r_0}\right)\ln\kappa^{-1}. \tag{5.28}$$

Here μ_d in d dimensions is given as follows:

$$\mu_d = \frac{2}{\pi\nu v_F r_0^{d-1}} = \begin{cases} 2, & d=1, \\ 4(p_F r_0)^{-1}, & d=2, \\ 4\pi(p_F r_0)^{-2}, & d=3, \end{cases} \tag{5.29}$$

and

$$\bar{f}_{\perp}\left(\frac{\mathbf{r}_{\perp}}{r_0}\right) = r_0^{d-1}\int\frac{d^{d-1}p_{\perp}}{(2\pi)^{d-1}}e^{i\mathbf{p}_{\perp}\mathbf{r}_{\perp}}f(\mathbf{p}_{\perp}). \tag{5.30}$$

Therefore the amplitudes can be written in a scaling form

$$\Gamma_i(\theta;u,u_1;\mathbf{r}_1) = \gamma_i[\xi(\theta;u,u_1;\mathbf{r}_{\perp});\gamma_i^0(\theta)],$$

$$\mathcal{B}_i^{\sigma}(\theta;u,u_1;\mathbf{r}_{\perp}) = \beta_i^{\sigma}[\xi(\theta;u,u_1;\mathbf{r}_{\perp});\gamma_i^0(\theta)],$$

$$\mathbf{\Delta}_i^{\sigma_1\sigma_2}(\theta;u,u_1;\mathbf{r}_\perp)=\Delta_i^{\sigma_1\sigma_2}[\xi(\theta;u,u_1;\mathbf{r}_\perp);\gamma_i^0(\theta)], \tag{5.31}$$

where $\gamma_1^0=\gamma_2^0=\gamma_f$, $\gamma_3^0=\gamma_4^0=\gamma_b$ give the initial conditions for the flow. The flow stops at $\max(\theta,T/\varepsilon_\infty)$, so that

$$\xi(\theta;u,u_1;\mathbf{r}_\perp)=-uu_1\mu_d\bar{f}_\perp\left(\frac{\mathbf{r}_\perp}{r_0}\right)\ln\left(\max\left[\theta,\frac{T}{\varepsilon_\infty}\right]\right). \tag{5.32}$$

For any perturbative calculation of the spin susceptibility the amplitudes $\gamma_1,\beta_1^\pm,\Delta_1^{\pm,\pm}$ cannot enter. This immediately follows from the relation $\Lambda_1\tau_3\mathcal{F}_0=-\mathcal{F}_0$, which means that the matrix structure of the Green's function in H space becomes trivial for every closed loop in perturbation theory. Since in this paper we are interested in the perturbative sector of the model only we give here the relevant renormalization group (RG) equations for the backward scattering components

$$\frac{d\gamma_3(\xi)}{d\xi}=-[\gamma_3(\xi)]^2; \tag{5.33}$$

$$\frac{d\beta_3^+(\xi)}{d\xi}=-2\gamma_3(\xi)\beta_3^+(\xi);\quad \frac{d\beta_3^-(\xi)}{d\xi}=-\gamma_3(\xi)\beta_3^-(\xi); \tag{5.34}$$

$$\frac{d\Delta_3^{++}(\xi)}{d\xi}=-2\Delta_3^{++}(\xi)\gamma_3(\xi)-2[\beta_3^+(\xi)]^2; \tag{5.35}$$

$$\frac{d\Delta_3^{-+}(\xi)}{d\xi}=\frac{d\Delta_3^{+-}(\xi)}{d\xi}=-2\beta_3^-(\xi)\beta_3^+(\xi). \tag{5.36}$$

There is a subtle point related to the amplitude Δ_3^{--}. Instead of a flow equation the relation

$$\Delta_3^{--}(\xi)\gamma_3(\xi)=[\beta_3^-(\xi)]^2 \tag{5.37}$$

was fixed in Ref. 1 to cancel ultraviolet divergencies that would otherwise develop under a change in the cutoff. We will come back to this point in Sec. VI below.

Appropriate boundary conditions have already been specified when introducing the model above. The solutions of the flow equations are

$$\gamma_3(\xi)=\beta_3^-(\xi)=\Delta_3^{--}(\xi)=\frac{1}{\xi_b^*+\xi}; \tag{5.38}$$

$$\beta_3^+(\xi)=\Delta_3^{+-}(\xi)=\Delta_3^{-+}(\xi)=\frac{\xi_b^*}{(\xi_b^*+\xi)^2}; \tag{5.39}$$

$$\Delta_3^{++}(\xi)=\frac{2\xi_b^{*2}}{(\xi_b^*+\xi)^3}-\frac{\xi_b^*}{(\xi_b^*+\xi)^2}, \tag{5.40}$$

where we introduced the notation

$$\xi_b^*(\theta)\equiv\frac{1}{\gamma_b(\theta)}>0, \tag{5.41}$$

and the backscattering amplitude γ_b^0 is defined in Eq. (4.40).

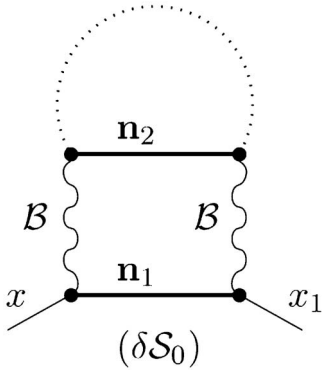

FIG. 5. Logarithmic correction to $\mathcal{S}_0$ in $d=1$.

D. Renormalization of $\mathcal{S}_0$, $\mathcal{S}_{b0}$, $\mathcal{S}_{b1}$, and $\mathcal{S}_{b2}$

In this section we consider the renormalization of the terms $\mathcal{S}_0$, $\mathcal{S}_{b0}$, $\mathcal{S}_{b1}$, and $\mathcal{S}_{b2}$, Eqs. (4.35) and (4.46)–(4.48). It will be shown that for the one-loop RG considered in this paper vertex corrections cancel in dimensions $d=2,3$, and, as a result, these terms are not renormalized. This is no longer true for $d=1$. Unlike in higher dimensions no angular integration is performed in one spatial dimension and this fact is responsible for the appearing of additional logarithmic corrections as will be shown below. When selecting the relevant corrections in $1d$, we have in mind a comparison to the well known result of Dzyaloshinskii and Larkin.[38]

1. Corrections to $\mathcal{S}_0$

This contribution has been noticed before[1] but was discarded, since for the renormalization of the interaction amplitudes this term was already beyond the desired accuracy. The relevant diagram is shown in Fig. 5. In the presence of external vertices this term should be considered.

After expanding field $\Psi(x_1)$ around a point x one obtains an expression containing the following integral:

$$T\sum_\omega\int d\mathbf{p}\frac{v_F\mathbf{n}_1\mathbf{p}+i\omega}{v_F\mathbf{n}_1\mathbf{p}-i\omega}\frac{1}{(v_F\mathbf{n}_2\mathbf{p}+i\omega)^2}A(\mathbf{p}), \tag{5.42}$$

where A is a product of the vertex parts $\mathcal{B}$ and cutoff functions f appearing in the expression. The crucial point is that there is a free integration over the vector $\mathbf{n}_2$ and unlike the contributions from diagrams shown in Fig. 4 a logarithm can be obtained only in $d=1$. The result can be written

$$\delta\mathcal{S}_0=-2i\nu\int dX\bar{\boldsymbol{\psi}}_X\mathcal{H}_0\delta\mathcal{R}\boldsymbol{\psi}_X, \tag{5.43}$$

where

$$\delta\mathcal{R}=u\int du_1\delta\xi(u,u_1)[(1+\Pi_3)\beta_1^+\beta_1^-+(1-\Pi_3)\beta_3^+\beta_3^-]. \tag{5.44}$$

It seems natural to interpret this term as a correction to $\mathcal{S}_0$. For our purposes it is more convenient, however, not to allow $\mathcal{S}_0$ to change. This can be achieved by rescaling fields $\boldsymbol{\psi}$ after each renormalization step in such a way that $\delta\mathcal{R}$ is

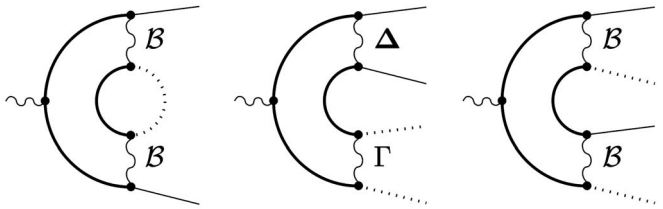

FIG. 6. Diagrams for the corrections to $\mathcal{S}_{b2}$. Logarithmic corrections from $\delta\mathcal{S}_{b2}^{(1)}$ and $\delta\mathcal{S}_{b2}^{(2)}$ cancel each other in any dimension. $\delta\mathcal{S}_{b2}^{(3)}$ gives a logarithmic correction in $d=1$, but for the susceptibility it gives corrections beyond our accuracy.

removed from $\mathcal{S}_0$. This is why we do not write additional RG equations here. In turn, this rescaling of the fields can lead to additional corrections in the flow equations for the interaction amplitudes or external vertices. For the interaction amplitudes, it is in fact easily seen that taking these corrections into account would be an overstepping of accuracy. This is no longer true for the external field vertices as will be discussed below.

2. Corrections to $\mathcal{S}_{b2}$

Relevant contributions to the term $\mathcal{S}_{b2}$ are represented in Fig. 6. The correction $\delta\mathcal{S}_{b2}^{(2)}$ is determined by the vertices Δ and Γ and can be written in the form

$$\delta\mathcal{S}_{b2}^{(2)} = 8\nu\eta\varepsilon_{\alpha\beta\gamma}\int dXdX_1uu_1b_\alpha(x) \times \gamma_3\Delta_3^{+-}\delta\xi\bar{\psi}_{X,\beta}\mathbb{K}_3^{++}\psi_{X_1,\gamma}\tilde{f}(x-x_1), \quad (5.45)$$

where

$$\mathbb{K} = \mathcal{F}_0\bar{\mathcal{F}}_0, \quad \mathbb{K}^{++} = \tau_+\mathbb{K}\tau_+, \quad (5.46)$$

and

$$\tilde{f}(x-x_1) = \bar{f}(\mathbf{r}-\mathbf{r}_1)\delta(\tau-\tau_1). \quad (5.47)$$

The correction is logarithmic in any dimension. However, the form of $\mathcal{S}_{b2}^{(2)}$ is different from that of $\mathcal{S}_{b2}$ because it contains integration over both $\mathbf{n}$ and $\mathbf{n}_1$, which contrast the bare form $\mathcal{S}_{b2}$, Eq. (4.48). Moreover, the matrix $\mathbb{K}$ breaks the symmetry in g space (superspace) and, at first glance, one should introduce additional renormalization coupling constants.

Fortunately there is another diagram that exactly cancels the previous one. It is also shown in Fig. 6 and its contribution equals

$$\delta\mathcal{S}_{b2}^{(1)} = -8\nu\eta\varepsilon_{\alpha\beta\gamma}\int dXdX_1uu_1b_\alpha(x) \times \beta_3^+\beta_3^-\delta\xi\bar{\psi}_{X,\beta}\mathbb{K}_3^{++}\psi_{X_1,\gamma}\tilde{f}(x-x_1). \quad (5.48)$$

In fact, one comes to the exact cancellation

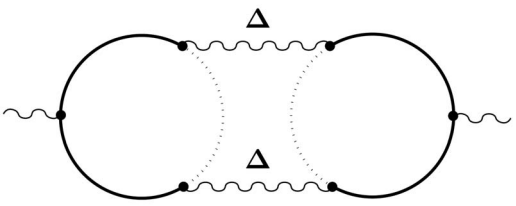

FIG. 7. Logarithmic correction to $\mathcal{S}_{b0}$ in $d=1$.

$$\delta\mathcal{S}_{b2}^{(2)} = -\delta\mathcal{S}_{b2}^{(1)} \quad (5.49)$$

by virtue of the relation

$$\gamma_3\Delta_3^{+-} = \beta_3^+\beta_3^- \quad (5.50)$$

that follows immediately from Eqs. (5.38)–(5.40).

Finally there is an additional logarithmic contribution in $1d$, $\delta\mathcal{S}_{b2}^{(3)}$, represented in Fig. 6. It has a similar form as δS_0, but taking this correction into account would mean overstepping the accuracy for our problem. The reason is that due to the supersymmetry no diagram for the susceptibility can be formed with the help of the vertex $\mathcal{S}_{b2}$ without including additional interaction amplitudes. As a consequence, the leading correction to the spin susceptibility in $d=1$ resulting from this contribution would be $\delta\chi \propto \gamma^3\ln(\cdots)$, which is beyond our accuracy. For the same reason the rescaling of the fields, which is necessary to bring $\mathcal{S}_0$ to its bare form after the renormalization, need not be considered here. This result relies on the symmetry in the g space and this is why it was important to check the cancellation of the terms violating the supersymmetry.

3. Corrections to $\mathcal{S}_{b0}$

The analytic expression corresponding to the diagram shown in Fig. 7 is quadratic in $\mathbf{b}$ but does not contain any slow field $\mathbf{\Psi}$. Therefore, we attribute the corresponding contribution to the renormalization of $\mathcal{S}_{b0}$, Eq. (4.46). Clearly, in this case the rescaling of the fields is not important. Since there is a free integration over both the vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ for $d>1$, the correction is logarithmic only in $d=1$. This is similar to what happens when calculating the correction to $\mathcal{S}_0$.

In $d=1$, we write $\mathcal{S}_{b0}$ in the form

$$S_{b0} = -\frac{1}{2}\nu\eta\int dxdu_1du_2\mathbf{b}^2(x)\sigma(\xi,u_1,u_2) \quad (5.51)$$

and set

$$\sigma(\xi=0,u_1,u_2) = 1. \quad (5.52)$$

Then the correction $\delta\sigma$ to this quantity takes the form

$$\delta\sigma = -\frac{1}{2}\eta u_1u_2\delta\xi((\Delta^{+-})^2 + (\Delta_3^{-+})^2 + 2\Delta_3^{++}\Delta_3^{--}). \quad (5.53)$$

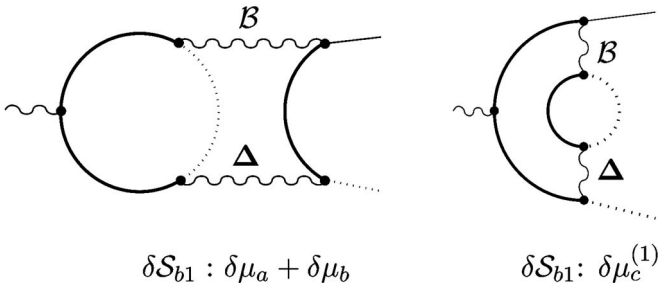

FIG. 8. Logarithmic corrections to $\delta\mathcal{S}_{b1}$ in $d=1$.

4. Correction to $\mathcal{S}_{b1}$

There are two separate contributions to $\mathcal{S}_{b1}$, Eq. (4.47), represented in Fig. 8. In $d>1$ the slow field $\mathbf{\Psi}$ fixes the vector $\mathbf{n}$ in one of the Green's functions $\mathcal{G}$ only, while the other vector $\mathbf{n}'$ is integrated over. As a consequence, a logarithmic correction is obtained only in $d=1$. In one dimension the rescaling of the fields (cf. Sec. V D 1) is also important and gives an additional contribution.

Considering the case $d=1$ we present $\mathcal{S}_{b1}$ in the form

$$\mathcal{S}_{b1} = \sqrt{-2i}\nu\eta \int dx dz_1 dz_2 b_\alpha(x) \bar{\mathcal{F}}_0 \tau_3 \bar{\mathcal{D}}(\xi,u_1,u_2)\psi_\alpha(x,u_2). \tag{5.54}$$

Trivial "angular" integration in $1d$ (weighted summation over the directions) has been performed. The operator $\bar{\mathcal{D}}$ is defined

$$\bar{\mathcal{D}}(\xi,u_1,u_2) = \begin{pmatrix} \frac{u_2}{2}(\mu_a i v_F \Sigma_3 \nabla + \mu_b \partial_\tau) & 0 \\ 0 & \mu_c \end{pmatrix}. \tag{5.55}$$

Here $\mu_i = \mu_i(\xi,u_1,u_2)$, and initially $\mu_i(\xi=0,u_1,u_2)=1$, $i=a,b,c$. In this case

$$\bar{\mathcal{D}}(\xi=0,u_1,u_2) = \bar{\partial}(x,u_2) \tag{5.56}$$

and we come back to the original form displayed in Eq. (4.47).

The diagrams in Fig. 8 represent corrections to μ_i. The left diagram determines corrections $\delta\mu_a$ and $\delta\mu_b$,

$$\delta\mu_a = -\frac{u_1}{2}(\beta_3^- \Delta_3^{+-} + \beta_3^+ \Delta_3^{--})\delta\xi = -\delta\mu_b. \tag{5.57}$$

The contribution $\delta\mu_c$ consists of two parts

$$\delta\mu_c = \delta\mu_c^{(1)} + \delta\mu_c^{(2)}. \tag{5.58}$$

The correction $\delta\mu_c^{(1)}$ is represented by the right diagram in Fig. 8 and reads

$$\delta\mu_c^{(1)} = -u_2(\beta_3^+ \Delta^{-+} + \beta_3^- \Delta_3^{+-})\delta\xi. \tag{5.59}$$

The correction $\delta\mu_c^{(2)}$ due to the rescaling of the fields has to be performed at each RG step to keep the form of $\mathcal{S}_0$ fixed

$$\delta\mu_c^{(2)} = -2u_2\beta_3^+\beta_3^-\delta\xi. \tag{5.60}$$

Note that the forward scattering components drop out as could be expected.

5. RG equations and their solution

We found logarithmic corrections to the vertices $\mathcal{S}_{b0}$, $\mathcal{S}_{b1}$, and $\mathcal{S}_{b2}$ in dimensionality $d=1$ only. This means that these terms are not renormalized in the first order in the dimensionalities $d=2,3$ and the vertices γ, β, and Δ given by Eqs. (5.38)–(5.40) are sufficient to determine the susceptibitlity.

At the same time, the renormalization of the vertices $\mathcal{S}_{b0}$, $\mathcal{S}_{b1}$, and $\mathcal{S}_{b2}$ is very important in $d=1$. Both functions μ_i and σ from Eqs. (5.51) and (5.55) do not have a simple form and one should write and solve proper RG equations. For the function σ related to $\delta\mathcal{S}_{b0}$, we write $\sigma=\sigma(\xi,u_1,u_2)$ and using the correction $\delta\sigma$, Eq. (5.53), obtain the following differential equation:

$$\frac{\partial\sigma}{\partial\xi} = -\frac{1}{2}\eta u_1 u_2 \left(\frac{6\xi_b^{*2}}{(\xi+\xi_b^*)^4} + \frac{2\xi_b^*}{(\xi+\xi_b^*)^3}\right). \tag{5.61}$$

With the boundary condition $\sigma(\xi=0,u_1,u_2)=1$ we obtain

$$\sigma(\xi,u_1,u_2) = 1 + \frac{1}{2}u_1 u_2 \eta \left(\frac{2\xi_b^{*2}}{(\xi+\xi_b^*)^3} - \frac{\xi_b^*}{(\xi+\xi_b^*)^2} - \frac{1}{\xi_b^*}\right). \tag{5.62}$$

The corresponding differential equations for μ_i are to be obtained from the forms of the corrections, Eqs. (5.57)–(5.60), and can be written

$$\frac{\partial\mu_a}{\partial\xi} = -u_1\frac{\xi_b^*}{(x+\xi_b^*)^3} = -\frac{\partial\mu_b}{\partial\xi}, \tag{5.63}$$

$$\frac{\partial\mu_c}{\partial\xi} = -u_2\left(\frac{3\xi_b^{*2}}{(\xi+\xi_b^*)^4} + \frac{\xi_b^*}{(\xi+\xi_b^*)^3}\right) \tag{5.64}$$

with the boundary conditions $\mu_i(\xi=0)=1$. Integrating these equations we obtain (only μ_a and μ_c will enter our results)

$$\mu_a = 1 + \frac{u_1}{2}\left(\frac{\xi_b^*}{(\xi+\xi_b^*)^2} - \frac{1}{\xi_b^*}\right), \tag{5.65}$$

$$\mu_c = 1 + \frac{u_2}{2}\left(\frac{2\xi_b^{*2}}{(\xi+\xi_b^*)^3} + \frac{\xi_b^*}{(\xi+\xi_b^*)^2} - \frac{3}{\xi_b^*}\right). \tag{5.66}$$

The calculations presented in this subsection allowed us to obtain all effective vertices entering the RG scheme. This gives us the possibility to calculate the susceptibility for all dimensions $d=1,2,3$. The result for $d=1$ is well known[38] from a renormalization group treatment for the initial electron model. We will reproduce now this result using the derived equations in order to check the formalism of the bosonization used here. Only after that we will concentrate on calculating the susceptibility in the higher dimensionalities $d=2,3$.

VI. SPIN SUSCEPTIBILITY IN $d=1$

We can now determine the temperature dependent correction to the static spin susceptibility in $d=1$.

We write the correction to the susceptibility as follows:

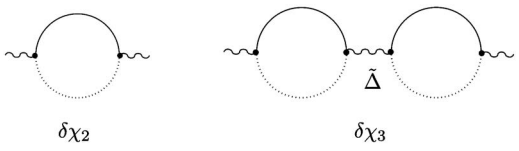

FIG. 9. These two diagrams with renormalized external vertices ($\delta\chi_2$) and interaction amplitude ($\delta\chi_3$) determine the correction to the susceptibility in $d=1$ together with the diagram of Fig. 7.

$$\delta\chi = \delta\chi_1 + \delta\chi_2 + \delta\chi_3 \tag{6.1}$$

and denote the contribution from $\delta\mathcal{S}_{b0}$, Fig. 7, as $\delta\chi_1$. The second term $\delta\chi_2$ in Eq. (6.1) is the contribution that corresponds to the diagram shown in Fig. 9 on the left-hand side.

Here only the renormalized vertex of $\mathcal{S}_{b1}$ enters but no additional interaction amplitude. A diagram with this property does not exist for vertex $\mathcal{S}_{b2}$. Finally, the diagram shown on the right-hand side of Fig. 9 gives a correction termed $\delta\chi_3$. It involves a renormalized interaction amplitude. The corresponding expressions take the form

$$\delta\chi_1 = \nu \int du_1 du_2 \sigma(\xi,u_1,u_2), \tag{6.2}$$

$$\delta\chi_2 = 2\nu \int du_1 du_2 du_3 u_2 \mu_c(\xi,u_3,u_2)\mu_a(\xi,u_1,u_2), \tag{6.3}$$

$$\delta\chi_3 = \frac{1}{2}\nu \int du_1 du_2 u_1 u_2 \sum_{\alpha,\beta=\pm} \tilde{\Delta}_3^{\alpha\beta}(\xi,u_1,u_2). \tag{6.4}$$

We do not write factors of η in $d=1$, since we want to avoid unnecessary complications while focusing on the leading temperature dependent corrections. In Eq. (6.4) we introduced the interaction amplitudes $\tilde{\Delta}_3^{\alpha\beta}$. Naively one would expect amplitudes $\Delta_3^{\alpha\beta}$ Eqs. (5.38)–(5.40) to enter here but this would not be correct. In fact, this question is intimately related to a subtle point related to the renormalization of Δ_3^{--} already alluded to in Sec. V C.

When calculating corrections to Δ_3^{--} within the renormalization scheme, the authors of Ref. 1 found ultraviolet divergencies that could be cancelled only provided the condition $\Delta_3^{--}\gamma_3=(\beta_3^-)^2$ is imposed. Since β_3^- and γ_3 can be determined independently, this condition fixes Δ_3^{--}. For large temperatures, where one can use the bare values of these amplitudes, this relation is automatically fulfilled. It is crucial to note now that it was necessary to fix Δ_3^{--} only because this amplitude itself enters $\mathcal{S}_2[\boldsymbol{\phi}]$, where fields $\boldsymbol{\phi}$ are the fast modes.

Returning to the diagram for $\delta\chi_3$, Fig. 9, we see that the frequencies flowing through the Green's functions are determined by the external vertices and therefore are vanishingly small. In particular, they are smaller than any frequency considered in the renormalization scheme. We argue that the part of $\mathcal{S}_2$ that contains the fields at vanishingly small frequencies should be split off from the beginning and when separating fast and slow modes, it must always contain slow fields only. Correspondingly, the interaction amplitudes, termed $\tilde{\Delta}_3^{\alpha\beta}$, are renormalized but do not play any role when calculating corrections to the interaction vertices. In such a situation, there is no reason to fix $\tilde{\Delta}_3^{--}$ as was done previously for Δ_3^{--}. Instead, one should follow the renormalization group scheme and derive a proper RG equation for $\tilde{\Delta}_3^{--}$.

The relevant diagrams have been already presented in Fig. 4 and the result of the RG procedure can be expressed by the equation

$$\frac{\partial\tilde{\Delta}_3^{--}}{\partial\xi} = -2(\beta_3^-)^2 = -\frac{2}{(\xi+\xi_b^*)^2}. \tag{6.5}$$

The solution of Eq. (6.5) with the initial condition $\tilde{\Delta}_3^{--}(\xi=0)=1/\xi_b^*$ takes the form

$$\tilde{\Delta}_3^{--} = \frac{2}{\xi+\xi_b^*} - \frac{1}{\xi_b^*}. \tag{6.6}$$

This should be contrasted with

$$\Delta_3^{--} = \frac{1}{\xi+\xi_b^*}. \tag{6.7}$$

We checked our reasoning by a perturbative calculation at order γ^2, where the difference between $\tilde{\Delta}_3^{--}$ and Δ_3^{--} is already noticeable.

Finally, we use the identities

$$\frac{1}{1+X} = \int_0^1 du_1 du_2 u_1 u_2 (z_{12} + z_{12}^2 + 2z_{12}^3) \tag{6.8}$$

$$= \int_0^1 du_1 du_2 u_1^2 (z_{12}^2 + 2z_{12}^3), \tag{6.9}$$

where $z_{12}=1/(1+u_1u_2X)$, that can be checked by a direct computation of the integrals. Then, recalling that $X=2\gamma_b\ln(\varepsilon_F/T)$ we come to the following temperature dependent correction to the spin susceptibility:

$$\delta\chi(T) = \frac{2\nu\gamma_b}{1+2\gamma_b\ln(\varepsilon_F/T)}. \tag{6.10}$$

This result has first been obtained by Dzyaloshinskii and Larkin.[38] Equation (6.10) serves as a good check of the bosonization approach used here. Actually, the calculations within the framework of the bosonization method of Ref. 1 are most difficult in $d=1$. It is clear that this method is less convenient for calculations in $1d$ than the other well developed ones.[39] However, calculations in $d=2,3$ are somewhat less involved and the present approach is the most convenient tool for calculations in these dimensionalities. In the next section we concentrate on such calculations.

VII. NONANALYTIC CORRECTIONS TO SPIN SUSCEPTIBILITY IN $d=2,3$

Nonanalytic corrections to the spin susceptibility have been considered in several works before.[11–18,22] A linear in T behavior at order γ^2 was obtained in $2d$, while the potential analog in $3d$, a $T^2\ln T$ behavior, was found to be absent and the first correction in $3d$ was proportional to T^2. We will

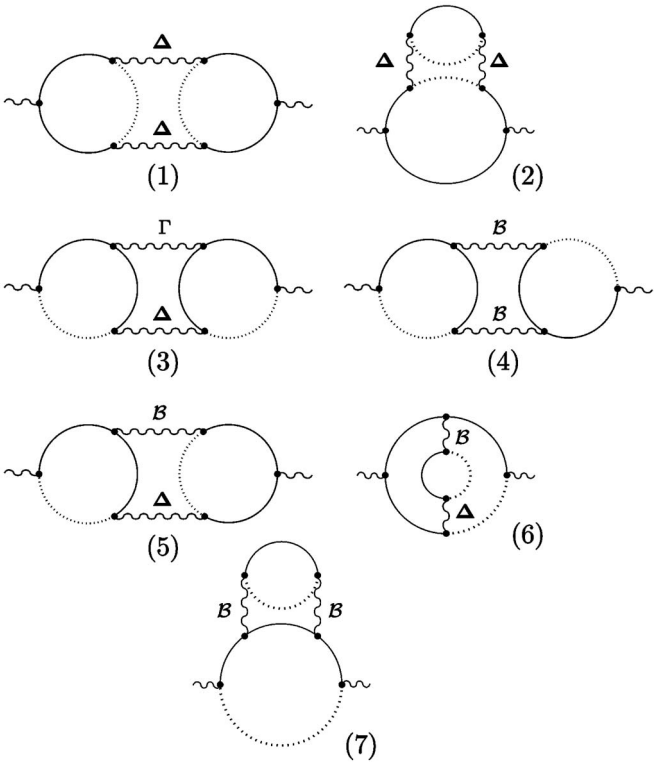

FIG. 10. These are the diagrams responsible for the nonanalytic temperature dependence of the spin susceptibility.

show now that there are logarithmic corrections to these results and sum up the leading logarithms.

Let us repeat that, as it has been demonstrated in Sec. V D, the terms $\mathcal{S}_{b0}$, $\mathcal{S}_{b1}$, and $\mathcal{S}_{b2}$, Eqs. (4.46)–(4.48) are not renormalized in dimensions $d>1$. Therefore we can perform a perturbative analysis with the renormalized interaction vertices γ, β, and Δ, Eqs. (5.38)–(5.40), keeping the bare values of $\mathcal{S}_{b0}$, $\mathcal{S}_{b1}$, and $\mathcal{S}_{b2}$.

The relevant diagrams leading to T^{d-1} corrections are displayed in Fig. 10. The solid lines carry the frequencies ω and the momenta k of the order of T and T/v_F, respectively. They are smaller than characteristic energies in the Green's function entering the vertices because the latter are responsible for the logarithmic contributions. This means that the vertices can be taken at zero external frequencies and momenta and this is the reason why one may just take the values of the vertices from Eqs. (5.38)–(5.40). The same procedure has been used in Ref. 1 for calculation of the specific heat. Putting the bare values for the vertices γ, β, and Δ would give the peturbative results of Refs. 11–14 in $d=2,3$. In this limit, the diagrams of Fig. 10 correspond to the conventional diagrams considered in those works.

As concerns diagrams containing the amplitude γ_f of the forward scattering, we did not find any logarithmic contributions. This is because one obtains integrals of products of Green's functions containing poles in the same half plane of complex variables **kn**.

Considering the contributions of the diagrams in Fig. 10 and comparing them with self-energy and vertex corrections in $1d$, Fig. 9, one can see that there is a close analogy between the terms responsible for the logarithmic corrections in one dimension and those responsible for the nonanalytic behavior in higher dimensions. Within our formalism, the main difference between the two cases is the additional angular integration in dimensions $d>1$.

Let us now turn to the computation of the diagrams in two and three dimensions. Calculating the terms of the perturbation theory corresponding to the diagrams displayed in Fig. 10 one finds that some of them show unphysical divergencies in the limit of vanishing momenta and frequencies. Therefore one should sum up certain diagrams first before taking the limit.

To demonstrate this feature explicitly, let us consider the backscattering contribution for the diagram 3 of Fig. 10. When evaluating the term $\chi_3(\mathbf{q},\varepsilon)$ corresponding to this diagram one finds terms containing the product

$$\mathcal{G}_{\mathbf{n}_1}(\mathbf{q},\varepsilon)\mathcal{G}_{\mathbf{n}_2}(\mathbf{q},-\varepsilon), \tag{7.1}$$

where $\mathbf{q}$ and ε are the external momentum and frequency.

The integral over the internal momenta is ultraviolet divergent and must be cut with the help of the function f, Eq. (3.6). At the same time, the limits $\mathbf{q}\to 0$, $\varepsilon\to 0$ in the diagram 3, Fig. 10, cannot be taken unambiguously.

In order to get rid of such unphysical divergencies, we note that this term contains the product of the interaction amplitudes $\Delta_3^{--}\Gamma_3$. A closer inspection reveals that this term is intimately related to the renormalization of Δ and the ultraviolet divergence encountered during the renormalization (see Fig. 4). It follows from the results of the renormalization that this divergence must be cancelled with the help of diagram 4 using the relation $\Delta_3^{--}\gamma_3=(\beta_3^{-})^2$.

The relation to the conventional perturbation theory, which is made obvious by including the dotted lines in the diagrams, in fact strongly suggests first to group several different diagrams before evaluating them. These are the groups

$$\chi_a=\chi_1+\chi_3+\chi_4+\chi_5, \tag{7.2}$$

$$\chi_b=\chi_2+\chi_6+\chi_7. \tag{7.3}$$

Strictly speaking, diagram 6 differs topologically from diagrams 2 and 7. It nevertheless turns out to be advantageous to combine them, since the expressions are similar at low energies that are considered in the model we use.

For the convenience of the reader, and since the algebra is rather tedious, explicit expressions for the diagrams are included in Appendix B. Using the mutual relations between the seven interaction amplitudes $\Delta_3^{\pm,\pm}$, $\beta_3^{\pm}$, and γ_3, one finds rather simple expressions for χ_a and χ_b, that allow one to easily take the limit $\mathbf{q}\to 0$, $\varepsilon\to 0$. After taking this limit they read

$$\chi_a=-32\eta^2T\sum_{\omega}\int d\mathbf{p}\int d\mathbf{n}_1 d\mathbf{n}_2\frac{\omega^2}{(v_F\mathbf{n}_1\mathbf{p}-i\omega)^2(v_F\mathbf{n}_2\mathbf{p}+i\omega)^2}\times Y(\mathbf{p},\widehat{\mathbf{n}_1\mathbf{n}_2}), \tag{7.4}$$

$$\chi_b = 32\eta^2 T\sum_\omega \int d\mathbf{p}\int d\mathbf{n}_1 d\mathbf{n}_2 \frac{i\omega(v_F\mathbf{n}_2\mathbf{p}-i\omega)}{(v_F\mathbf{n}_1\mathbf{p}-i\omega)^3(v_F\mathbf{n}_2\mathbf{p}+i\omega)} \times Y(\mathbf{p},\widehat{\mathbf{n}_1\mathbf{n}_2}), \tag{7.5}$$

where

$$Y(\mathbf{p},\theta) = \left[\int d^d\mathbf{r}e^{-i\mathbf{pr}}\frac{\gamma_b(\theta)}{1+\bar{f}_\perp\left(\frac{\mathbf{r}_\perp}{r_0}\right)X(\theta)}\bar{f}(\mathbf{r})\right]^2, \tag{7.6}$$

and

$$X(\theta) = -\mu_d\gamma_b(\theta)\ln(\max\{\theta,T/\varepsilon_0\}). \tag{7.7}$$

The numerical coefficient μ_d was introduced in Eq. (5.29). The integration over u_1, u_2 was performed with the help of the following relations:

$$\int du_1du_2u_1^2u_2^2(z_1z_2+2(z_1^3z_2+z_1z_2^3)+(z_1^2z_2+z_1z_2^2)+2z_1^2z_2^2) = \frac{1}{(1+x_1)(1+x_2)}, \tag{7.8}$$

$$\int du_1du_2u_1^3u_2(2z_1^2z_2^2+2(z_1^3z_2+z_1z_2^3)+(z_1^2z_2+z_1z_2^2)) = \frac{1}{(1+x_1)(1+x_2)}, \tag{7.9}$$

where

$$z_i = \frac{1}{1+u_1u_2x_i}. \tag{7.10}$$

The nonanalytic contribution to the spin susceptibility is found from the small region of phase space, for which the angles $\mathbf{n}_1$ and $\mathbf{n}_2$ are close to each other, $|\mathbf{n}_1-\mathbf{n}_2|\ll 1$. We therefore introduce

$$\mathbf{n} = (\mathbf{n}_1+\mathbf{n}_2)/2, \quad \mathbf{m} = \mathbf{n}_1-\mathbf{n}_2, \tag{7.11}$$

$$p_\parallel = \mathbf{pn}, \quad \mathbf{p}_\perp = \mathbf{p}-p_\parallel\mathbf{n} \tag{7.12}$$

and perform the integration in $p_\parallel$ in Eqs. (7.4) and (7.5).

As a result, we obtain the following formula for the nonanalytic correction to the spin-susceptibility in dimension $d>1$,

$$\delta\tilde{\chi}(T) = \chi_a(T)+\chi_b(T) = \frac{256}{v_F}\eta^2T\sum_\omega|\omega|^3\int\frac{d^{d-1}\mathbf{p}_\perp}{(2\pi)^{d-1}}\int d\mathbf{n}_1d\mathbf{n}_2 \times\frac{3(v_F\mathbf{mp}_\perp)^2-4\omega^2}{((v_F\mathbf{mp}_\perp)^2+4\omega^2)^3}Y(p_\parallel\sim 0,\mathbf{p}_\perp,|\mathbf{m}|) \tag{7.13}$$

The main contribution to the integrals in Eqs. (7.4) and (7.5) comes from $p_\parallel$ of the order of T/v_F and this is why we can set $p_\parallel\sim 0$ in the argument of the function Y.

Equation (7.13) contains a sum over bosonic Matsubara frequencies and we write this sum symbolically as follows:

$$\delta\tilde{\chi}(T) = T\sum_{\omega_n}\mathbb{R}(\omega_n). \tag{7.14}$$

Technically it is more convenient to calculate the deviation from the zero-temperature limit instead of computing the sum, i.e., to calculate the quantity

$$\delta\chi(T) = \delta\tilde{\chi}(T)-\delta\tilde{\chi}(T=0). \tag{7.15}$$

Using the Poisson formula the temperature dependent correction to the susceptibility $\delta\chi(T)$ can be represented as follows:

$$\delta\chi(T) = \left(T\sum_\omega-\int\frac{d\omega}{2\pi}\right)\mathbb{R}(\omega) \tag{7.16}$$

$$= \sum_{l\neq 0}\int\frac{d\omega}{2\pi}\mathbb{R}(\omega)\exp\left(-i\frac{l\omega}{T}\right). \tag{7.17}$$

The further evaluation is slightly different in dimensions $d=2$ and $d=3$ and we discuss the two cases separately.

A. Nonanalytic correction in two dimensions

Rescaling the momentum and integrating over the angle $\mathbf{n}$ we obtain

$$\delta\chi(T) = \frac{32}{v_F^2}\eta^2\left(T\sum_\omega-\int\frac{d\omega}{2\pi}\right)\int_0^1\frac{d|\mathbf{m}|}{2\pi}\frac{1}{|\mathbf{m}|} \times\int\frac{dk}{2\pi}\frac{3k^2-1}{(k^2+1)^3}Y\left(p_\parallel=0,p_\perp=\frac{2|\omega|k}{v_F|\mathbf{m}|},|\mathbf{m}|\right). \tag{7.18}$$

The integral over $|\mathbf{m}|$ is logarithmic and therefore not very sensitive to the upper limit that can safely be set to 1. One notices that the momentum dependence of Y is crucial here coupling the integrals in $|\mathbf{m}|$ and k. If Y were independent of the momentum, the k integral would be equal to zero, whereas, at the same time, the integral over $\mathbf{m}$ would diverge at the lower limit.

Fortunately, this uncertainty can easily be avoided taking into the momentum dependence of the function Y.

After introducing the Fourier transform of Y,

$$\bar{Y}(|r|;\theta) = \int\frac{dp_\perp}{2\pi}e^{ip_\perp r}Y(p_\perp,p_\parallel=0,\theta) \tag{7.19}$$

the momentum integration can be performed with the help of the identity

$$\int_{-\infty}^{\infty}\frac{dk}{(2\pi)}\frac{3k^2-1}{(1+k^2)^3}e^{-ikb} = -\frac{1}{4}|b|^2e^{-|b|}, \tag{7.20}$$

where $b=2r|\omega|/(v_F|\mathbf{m}|)$.

Then, we use Eq. (7.17) and obtain the following expression:

$$\left(T\sum_{\omega} - \int \frac{d\omega}{2\pi}\right)\beta^2\omega^2 e^{-\beta|\omega|} = T\left(2x^2\frac{\coth x}{\sinh^2 x} - \frac{2}{x}\right), \tag{7.21}$$

where $x=2\pi T|r|/(v_F|\mathbf{m}|)=\pi T\beta$, and change the integration variable from $|\mathbf{m}|$ to x. As a result, we find

$$\delta\chi(T) = -\frac{4T}{\pi v_F^2}\eta^2\int_{-\infty}^{\infty} dr\mathcal{I}_2(a)\bar{Y}\left(|r|,\theta=\frac{2\pi T}{\varepsilon_\infty}\frac{r}{r_0}\frac{1}{x}\right), \tag{7.22}$$

where

$$\mathcal{I}_2(a) = \int_a^{\infty} dx\left(\frac{2x\coth x}{\sinh^2 x} - \frac{2}{x^2}\right) \tag{7.23}$$

and $a=\frac{2\pi T}{\varepsilon_\infty}\frac{r}{r_0}\frac{1}{\Delta\phi}$.

We reintroduced formally an upper cutoff $\Delta\phi$ for the integration over $|\mathbf{m}|$ but it will drop out from the final result. One can see that the essential r as controlled by the function Y (and thus f) are small, $r\lesssim r_0$, while essential x in the integral are large, $x>1$. This means that the main contribution in the integral over the angles θ comes from θ of the order of T/ε_∞. Therefore we can with logarithmic accuracy set $\theta=0$ in the argument of $\bar{Y}$. In turn, it means that the integral over x is rather insensitive to the lower bound as long as $x<1$ and we may safely extend the integration range in x to the interval $(0,\infty)$. Then, the integrations over x and r can be easily performed. To this end we note that $\mathcal{I}_2(0)=-1$ and introduce the notation

$$Y(\theta) = Y(\theta,\mathbf{p}=0) \tag{7.24}$$

to formulate our result for the susceptibility $\delta\chi$,

$$\delta\chi^{2d}(T) = 2\eta^2\frac{T}{\varepsilon_F}\chi_0^{2d}Y(\theta=0). \tag{7.25}$$

In Eq. (7.25), $\chi_0^{2d}=m/\pi$. The vertex part η, Eq. (4.49), should be understood as $\eta_{\alpha=1}$ and it is a result of an additional summation of ladder diagrams including $\mathcal{S}_2$, in close analogy to the discussion in Sec. IV F. The limit $\theta=0$ corresponds to the backward scattering. Before further discussing this result in Sec. VII C we turn to the three-dimensional case.

B. Nonanalytic correction in three dimensions

In $3d$ one obtains from Eq. (7.13) after rescaling of momenta and integration over $|\mathbf{n}|$ the following expression:

$$\delta\chi(T) = \frac{32\pi}{v_F^3}\eta^2\left(T\sum_{\omega} - \int\frac{d\omega}{2\pi}\right) \times|\omega|\int_0^1\frac{d|\mathbf{m}|}{2\pi}\frac{1}{|\mathbf{m}|}\int\frac{d^2k}{(2\pi)^2}\frac{3(e_{\mathbf{m}}\mathbf{k})^2-1}{((e_{\mathbf{m}}\mathbf{k})^2+1)^3} \times Y\left(p_\parallel=0,\mathbf{p}_\perp=\frac{2|\omega|\mathbf{k}}{v_F|\mathbf{m}|},|\mathbf{m}|\right), \tag{7.26}$$

where $e_{\mathbf{m}}=\mathbf{m}/|\mathbf{m}|$.

We see that the integral over $\mathbf{m}$ in Eq. (7.26) is logarithmic. However, corrections of the form $\delta\chi\propto\gamma^2T^2\ln T$ are absent and this is due to the fact that the integral over k vanishes provided the momentum dependence of Y is neglected. Nevertheless, if the function Y depends on the momentum $\mathbf{p}_\perp$ the entire integral is finite.

After introducing the Fourier transform of Y,

$$\bar{Y}(|\mathbf{r}|,\theta) = \int\frac{d^2p}{(2\pi)^2}e^{i\mathbf{p}_\perp\mathbf{r}}Y(p_\parallel=0,\mathbf{p}_\perp,\theta) \tag{7.27}$$

it is convenient to decompose the vectors $\mathbf{k},\mathbf{p}_\perp,\mathbf{r}$ into components parallel and perpendicular to $e_{\mathbf{m}}$, such that $\mathbf{r}=(\tilde{r}_\parallel,\tilde{r}_\perp)$. Then, one can then proceed in close analogy to the calculation in $2d$. Using Eq. (7.20) and Eq. (7.21) one arrives at

$$\delta\chi(T) = -\frac{4T^2}{v_F^3}\eta^2\int d^2r\mathcal{I}_3(a)\bar{Y}\left(|\mathbf{r}|,\theta=\frac{2\pi T}{\varepsilon_\infty}\frac{|\mathbf{r}|}{r_0}\frac{1}{x}\right), \tag{7.28}$$

where

$$\mathcal{I}_3(a) = \int_a^{\infty} dx\left(2\frac{\coth x}{\sinh^2 x} - \frac{2}{x^3}\right) \tag{7.29}$$

and $a=\frac{2\pi T}{\varepsilon_\infty}\frac{|\mathbf{r}|}{r_0}\frac{1}{\Delta\phi}$. Again, we reintroduced the upper cutoff $\Delta\phi$ for the $\mathbf{m}$ integration.

The integral over x in Eq. (7.28) shows a somewhat stronger dependence on the lower cutoff a as compared to the two-dimensional case with $\mathcal{I}_3(0)=-1/3$, $\mathcal{I}_3(1)\sim-0.28$. This means that angles larger than T/ε_∞ start contributing more significantly. Still, the dominant contribution to the integral comes from $x>1$, so that we can set $\theta=0$ in the argument of $\bar{Y}$ with logarithmic accuracy. Then, we come to the following result for the temperature dependent correction to the spin susceptibility:

$$\delta\chi^{3d}(T) = \frac{\pi^2}{3}\eta^2\frac{T^2}{\varepsilon_F^2}\chi_0^{3d}Y(\theta=0), \tag{7.30}$$

where $\chi_0^{3d}=mp_F/\pi^2$, and $Y(\theta)=Y(\mathbf{p}=0,\theta)$, and $\eta=\eta_{\alpha=1}$, see the remarks below Eq. (7.25).

We see that, as in the $2d$ case, this correction is determined completely by the backward scattering ($\theta=0$).

C. Final results in $d=2,3$

1. General results

The quantity $Y(\theta=0)$ can be considered as the square of an effective temperature dependent backward scattering amplitude $\gamma_b(T)$, and we write it in the form $Y(\theta=0)=\gamma_b^2(T)$, where

$$\gamma_b(T) = \gamma_b \int \frac{d^{d-1}\mathbf{r}_\perp}{r_0^{d-1}} \frac{\bar{f}_\perp\left(\frac{\mathbf{r}_\perp}{r_0}\right)}{1+\bar{f}_\perp\left(\frac{\mathbf{r}_\perp}{r_0}\right)X(T)}, \tag{7.31}$$

$\gamma_b = \gamma_b(\theta=0)$, $X(T) = \mu_d \gamma_b \ln(\varepsilon_\infty/T)$ and $\bar{f}_\perp(\mathbf{r}_\perp/r_0)$ was defined in Eq. (5.30).

So, our results can be written in the most general form as follows:

$$\delta\chi^{2d}(T) = 2\eta^2\gamma_b^2(T)\frac{T}{\varepsilon_F}\chi_0^{2d}, \tag{7.32}$$

$$\delta\chi^{3d}(T) = \frac{\pi^2}{3}\eta^2\gamma_b^2(T)\frac{T^2}{\varepsilon_F^2}\chi_0^{3d}, \tag{7.33}$$

and we remind the reader that $\eta = \eta_{\alpha=1}$, where η is determined by Eq. (4.49).

If we replaced $\gamma_b(T)$ in Eq. (7.32) by the bare coupling constant γ_b for $d=2$, we would obtain the previously reported linear T dependence of the nonanalytic corrections.[11–14] This replacement means neglecting the renormalization of the interaction constants discussed in Sec. V C. If we set the function $\gamma_b(T)$ equal to the bare value γ_b in $d=3$ we would obtain the correction $\delta\chi^{3d}(T)$ proportional to T^2, which is regular in T^2. This means that the first nonanalytical $T^2\ln(\varepsilon_\infty/T)$ term in $3d$ is of the order γ^3.

In the limit of small $X(T) \ll 1$ the temperature dependence of $\gamma_b^2(T)$ takes the form

$$\gamma_b^2(T) \sim \gamma_b^2 - 2\gamma_b^3 c_d \ln\frac{\varepsilon_\infty}{T}, \quad X(T) \ll 1, \tag{7.34}$$

where

$$c_d = \mu_d \int \frac{d^{d-1}\mathbf{r}_\perp}{r_0^{d-1}} \bar{f}_\perp^2\left(\frac{\mathbf{r}_\perp}{r_0}\right). \tag{7.35}$$

The factor c_d depends on the precise form of the cutoff and can be estimated only. It is roughly of the order of unity. Equation (7.34) shows that the first logarithmic in temperature corrections contain the prefactor γ_b^3 both in two and three dimensions. This rather high order in the coupling constant γ_b is, apparently, the reason why the logarithmic corrections to the susceptibility have not been noticed previously in the diagrammatic expansions[11–14] (see, however, Ref. 40 for $2d$).

In the limit of large $X(T) \gg 1$ one finds the following asymptotic temperature dependence of $\gamma_b^2(T)$,

$$\gamma_b^2(T) \propto \left(\ln\frac{\varepsilon_\infty}{T}\right)^{-2}, \quad X(T) \gg 1. \tag{7.36}$$

More explicit formulae can only be written using a model cutoff function and this will be done in the next section.

2. Results for a model cutoff function

We choose the following model cutoff function $\bar{f}_\perp(r_\perp/r_0)$:

$$\bar{f}_\perp\left(\frac{r_\perp}{r_0}\right) = \frac{1}{\Omega_{d-1}}\exp\left(-\frac{r_\perp}{r_0}\right), \tag{7.37}$$

where Ω_{d-1} is the d-1 dimensional solid angle.

Performing the remaining integration for this case one obtains the following temperature dependence for the effective backward scattering constants

$$\gamma_b^{d=2}(T) = \frac{2\gamma_b \ln[1+X(T)/2]}{X(T)}, \tag{7.38}$$

$$\gamma_b^{d=3}(T) = -\frac{2\pi\gamma_b \mathrm{Li}_2[-X(T)/2\pi]}{X(T)}, \tag{7.39}$$

where $\mathrm{Li}_2(x) = \Sigma_{k=1}^{\infty} x^k/k^2$ is the polylogarithm function.

In the limit of small $X(T) \ll 1$ the temperature dependence of the susceptibility computed with the model cutoff function takes the form

$$\delta\chi^{2d}(T) = 2\eta^2\gamma_b^2\frac{T}{\varepsilon_F}\chi_0^{2d}\left(1 - 2\gamma_b \ln\frac{\varepsilon_\infty}{T}\right), \tag{7.40}$$

$$\delta\chi^{3d}(T) = \frac{\pi^2}{3}\eta^2\gamma_b^2\frac{T^2}{\varepsilon_F^2}\chi_0^{3d}\left(1 - \gamma_b \ln\frac{\varepsilon_\infty}{T}\right), \tag{7.41}$$

where we put $r_0^{-1} \sim p_F$ for simplicity. It should be stressed once again that the coefficient of the logarithmic correction cannot be determined rigorously within our model.

In the opposite limit of very low temperatures, $X(T) \gg 1$, asymptotic expressions for the corrections to the susceptibility can be written using the model cutoff function of Eq. (7.37),

$$\delta\chi^{2d}(T) = \frac{1}{2}\eta^2\frac{T}{\varepsilon_F}\chi_0^{2d}\frac{\ln^2\left(4\gamma_b \ln\frac{\varepsilon_\infty}{T}\right)}{\ln^2\frac{\varepsilon_\infty}{T}}, \tag{7.42}$$

$$\delta\chi^{3d}(T) = \frac{\pi^2}{48}\eta^2\frac{T^2}{\varepsilon_F^2}\chi_0^{3d}\frac{\ln^4\left(4\pi\gamma_b \ln\frac{\varepsilon_\infty}{T}\right)}{\ln^2\frac{\varepsilon_\infty}{T}}. \tag{7.43}$$

Again we used $r_0^{-1} \sim p_F$ for simplicity. The asymptotic behavior $1/[\ln(\varepsilon_\infty/T)]^2$ in these equations is not very sensitive to the form of the function $f(\mathbf{k})$ as can be seen from Eq. (7.31).

VIII. DISCUSSION

We have calculated nonanalytical logarithm in temperature contributions to the spin susceptibility of a d-dimensional electron gas for $d=1,2,3$. We used the bosonization method recently developed in Ref. 1 and demonstrated that it can give results not only for the specific heat as in Ref. 1 but also for the spin susceptibility.

The main contribution to this quantity comes from effective spin modes that interact with each other, which leads to

the nonanalytic logarithmic contributions. Although we consider isotropic systems, the low temperature behavior is determined by spin excitations moving antiparallel to each other. As a result, the nonanalytic contributions are determined by the backward scattering showing that there are one dimensional features also in the dimensions $d=2,3$.

The final form of the temperature corrections to the susceptibility in two and three dimensions is given by Eqs. (7.31)–(7.33). Although in $2d$ the correction to the susceptibility χ is very similar to the correction to the quantity $C(T)/T$, where $C(T)$ is the specific heat,[1] they are quite different in $3d$. The first logarithmic contribution to $C(T)/T$ is of the order γ_b^2, which is a well known result for $3d$ (Refs. 3–7). At the same time, the expansion of the susceptibility in the logarithms starts with the term of the order of γ_b^3, which shows that the nonanalytical temperature corrections exist for this quantity in three dimensions, too.

Using the bosonization scheme of Ref. 1 we have also reproduced the temperature dependent correction in one dimension, Eq. (6.10), that has been obtained long ago[38] using a renormalization group approach for the initial electron model.

The temperature dependent correction to the susceptibility in $2d$ was calculated recently by Shekhter and Finkelstein[40] using direct diagrammatic expansions for the initial electron model. In the approach of Ref. 40, which was tailored for the calculation of the spin susceptibility in $d=2$, the renormalization of the effective backward scattering amplitude is attributed to all Cooper channel harmonics, while no cutoff function was used. In the formalism of Ref. 1 which we studied no decoupling in the Cooper channel is introduced in addition to the particle-hole channel in order to avoid overcounting in the region of phase space close to backward scattering, which turned out to be most important (for a more detailed discussion of the role of the Cooper channel in the bosonization approach see Sec. VII C of Ref. 1). In fact, the renormalization of the backward scattering amplitude is obtained in this way as well, nonzero angular harmonics are, however, not included.

It is important to mention that in some cases not all nonanalytical corrections are accounted for by the backward scattering. Interesting contributions of the type $T^3 \ln T$ to, e.g., specific heat in three dimensions, are given by three-loop diagrams in the language of the electronic Green's functions (Ref. 7) and they cannot be expressed in terms of the backward or forward scattering. However, these corrections are proportional to higher powers of the interaction constant and are smaller than those given by the backward scattering unless the temperature is very low. In the latter regime the effective backscattering amplitude is very small, Eq. (7.36), and the contribution of three and more loop diagrams can become the most important one. Contributions that are not reduced to the backward scattering are discussed in Ref. 40 for the susceptibility in two dimensions.

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APPENDIX A: DERIVATION OF FORMULA EQ. (4.29) FOR $\mathcal{Z}_s[\mathbf{h}]$ BY EXPLICIT CONSTRUCTION

In this appendix we explicitly construct the supersymmetric representation of $\mathcal{Z}_s[\mathbf{h}]$ in Eq. (4.29).

It is a straightforward application of the results of Ref. 37 that $\mathcal{Z}_s[\mathbf{h}]$ of Eq. (4.1) in the main text can be rewritten

$$\mathcal{Z}_s[\mathbf{h}] = \exp\left(2\nu \int_{\hat{X},\hat{X}'} u\mathbf{h}_{\mathbf{n}}(x) L^{-1}_{X,X'} \partial_{\tau'} \mathbf{h}_{\mathbf{n}'}(x')\right), \quad \text{(A1)}$$

where the form of $L^{-1}_{X,X'}$ will be specified in the following. Here $X=(\mathbf{r},\tau,\mathbf{n},u)$ and the hat in $\int_{\hat{X}}$ indicates that integration $\int_{\hat{n}}$ in this formula is over the full solid angle.

$$(L^{-1}_{X,X'})_{\alpha,\beta} = -\frac{i}{2}[\langle \mathbf{S}^2_{\alpha,X} \mathbf{S}^{1*}_{\beta,X'}\rangle + \langle \mathbf{S}^1_{\alpha,X} \mathbf{S}^{1*}_{\beta,X'}\rangle - \langle \mathbf{S}^2_{\alpha,X} \mathbf{S}^{2*}_{\beta,X'}\rangle - \langle \mathbf{S}^1_{\alpha,X} \mathbf{S}^{2*}_{\beta,X'}\rangle]. \quad \text{(A2)}$$

Here α,β are spin indices and averaging is defined

$$\langle \cdots \rangle = \int \mathcal{D}(\boldsymbol{\varphi},\bar{\boldsymbol{\varphi}})(\cdots) e^{-\mathcal{L}[\bar{\boldsymbol{\varphi}},\boldsymbol{\varphi}]}, \quad \text{(A3)}$$

where

$$\mathcal{L}[\bar{\boldsymbol{\varphi}},\boldsymbol{\varphi}] = -i \int_{\hat{X}} \bar{\boldsymbol{\varphi}}_X \Lambda[\hat{M} + i\delta] \boldsymbol{\varphi}_X \quad \text{(A4)}$$

and supervector $\boldsymbol{\varphi}$ has been defined in Eqs. (4.14) and (4.15).

$$\bar{\boldsymbol{\varphi}} = \boldsymbol{\varphi}^\dagger \Lambda, \quad \Lambda = \sigma_3^{(H)}. \quad \text{(A5)}$$

The fermionic part of $\boldsymbol{\varphi}$, $\bar{\boldsymbol{\varphi}}$ takes care of the normalization via identity

$$\int \mathcal{D}(\boldsymbol{\chi},\bar{\boldsymbol{\chi}}) e^{-\mathcal{L}[\bar{\chi},\chi]} = \left[\int \mathcal{D}(\mathbf{S},\bar{\mathbf{S}}) e^{-\mathcal{L}[\bar{\mathbf{S}},\mathbf{S}]}\right]^{-1}. \quad \text{(A6)}$$

Finally

$$\Lambda \hat{M}_{\mathbf{n}} = \begin{pmatrix} \hat{L}'_{\mathbf{n}} & i\hat{L}''_{\mathbf{n}} \\ -i\hat{L}''_{\mathbf{n}} & -\hat{L}'_{\mathbf{n}} \end{pmatrix}_H. \quad \text{(A7)}$$

We repeat that $\hat{L}'=(\hat{L}+\hat{L}^\dagger)/2$ and $\hat{L}''=-i(\hat{L}-\hat{L}^\dagger)/2$ are Hermitian. The explicit form of $\Lambda\hat{M}$ is

$$\Lambda \hat{M}_{\mathbf{n}} = iv_0 \mathbf{n}\nabla - \Lambda_1 \partial_\tau + 2iu\hat{h}_{\mathbf{n}}. \quad \text{(A8)}$$

The matrix Λ_1 acts in H space and is written in Eq. (4.24). Restricting the angular integration to just one half sphere we can cast formula Eq. (A4) in another form by introducing the supervector $\boldsymbol{\phi}$ as follows:

$$\boldsymbol{\phi}(\mathbf{n}) = \begin{pmatrix} \boldsymbol{\varphi}(\mathbf{n}) \\ \boldsymbol{\varphi}(-\mathbf{n}) \end{pmatrix}_n, \quad \overline{\boldsymbol{\phi}(\mathbf{n})} = \boldsymbol{\phi}^\dagger(\mathbf{n})\Lambda, \tag{A9}$$

$\mathcal{L}$ has to be modified accordingly.

$$\mathcal{L} \to \mathcal{L}[\bar{\boldsymbol{\phi}}, \boldsymbol{\phi}] = -i\int_X \bar{\boldsymbol{\phi}}_X \Lambda[\hat{\mathbb{M}} + i\delta]\boldsymbol{\phi}_X, \tag{A10}$$

where

$$\hat{\mathbb{M}}_{\mathbf{n}} = \begin{pmatrix} \hat{M}_{\mathbf{n}} & 0 \\ 0 & \hat{M}_{-\mathbf{n}} \end{pmatrix}. \tag{A11}$$

The explicit form of $\Lambda\hat{\mathbb{M}}$ is

$$\Lambda\hat{\mathbb{M}}_{\mathbf{n}} = iv_0\mathbf{n}\Sigma_3\nabla - \Lambda_1\partial_\tau + 2iu\hat{\mathbb{H}}_{\mathbf{n}}. \tag{A12}$$

Here we introduced $\Sigma_3 = \sigma_3^{(n)}$ and $\mathbb{H}_{\mathbf{n}}(x)$ of Eq. (4.25).

Finally the number of field components in $\boldsymbol{\phi}$, $\bar{\boldsymbol{\phi}}$ is doubled once more by introducing the electron-hole (eh) sector. This can be done by introducing the vector

$$\boldsymbol{\psi} = \frac{1}{\sqrt{2}}\begin{pmatrix} \boldsymbol{\phi}^* \\ \boldsymbol{\phi} \end{pmatrix}_{eh}, \quad \bar{\boldsymbol{\psi}} = \boldsymbol{\psi}^\dagger\Lambda. \tag{A13}$$

Now

$$\mathcal{L} \to \mathcal{L}[\bar{\boldsymbol{\psi}}, \boldsymbol{\psi}] = -i\int_X \bar{\boldsymbol{\psi}}_X \Lambda[\hat{\mathcal{M}} + i\delta]\boldsymbol{\phi}_X, \tag{A14}$$

where

$$\hat{\mathcal{M}}_{\mathbf{n}} = \begin{pmatrix} \hat{\mathbb{M}}_{\mathbf{n}} & 0 \\ 0 & \hat{\mathbb{M}}_{\mathbf{n}}^T \end{pmatrix}_{TR}. \tag{A15}$$

Note that the transposition for $\hat{\mathbb{M}}^T$ includes derivatives. Here matrix $\tau_3 = \sigma_3^{(eh)}$ acts in eh space. The explicit form is

$$\Lambda\hat{\mathcal{M}}_{\mathbf{n},u} = -iv_0\tau_3\Sigma_3\mathbf{n}\boldsymbol{\nabla} - \Lambda_1\partial_\tau - 2i\tau_3\hat{\mathbb{H}}_{\mathbf{n}}. \tag{A16}$$

Now we can write an appropriate generalization of Eq. (A1), Eq. (A3) and make contact to formula Eq. (4.29) in the main text. We write

$$\mathcal{L}[\bar{\boldsymbol{\psi}}, \boldsymbol{\psi}] = -i2\nu\int_X \bar{\boldsymbol{\psi}}_X(\mathcal{H} + i\delta\Lambda)\boldsymbol{\psi}_X, \tag{A17}$$

where $\mathcal{H} = \Lambda\hat{\mathcal{M}}$ (factor of 2ν is introduced for convenience) and the averaging with respect to this Lagrangian is defined

$$\langle\cdots\rangle = \int \mathcal{D}(\boldsymbol{\psi}, \bar{\boldsymbol{\psi}})(\cdots)e^{-\mathcal{L}[\bar{\boldsymbol{\psi}}, \boldsymbol{\psi}]}. \tag{A18}$$

Using

$$\mathcal{F}_{\mathbf{h}}(X) = \partial_X\mathbb{H}_{\mathbf{n}}(x)\mathcal{F}_0, \quad \overline{\mathcal{F}_{\mathbf{h}}(X)}[C\mathcal{F}_{\mathbf{h}}(X)]^T, \tag{A19}$$

where $\mathcal{F}_0$ is defined in Eq. (4.25) and

$$\partial_X(\alpha) = \begin{pmatrix} 1 & 0 \\ 0 & u[\alpha\partial_\tau + (1-\alpha)iv_0\mathbf{n}\nabla\Sigma_3] \end{pmatrix} \tag{A20}$$

one verifies that

$$\bar{\boldsymbol{\psi}}\mathcal{F}_{\mathbf{h}} = \overline{\mathcal{F}_{\mathbf{h}}}\boldsymbol{\psi} = \frac{1}{2}(\hat{\mathcal{O}}_{\mathbf{n}}(\alpha)\mathbf{h}_{\mathbf{n}}(\mathbf{S}_{\mathbf{n}}^{1*} - \mathbf{S}_{\mathbf{n}}^{2*}) + \mathbf{h}_{\mathbf{n}}(\mathbf{S}_{\mathbf{n}}^1 + \mathbf{S}_{\mathbf{n}}^2) + (\mathbf{n} \leftrightarrow -\mathbf{n})), \tag{A21}$$

where

$$\hat{\mathcal{O}}_{\mathbf{n}}(\alpha) = u[\alpha\partial_\tau + (1-\alpha)iv_0\mathbf{n}\nabla]. \tag{A22}$$

Using $\langle S_{\alpha,X}^i S_{\beta,X'}^{j*}\rangle \propto \delta_{\mathbf{n},\mathbf{n}'}\delta_{u,u'}$ one obtains

$$\mathcal{I}[\mathbf{h}] = \int_{XX'} \overline{\mathcal{F}_{\mathbf{h}}(X)}\langle\boldsymbol{\psi}_X\overline{\boldsymbol{\psi}_{X'}}\rangle\mathcal{F}_{\mathbf{h}}(X') = \frac{1}{2}\int_{XX'} h_{\mathbf{n}}^\gamma(x)\langle(S_{\gamma,X}^1 + S_{\gamma,X}^2)(S_{\beta,X}^{1*} - S_{\beta,X}^{2*})\rangle\hat{\mathcal{O}}_{\mathbf{n}}(\alpha)h_{\mathbf{n}}^\beta(x'). \tag{A23}$$

Summation over spin indices γ, β is implied. Using further Eq. (A2) one finds

$$\mathcal{I}[\mathbf{h}] = \frac{i}{2\nu}\int dudx(d\mathbf{n})[\mathbf{h}_{\mathbf{n}}(x)(u\hat{L}_{\mathbf{n},u}^{-1}(\partial_\tau\mathbf{h}_{\mathbf{n}})(x)) + \mathbf{h}_{\mathbf{n}}(x)(u(1-\alpha)\hat{L}_{\mathbf{n},u}^{-1}(iv_0\mathbf{n}\nabla - \partial_\tau)\mathbf{h}_{\mathbf{n}}(x))]. \tag{A24}$$

The last line can be simplified by noticing

$$\hat{L}_{\mathbf{n},u}^{-1}(iv_0\mathbf{n}\boldsymbol{\nabla} - \partial_\tau)\mathbf{h}_{\mathbf{n}}(x) = \mathbf{h}_{\mathbf{n}}(x). \tag{A25}$$

This equality holds, since $\hat{h}\mathbf{h} = \mathbf{h}\times\mathbf{h} = 0$. The result is

$$\mathcal{Z}_s[\mathbf{h}] = \exp\left(-4i\nu^2\int_{XX'} \overline{\mathcal{F}_{\mathbf{h}}(X)}\langle\boldsymbol{\psi}_X\overline{\boldsymbol{\psi}_{X'}}\rangle\mathcal{F}_{\mathbf{h}}(X')\right) \times\exp\left(-\nu(1-\alpha)\int_{\hat{\mathbf{n}},x}\mathbf{h}_{\mathbf{n}}^2(x)\right). \tag{A26}$$

This formula is used in the main text, Eq. (4.29).

APPENDIX B: DIAGRAMS OF FIG. 10

In this appendix we give analytic expressions for the diagrams displayed in Fig. 10. We introduce

$$\boldsymbol{\Delta}_i^{\sigma_1\sigma_2}(z_1, z_2, \mathbf{k}) = \int d^d\mathbf{r}e^{-i\mathbf{k}\mathbf{r}}\boldsymbol{\Delta}_i^{\sigma_1\sigma_2}(\widehat{\mathbf{n}_1\mathbf{n}_2}, u_1, u_2, \mathbf{r}_\perp)\bar{f}(\mathbf{r}) \tag{B1}$$

for $\boldsymbol{\Delta}$ and use similar notation for amplitudes $\mathcal{B}$ and Γ. To simplify expressions let us write

$$\boldsymbol{\Delta}_{\mathcal{F}}(z_1, z_2, \mathbf{k}) = \sum_{i=1}^4 \sum_{\sigma_k=\pm} \boldsymbol{\Delta}_i^{\sigma_1\sigma_2}(z_1, z_2, \mathbf{k})(\mathcal{F}_{\sigma_1}\overline{\mathcal{F}_{\sigma_2}})^i \tag{B2}$$

and suppress the trivial dependence on (z_1, z_2). Then the relevant expressions read

$$\delta\chi_1(\mathbf{q},\varepsilon) = -4\eta^2 T\sum_\omega \int d\mathbf{p}dz_1dz_2u_1^2u_2^2\mathrm{str}(\Delta_{\mathcal{F}}(\mathbf{p}) \times \mathbb{T}^{(1)}_{\mathbf{n}_1}(p,q)\Delta_{\mathcal{F}}(\mathbf{p}-\mathbf{q})\overline{\mathbb{T}}^{(1)}_{\mathbf{n}_2}(p,q)), \tag{B3}$$

$$\delta\chi_2(\mathbf{q},\varepsilon) = -8\eta^2 T\sum_\omega \int d\mathbf{p}dz_1dz_2u_1^3u_2\mathrm{str}(\mathbf{\Delta}_{\mathcal{F}}(\mathbf{p}) \times \mathbb{T}^{(2a)}_{\mathbf{n}_1}(p,q)\mathbf{\Delta}_{\mathcal{F}}(\mathbf{p})\mathbb{T}^{(2b)}_{\mathbf{n}_2}(\mathbf{p})), \tag{B4}$$

$$\delta\chi_3(\mathbf{q},\varepsilon) = 8\eta^2\sum_{i=1}^{4} T\sum_\omega \int d\mathbf{p}dz_1dz_2u_1^2u_2^2\Gamma_i(\mathbf{p}-\mathbf{q})\overline{\mathcal{F}_0} \times \mathbb{T}^{(3)}_{\mathbf{n}_1}(p,q)\mathbf{\Delta}^i_{\mathcal{F}}(\mathbf{p})\overline{\mathbb{T}}^{(3)}_{\mathbf{n}_2}(p,q)\mathcal{F}_0, \tag{B5}$$

$$\delta\chi_4(\mathbf{q},\varepsilon) = -8\eta^2\sum_{i=1}^{4} T\sum_\omega \int d\mathbf{p}dz_1dz_2u_1^2u_2^2\mathcal{B}_i^{\sigma_1}(\mathbf{p}-\mathbf{q})\mathcal{B}_i^{\sigma_2}(\mathbf{p}) \times \overline{\mathcal{F}_0}\mathbb{T}^{(4a)}_{\mathbf{n}_1}(p,q)(\mathcal{F}_{\sigma_1}\overline{\mathcal{F}_{\sigma_2}})^i\mathbb{T}^{(4b)}_{\mathbf{n}_2}(p,q)\mathcal{F}_0, \tag{B6}$$

$$\chi_5(\mathbf{q},\varepsilon) = -16\eta^2\sum_{i=1}^{4}\sum_{\sigma_1} T\sum_\omega \int d\mathbf{p}dz_1dz_2u_1^2u_2^2\mathcal{B}_i^{\sigma_1}(\mathbf{p}-\mathbf{q}) \times \overline{\mathcal{F}_{\sigma_1}}\mathbb{T}^{(5a)}_{\mathbf{n}_1}(p,q)\mathbf{\Delta}^i_{\mathcal{F}}(\mathbf{p})\mathbb{T}^{(5b)}_{\mathbf{n}_2}(p,q)\mathcal{F}_0, \tag{B7}$$

$$\chi_6(\mathbf{q},\varepsilon) = -16\eta^2\sum_{\sigma_1}\sum_{i=1}^{4} T\sum_\omega \int d\mathbf{p}dz_1dz_2u_1u_2^3\mathcal{B}_i^{\sigma_1}(\mathbf{p}-\mathbf{q}) \times \overline{\mathcal{F}_{\sigma_1}}\mathbb{T}^{(6a)}_{\mathbf{n}_1}(p)\mathbf{\Delta}^i_{\mathcal{F}}(\mathbf{p})\mathbb{T}^{(6b)}_{\mathbf{n}_2}(p,q)\mathcal{F}_0, \tag{B8}$$

$$\chi_7(\mathbf{q},\varepsilon) = 8\eta^2\sum_{\sigma_1,\sigma_2=\pm}\sum_{i=1}^{4} T\sum_\omega \int d\mathbf{p}dz_1dz_2u_1^3u_2 \times \mathcal{B}_i^{\sigma_1}(\mathbf{p})\mathcal{B}_i^{\sigma_2}(\mathbf{p})\bar{\mathcal{F}}_0\mathbb{T}^{(7a)}_{\mathbf{n}_1}(p,q)(\mathcal{F}_0\overline{\mathcal{F}_{\sigma_1}})^i\mathbb{T}^{(7b)}_{\mathbf{n}_2}(p)\mathcal{F}_{\sigma_2}, \tag{B9}$$

where

$$\mathbb{T}^{(1)}_{\mathbf{n}_1}(p,q) = [(i\omega+\mathbf{v}_1\mathbf{p}\Sigma_3)-\tau_+(i\varepsilon+\mathbf{v}_1\mathbf{q}\Sigma_3)]\mathcal{G}_{\mathbf{n}_1}(p)\mathcal{G}_{\mathbf{n}_1}(p-q), \tag{B10}$$

$$\mathbb{T}^{(2a)}_{\mathbf{n}_1}(p,q) = \tau_3(i\omega+\mathbf{v}_1\mathbf{p}\Sigma_3)\mathcal{G}^2_{\mathbf{n}_1}(p)\mathcal{G}_{\mathbf{n}_1}(p+q), \tag{B11}$$

$$\mathbb{T}^{(2b)}_{\mathbf{n}_2}(p) = \tau_3(i\omega+\mathbf{v}_2\mathbf{p}\Sigma_3)\mathcal{G}_{\mathbf{n}_2}(p), \tag{B12}$$

$$\mathbb{T}^{(3)}_{\mathbf{n}_1}(p,q) = [\tau_+(i\varepsilon+\mathbf{v}_1\mathbf{q}\Sigma_3)+\tau_-(i\omega+\mathbf{v}_1\mathbf{p}\Sigma_3)] \times \mathcal{G}_{\mathbf{n}_1}(p)\mathcal{G}_{\mathbf{n}_1}(q), \tag{B13}$$

$$\mathbb{T}^{(4a)}_{\mathbf{n}_1}(p,q) = [(i\varepsilon+\mathbf{v}_1\mathbf{q}\Sigma_3)-\tau_-(i\omega+\mathbf{v}_1\mathbf{p}\Sigma_3)] \times \mathcal{G}_{\mathbf{n}_1}(q)\mathcal{G}_{\mathbf{n}_1}(q-p), \tag{B14}$$

$$\mathbb{T}^{(4b)}_{\mathbf{n}_2}(p,q) = [\tau_+(i\omega+\mathbf{v}_2\mathbf{p}\Sigma_3)+\tau_-(i\varepsilon+\mathbf{v}_2\mathbf{q}\Sigma_3)] \times \mathcal{G}_{\mathbf{n}_2}(p)\mathcal{G}_{\mathbf{n}_2}(q), \tag{B15}$$

$$\mathbb{T}^{(5a)}_{\mathbf{n}_1}(p,q) = [(i\omega+\mathbf{v}_1\mathbf{p}\Sigma_3)-\tau_+(i\varepsilon+\mathbf{v}_1\mathbf{q}\Sigma_3)] \times \mathcal{G}_{\mathbf{n}_1}(p-q)\mathcal{G}_{\mathbf{n}_2}(p), \tag{B16}$$

$$\mathbb{T}^{(5b)}_{\mathbf{n}_2}(p,q) = [\tau_+(i\omega+\mathbf{v}_2\mathbf{p}\Sigma_3)+\tau_-(i\varepsilon+\mathbf{v}_2\mathbf{q}\Sigma_3)] \times \mathcal{G}_{\mathbf{n}_2}(p)\mathcal{G}_{\mathbf{n}_2}(q), \tag{B17}$$

$$\mathbb{T}^{(6a)}_{\mathbf{n}_1}(p) = \tau_3(i\omega+\mathbf{v}_1\mathbf{n}_1\mathbf{p}\Sigma_3)\mathcal{G}_{\mathbf{n}_1}(p), \tag{B18}$$

$$\mathbb{T}^{(6b)}_{\mathbf{n}_2}(p,q) = [\tau_+(i\omega+\mathbf{v}_2\mathbf{p}\Sigma_3)+\tau_-(i\varepsilon+\mathbf{v}_2\mathbf{q}\Sigma_3)] \times \tau_3\mathcal{G}_{\mathbf{n}_2}(p)\mathcal{G}_{\mathbf{n}_2}(p+q)\mathcal{G}_{\mathbf{n}_2}(q), \tag{B19}$$

$$\mathbb{T}^{(7a)}_{\mathbf{n}_1}(p,q) = \tau_3(i\varepsilon+\mathbf{v}_1\mathbf{p}\Sigma_3)\mathcal{G}^2_{\mathbf{n}_1}(q)\mathcal{G}_{\mathbf{n}_1}(p+q), \tag{B20}$$

$$\mathbb{T}^{(7b)}_{\mathbf{n}_2}(p) = \tau_3(i\omega+\mathbf{v}_2\mathbf{p}\Sigma_3)\mathcal{G}_{\mathbf{n}_2}(p). \tag{B21}$$

Four dimensional notation was used $p=(\omega,\mathbf{p})$, $q=(\varepsilon,\mathbf{q})$.

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