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Wiedemann-Franz law in the SU(N) Wolff model

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We study the electrical and thermal transport through the SU(N) Wolff model with the use of bosonization. The Wilson ratio reaches unity as N grows to infinity. The electric conductance is dominated by the charge channel, and decreases monotonically with increasing interaction. The thermal conductivity enhances in the presence of a local Hubbard U. The Wiedemann-Franz law is violated; the Lorentz number depends strongly on the interaction parameter, which can be regarded as a manifestation of spin-charge separation.

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The Wiedemann-Franz law is one of the basic properties of a Fermi liquid.[1] Simply stated, it reflects the fact that the ability of quasiparticles to carry charge is the same as to transport heat, and is given by

$$L = \frac{\kappa}{\sigma T} = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2, \tag{1}$$

where κ and σ are the heat and electrical conductivity, e is the electron charge, k_B is the Boltzmann constant, and L is called the Lorentz number. A possible breakdown of this relation is interpreted in terms of spin-charge separation. Spinons can carry heat in much the same way as charges do, and contribute to heat transport. On the other hand, they fail in electric transport, while charge excitations contribute to both. Another explanation requires inelastic scattering.[1,2] Elastic scattering affects both charge and heat transport, while inelastic processes influence only the latter. Electrons supplemented with deeply inelastic forward scattering readily show the violation of Eq. (1).[1]

Since the universal Lorentz number appears to be a defining characteristic of Landau's Fermi-liquid phases, deviations from this are expected mainly in non-Fermi-liquid systems. An ideal model is a one-dimensional interacting electron gas, in which spin and charge excitations propagate with different velocities.[3] Indeed, the violation of the Wiedemann-Franz law has been found in such systems.[4] Beyond one-dimensional models, the large-N limit (N stands for the spin degeneracy) of the t-J model in two dimensions also provides us with the breaking of the Wiedemann-Franz law. Its ground state in a certain parameter range is the staggered flux phase,[5] in which the Lorentz number strongly deviates from its universal value.[5,6] On the other hand, other properties of this phase are Fermi-liquid-like. The breakdown of the Wiedemann-Franz law originates from the fractionalization of quasiparticles into separate spin and charge.[6] Finally, s-wave superconductors also show this violation, since Cooper pairs carry charge, but the condensate has no entropy.

The simplest models exhibiting spin-charge separation belong to the family of Kondo models.[3] The dc electric transport was exhaustively explored in these models, but less is known about the ac response, and the heat transport remained completely unexplored. Here we undertake the job of studying heat transport in an N-fold degenerate model capable of the Kondo phenomenon. This model not only shows the violation of the Wiedemann-Franz law, but enables us to connect different parts of the discussion above. We can study explicitly spin-charge separation for arbitrary N, and see how the mean field behavior in the large-N limit emerges.

The model we have chosen is the SU(N) Wolff model.[7–10] It consists of N species of electrons interacting with each other only at a single site. The model is studied with Abelian bosonization[3] for arbitrary N. We consider fermions with SU (N) spin index or alternatively we take the ensemble of spin and orbital degrees of freedom into account by the N index.[11,12] These additional degrees of freedom can be realized through orbital degeneracy, for example, as in Mn oxides.[13] As a result, the additional degrees of freedom can be called the flavor or color index. The Wolff model is one of the simplest impurity models, where electron correlation is still present.[7,9,14] Also it is the basis for studying the effect of Coulomb interaction on resonant tunneling through a single quantum level.[15,16] In spite of several efforts to explore this model, its frequency-dependent electric conductance and heat transport has not been studied before to our knowledge.

The Wilson ratio approaches unity as N grows; it crosses over to noninteracting (mean-field) behavior as N grows to infinity, a phenomenon characteristic of SU(N) models.[17,18] The electric conduction involves only the charge sector, and increasing interaction increases the resistivity. Heat is transported by both spin and charge excitation, and heat conduction is favored by any finite value of U, regardless of its sign. The respective conductivities depend strongly on the local interaction, leading to the violation of the Wiedemann-Franz law even in the large-N limit. This suggests that spin-charge separation is the origin of this breakdown.

The Hamiltonian describing N different species of electrons interacting only at the origin is given by

$$H = \sum_{m=1}^{N}\left[-iv\int_{-V/2}^{V/2} dx\, \Psi_m^\dagger(x)\partial_x\Psi_m(x) + E\rho_m(0) + \frac{U}{2}\sum_{n=1,n\neq m}^{N}\rho_m(0)\rho_n(0)\right], \tag{2}$$

and only the radial motion of the particles is accounted for by chiral (right moving) fermion fields,[19] $\rho_m(x) = :\Psi_m^\dagger(x)\Psi_m(x):$, V is the length of the system, v is the Fermi velocity, E describes potential scattering at the origin, and U stands for the local electron-electron interaction. The model can be bosonized via[3,20]

$$\Psi_m(x) = 1/\sqrt{2\pi\alpha}e^{i\sqrt{4\pi}\Phi_m(x)}, \tag{3}$$

and after introducing charge and spin fields as[12]

$$\Phi_c(x) = \frac{1}{\sqrt{N}}\sum_{m=1}^{N}\Phi_m(x), \tag{4}$$

$$\Phi_{n,s}(x) = \frac{1}{\sqrt{n(n+1)}}\left(\sum_{m=1}^{n}\Phi_m(x) - n\Phi_{n+1}(x)\right), \tag{5}$$

$n=1,\ldots,N-1$, the Hamiltonian separates into different sectors: the spin sector is described by $N-1$ identical decoupled bosonic modes, and the charge sector transforms into a similar massless bosonic mode as

$$H_s = \sum_{m=1}^{N-1}\left(v\int_{-V/2}^{V/2}dx[\partial_x\Phi_{m,s}(x)]^2 - \frac{U}{2\pi}[\partial_x\Phi_{m,s}(0)]^2\right), \tag{6}$$

$$H_c = v\int_{-V/2}^{V/2}dx[\partial_x\Phi_c(x)]^2 + E\sqrt{\frac{N}{\pi}}\partial_x\Phi_c(0) + \frac{(N-1)U}{2\pi}[\partial_x\Phi_c(0)]^2. \tag{7}$$

These can readily be diagonalized by introducing the pure bosonic representation of the $\Phi(x)$ fields as

$$\Phi(x) = \sum_{q>0}\frac{1}{\sqrt{2Vq}}(e^{iqx}b_q + e^{-iqx}b_q^\dagger)e^{-\alpha q/2}, \tag{8}$$

where α is the ultraviolet cutoff. Usually $x\in[-V/2,V/2]$, but $V\to\infty$ in actual calculations. The charge Hamiltonian is rewritten as

$$H_c = \sum_{q>0}\left(vqb_q^\dagger b_q + iE\sqrt{\frac{qN}{2V\pi}}(b_q - b_q^\dagger) + \frac{(N-1)U}{2\pi}\sum_{k>0}\frac{-\sqrt{qk}}{2V}(b_q - b_q^\dagger)(b_k - b_k^\dagger)\right), \tag{9}$$

which describes spinless bosons scattered by U, and a source term E. The latter can be transformed out by a linear shift of the bosonic field, while the former can be considered exactly via Dyson equation. Similar equations describe the spin sector. Within the realm of bosonization, U is restricted to $(-U_0/(N-1),U_0)$ with $U_0=\pi v/n_0$, where n_0 is the average density per spin in the homogeneous case.[10]

As a first step to understand the response of our model, it is instructive to investigate the Wilson ratio. The Wilson ratio characterizes to what extent the impurity interaction influences the conduction electron properties. Using Ref. 10, it is calculated for the SU(N) Wolff model as

$$R = \frac{\chi_{imp}^s}{\chi_0^s}\frac{\partial C_0/\partial T}{\partial C_{imp}/\partial T} = 1 + \frac{U}{U_0 + (N-2)U}, \tag{10}$$

where χ_{imp}^s and χ_0^s are the local spin susceptibilities with and without the impurity, and C_{imp} and C_0 are the specific heat with and without the impurity, respectively. It interpolates smoothly between zero in the attractive case and $N/(N-1)$ in the repulsive case. Interestingly, the latter value was calculated in the N-fold degenerate Coqblin-Schrieffer model[21] as well. This might suggest that the $U\to U_0$ limit of the Wolff model found by bosonization may describe the $U\to\infty$ limit of the original model. The shortcoming of bosonization is to underestimate the strength of interaction as it does for simple impurity scattering.[3]

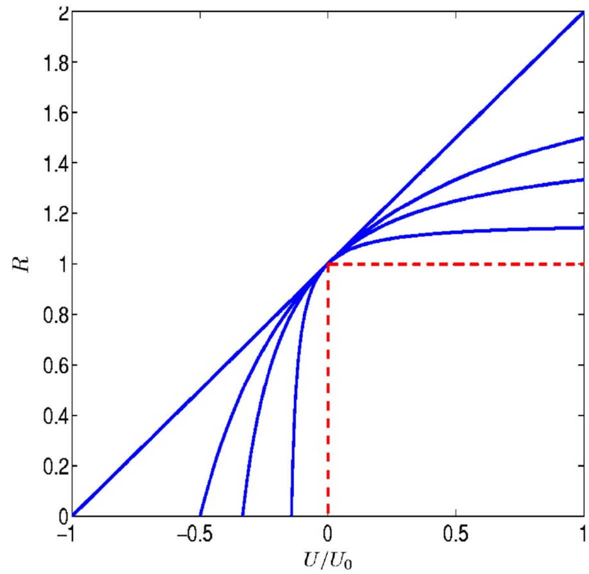

FIG. 1. (Color online) The Wilson ratio of the SU(N) Wolff model is plotted for N=2, 3, 4, 8, and ∞ (red dashed line) from top to bottom.

As N grows, R approaches unity (i.e., the noninteracting limit). Indeed, the ground state of the $N\to\infty$ limit of impurity models is the mean-field solution,[21] because fluctuations around the saddle point are suppressed by $1/N$, and vanish as N reaches infinity. As a result, both quantities determining the Wilson ratio are proportional to the density of states at the Fermi energy,[10] and hence their ratio is 1. The general behavior of the Wilson ratio for various N's is shown in Fig. 1. The SU(2) case was also investigated in Ref. 14.

The calculation of the electric conductivity relies on the physical assumption that the whole voltage drop occurs at the impurity site.[3,22] From this, the linear conductivity can be calculated by the Kubo formula. From the continuity equation ($\partial_t n + \partial_x j = 0$), the conductance is obtained, if we proceed following Refs. 16 and 23. The local density is determined by

$$n(x) = \sqrt{\frac{N}{\pi}}\partial_x\Phi_c(x), \tag{11}$$

from which the local current operator is calculated as

$$j(x) = -e\sqrt{\frac{N}{\pi}}v\partial_x\Phi_c(x) = -evn(x). \tag{12}$$

Only charge excitations participate in electric transport; spinons transport only heat, not charge, as will be demonstrated below. The expectation value of $\partial_x\Phi_c(x)$ is finite as seen in Ref. 10, which is a natural consequence of having the

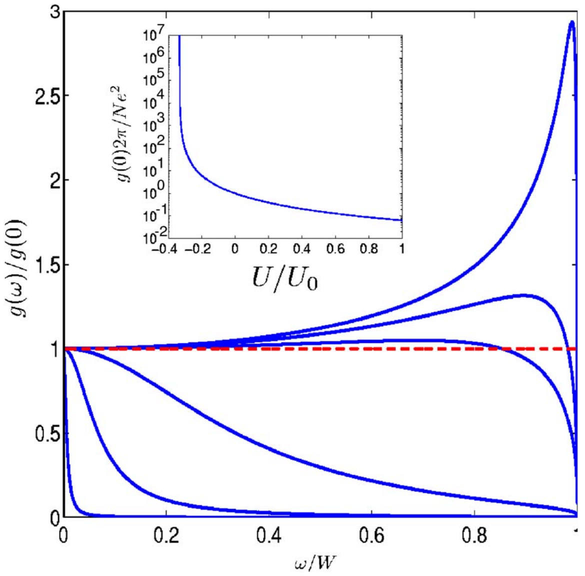

FIG. 2. (Color online) The frequency-dependent conductance is shown for $N=4$ for $U/U_0=-0.33$, -0.3, -0.2 below the red dashed line indicating $U=0$, from bottom to top. The other curves correspond to $U/U_0=0.2$, 0.5, and 0.9 from top to bottom. W is a sharp momentum space cutoff. The inset shows the dc conductance as a function of the Hubbard interaction.

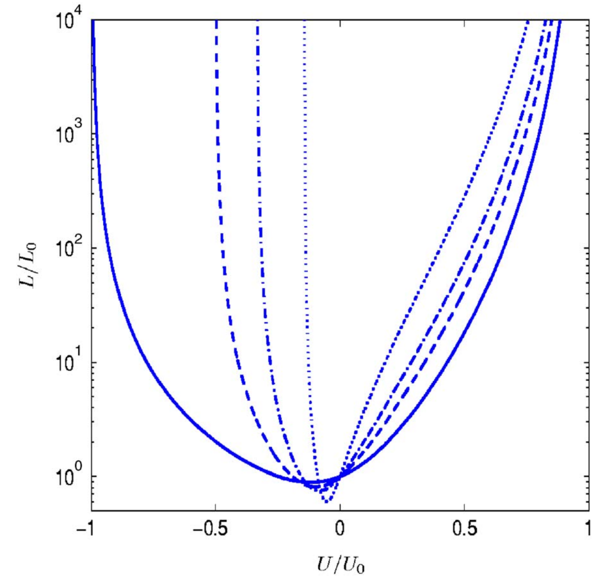

FIG. 3. (Color online) The Lorentz number in the SU(N) Wolff model is plotted for $N=2$ (solid line), 3 (dashed line), 4 (dash-dotted line), and 8 (dotted line) on a semilogarithmic scale.

same chirality for all N channels. The current-current correlation function can be evaluated from the bosonic representation of the $\Phi_c(x)$ field through the equation of motion method. After solving the closed set of equations using Eq. (9), the frequency-dependent ac conductance is evaluated as

$$g(\omega) = (ev)^2 \mathrm{Re}\frac{N\chi_0(\omega)}{i\omega[1+(N-1)U\chi_0(\omega)]}, \tag{13}$$

where $\chi_0(\omega)$ is the local charge correlator per spin at $U=0$. An identical result was obtained for the charge correlator using the generating functional.[10] It is shown in Fig. 2 for $N=4$. Similar curves describe the $N\neq 4$ behavior as well. Its dc part yields

$$g(0) = \frac{e^2}{2\pi}\frac{NU_0^2}{[U_0+(N-1)U]^2}, \tag{14}$$

which decreases smoothly for repulsive interaction, but increases rapidly in the attractive sector. Throughout the calculations we set $\hbar=1$, which explains the $e^2/2\pi$ factor in front of the right-hand side of Eq. (14). By restoring the original units, this gets replaced by the universal channel conductance unit e^2/h. Qualitatively similar behavior was identified in the conductance of a one-dimensional interacting electron gas (Luttinger liquid) through a weak link.[24] Repulsive interaction caused perfect reflection; attractive interaction resulted in perfect transmission. Although the details are different, our results exhibit the same phenomenon as seen from Eq. (14) and from the inset of Fig. 2. In the $N\to\infty$ limit, the dc conductance is zero for any finite U. The charge sector involved in electric transport is governed by Eq. (7), where the scattering term ($\propto U$) grows with N, hindering any transport in the mean-field limit.

The ac conductance of another member of the Kondo family, the Anderson impurity model, was investigated in Ref. 25, with special emphasis on the relation of the ac conductance and equilibrium spectral function. This is an effective two-band model, with interacting local f electrons weakly hybridized with the conduction band, ideal for Kondo physics. As opposed to this, our model consists of a single band with no distinction between conduction and localized electrons, and supplemented with weak interaction, where our approach is restricted to that of Ref. 10; hence a direct comparison of the results is difficult.

The heat transport can be studied similarly to the electric one. The heat-current operator is obtained from the energy conservation, following the steps outlined in Refs. 4 and 5, and reads as

$$J_Q = v^2\sum_{m=1}^{N}[\partial_x\Phi_m(x)]^2 = v^2\left([\partial_x\Phi_c(x)]^2 + \sum_{n=1}^{N-1}[\partial_x\Phi_{n,s}(x)]^2\right). \tag{15}$$

The Hamiltonians in the different sectors commute with each other; hence their contributions can be determined independently. The thermal conductivity can be calculated from the energy current-current correlation function given by

$$\Pi(i\omega_n) = -\int_0^\beta d\tau e^{i\omega_n\tau}\langle T_\tau J_Q(\tau)J_Q(0)\rangle, \tag{16}$$

and the retarded response function is obtained after analytic continuation as

$$\frac{\kappa}{T} = -\frac{\mathrm{Im}[\Pi(i\omega_n\to\omega+i\delta)]}{T^2\omega}. \tag{17}$$

Its calculation amounts to determining the $\langle T_\tau[\partial_x\Phi(0,\tau)]^2[\partial_x\Phi(0,0)]^2\rangle$ correlator. Possible pairings are of $\langle T_\tau\partial_x\Phi(0,\tau)\partial_x\Phi(0,0)\rangle$ type in all sectors, which equals the charge and spin correlation functions obtained in Ref. 10. The Hamiltonians describe scattering of free bosons on a localized "impurity"; hence only self-energy corrections can be taken into account (vertex corrections are absent). The calculation can be carried out in a straightforward manner, and by taking all the channels into account, we find

$$\frac{\kappa}{T}=\frac{1}{4T^3\pi}\int_{-\infty}^{\infty}dx\frac{1}{\sinh^2(\beta x/2)}\times\{[\mathrm{Im}G_c(x)]^2+(N-1)[\mathrm{Im}G_s(x)]^2\}, \quad (18)$$

where $G_c(x)=v^2\pi\chi_0(x)/[1+(N-1)U\chi_0(x)]$ and $G_s(x)=v^2\pi\chi_0(x)/[1-U\chi_0(x)]$. As $T\to 0$, this can readily be evaluated to yield

$$\frac{\kappa}{T}=\frac{\pi}{6}\left(\frac{U_0^4}{[U_0+(N-1)U]^4}+\frac{(N-1)U_0^4}{(U_0-U)^4}\right). \quad (19)$$

In the absence of U, it gives the pure result $\kappa=(\pi/6)NT$ (or $\pi^2k_B^2NT/3h$ upon restoring original units). In the presence of finite U, it increases regardless of the sign of the interaction. In other words, local Coulomb repulsion or attraction facilitates heat transport. From these, the Lorentz number reads as

$$L=\frac{\kappa}{Tg(0)}=\frac{L_0}{N}\left(\frac{U_0^2}{[U_0+(N-1)U]^2}+(N-1)\frac{U_0^2[U_0+(N-1)U]^2}{(U_0-U)^4}\right), \quad (20)$$

where $L_0=\pi^2k_B^2/3e^2$ is the noninteracting result. For any finite U and N, the Wiedemann-Franz law is broken, except a single value of attractive U for any given N. This breakdown is a natural consequence of spin-charge separation, as seen in Eqs. (6) and (7). Had we chosen $N=1$, the Wiedemann-Franz law would hold. As U increases in the repulsive regime, it is enhanced strongly. In the attractive case, a minimum value is reached for decreasing U before the divergence at $-U_0/(N-1)$. For finite T, the Lorentz number decreases monotonically. Violation of the Wiedemann-Franz law was reported in a one-dimensional electron gas (Luttinger liquid) with repulsive interaction. For any finite interaction, L increased from the Fermi-liquid value, similarly to our case. From these similarities in transport and in x-ray response,[10] we conclude that the physical quantities evaluated within our model and its lattice version (Hubbard model) exhibit the same type of behavior as a function of interaction. Other failures of the Wiedemann-Franz law have been seen in mean-field theories as well.[5,6] This is plotted in Fig. 3 for various N's. Although the long-time asymptotics of the Green's function is of Fermi-liquid character, still spin-charge separation occurs on the microscopic level, as seen in Eqs. (6) and (7), causing the breakdown of the Wiedemann-Franz law. A similar phenomenon is expected to occur in the single- and multichannel realizations of the Kondo effect.[3]

In conclusion, we have studied the electric and heat transport properties of the SU(N) Wolff model. As N grows to infinity, the system crosses over to noninteracting behavior. At the same time, the Wiedemann-Franz law remains broken due to spin-charge separation.

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