

Enhancing Kerr nonlinearity in an asymmetric double quantum well via Fano interference

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We investigate the enhancement of Kerr nonlinearity in an asymmetric GaAs double quantum well via Fano interference, which is caused by tunneling from the excited subband to the continuum. In our structure, owing to Fano interference, the Kerr nonlinearity can be enhanced by appropriately choosing the values of the detunings and the intensity of the pump field, while cancel the linear and nonlinear absorptions.

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I. INTRODUCTION

In the past few decades, there have attracted tremendous interests in the study of electromagnetically induced transparency (EIT),¹ which results from quantum coherence and interference. This coherence is due to the population trapping in the dark state. EIT has been studied extensively because it can be applied to achieve lasing without inversion (LWI),² produce subluminal and superluminal propagation of light pulses,³ and enhance the efficiency of four-wave mixing (FWM) (Ref. 4) and frequency conversion,⁵ etc. In the conduction band of semiconductor quantum wells (QWs), the two-dimensional electron gas behaves atomlike optical responses. Quantum interference and coherence⁶ in QWs can also produce some interesting phenomena such as strong EIT,⁷ CPT,⁸ LWI,⁹ enhancement of refractive index,¹⁰ all-optical switching,¹¹ and so on.^{12–18}

In nonlinear optics,¹⁹ since the third-order Kerr nonlinearity $\chi^{(3)}$ plays an important role in cross-phase modulation for optical shutters,²⁰ generation of optical solitons,²¹ etc., it is desirable to achieve giant Kerr nonlinearity with low light powers.²² In recent years, both theoretically^{20,23–26} and experimentally,²⁷ the study of giant third-order nonlinear susceptibility with reducing, especially with vanishing linear absorption has attracted considerable interests. Matsko *et al.*, based on multilevel atomic coherence, proposed a new method of resonant enhancement of Kerr nonlinearity where one-photon resonant absorption is suppressed by coherence effects. In a generic four-level system, coherence perturbation leads to double-dark resonance, which as a whole is the definite signature of a new type of quantum interference effect.²⁸ Using the interaction of double dark resonances, Kerr nonlinearity can be enhanced several orders of magnitude accompanied by vanishing linear absorption.²⁵ More recently, Niu *et al.* suggested, due to the spontaneously generated coherence (SGC), that the Kerr nonlinearity can be enhanced with vanishing linear and nonlinear absorptions.²⁶ However, for the bound four-level atom,²⁰ $\text{Im}[\chi_{\text{XPM}}^{(3)}]$ cannot be made completely zero. In Ref. 22, Nakajima considered a quasi-three-level autoionizing scheme and proved that, when both the probe and the pump fields were weak, proper choice of the detunings could lead to enhanced nonlinearities, while canceling the linear and nonlinear absorptions simulta-

neously. In this autoionizing scheme, the quantum interference between the discrete state and continuum component gives rise to asymmetric line profiles.²⁹ This type interference is called Fano interference,³⁰ which will lead to nonreciprocal absorptive and dispersive profiles. Substantial effort has been made to investigate the linear and nonlinear optical properties on this type interference^{11,31–39} since it can lead to tunneling induced transparency,³¹ reduction of the group velocity³² and high-efficiency nonlinear optical processes,³³ etc. Fano interference has been observed experimentally in atomic systems³⁴ and in semiconductor interband transitions.¹⁵ In intersubband transitions, we want to remark the original paper³⁵ proposed by Faist *et al.* In that experiment, tunneling induced transparency was observed through Fano interference in a structure where a single excited state is coupled to a continuum. Interference occurred through the simultaneous coupling of the ground state to the excited state and the continuum. The sign of quantum interference (constructive or destructive) in optical absorption from the ground state can be reversed by reversing the direction of tunneling from a doublet of excited energy levels to a common continuum.³⁶

In atomic system, giant Kerr nonlinearity with vanishing linear and nonlinear absorptions have been pointed out, yet there appears to be few investigations in semiconductor QW. In the present paper, we consider the potential of enhancing the Kerr nonlinearity in an *n*-type asymmetric semiconductor GaAs double QWs. The studies of the linear and nonlinear responses in this structure show that Kerr nonlinearity can be enhanced due to Fano interference, while maintaining vanishing the linear and nonlinear absorptions by appropriately choosing the values of the detunings and the intensity of the pump field. Analysis show that this is a result of Fano interference.

II. THE MODEL AND BASIC EQUATIONS

We consider an asymmetric double quantum well structure such as that shown in Fig. 1. This structure consists of two GaAs layers with thickness of 6.4 and 3.5 nm, and they are separated by a 1.5 nm $\text{Al}_{0.32}\text{Ga}_{0.68}\text{As}$ tunnel barrier. On the left-hand side of the wide well is a 1.0 nm $\text{Al}_{0.32}\text{Ga}_{0.68}\text{As}$ barrier, which is followed by the thick layer of $\text{Al}_{0.24}\text{Ga}_{0.76}\text{As}$

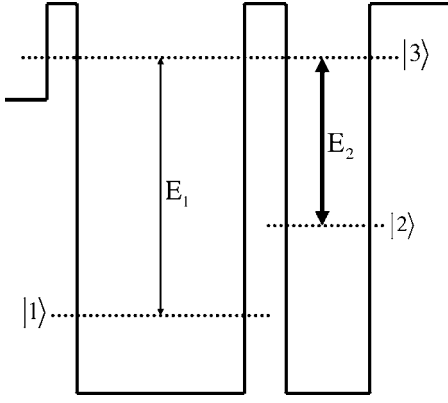


FIG. 1. Conduction subband of the asymmetric double quantum well in the bare state picture. The ground subbands of the wide well ($|1\rangle$) and the narrow one ($|2\rangle$) are coupled, near resonantly, to the subband $|3\rangle$ via the probe and pump fields $E_1(t)$ and $E_2(t)$, respectively. The transition energy between $|1\rangle$ and $|3\rangle$ is ~ 171 meV and that between $|2\rangle$ and $|3\rangle$ is ~ 110 meV. The subband ($|3\rangle$) is ~ 36 meV below the potential barrier edge.

on the left. Finally, $\text{Al}_{0.32}\text{Ga}_{0.68}\text{As}$ potential barrier is on the right of the narrow well. In this structure, one would observe a subband ($|2\rangle$) in the narrow well, and in the wide one, the subbands ($|1\rangle$) and ($|3\rangle$) can be observed. The transition energy between $|1\rangle$ and $|3\rangle$ is ~ 171 meV and that between $|2\rangle$ and $|3\rangle$ is ~ 110 meV. The subband ($|3\rangle$) is ~ 36 meV below the barrier edge. The QW is taken to be n doped with a carrier density $3 \times 10^{11} \text{ cm}^{-2}$. In order to study the linear and nonlinear optical responses in this structure, we apply a pump field to couple the transition $|2\rangle \leftrightarrow |3\rangle$. A probe field for measuring the optical response is applied to the transition $|1\rangle \leftrightarrow |3\rangle$. Their carrier frequencies are ν_1 and ν_2 , respectively. The presence of the continuum for the decay of the excited state $|3\rangle$ by tunneling gives rise to asymmetric line shapes due to Fano interference.³⁷ This structure can be simplified to a three-level Λ -type scheme. Following the standard procedure,³³ we find that this structure is governed by a set of density matrix equations given below,

$$\dot{\sigma}_{11} = -\gamma_1 \sigma_{11} + 2 \text{Im}(\tilde{\Omega}_1 \sigma_{31}) + \lambda_2 \sigma_{22} + \lambda_3 \sigma_{33} + \lambda_c \sigma_{cc}, \quad (1)$$

$$\dot{\sigma}_{22} = -(\gamma_2 + \lambda_2) \sigma_{22} + 2 \text{Im}(\tilde{\Omega}_2 \sigma_{32}), \quad (2)$$

$$\dot{\sigma}_{33} = -(\Gamma_3 + \lambda_3) \sigma_{33} - 2 \text{Im}(\tilde{\Omega}_1^* \sigma_{31}) - 2 \text{Im}(\tilde{\Omega}_2^* \sigma_{32}), \quad (3)$$

$$\dot{\sigma}_{21} = -[i(\Delta_1 - \Delta_2) + (\gamma_{21}^p + \gamma_{21}^{e-e})] \sigma_{21} + i\tilde{\Omega}_1 \sigma_{23} - i\tilde{\Omega}_2 \sigma_{31}, \quad (4)$$

$$\dot{\sigma}_{31} = -[i\Delta_1 + (\gamma_{31}^p + \gamma_{31}^{e-e})] \sigma_{31} - i\tilde{\Omega}_2 \sigma_{21} - i\tilde{\Omega}_1 \sigma_{11} + i\tilde{\Omega}_1^* \sigma_{33}, \quad (5)$$

$$\dot{\sigma}_{32} = -(i\Delta_2 + \gamma_{32}^p) \sigma_{32} + i\tilde{\Omega}_2^* \sigma_{33} - i\tilde{\Omega}_2 \sigma_{22} - i\tilde{\Omega}_1 \sigma_{12}, \quad (6)$$

where $\Delta_1 = \omega_{31} - \nu_1$ and $\Delta_2 = \omega_{32} - \nu_2$ are the detunings of the probe and pump fields, respectively. Here $\omega_{3l} (l=1,2)$ is the

transition frequency between subbands $|3\rangle$ and $|l\rangle$. The γ_1 and γ_2 are the electron transition rates from subbands $|1\rangle$ and $|3\rangle$ to the continuum by the probe and pump fields present, respectively. The terms γ_1 and γ_2 can also include other broadening mechanisms such as collisional broadening and laser bandwidth. Γ_3 is the decay rate from the subband $|3\rangle$ to the continuum by tunneling. Those from $|2\rangle$, $|3\rangle$ and continuum back to $|1\rangle$ are denoted by λ_2 , λ_3 , and λ_c , respectively. In Eqs. (4)–(6), considering the contribution of the electron-photon scattering process, the dephasing rates of the nondiagonal elements of the density matrix are denoted by γ_{ij}^p ($i, j=1,2,3$ and $i \neq j$), and they are given by $\gamma_{21}^p = (\gamma_1 + \gamma_2 + \lambda_2)/2$, $\gamma_{31}^p = (\gamma_1 + \Gamma_3 + \lambda_3)/2$, and $\gamma_{32}^p = (\gamma_2 + \Gamma_3 + \lambda_2 + \lambda_3)/2$, respectively. The polarization dephasing rates of this structure are strongly dependent upon the electron-electron scattering process. We introduce γ_{21}^{e-e} and γ_{31}^{e-e} in Eqs. (4) and (5) in a phenomenological way. It is not included in Eqs. (1)–(3) and (6) because what we are interested in is the steady state case and electrons are mostly in subband $|1\rangle$. Due to Fano interference, the Rabi frequencies associated with the probe and pump fields become complex quantities and are defined by

$$\tilde{\Omega}_1 = \Omega_1 \left(1 - \frac{i}{q_1}\right), \quad \tilde{\Omega}_2 = \Omega_2 \left(1 - \frac{i}{q_2}\right), \quad (7)$$

where q_1 and q_2 are asymmetry parameters, and they are given by³²

$$q_1 = \frac{2\Omega_1}{\sqrt{\gamma_1 \Gamma_3}}, \quad q_2 = \frac{2\Omega_2}{\sqrt{\gamma_2 \Gamma_3}}, \quad (8)$$

where Ω_1 and Ω_2 are given by $\Omega_1 = \mu_{13} \mathcal{E}_1 / \hbar$ and $\Omega_2 = \mu_{23} \mathcal{E}_2 / \hbar$. Where μ_{13} and μ_{23} denote the electric dipole matrix elements, and \mathcal{E}_1 and \mathcal{E}_2 are the slowly varying amplitudes of the probe and pump fields, respectively.

III. RESULTS AND DISCUSSIONS

The polarization components with frequency ν_1 can be obtained in terms of the density matrix elements σ_{31} and σ_{c1} [already eliminated in Eqs. (1)–(6)]

$$\begin{aligned} P &= \left(\mu_{13} \sigma_{31} + \sum_c \mu_{1c} \sigma_{c1} \right) + \text{c.c.} \\ &= \left(s'_1 - \frac{i}{2} \gamma'_1 \right) \mathcal{E}_1 \sigma_{11} + \mu_{13} \left(1 - \frac{i}{q_1} \right) \sigma_{31} + \text{c.c.}, \end{aligned} \quad (9)$$

where μ_{1c} is the dipole matrix element between the ground subband of the wide well and the left continuum. s'_1 and γ'_1 are defined by

$$s'_1 = P \sum_c \frac{|\mu_{1c}|^2}{\hbar^2 (\nu_1 - \omega_{c1})}, \quad (10)$$

$$\gamma'_1 = \frac{4\mu_{13}^2}{q_1^2 \Gamma_3}, \quad (11)$$

where s'_1 is the level shift of the excited subband $|3\rangle$ due to Fano interference. P denotes the principal value.

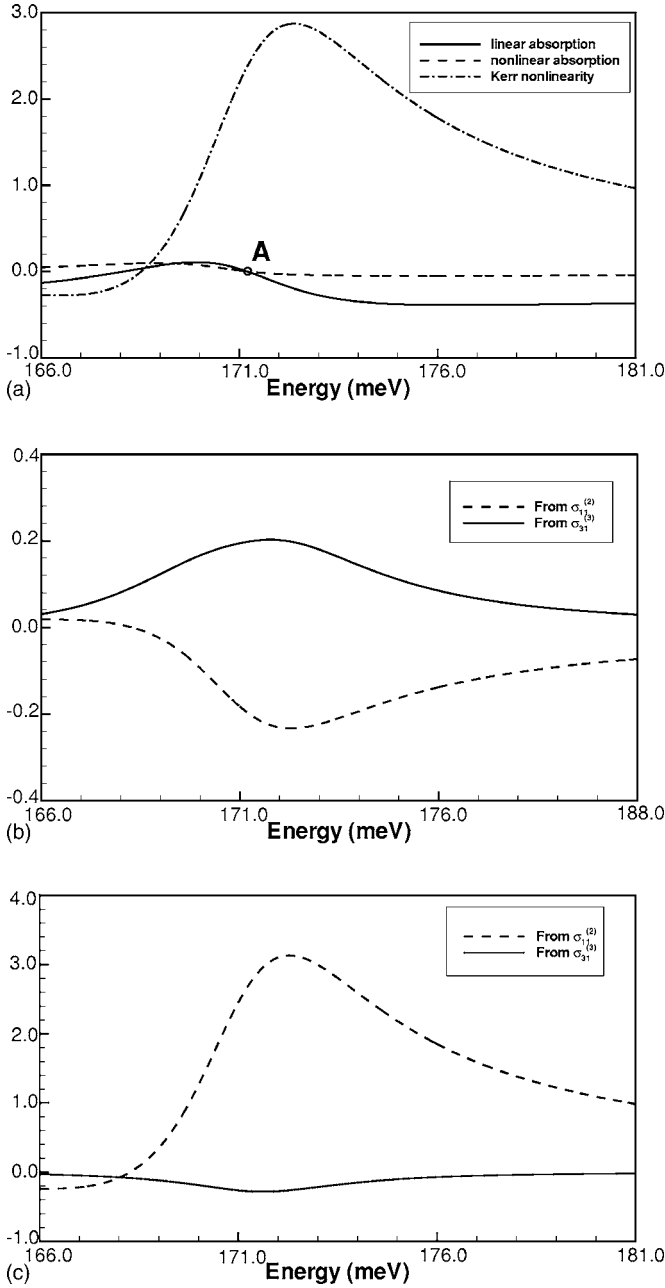


FIG. 2. (a) The variations of the linear (solid curve) and nonlinear absorptions (dashed curve) and the Kerr nonlinearity (dashed-dotted curve) versus the probe photon energy. (b) and (c) Contributions of $\sigma_{11}^{(2)}$ (together with the level shift owing to Fano interference) and $\sigma_{31}^{(3)}$ to the nonlinear absorption and Kerr nonlinearity as a function of the probe photon energy. $\Omega_2=3.20$ meV, $\Delta_2=-6.80$ meV.

The expressions of $\chi^{(1)}$ and $\chi^{(3)}$ can be obtained by substituting the solutions of Eqs. (1)–(6) into Eq. (9). The imaginary part of $\chi^{(1)}$ accounts for the linear absorption. The real and imaginary parts of $\chi^{(3)}$, respectively, yield the Kerr nonlinearity and the nonlinear absorption. In order to derive the equations for the linear and nonlinear susceptibilities, we need to solve Eqs. (1)–(6) in steady state. For this purpose, we assume the pump field to be much stronger than the probe field and $\sigma_{11}^{(0)}=1$, $\sigma_{22}^{(0)}=\sigma_{33}^{(0)}=0$, i.e., all the electrons are ini-

tially in the ground subband $|1\rangle$. Combined with $\chi=\chi^{(1)}+3|E_1|^2\chi^{(3)}$ and Eq. (11), the imaginary part of $\chi^{(1)}$ and $\chi^{(3)}$ are, respectively, given by³²

$$\text{Im}(\chi^{(1)}) = -\frac{N\mu_{13}^2}{\hbar\epsilon_0} \left(\frac{2}{\Gamma_3} + \frac{AD-BC}{C^2+D^2} \right), \quad (12)$$

$$\chi^{(3)}\mathcal{E}_1^3 = \frac{N}{3\epsilon_0} \left[\left(s'_1 - \frac{i}{2}\gamma'_1 \right) \mathcal{E}_1 \sigma_{11}^{(2)} + \mu_{13} \left(1 - \frac{i}{q_1} \right) \sigma_{31}^{(3)} \right], \quad (13)$$

where $\sigma_{11}^{(2)}$, $\sigma_{31}^{(3)}$ are the second- and third-order steady-state solutions of the density matrix elements σ_{11} and σ_{31} . They can be obtained from Eqs. (1)–(6). However, it is so tedious that we will not present it here. \mathcal{A} , \mathcal{B} , \mathcal{C} , and \mathcal{D} are

$$\mathcal{A} = \frac{2}{q_1} (\gamma_{21}^p + \gamma_{21}^{e-e}) - \left(1 - \frac{1}{q_1} \right),$$

$$\mathcal{B} = \left(1 - \frac{1}{q_1} \right) (\gamma_{21}^p + \gamma_{21}^{e-e}) + \frac{2}{q_1} (\Delta_1 - \Delta_2),$$

$$\mathcal{C} = \Delta_1(\Delta_1 - \Delta_2) - (\gamma_{21}^p + \gamma_{21}^{e-e})(\gamma_{31}^p + \gamma_{31}^{e-e}) - \Omega_2^2 \left(1 - \frac{1}{q_2} \right),$$

$$\mathcal{D} = \frac{2\Omega_2^2}{q_2} - [\Delta_1(\gamma_{21}^p + \gamma_{21}^{e-e}) + (\Delta_1 - \Delta_2)(\gamma_{31}^p + \gamma_{31}^{e-e})].$$

Combined with the definitions of the Rabi frequencies, we can know the effect of Fano interference on the linear and nonlinear optical properties of this structure clearly from Eqs. (12) and (13). It not only changes the Rabi frequencies to complex quantities, but also shifts the energy level, which embedded in the continuum by tunneling. This energy shift influences the linear and nonlinear optical responses of this structure together with $\sigma_{11}^{(0)}$ and $\sigma_{11}^{(2)}$, respectively. These are very different from bound state.

In what follows we focus our attention on enhancing Kerr nonlinearity with nonabsorption (including the linear and nonlinear absorptions). In this structure under consideration, the asymmetric parameters and the values of electron decay rates are uncontrollable once the sample is prepared. However, the linear and nonlinear properties can be controlled through the intensity Ω_2 , the detunings Δ_1 and Δ_2 . With the temperature 77 K and carrier density 3×10^{11} cm⁻², we get $q_1=1.44$ and $q_2=1.08$.³⁸ By solving the effective mass Schrödinger equation, we obtain a set of complex eigenvalues whose real and imaginary parts account for the quasi-bound state energy level and resonance widths, respectively. For the structure under consideration, we have $\Gamma_3=3.60$ meV, $\lambda_3=3.15$ meV, and $\lambda_c=1.07$ meV. λ_2 is very small and can be neglected. The direct electron transition rates from subbands $|1\rangle$ and $|2\rangle$ to continuum are taken as $\gamma_1=\gamma_2=1.5 \times 10^{-5}\Gamma_3$ since they are much smaller than the decay rate from state $|3\rangle$ to continuum, which is usually the case.³³ For the carrier density considered here, we take $\gamma_{21}^{e-e}=\gamma_{31}^{e-e}=2.07$ meV.¹⁸

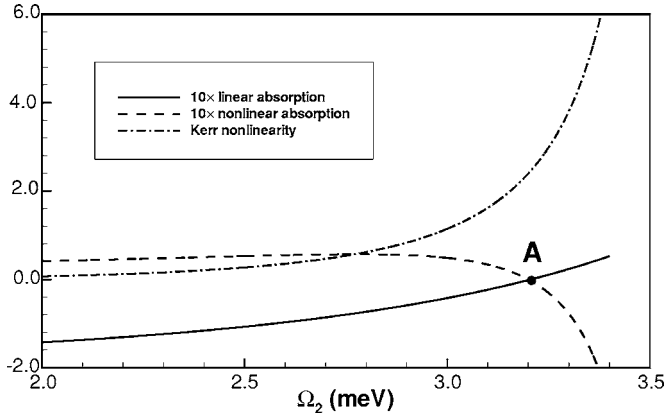


FIG. 3. The variations of the linear (solid curve) and nonlinear (dashed curve) absorptions and the Kerr nonlinearity (dashed-dotted curve) versus the intensity of the pump field Ω_2 . The probe photon energy is taken as ≈ 171.21 meV. The others are the same as those in Fig. 2(a).

Starting from $\text{Im}(\chi^{(1)})=0$ and $\text{Im}(\chi^{(3)})=0$, the relations of Ω_2 , Δ_1 , and Δ_2 satisfying the nonabsorptions (both the linear and nonlinear absorptions) conditions can be derived. However, their analytical expressions are severely tedious, we present a numerical example here for illustration. Once the intensity of the pump field is given, the values of the detunings Δ_1 and Δ_2 are determined by the nonabsorptions conditions. When $\Omega_2=3.20$ meV, the values of the detunings are $\Delta_1 \approx 0.21$ meV and $\Delta_2 \approx -6.80$ meV, respectively. We plot the representative profiles of $\text{Im}[\hbar\varepsilon_0\chi^{(1)}/(2N|\mu_{13}|^2)]$ (solid curve), $\text{Im}[3\hbar^3\varepsilon_0\chi^{(3)}/(2N|\mu_{13}|^4)]$ (dashed curve) and $\text{Re}[3\hbar^3\varepsilon_0\chi^{(3)}/(2N|\mu_{13}|^4)]$ (dashed-dotted curve) as a function of the probe photon energy in Fig. 2(a). This figure exhibits that the imaginary parts of both the first- and the third-order susceptibilities look like the dispersive shape, and the real part of the third-order susceptibility looks like the absorptive shape. When the probe photon energy is equal to 171.21 meV, i.e., $\Delta_1 \approx 0.21$ meV (Dot A), the linear and nonlinear absorptions are approximately equal to zero. While the Kerr nonlinearity is large (≈ 2.385). These results indicate that the Kerr nonlinearity can be enhanced, while the linear and nonlinear absorptions are canceled.

Equation (13) exhibits that the contributions to the third-order susceptibility consist of two parts: the first term, which is the influence of the energy level shift of the excited subband together with $\sigma_{11}^{(2)}$; the second term, which is the background of the third-order susceptibility except the Rabi frequencies are replaced by complex quantities. So, in Figs. 2(b) and 2(c), we plot their contributions to the nonlinear absorption and the Kerr nonlinearity in dashed and solid curves, respectively. As we can see from Fig. 2(b) that, when the probe energy is approximately 171.21 meV, the contribution of the first term is negative (≈ -0.198) whereas that of the second term is positive (≈ 0.198). This indicates that the energy level shift of the excited state $|3\rangle$ causes nonlinear gain on the probe beam, while the second term causes nonlinear absorption. The destruction of interference from these two terms produces vanishing nonlinear absorption. Calculations of these two terms to the linear absorption (not shown

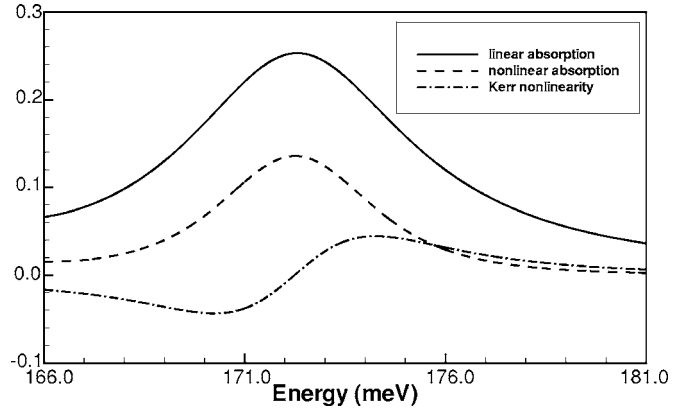


FIG. 4. The variations of the linear (solid curve) and nonlinear (dashed curve) absorptions and the Kerr nonlinearity (dashed-dotted curve) as a function of the probe photon energy. Parameters are $\gamma_1=\gamma_2=\lambda_c=\Gamma_3=s'_1=\gamma'_1=0.0$, $q_1=q_2=10^6$. The others are the same as those in Fig. 2(a).

in this paper) show that the linear absorption is also approximately zero because of the destructive quantum interference. Their contributions to the Kerr nonlinearity are ≈ 2.659 and ≈ -0.274 , respectively [as shown in Fig. 2(c)]. The former's contribution is much larger than that from the latter one. This suggests that the Kerr nonlinearity can be enhanced by choosing the intensity of the pump field appropriately, while the linear and nonlinear absorptions are canceled. Now one may ask what happens when the intensity of the pump field varies. To see this, we plot the imaginary part of the first-order susceptibility and the real and imaginary parts of the third-order nonlinear susceptibility as a function of Ω_2 in Fig. 3 as Ω_2 increases from 2.0 meV to 3.4 meV. For the sake of clarity, we amplify the linear and nonlinear absorption for 10 times. This figure shows that, when the intensity of the pump field increases, the linear absorption varies from negative to positive, and the nonlinear absorption becomes from positive to negative. When $\Omega \approx 3.20$ meV (Dot A), we note that the linear and nonlinear absorptions are both approximately zero, while the Kerr nonlinearity is very large.

To understand the effect of Fano interference on the linear and nonlinear susceptibilities more clearly, in Fig. 4, we plot the representative profiles of $\text{Im}[\hbar\varepsilon_0\chi^{(1)}/(2N|\mu_{13}|^2)]$ (solid curve), $\text{Im}[3\hbar^3\varepsilon_0\chi^{(3)}/(2N|\mu_{13}|^4)]$ (dashed curve), and $\text{Re}[3\hbar^3\varepsilon_0\chi^{(3)}/(2N|\mu_{13}|^4)]$ (dashed-dotted curve) with $\gamma_1=\gamma_2=\lambda_c=\Gamma_3=s'_1=\gamma'_1=0.0$, and $q_1=q_2=10^6$. The other parameters are the same as those in Fig. 2(a). As expected, they look like those in a bound state. This figure shows that, within the region when the Kerr nonlinearity is enhanced, the linear and nonlinear absorptions are always very large. Comparing with Fig. 2(a), it can be concluded that the enhanced Kerr nonlinearity with vanishing linear and nonlinear absorptions is a result of Fano interference. The line shapes are asymmetric because the pump field is not on-resonant with the optical transition.

IV. CONCLUSIONS

In conclusion, we have presented a class of semiconductor nonlinear optical devices based on absorption transpar-

ency induced by Fano interference, and studied the linear and nonlinear optical responses. The results of the first- and third-order susceptibilities have shown that in this structure appropriate choice of the detunings and the intensity of the pump field lead to enhanced Kerr nonlinearity, while canceling the linear and nonlinear absorptions. This structure has the potential applications in all-optical switching and other information processes.

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