Vortex flow and shot noise in the quantum-liquid phase of thick amorphous superconducting films

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We measure the fluctuations (noise) of the vortex-flow voltage in the low-temperature liquid phase of thick amorphous (a-)Mg_xB_{1-x} and a-Mo_xSi_{1-x} films with different transition temperatures and widths of the quantum-vortex-liquid (QVL) phase. An anomalous vortex flow with an asymmetric distribution of (real-time) voltage fluctuations is commonly observed in the QVL phase, independent of material as well as strength of quantum fluctuations. In the QVL phase we observe the Lorentzian-type noise spectra indicative of shot noise, whose origin is attributed to the vortex bundles that are depinned and pinned randomly in the stationary flow of QVL.

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It has been widely accepted that thermal fluctuations cause the melting of the vortex solid. Even at zero temperature ($T=0$) the vortex solid can melt into the liquid in the presence of strong quantum fluctuations.[1,2] This is called the quantum-vortex liquid (QVL). The QVL has been reported in a variety of type-II superconductors, such as high-T_c cuprates,[3] organic superconductors,[4,5] magnesium diborides,[6] doped diamonds,[7] and amorphous films.[1,8–11] In thin amorphous films with strong disorder, the issue on QVL is closely related to the field (B)-driven "superconductor-insulator" transition for two dimensions.[8,9,11] While the QVL has been extensively studied to elucidate the equilibrium phase diagram, the dynamic properties of vortices in QVL have not yet been clarified experimentally or theoretically.

For the thick amorphous (a-)Mo_xSi_{1-x} films we have obtained evidence for the QVL phase at low T based on the dc and ac complex resistivities[12,13] and constructed the equilibrium vortex phase diagram over the whole T and B regime. By measuring the time (t)-dependent component $\delta V(t)$ of the vortex-flow voltage $V(t)$ about the average V_0 we have also found anomalous vortex flow in the QVL phase of a thick film (film 5) with high resistivity (low T_c), where the probability distribution of $\delta V(t)$, $P(\delta V)$, is anomalously asymmetric, indicative of large velocity and/or number fluctuations of driven vortices.[14] We have suggested that the vortex dynamics in the QVL phase is dominated by quantum fluctuations. However, there is no theory that accounts for the finding comprehensively. Experimentally, it is important to clarify (i) whether the anomalous vortex flow in the QVL phase is a phenomenon commonly observed irrespective of the strength of quantum fluctuations (T_c) and material, and (ii) how the vortices flow in the QVL phase.

Here, we present the measurements of fluctuations (noise) of flow voltage in the QVL phase of a thick a-Mo_xSi_{1-x} film (film 1) with *lower* resistivity (*higher* T_c) and of a thick a-Mg_xB_{1-x} film. In particular, we measure the (excess) voltage noise spectrum $S_V(f)$ (f is a frequency), in addition to $P(\delta V)$, induced by the dc current I for film 1. A large asymmetry of $P(\delta V)$, accompanied by the increase in the amplitude of voltage fluctuations, is commonly observed in the QVL phase of all the a-Mo_xSi_{1-x} and a-Mg_xB_{1-x} films. The spectral shape of $S_V(f)$ in the QVL phase is Lorentzian-type indicative of shot noise (SN),[15,16] whose origin is attributed to the vortex bundles that are depinned and pinned randomly in the stationary flow of QVL.

The a-Mo_xSi_{1-x} film used in this study is the most conductive film 1 ($x=0.58$) among five thick (100 nm) films studied in Ref. 13, which were prepared by the coevaporation of pure Mo and Si. The thick (100 nm) a-Mg_xB_{1-x} film ($x=0.31$) was prepared in a similar manner.[17] The mean-field transition temperature T_{c0} defined by a criterion that the linear resistivity ρ decreases to 95% of the normal-state resistivity ρ_n and ρ_n just above T_{c0} are, respectively, 3.86 K and 3.7 $\mu\Omega$ m for film 1, and 5.58 K and 1.5 $\mu\Omega$ m for the a-Mg_xB_{1-x} film. All the data were taken in our dilution refrigerator. ρ, $V(t)$, and $S_V(f)$ induced by dc I were measured using a four-terminal method. In measuring $V(t)$ with a time resolution of 390 μs and $S_V(f)$ in a frequency range $f=1$ Hz–5 kHz, the voltage enhanced with a preamplifier (placed at 300 K) was recorded and analyzed, using a fast-Fourier transform spectrum analyzer.[14,18] The setup of S_V measurement is nearly the same as that used in Ref. 18. The field B was applied perpendicular to the film plane.

First, we show the vortex phase diagram at low T and high B for film 1 in Fig. 1(a).[13] The crossover temperature $T_Q(B)$ between the thermal-vortex-liquid (TVL) and QVL phases, which is defined as a temperature at which $d^2 \log \rho / dx^2=0$ (where $x \equiv 1/T$), is around $T=0.12$ K ($T/T_{c0}=0.03$). The relative widths of the QVL phase, $\Delta T/T_{c0}(\equiv T_Q/T_{c0})$ and $\Delta B/B_{c2}(0)[\equiv 1-B_g/B_{c2}(0)]$, respectively, along the T and B axes, are approximately 50–60 % smaller than those [Fig. 1(c)] for the most resistive film 5 studied previously.[14] Here, B_g is the vortex-glass (VG) transition and $B_{c2}(0)$ is an upper critical field (defined by the 95% criterion) at $T \to 0$; 8.77 and 3.43 T for films 1 and 5, respectively.

We measure $\delta V(t)[\equiv V(t)-V_0]$ for film 1 at different (B,T) points [open circles in Fig. 1(a)] in the liquid phase yielding $V_0/V_n \approx 0.71$ at $I \to 0$, where V_n is the voltage in the normal state. For $I=0$, which corresponds to the background contribution due to external noise, the amplitude of $\delta V(t)$, $|\delta V(t)_{BG}|$, is as small as 5 μV and its distribution $P(\delta V)$ is symmetric. In the TVL phase both $\delta V(t)$ and $P(\delta V)$ are similar to the background data for all the I studied, while in the

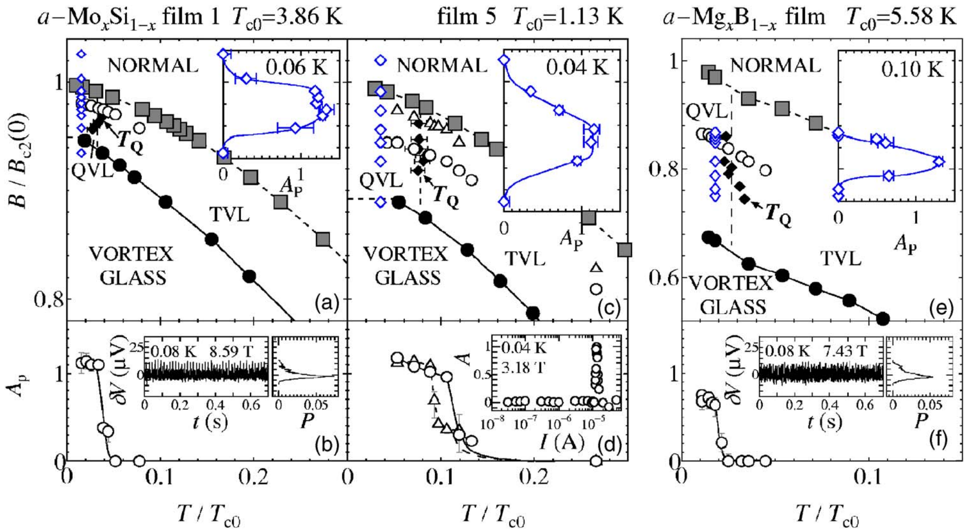

FIG. 1. (Color online) (a) B-T phase diagram at low T in high B for a-$\mathrm{Mo}_x\mathrm{Si}_{1-x}$ film 1 (T_{c0}=3.86 K), (c) film 5 (T_{c0}=1.13 K), and (e) a-$\mathrm{Mg}_x\mathrm{B}_{1-x}$ film (T_{c0}=5.58 K). Full circles, gray squares, and full diamonds represent $B_g(T)$, $B_{c2}(T)$, and $T_Q(B)$, respectively. Open symbols indicate (B,T) where $\delta V(t)$ is measured. Horizontal dotted lines in (a) and (c) mark the upper bound of the VG phase and other lines are guides for the eye. (b), (d), (f) A_p vs T/T_{c0}. Open circles and triangles in (d) correspond to the data measured at (B,T) indicated with the open circles and triangles in (c), respectively. Inset: (a), (c), (e) A_p vs $B/B_{c2}(0)$ measured at fixed T (open diamonds in the main panels). (b) $\delta V(t)$ and $P(\delta V)$ at 0.08 K in 8.59 T (V_0=35.4 μV) and (f) at 0.08 K in 7.43 T (V_0=43 μV). (d) A vs I at 0.04 K in 3.18 T just above B_g(=3.06 T). Some of the data points shown in (a) and (c) are from Ref. 13 and those in (d) are from Ref. 14.

QVL phase the contribution of $\delta V(t)$ from the flux motion is clearly visible.[19] Representatively shown in the inset of Fig. 1(b) are $\delta V(t)$ and $P(\delta V)$ measured at 0.08 K in 8.59 T (in the QVL phase) for I=3.0 μA (V_0=35.4 μV). The amplitude of $\delta V(t)$ measured at $V_0>0$ is remarkably larger than $|\delta V_{BG}|$ and the shape of $P(\delta V)$ is highly asymmetric having a tail that extends to the direction of flux motion $[\delta V(t)>0]$. The contribution of $|\delta V(t)|$ from flux motion is roughly estimated to be $|\delta V(t)_{fl}| \equiv |\delta V| - |\delta V_{BG}| \approx 2$ μV, which corresponds to 6–7 % of V_0.

In Fig. 2(a) we plot I dependences of V/I (full circles) and the asymmetry (skewness) A (Ref. 20) (open circles) of $P(\delta V)$ at 0.08 K in 8.53 T (in the QVL phase). With increasing I, A gradually grows and exhibits a large broad peak A_p at large I in the linear regime above which the nonlinearity occurs. As I becomes large enough to make the film nearly normal, both $\delta V(t)$ and $P(\delta V)$ approach the background data. In Fig. 1(b) we plot the peak value of $A(I)$, A_p, as a function of reduced temperature T/T_{c0}. One can see that a temperature T_q at which $A_p(T)$ starts to rise upon cooling nearly coincides with T_Q. These results are very similar to what have been observed for the most resistive film 5 [Fig. 1(d)].[14]

For the a-$\mathrm{Mg}_x\mathrm{B}_{1-x}$ film the T dependence of ρ in different B and the phase diagram [Fig. 1(e)] obtained from $\rho(T,B)$ are qualitatively similar to those for the a-$\mathrm{Mo}_x\mathrm{Si}_{1-x}$ films.[21] Also, $\delta V(t)$ and $P(\delta V)$ as functions of I, T, and B are similar to those for the a-$\mathrm{Mo}_x\mathrm{Si}_{1-x}$ films. In the inset of Fig. 1(f) we representatively show $\delta V(t)$ and $P(\delta V)$ measured at 0.08 K in 7.43 T (in the QVL phase) for I=3 μA (V_0=43 μV), where $\delta V(t)$ with asymmetric $P(\delta V)$ is again visible. Furthermore, $A(I)$ takes a large broad peak A_p at large I in the linear regime, as observed for the a-$\mathrm{Mo}_x\mathrm{Si}_{1-x}$ films [Fig. 2(a)]. In Fig. 1(f) we plot the T/T_{c0} dependence of A_p extracted from $\delta V(t)$ measured at different (B,T) points [open circles in Fig. 1(e)] in the liquid phase (V_0/V_n=0.48 at $I\rightarrow 0$). Upon cooling, A_p again exhibits a steep rise at around $T_Q/T_{c0}(\approx 0.025)$. All of the results suggest that the anomalous vortex flow with asymmetric $P(\delta V)$ is a phenomenon commonly observed in the QVL phase of disordered amor-

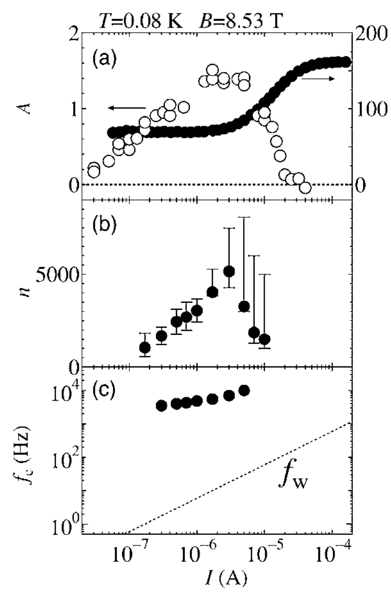

FIG. 2. (a) I dependences of V/I (full circles) and A (open circles) for a-$\mathrm{Mo}_x\mathrm{Si}_{1-x}$ film 1 measured at 0.08 K in 8.53 T. A dotted line indicates V/I=A=0. (b) The bundle size n and (c) characteristic frequency f_c extracted from $S_V(f)$ plotted against I. A dotted line in (c) represents an inverse of the transit time f_w calculated in the linear regime.

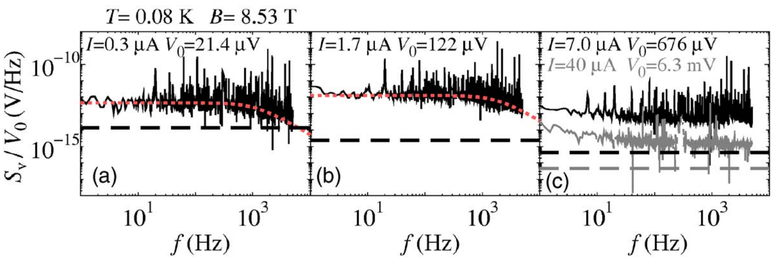

FIG. 3. (Color online). (a) Noise spectra $S_V(f)/V_0$ for a-Mo_xSi_{1-x} film 1 measured at 0.08 K in 8.53 T for I=0.3 μA (V_0=21.4 μV), (b) 1.7 μA (V_0=122 μV), (c) 7.0 μA (V_0=676 μV), and 40 μA (V_0=6.3 mV) (a gray line). Horizontal dashed lines indicate the experimental resolutions and the dotted curves represent the fit of the data to the Lorentzian form.

phous films, independent of material as well as strength of quantum fluctuations.

Let us next focus on $S_V(f)$ induced by vortex motion, which provides us with important information on the characteristic time and/or size of driven vortices giving rise to noise. In the TVL phase we do not detect appreciable $S_V(f)$ for any I studied within our experimental resolutions ($S_V \sim 10^{-19}-10^{-18}$ V^2/Hz). This is reasonable, considering that in the ordinary flow of vortex liquid, the density of vortices is so high that voltage noise produced by individual vortex motion is averaged out, leading to negligibly small $S_V(f)$. In contrast, in the particular T, B, and I regime (in the QVL phase) where the skewness A of $P(\delta V)$ and (normalized) amplitude of voltage fluctuations, $|\delta V(t)_{fl}|/V_0$, due to vortex motion take large values, large broadband noise is observed. Representatively shown in Figs. 3(a)–3(c) are I-induced noise spectra $S_V(f)/V_0$ (S_V divided by V_0) measured at 0.08 K in 8.53 T for different I, where each horizontal dashed line indicates the experimental resolution. For small I=0.3 μA (V_0=21.4 μV) in the linear regime the spectral shape is fit by a Lorentzian form, $S_V(f) \propto 1/[1+(\pi f/f_c)^2]$, with a corner frequency $f_c \approx 3$ kHz [Fig. 3(a)] (a dotted curve).

This type of spectrum is markedly different from the $1/f$-type spectrum widely observed both in the VG and Meissner phases at relatively high T.[18,22] Moreover, the I dependence of $S_V(f)/V_0$ in the QVL phase is different from that in the VG and Meissner phases, where $S_V(f)/V_0$ (at fixed f) decreases monotonically with increasing I. In the QVL phase, as I is increased, e.g., from 0.3 to 1.7 μA [Fig. 3(b)], both $S_V(f\to 0)/V_0$ and f_c first increase (though the increase in f_c is slight). With further increasing I, $S_V(f\to 0)/V_0$ then decreases and approaches the experimental resolution [see, e.g., $S_V(f)/V_0$ for 7.0 and 40 μA in Fig. 3(c)], while f_c increases monotonically and eventually exceeds our f window ($>$5 kHz).

Except for the large I region, there are relatively large fluctuations in the spectra at higher frequencies due to unspecified reasons. They may originate from the vortex motion, rather than simple external noise, e.g., from the current source. This is because, when we apply large enough currents (e.g., 40 μA) to make the film nearly a normal state, the fluctuations in the spectra as well as S_V/V_0 decrease [Fig. 3(c)].

We analyze the spectra in terms of the SN model for vortex motion.[15,16] According to the simple SN model, which assumes the vortex "bundles" of a similar size moving with nearly the same velocity in the same direction, the magnitude of $S_V(f)/V_{dc}$ at low $f(\ll f_c)$ and f_c give a bundle size n $=S_V(f\to 0)/2V_{dc}\Phi_0$ and a characteristic time $\tau_c(=1/f_c)$ of the motion, respectively. Here, Φ_0 is the flux quantum and V_{dc} is the dc voltage originating from the driven vortices (bundles) giving rise to SN. Thus V_{dc} should be taken as $|\delta V(t)_{fl}|$, typically, $\approx(0.01-0.1)\times V_0$, rather than V_0, which originates from the underlying flow of individual vortices that do not contribute to SN. The bundle size n thus estimated is plotted against I in Fig. 2(b). With increasing I in the linear regime, n first goes up and exhibits a large peak $n \approx 5\times 10^3$ at $I \approx 2-3$ μA. As I increases further and the nonlinear regime is entered, n falls abruptly. The I dependence of n looks similar to that of A shown in Fig. 2(a). This reflects the fact that the increase in broadband noise accompanies the increase in the asymmetry of $P(\delta V)$. The origin of the anomalous $\delta V(t)$ is therefore attributed to the random vortex-bundle motion giving rise to SN.

In Fig. 2(c) we plot f_c as a function of I. It is found that f_c increases monotonically against I and the dependence of f_c on I is weaker than the linear relation. A dotted line represents an inverse of the transit time τ_w for each single vortex to travel across the sample width, which is calculated as $f_w(\equiv 1/\tau_w)=v(I)/w=5\times 10^6 I$, where $v(I)\approx 3\times 10^3 I$ m/s is a velocity of the vortex bundles estimated from the flow resistance and $w(=6\times 10^{-4}$ m) is the sample width. One can see that f_c is approximately three orders of magnitude larger than f_w. This result indicates the finite vortex mean free path $l_v(=v/f_c)$,[16] that is much smaller than w; e.g., $l_v \sim 0.5\times 10^{-3} w$ at small $I(\approx 0.3$ μA) and $\sim 3\times 10^{-3} w$ at larger $I(\approx 5$ μA).

The possible origin of the vortex bundles is due to some small vortex solids that may be present in the liquid. They are considered to be soft vortex droplets rather than hard crystallites, because they are in the liquid phase well above the VG phase. At small I each soft bundle is effectively pinned by point defects in the amorphous films. With increasing I (or v), the vortex bundles with larger sizes (n) are depinned successively and take part in the flow. With further increasing I, the number of the depinned vortices (bundles) increases progressively and the large vortex bundles may split into the small ones, thus resulting in the peak in $n(I)$, as seen in Fig. 2(b). The I dependence of f_c shown in Fig. 2(c) indicates that l_v increases monotonically, though slowly, with increasing I. This reflects the fact that the faster vortices feel the pinning potential less effectively.

We have also measured the B dependence of $\delta V(t)$ at fixed T [open diamonds in Figs. 1(a), 1(c), and 1(e)] below T_Q (in the QVL phase). It is commonly observed for all three films that at *low B* just above B_g, with increasing I, $A(I)=0$ at small I and rises at large I, exhibiting a sharp peak, as representatively shown in the inset of Fig. 1(d) (the data was taken at 3.18 T just above B_g=3.06 T). This is in contrast to the result at higher B [e.g., Fig. 2(a)]. We plot A_p vs $B/B_{c2}(0)$ for a-Mo_xSi_{1-x} films 1 and 5, and for the a-Mg_xB_{1-x} film in the insets of Figs. 1(a), 1(c), and 1(e), respectively. These results are explained as follows: At low B above B_g, the size of vortex bundles is relatively large and most of them are strongly pinned. Thus they do not contribute to noise (A) until sufficiently large I is applied [inset of Fig. 1(d)]. As B is increased, the bundle size decreases. The small vortex bundles are easily depinned and contribute to noise even at small I [Fig. 2(a)]. As B is increased further up to $\sim B_{c2}$, the size of bundles approaches one (Φ_0) and their number increases significantly. As a result, noise becomes diminishingly small.

We propose a picture that the vortex bundles that are depinned and pinned randomly in the stationary flow of the liquid give rise to SN and anomalous fluctuations of $\delta V(t)$. The most striking feature is that this phenomenon is visible only in the QVL phase, which is revealed by the proximity of T_q to T_Q for all the films. At present, there is no available theory to account for the finding. One of the possible explanations is that the QVL phase denoted in our paper is identified with the vortex-slush (VS) phase,[23,24] which has been reported in the high-T liquid just above the solid phase for high-T_c cuprates[23,25,26] and in the low-T high-B regime for organic superconductors.[4] We note, however, that the origin of the QVL phase is due to quantum fluctuations,[1,2] as verified experimentally,[8,13] while that of the VS phase is independent of quantum effects.[27] In addition, the dynamic properties of VS to be compared with the present results have not yet been reported. Alternatively, we assume that the soft vortex bundles may survive even in the high-T liquid phase above $T_q(\approx T_Q)$ and most of them are pinned. Upon cooling below T_q, quantum tunneling[9] may become remarkable in the presence of I, which assists the vortex bundles to depin from their pinning sites. Within the interpretation, the liquid phase at $T<T_q$ (QVL phase) is regarded as a regime where quantum tunneling of a large number of vortices ($<5\times10^3\Phi_0$) may take place.

To summarize, we measure the fluctuations (noise) of the flow voltage in the liquid phase of thick a-Mg_xB_{1-x} and a-Mo_xSi_{1-x} films with different T_{c0} (quantum fluctuations) and widths of the QVL phase. The results indicate that the anomalous vortex flow with asymmetric distribution of voltage fluctuations $\delta V(t)$ is a phenomenon commonly observed in the QVL phase of disordered amorphous films, independent of material as well as strength of quantum fluctuations. In the flow state in the QVL phase we observe the Lorentzian-type noise spectra indicative of shot noise, whose origin is attributed to the vortex bundles (soft droplets) that are depinned and pinned randomly in the stationary flow of QVL.

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[21] Quantitatively, $\Delta T/T_{c0}$ is 0.03, which is smaller than 0.05 for film 1, while $\Delta B/B_{c2}(0)$ is 0.3 [where $B_{c2}(0)$=8.6 T], which is even larger than 0.11 for film 5.
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