Chiral symmetry restoration in (2+1)-dimensional QED with a Maxwell-Chern-Simons term at finite temperature

Raoul Dillenschneider* and Jean Richert†

Laboratoire de Physique Théorique, UMR 7085 CNRS/ULP, Université Louis Pasteur, 67084 Strasbourg Cedex, France

(Received 29 June 2006; published 4 October 2006)

We study the role played by a Chern-Simons contribution to the action in the QED_3 formulation of a two-dimensional Heisenberg model of quantum spin systems with a strictly fixed site occupation at finite temperature. We show how this contribution affects the screening of the potential that acts between spinons and contributes to the restoration of chiral symmetry in the spinon sector. The constant that characterizes the Chern-Simons term can be related to the critical temperature T_c above which the dynamical mass goes to zero.

DOI: 10.1103/PhysRevB.74.144404 PACS number(s): 75.10.Jm, 11.10.Kk, 11.10.Wx, 11.30.Rd

I. INTRODUCTION

Quantum electrodynamics $QED_{(2+1)}$ is a common framework aimed to describe strongly correlated systems such as quantum spin systems in two space and one time dimension, as well as related specific phenomena like high-T_c superconductivity.[1–4] Indeed, a gauge field formulation of the antiferromagnetic Heisenberg model in $d=2$ dimensions leads to a QED_3 action for spinons; see, e.g., Ghaemi and Senthil,[2] Morinari,[3] and also Ref. 5.

In the framework of this approach the spinon field acquires a dynamical mass at finite temperature. This mass is generated by an $U(1)$ gauge field. It depends on temperature T and disappears above a critical value $T=T_c$.

We consider here the π-flux state approach introduced by Affleck and Marston.[6,7] In this description it was shown by Marston[8] that the flux through a plaquette formed by spin sites must be equal to multiples of π, otherwise the projection properties of the loop operator are broken. The flux can be strictly fixed to $k\pi$ where k is an integer by means of a Chern-Simons (CS) term.

We introduce such a CS term here. Solving the Schwinger-Dyson equation for the spinon field, we show below how the critical temperature T_c (above which the dynamical mass vanishes) is affected by this term. Furthermore we guarantee a strict site occupation of the spin system by a single spin 1/2 per lattice site by means of a rigorous procedure suggested by Popov and Fedotov.[9,5,10]

The outline of the paper is as follows. In Sec. II we sketch the main steps of the QED_3 formulation of the two-dimensional (2D) Heisenberg model. Section III introduces the CS term and justifies its presence. In Sec. IV we show and comment on the chiral symmetry restoration of the spinon field in the presence of this term.

II. FROM THE HEISENBERG INTERACTION TO THE π-FLUX DIRAC ACTION

Heisenberg quantum spin Hamiltonians of the type

$$H = -\frac{1}{2}\sum_{i,j} J_{ij}\vec{S}_i\vec{S}_j \tag{1}$$

with antiferromagnetic coupling $\{J_{ij}\}<0$ can be mapped onto Fock space by means of the transformation $S_i^+ = f_{i,\uparrow}^\dagger f_{i,\downarrow}$, $S_i^- = f_{i,\downarrow}^\dagger f_{i,\uparrow}$, and $S_i^z = \frac{1}{2}(f_{i,\uparrow}^\dagger f_{i,\uparrow} - f_{i,\downarrow}^\dagger f_{i,\downarrow})$ where $\{f_{i,\sigma}^\dagger, f_{i,\sigma}\}$ are anticommuting fermion operators which create and annihilate spinons with $\sigma=\pm 1/2$. The projection onto Fock space is exact when the number of fermions per lattice site verifies $\Sigma_{\sigma=\pm 1/2} f_{i,\sigma}^\dagger f_{i,\sigma}=1$. This can be enforced by using the Popov and Fedotov procedure[9,5,10] which introduces the imaginary chemical potential $\mu = i\pi/2\beta$ at temperature β^{-1}, adding the term μN to the expression given by (1):

$$H = -\frac{1}{2}\sum_{i,j} J_{ij}\vec{S}_i\vec{S}_j - \mu N,$$

where $N=\Sigma_{i,\sigma} f_{i,\sigma}^\dagger f_{i,\sigma}$ counts the number of fermions in the spin system.

In 2D space the Heisenberg Hamiltonian given by Eq. (1) can be written[9,11] in terms of composite nonlocal operators $\{\mathcal{D}_{ij}\}$ ("diffusions") defined as

$$\mathcal{D}_{ij} = f_{i,\uparrow}^\dagger f_{j,\uparrow} + f_{i,\downarrow}^\dagger f_{j,\downarrow}.$$

If the coupling strengths are fixed as

$$J_{ij} = J\sum_{\vec{\eta}} \delta(\vec{r}_i - \vec{r}_j \pm \vec{\eta}),$$

where $\vec{\eta}$ is a lattice vector $\{a_1, a_2\}$ in the $\vec{Ox}$ and $\vec{Oy}$ directions, the Hamiltonian takes the form

$$H = -J\sum_{\langle ij\rangle}\left(\frac{1}{2}\mathcal{D}_{ij}^\dagger\mathcal{D}_{ij} - \frac{n_i}{2} + \frac{n_i n_j}{4}\right) - \mu N, \tag{2}$$

where i and j are nearest neighbor sites.

The number operator products $\{n_i n_j\}$ in Eq. (2) are quartic in terms of creation and annihilation operators in Fock space. In principle the formal treatment of these terms requires the introduction of a Hubbard-Stratonovich (HS) transformation. One can, however, show that the presence of this term has no influence on the results obtained from the partition function. Indeed both $\{n_i\}$ and $\{n_i n_j\}$ lead to constant contributions under the exact site occupation constraint and hence are of no importance for the physics described by the Hamiltonian (2). As a consequence we leave them out from the beginning.

Using a HS transformation in order to reduce the first term in Eq. (2) from quartic to quadratic order in the fermion operators $f^\dagger$ and f, the Heisenberg Hamiltonian reads

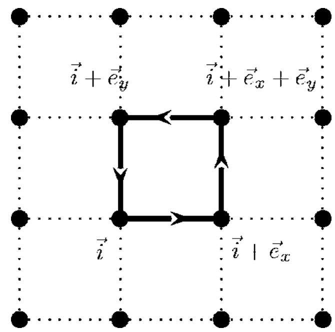

FIG. 1. Plaquette (□) on a two-dimensional spin lattice. $\vec{e}_x$ and $\vec{e}_y$ are the unit vectors along the directions $\vec{Ox}$ and $\vec{Oy}$ starting from site $\vec{i}$ on the lattice.

$$\mathcal{H} = \frac{2}{|J|}\sum_{\langle ij\rangle} \bar{\Delta}_{ij}\Delta_{ij} + \sum_{\langle ij\rangle}[\bar{\Delta}_{ij}\mathcal{D}_{ij} + \Delta_{ij}\mathcal{D}_{ij}^{\dagger}] - \mu N, \qquad (3)$$

where $\{\Delta_{ij}\}$ are the HS auxiliary fields. At this point no approximation has been made and Eq. (3) is exact.

The fields Δ_{ij} can be chosen as complex quantities $\Delta_{ij} = |\Delta| e^{i\phi_{ij}}$. This parametrization introduces gauge fields ϕ_{ij} which are defined on the square plaquette shown in Fig. 1 and can be decomposed into a mean-field part and a fluctuation contribution, $\phi_{ij}=\phi_{ij}^{mf}+\delta\phi_{ij}$. The amplitude $|\Delta|$ too may contain a mean-field and a fluctuating contribution. In the following we assume that at low energy the essential quantum fluctuation contributions are generated by the gauge field $\delta\phi_{ij}$ and neglect the amplitude fluctuations from now on.

The ϕ_{ij}^{mf}'s are fixed on the plaquette in such a way that

$$\phi^{mf} = \sum_{(ij)\in\square} \phi_{ij}^{mf},$$

where ϕ^{mf} is taken to be constant.

In order to implement the SU(2) invariance in (1) at the level of the mean-field Hamiltonian (3) we follow[6,7,12–14] and introduce the configuration

$$\phi_{ij}^{mf} = \begin{cases} \frac{\pi}{4}(-1)^i & \text{if } \vec{r}_j = \vec{r}_i + \vec{e}_x, \\ -\frac{\pi}{4}(-1)^i & \text{if } \vec{r}_j = \vec{r}_i + \vec{e}_y. \end{cases}$$

Then the total flux through the fundamental plaquette is such that $\phi^{mf}=\pi$, which guarantees the SU(2) symmetry.[8]

Under these conditions the Hamiltonian (3) goes over to the π-flux mean-field Hamiltonian

$$\mathcal{H}_{MF}^{(PFP)} = \mathcal{N}z\frac{\Delta^2}{|J|} + \sum_{\vec{k}\in SBZ}\sum_{\sigma}(f_{\vec{k},\sigma}^{\dagger} f_{\vec{k}+\vec{\pi},\sigma}^{\dagger})[\tilde{H}]\begin{pmatrix} f_{\vec{k},\sigma} \\ f_{\vec{k}+\vec{\pi},\sigma}\end{pmatrix} \qquad (4)$$

(SBZ is the surface Brillouin zone) with

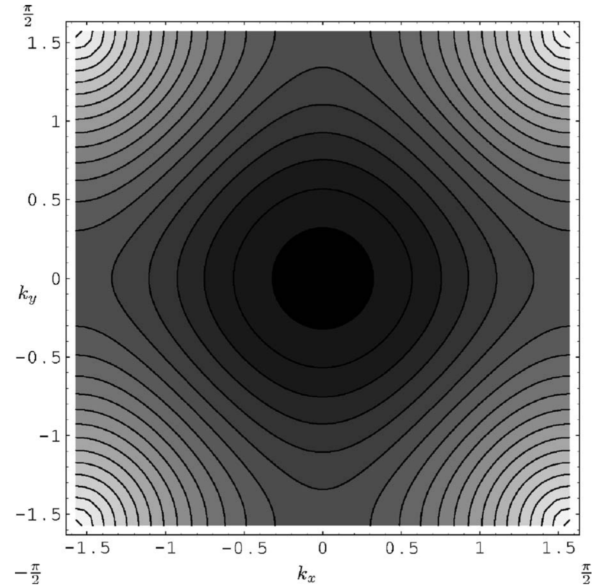

FIG. 2. The contour representation of the energy spectrum $\omega_{(-)\vec{k},\sigma}^{(PFP)}+\mu=-2\Delta\sqrt{\cos^2(k_x)+\cos^2(k_y)}$ for k_x and k_y belonging to $[-\pi/2,\pi/2]$ and showing the presence of the nodal points $(\pm\pi/2,\pm\pi/2)$ where the energy is equal to zero.

$$[\tilde{H}] = \begin{bmatrix} -\mu + \Delta\cos\left(\frac{\pi}{4}\right)\gamma_{k_x,k_y} & -i\Delta\sin\left(\frac{\pi}{4}\right)\gamma_{k_x,k_y+\pi} \\ +i\Delta\sin\left(\frac{\pi}{4}\right)\gamma_{k_x,k_y+\pi} & -\mu - \Delta\cos\left(\frac{\pi}{4}\right)\gamma_{k_x,k_y} \end{bmatrix},$$

where the $\gamma_{\vec{k}}$'s are defined by $\gamma_{\vec{k}}=\sum_{\vec{\eta}}e^{i\vec{k}\cdot\vec{\eta}}=2(\cos k_x a_1 + \cos k_y a_2)$. The eigenvalues of $\mathcal{H}_{MF}^{(PFP)}$ read $\omega_{(\pm),\vec{k},\sigma}^{(PFP)}=-\mu \pm 2\Delta\sqrt{\cos^2(k_x)+\cos^2(k_y)}$.

We are interested in the low-energy behavior of the quantum spin system described by (4) in the neighborhood of the nodal points $(k_x=\pm\frac{\pi}{2},k_y=\pm\frac{\pi}{2})$ where the energy gap $(\omega_{(+),\vec{k},\sigma}^{(PFP)}-\omega_{(-),\vec{k},\sigma}^{(PFP)})$ vanishes. Figure 2 shows the contour plot of the energy spectrum $\omega_{(-),\vec{k},\sigma}^{(PFP)}$ and locates the nodal points. We linearize the energies in the neighborhood of these points.

Following 2, 3, and 5 the spin liquid Hamiltonian (4) at low energy can be described in terms of four-component Dirac spinons in the continuum limit. The Dirac action of this spin liquid in (2+1) dimensions including the phase fluctuations $\delta\phi_{ij}$ around the π-flux mean-field phase ϕ_{ij}^{mf} has been derived in Ref. 5 and reads

$$S_E = \int_0^{\beta}\int d^2\vec{r}\left\{-\frac{1}{2}a_{\mu}\{[\square\,\delta^{\mu\nu}+(1-\lambda)\partial^{\mu}\partial^{\nu}]\}a_{\nu} + \sum_{\sigma}\bar{\psi}_{\vec{r}\sigma}[\gamma_{\mu}(\partial_{\mu}-iga_{\mu})]\psi_{\vec{r}\sigma}\right\}. \qquad (5)$$

ψ is the four-dimensional Dirac spinon field

$$\psi_{\vec{k}\sigma} = \begin{pmatrix} f_{1a,\vec{k}\sigma} \\ f_{1b,\vec{k}\sigma} \\ f_{2a\vec{k}\sigma} \\ f_{2b\vec{k}\sigma} \end{pmatrix},$$

where $f^{\dagger}_{1,\vec{k},\sigma}$ and $f_{1,\vec{k},\sigma}$ ($f^{\dagger}_{2,\vec{k},\sigma}$ and $f_{2,\vec{k},\sigma}$) are fermion creation and annihilation operators near the nodal points $(\frac{\pi}{2},\frac{\pi}{2})$ $[(-\frac{\pi}{2},\frac{\pi}{2})]$ of the momentum $\vec{k}$, and indices a and b characterize the rotated operators

$$f_{a,\vec{k},\sigma} = \frac{1}{\sqrt{2}}(f_{\vec{k},\sigma} + f_{\vec{k}+\vec{\pi},\sigma}), \quad f_{b,\vec{k},\sigma} = \frac{1}{\sqrt{2}}(f_{\vec{k},\sigma} - f_{\vec{k}+\vec{\pi},\sigma}).$$

The first term in (5) originates from the U(1) symmetry transformation $\psi \to e^{ig\theta}\psi$ which generates a gauge field $a_\mu = \partial_\mu \theta$. The constant g in (5) is the coupling strength between the gauge field a_μ and the Dirac spinons ψ. The first term corresponds to the "Maxwell" term $-\frac{1}{4}f_{\mu\nu}f^{\mu\nu}$ of the gauge field a_μ where $f^{\mu\nu} = \partial^\mu a_\nu - \partial^\nu a_\mu$, λ is the parameter of the Faddeev-Popov gauge fixing term $-\lambda(\partial^\mu a_\mu)^2$,[15] $\delta^{\mu\nu}$ the Kronecker δ, and $\Box = \partial_\tau^2 + \vec{\nabla}^2$ the Laplacian in Euclidean spacetime. This form of the action originates from a shift of the imaginary time derivation $\partial_\tau \to \partial_\tau + \mu$ where μ is the chemical potential introduced above. It leads to a new definition of the Matsubara frequencies of the fermion fields[10] ψ which then read $\tilde{\omega}_{F,n} = \omega_{F,n} - \mu/i = \frac{2\pi}{\beta}(n+1/4)$.

III. MAXWELL-CHERN-SIMONS ACTION AT FINITE TEMPERATURE

A. Justification and implementation

As shown by Marston,[8] only gauge configurations of the flux states belonging to Z_2 symmetry ($\pm\pi$) are allowed. Hence the flux through the plaquette is restricted to $\phi = \phi^{mf} + \delta\phi = \{0, \pm\pi\}$. This was derived in the following way.[8]

The loop operator $\Pi = f^{\dagger}_{\vec{i}} f_{\vec{i}+\vec{e}_x} f^{\dagger}_{\vec{i}+\vec{e}_x} f_{\vec{i}+\vec{e}_x+\vec{e}_y} f^{\dagger}_{\vec{i}+\vec{e}_x+\vec{e}_y} f_{\vec{i}+\vec{e}_y} f^{\dagger}_{\vec{i}+\vec{e}_y} f_{\vec{i}}$ verifies $\Pi^3 = \Pi$. Defining two quantum state $|u\rangle = \Pi^2|\varphi\rangle$ and $|v\rangle = (1-\Pi^2)|\varphi\rangle$where $|\varphi\rangle = |u\rangle + |v\rangle$ is a general quantum state, it is easy to see that $\langle v|\Pi|v\rangle = 0$ and $\Pi^2|u\rangle = |u\rangle$. From the last equality one deduces that $|u\rangle$ can be decomposed into the eigenstates of Π with eigenvalues ± 1. The loop operator can also be rewritten as $\Pi = |\Pi| e^{i\phi}$ where ϕ is the total flux through the plaquette. In order to guarantee the properties of Π the total flux through the plaquette has to verify $\phi = \pi k$ where k is an integer. Other values are thus "forbidden" gauge configurations.

In order to remove forbidden U(1) gauge configurations of the antiferromagnet Heisenberg model ($\phi \neq \pm\pi$) a CS term should be included in the QED_3 action in order to fix the total flux through a plaquette. This leads to the Maxwell-Chern-Simons (MCS) action in Euclidean space

$$S_E = \int_0^\beta \int d^2\vec{r}\left(-\frac{1}{2}a_\mu\{[\Box\delta^{\mu\nu} + (1-\lambda)\partial^\mu\partial^\nu] + i\kappa\varepsilon^{\mu\rho\nu}\partial_\rho\}a_\nu + \sum_\sigma \bar{\psi}_{\vec{r}\sigma}[\gamma_\mu(\partial_\mu - iga_\mu)]\psi_{\vec{r}\sigma}\right). \quad (6)$$

The implementation of the CS action

$$S_E^{CS} = \int_0^\beta d\tau \int d^2\vec{r}\left(i\frac{\kappa}{2}\varepsilon^{\mu\rho\nu}a_\mu\partial_\rho a_\nu\right) \quad (7)$$

introduces a new constant κ. We show below that this constant can be fixed to a definite value.

From the above action (6), the equation of motion of the gauge field in Minkowskian space

$$\partial_\mu f^{\nu\mu} - (\kappa/2)\varepsilon^{\mu\nu\rho}f_{\nu\rho} = -g\sum_\sigma \bar{\psi}_\sigma\gamma^\mu\psi_\sigma \quad (8)$$

leads to a relation between a magnetic field and the CS coefficient.[16,17] If $\mathcal{B} = \partial_1 a_2 - \partial_2 a_1$ is chosen to be constant in such a way that the whole system experiences a homogeneous magnetic field the equation of motion (8) of the gauge field becomes

$$\kappa\mathcal{B} = -g\sum_\sigma \langle\psi^\dagger_\sigma\psi_\sigma\rangle. \quad (9)$$

The gauge field a_μ is related to the phase $\theta(\vec{r})$ of the spinon at site $\vec{r}$ through the gauge transformation $f_{\vec{r},\sigma} \to e^{ig\theta(\vec{r})}f_{\vec{r},\sigma}$ which keeps the Heisenberg Hamiltonian (3) invariant. From the definition of ψ one gets $\psi_{\vec{r}\sigma} \to e^{ig\theta(\vec{r})}\psi_{\vec{r}\sigma}$. It is clear that $\theta(\vec{r})$ is the phase at the lattice site $\vec{r}$ and that $a_\mu(\vec{r}) = \partial_\mu\theta(\vec{r})$. Hence the magnetic field $\mathcal{B}$ is then directly related to the flux ϕ through the plaquette shown in Fig. 1:

$$\phi = g\sum_{\langle i,j\rangle\in\Box}[\theta(\vec{r}_i) - \theta(\vec{r}_j)] = g\int_\Box d\vec{l}\cdot\vec{a} = g\int_\Box d\vec{\Omega}_\Box\cdot\vec{\mathcal{B}} = g\Omega_\Box\mathcal{B} = \pi k \quad (10)$$

where $\Omega_\Box$ is the surface of the plaquette and k an integer. Here the flux is fixed to be to be equal to $\{0, \pm\pi\}$.[8]

Hence κ can be fixed by the flux through the plaquette using Eqs. (10) and (9). Defining $\rho = \Sigma_\sigma\langle\psi^\dagger_\sigma\psi_\sigma\rangle$ as the density of spinons, one can indeed rewrite Eq. (9) as

$$\kappa = -\frac{g\rho}{\mathcal{B}} = \frac{g^2}{\pi}\frac{\mathcal{N}}{k},$$

where $\mathcal{N}$ is the number of spinons on the plaquette and k is an integer [see Eq. (10)]. Recalling that

$$\nu = \frac{\text{number of particle}}{\text{number of flux quanta}} = \frac{\rho}{g|\mathcal{B}|/(2\pi)} = \frac{2\mathcal{N}}{k} \quad (11)$$

is similar to the filling factor of the Landau level[18] in the quantum Hall effect (QHE) one finally gets

$$\kappa = \frac{g^2}{2\pi}\nu. \quad (12)$$

The CS term breaks parity and time-reversal invariance and leads to a permanent pseudomagnetic field $\mathcal{B}$ through a plaquette. However, the effects of the gauge field involved in this term have not yet been observed experimentally. Indeed, a_μ is not necessarily related to a physical field. As a consequence the broken symmetries here have no connection up to now with physical observables.

The present analysis may suggest that the application of a real magnetic field to the spin system could allow one to detect the presence of spinons through the quantum Hall effect.[18] We leave this point for further investigations.

B. The photon propagator at finite temperature

In this section we construct the dressed photon propagator of QED_3 with a MCS term at finite temperature in order to gain information about the interaction between spinons ψ and the implication of the CS term on the dynamical mass generation.

Integrating (6) over the fermion fields ψ, the partition function of the spin system $\mathcal{Z}[\psi,a]=\int\mathcal{D}(\psi,a)e^{-S_E}$ with the action S_E given by (6) leads to the pure gauge partition function

$$\mathcal{Z}[a]=\int \mathcal{D}(a)e^{-S_{eff}[a]}, \tag{13}$$

where the effective pure gauge field action $S_{eff}[a]$ comes in the form

$$S_{eff}[a]=\int_0^\beta d\tau\int d^2\vec{r}\Big(-\frac{1}{2}a_\mu\{[\Box\,\delta^{\mu\nu}+(1-\lambda)\partial^\mu\partial^\nu] + i\kappa\varepsilon^{\mu\rho\nu}\partial_\rho\}a_\nu\Big)-\ln\det[\gamma_\mu(\partial_\mu-iga_\mu)]. \tag{14}$$

One can develop the last term in the effective gauge field action $S_{eff}[a]$ into a series and write

$$\ln\det[\gamma_\mu(\partial_\mu-iga_\mu)] =\ln\det G_F^{-1}-\sum_{n=1}^{\infty}\frac{1}{n}\,\mathrm{Tr}[iG_F\gamma^\mu a_\mu]^n, \tag{15}$$

where $G_F^{-1}(k-k')=i\frac{\gamma^\mu k_\mu}{(2\pi)^2\beta}\delta(k-k')$ is the fermion Green's function in the Fourier space-time with $k=(\tilde{\omega}_{F,n},\vec{k})$; hence $G_F=-i\frac{\gamma^\mu k_\mu}{k^2}(2\pi)^2\beta\delta(k-k')$. The first term on the right-hand side (r.h.s.) of Eq. (15) is independent of the gauge field $\{a_\mu\}$. It can be removed from the series since we focus our attention on pure gauge field terms. The first term proportional to the gauge field $n=1$ in the sum vanishes since tr $\gamma_\mu=0$. Keeping only second-order terms in order to stay with Gaussian contributions to the fluctuations one gets the pure gauge action

$$S_{eff}^{(2)}[a]=\int_0^\beta d\tau\int d^2\vec{r}\Big(-\frac{1}{2}a_\mu\{[\Box\,\delta^{\mu\nu}+(1-\lambda)\partial^\mu\partial^\nu] + i\kappa\varepsilon^{\mu\rho\nu}\partial_\rho\}a_\nu\Big)+\frac{g^2}{2\beta}\sum_\sigma\sum_{\omega_{F,1}}\int\frac{d^2\vec{k}_1}{(2\pi)^2}\frac{1}{\beta}\sum_{\omega_F''}\int\frac{d^2\vec{k}''}{(2\pi)^2} \times\mathrm{tr}\Big(\frac{\gamma^\rho k_{1,\rho}}{k_1^2}\gamma^\mu a_\mu(k_1-k'')\frac{\gamma^\eta k''_\eta}{k''^2}\gamma^\nu a_\nu[-(k_1-k'')]\Big). \tag{16}$$

The second term in Eq. (16) has been worked out in Ref. 5. The whole action can be put into the form

$$S_{eff}^{(2)}[a]=-\frac{g^2}{2\beta}\sum_{\omega_B}\int\frac{d^2\vec{q}}{(2\pi)^2}a_\mu(-q)[\Delta_{E\mu\nu}^{(0)\;-1}+\Pi_{\mu\nu}(q)]a_\nu(q), \tag{17}$$

where $\Delta_{E\mu\nu}^{(0)}=\frac{1}{q^2(q^2+\kappa^2)}[q^2\delta_{\mu\nu}-q_\mu q_\nu-\kappa\varepsilon_{\mu\nu\rho}q^\rho]+\frac{1}{\lambda}\frac{q_\mu q_\nu}{(q^2)^2}$ is the bare photon propagator in Euclidean space-time. The one-loop vacuum polarization term[5] reads

$$\Pi_{\mu\nu}=\Pi_A A_{\mu\nu}+\Pi_B B_{\mu\nu} =[\tilde{\Pi}_1(q_\mu)+\tilde{\Pi}_2(q_\mu)]A_{\mu\nu}+\tilde{\Pi}_3(q_\mu)B_{\mu\nu},$$

where $A_{\mu\nu}$ and $B_{\mu\nu}$ are Lorentz invariant tensors given in the Appendix and

$$\tilde{\Pi}_1(q_\mu)=\frac{\alpha q}{\pi}\int_0^1 dx\sqrt{x(1-x)}\frac{\sinh\beta q\sqrt{x(1-x)}}{D(X,Y)},$$

$$\tilde{\Pi}_2(q_\mu)=\frac{\alpha m}{\beta}\int_0^1 dx(1-2x)\frac{\cos 2\pi xm}{D(X,Y)},$$

$$\tilde{\Pi}_3(q_\mu)=\frac{\alpha}{\pi\beta}\int_0^1 dx\ln 2D(X,Y)$$

with $D(X,Y)=\cosh[\beta q\sqrt{x(1-x)}]+\sin(2\pi xm)$. Here the photon momentum $q_\mu=(\omega_{B,m}=\frac{2\pi m}{\beta},\vec{q})$ with $\mu=\{0,1,2\}$, m is an integer, and $\alpha=2g^2$ is the coupling constant.

The finite-temperature dressed photon propagator in Euclidean space verifies the Dyson equation

$$\Delta_{E\mu\nu}^{-1}=\Delta_{E\mu\nu}^{(0)\;-1}+\Pi_{\mu\nu}. \tag{18}$$

The inversion of Eq. (18) leads to the dressed photon propagator with the CS term at finite temperature:

$$\Delta_{E\mu\nu}=[(q^2+\Pi_A)A_{\mu\nu}+(q^2+\Pi_B)B_{\mu\nu}-\kappa\varepsilon_{\mu\nu\rho}q^\rho] \times[(q^2+\Pi_A)(q^2+\Pi_B)+(\kappa q)^2]+\frac{q_\mu q_\nu}{\lambda(q^2)^2}. \tag{19}$$

IV. "CHIRAL" SYMMETRY RESTORATION

The coupling of the gauge field a_μ to the spinon field generates a mass for this field.[5] Chiral symmetry in four dimensions requires fermions to be massless. In this space a mass term $m\bar{\psi}\psi$ changes sign under chiral transformations generated by means of the Dirac matrix γ_5. Hence fermions must be massless in order to keep the action invariant. In three dimensions no real γ_5 matrix can be defined. However, by embedding the (2+1)-dimensional space into a four-dimensional space two types of "chiral" symmetries can be defined from γ_3 and γ_5,[19–21] where γ_3 and γ_5 are 4×4 matrices

$$\gamma_3 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \quad \gamma_5 = i\begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix},$$

which induce "chiral" transformations $e^{ig\theta\gamma_3}$ and $e^{ig\theta\gamma_5}$. In (2+1) dimensions the algebra is completed by

$$\gamma^0 = \begin{pmatrix} \tau_3 & 0 \\ 0 & -\tau_3 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} \tau_1 & 0 \\ 0 & -\tau_1 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} \tau_2 & 0 \\ 0 & -\tau_2 \end{pmatrix},$$

where $\{\tau_i, i=1,2,3\}$ are the Pauli matrices and the Dirac matrices verify $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}$ in Euclidean space.

Appelquist *et al.*[19,22] showed that at zero temperature the originally massless fermion can acquire a dynamical mass when the number N $(=\Sigma_\sigma 1)$ of fermion flavors is lower than the critical value $N_c = 32/\pi^2$. Later Maris[23] confirmed this result with $N_c \simeq 3.3$. Since we consider only spin-1/2 systems, $N=2$ and hence $N<N_c$.

At zero temperature the dynamical mass term is renormalized by the CS term $\frac{m(\kappa\neq 0)}{m(\kappa=0)} = e\left[-\frac{4N}{N_c}\cdot\frac{\kappa^2}{(\alpha/16)^2}\right]$ and even the critical value N_c is affected as $\tilde{N}_c = N_c[1+(16\kappa/\alpha)^2]$ as shown by Hong and Park.[24]

Here we concentrate on the impact of the CS term on the dynamical mass generation and show that chiral symmetry can be restored at finite temperature. An explanation of the mechanism behind this symmetry restoration will also be given.

A. Effective potential at finite temperature

In the present theory mass is generated in two different ways. First, as shown earlier, the massless photon induces a mass for the spinon through the coupling of the two fields.[5] Second, the CS coefficient gives a mass to the "photon" (gauge field a_μ), $m_{MCS} = \kappa$. This can be seen from the pure gauge equation of motion for the dual field[16] $\tilde{f}_\mu \equiv \frac{1}{2}\varepsilon^{\mu\nu\rho} f_{\nu\rho}$

$$(\partial^\mu\partial_\mu + \kappa^2)\tilde{f}_\nu = 0.$$

The massive photon induces the same effect (dynamical mass generation) at zero temperature.[24]

We show now how the photon mass κ (the CS coefficient) affects the effective potential at finite temperature between two spinons.

The static effective potential between spinons with opposite charge g is given by

$$\begin{aligned} V(R) &= -g^2\int_0^\beta d\tau\,\Delta_{00}(\tau,R) \\ &= -\frac{g^2}{2\pi}\int \frac{d^2\vec{q}}{(2\pi)^2}\Delta_{00}(q^0=0,\vec{q}e^{i\vec{q}\cdot\vec{R}}) \\ &= -\frac{g^2}{2\pi}\int_0^\infty dq\, q\cdot J_0(qR)\cdot\Delta_{00}(0,\vec{q}). \end{aligned}$$

$J_0(qR)$ is the Bessel function of the first kind and

$$\Delta_{00} = \frac{1}{[q^2+\Pi_B(m=0)]+(\kappa q)^2/[q^2+\Pi_A(m=0)]}.$$

At large distances $q\to 0$ the one-loop vacuum polarization parts become $\Pi_A(m=0) =_{q\to 0} q^2\frac{\alpha\beta}{12\pi}$ and $\Pi_B(m=0) = q \to 0\frac{\alpha}{\pi\beta}\ln 2$ where the integer m is related to the photon energy [see above (18)]. Hence the longitudinal part of the photon propagator Δ_{00} leads to the definition of a correlation length ξ_κ,

$$\Delta_{00}(0,\vec{q}) = \frac{1}{q^2+\xi_\kappa^{-2}},$$

where ξ_κ is given by

$$\xi_\kappa^{-2} = \frac{\alpha}{\pi\beta}\ln 2 + \frac{\kappa^2}{1+\alpha\beta/12\pi}.$$

Integrating over the photon momentum q at large distance R, the effective potential at finite temperature reads

$$V(R,\beta) \simeq -\frac{g^2}{2\pi}\int_0^\infty dq\frac{qJ_0(qR)}{q^2+\xi_\kappa^{-2}} = -\frac{\alpha}{N}\sqrt{\frac{\xi_\kappa}{8\pi R}}e^{-R/\xi_\kappa},$$

which shows that the stronger κ, the shorter the correlation length ξ_κ. Hence variations of κ affect the correlation length between spinons. Moreover, the variation of the flux through the square plaquette also affects the correlation length since the CS coefficient is related to the flux through Eq. (9). If the flux ϕ through the plaquette increases the correlation length ξ_κ also increases; the larger κ, the shorter the interaction between spinons.

B. Dynamical mass generation

We show how the CS term affects the "chiral" restoring transition temperature of the dynamical mass generation. The Schwinger-Dyson equation for the spinon propagator at finite temperature reads

$$G^{-1}(k) = G^{(0)-1}(k) - \frac{g}{\beta}\sum_{\tilde{\omega}_{F,n}}\int\frac{d^2\vec{P}}{(2\pi)^2}\gamma_\mu G(p)\Delta_{\mu\nu}(k-p)\Gamma_\nu, \tag{20}$$

where $p=(p_0=\tilde{\omega}_{F,n},\vec{P})$, G is the spinon propagator, Γ_ν is the spinon-photon vertex, which will be approximated here by its bare value $g\gamma_\nu$ and $\Delta_{\mu\nu}$ is the dressed photon propagator (19). The second term in (20) is the fermion self-energy Σ $(G^{-1} = G^{(0)-1} - \Sigma)$. Performing the trace over the γ matrices in

Eq. (20) leads to a self-consistent equation for the self-energy

$$\Sigma(k) = \frac{g^2}{\beta}\sum_{\tilde{\omega}_{F,n}} \int \frac{d^2\vec{P}}{(2\pi)^2}\Delta_{\mu\mu}(k-p)\frac{\Sigma(p)}{p^2+\Sigma(p)^2}. \qquad (21)$$

In the low-energy and -momentum limit $\Sigma(k)=m(\beta,\kappa) \simeq \Sigma(0)$. Equation (21) simplifies to

$$1 = \frac{g^2}{\beta}\sum_{\tilde{\omega}_{F,n}} \int \frac{d^2\vec{P}}{(2\pi)^2}\Delta_{\mu\mu}(-p)\frac{1}{p^2+m(\beta,\kappa)^2}. \qquad (22)$$

If the main contribution comes from the longitudinal part $\Delta_{00}(0,-\vec{P})$ of the photon propagator, (22) goes over to

$$1 = \frac{g^2}{\beta}\sum_{\tilde{\omega}_{F,n}} \int \frac{d^2\vec{P}}{(2\pi)^2}\left([\vec{P}^2+\Pi_B(m=0)] + \frac{(\kappa\vec{P})^2}{[\vec{P}^2+\Pi_A(m=0)]}\right)^{-1}\frac{1}{[\tilde{\omega}_{F,n}^2+\vec{P}^2+m(\beta,\kappa)^2]}, \qquad (23)$$

where

$$\Pi_A(m=0) = \Pi_1(m=0)+\Pi_2(m=0) = \frac{\alpha P}{\pi}\int_0^1 dx\sqrt{x(1-x)}\tanh \beta P\sqrt{x(1-x)},$$

$$\Pi_B(m=0) = \Pi_3(m=0) = \frac{\alpha}{\pi\beta}\int_0^1 dx \ln 2[\cosh \beta P\sqrt{x(1-x)}].$$

Performing the summation over the modified fermion Matsubara frequencies $\tilde{\omega}_{F,n}=\frac{2\pi}{\beta}(n+1/4)$ (Refs. 9 and 5) the self-consistent equation takes the form

$$1 = \frac{(\alpha/\Lambda)}{4\pi N}\int_0^1 dP P \tanh\left[(\beta\Lambda)\sqrt{P^2+\left(\frac{m(\beta,\kappa)}{\Lambda}\right)^2}\right] \times\left[\left(P^2+\frac{\Pi_B(m=0)}{\Lambda^2}\right)+\frac{\left(\frac{\kappa}{\Lambda}P\right)^2}{\left(P^2+\frac{\Pi_A(m=0)}{\Lambda^2}\right)}\right]^{-1} \times\frac{1}{\sqrt{P^2+\left(\frac{m(\beta,\kappa)}{\Lambda}\right)^2}}. \qquad (24)$$

As defined above $\alpha=g^2N$ with $N=2$ since we have implemented the Popov-Fedotov procedure.[9,5] Here Λ is the uv cutoff and can be identified as the inverse spin lattice spacing. Equation (24) can be solved numerically.

Figure 3 shows the dependence of the dynamical mass on the temperature for different values of κ/Λ. This mass, which is different from zero for low temperature T, vanishes at some temperature T_c that depends on κ. As κ/Λ increases the chiral symmetry transition temperature T_c/Λ decreases, following the relation $\frac{T_c(\kappa\neq 0)}{T_c(\kappa=0)}=e^{-a(\alpha/\Lambda)\kappa^2/\Lambda^2}$ where $a(\alpha/\Lambda)$ is a coefficient depending on α/Λ. The behavior for a fixed α is shown in Fig. 4.

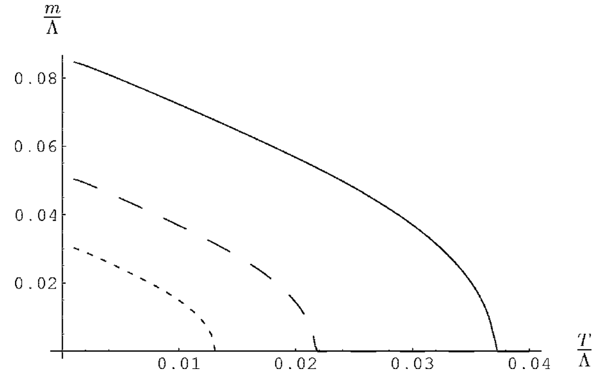

FIG. 3. Dependence of the dynamical mass $m(T)$ on the temperature. Full line, $\kappa/\Lambda=0$. Long-dashed line, $\kappa/\Lambda=5\times 10^3$. Dashed line, $\kappa/\Lambda=7\times 10^3$. All curves are obtained with $\alpha/\Lambda=10^5$.

One can understand the mechanism of chiral symmetry restoration as follows. The photon (gauge field) gives a mass to the fermions (spinons) through a dynamical mass generation mechanism. When the temperature increases, this mechanism is lowered by fluctuations, a fermions gain in mobility [the dynamical mass $m(\beta,\kappa)$ decreases]. This is similar to the situation in plasmas. When the temperature is high enough the charged particles composing the plasma are considered as free particles. Below some temperature these charged particles are screened; thus their mass is renormalized and gets larger than in the high-temperature plasma. The CS mass κ contributes also to the photon mass; the interaction $V(R,\beta)$ between fermions is weakened as κ increases for a fixed length R, the correlation length ξ_κ gets weaker, and thus the screening effect gets weaker. Finally, the chiral symmetry restoring temperature decreases with increasing κ since the screening effect is smaller and thus fermions gain mobility and their mass term is renormalized to a smaller value. At zero temperature the dynamical mass decreases as κ increases as $\frac{m(\kappa\neq 0)}{m(\kappa=0)}=e^{-(4N/N_c)[\kappa^2/(\alpha/16)^2]}$,[17] thus proving that the screening effect is lower as the photon mass κ increases and reduces the dynamical mass of the spinon.

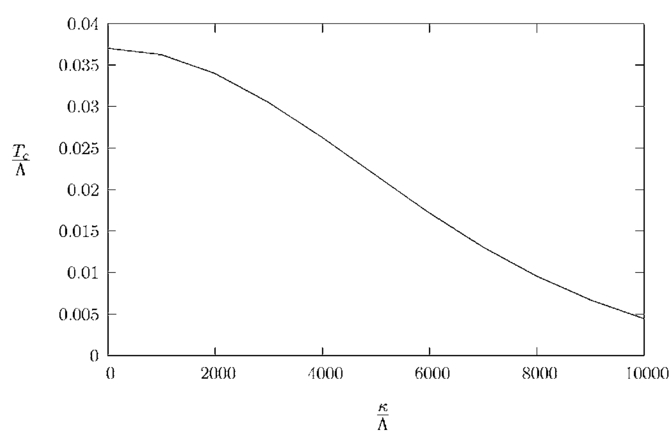

FIG. 4. Chiral symmetry transition temperature T_c/Λ depending on the CS coefficient κ/Λ. Here $\frac{\alpha}{\Lambda}=10^5$.

Going back to the Heisenberg model and looking for consequences of this chiral symmetry restoration on the energy spectrum of the spinons, one can see that when the CS coefficient differs from zero the gauge field a_μ gets a gap due to the mass term κ. As κ goes to ∞ the spinon energy gap $m(\beta,\kappa)$ decreases, leaving a gapless spinon spectrum for a transition temperature T_c going to zero.

For fixed α/Λ the knowledge of T_c fixes κ (Fig. 4) and consequently the generated mass $m(\beta,\kappa)$ (Fig. 3).

V. CONCLUSIONS

The low-energy spectrum of the Heisenberg model describing a two-dimensional antiferromagnet quantum spin system with an SU(2) symmetry has been mapped onto a (2+1)-dimensional quantum electrodynamics action with a U(1) gauge field symmetry.[5] In this framework we showed that the addition of a Chern-Simons term to the Maxwell term in the pure gauge field theory of QED_3 at finite temperature affects the spinon energy spectrum through a chiral symmetry restoration transition. The chiral symmetry transition temperature T_c above which the dynamical spinon mass is equal to zero follows the relation $\frac{T_c(\kappa\neq 0)}{T_c(\kappa=0)}=e^{-a(\alpha/\Lambda)\kappa^2/\Lambda^2}$ where $a(\alpha/\Lambda)$ is a coefficient depending on α/Λ and Λ is the uv cutoff.

The effective potential between two spinons with opposite charge g at finite temperature shows that the Chern-Simons term controls also the screening of this interaction through the photon mass, which is identified with the Chern-Simons coefficient κ. The correlation length ξ_κ decreases, showing that the screening effect gets weaker as κ increases.

The value of κ can be controlled by fixing the flux through a plaquette going around neighboring spin lattice sites. The gap in the spinon spectrum shrinks to zero with increasing κ for a fixed temperature.

Hence the Maxwell-Chern-Simons term at finite temperature, which is aimed to fix the correct U(1) gauge configuration, provides an interesting way to control the chiral symmetry restoration temperature and the effective potential between spinons. The present study has been done in the framework of a noncompact theory. One may ask how it will be influenced by the presence of instantons in a compact description of the gauge field. This point related to the confinement-deconfinement problem is still under discussion.[25–28]

ACKNOWLEDGMENTS

One of us (R.D.) would like to thank M. Rausch de Traubenberg for enlightening discussions and F. Stauffer for his encouragement.

APPENDIX

One may believe that a system at finite temperature breaks Lorentz invariance since the frame described by the heat bath already selects out a specific Lorentz frame. However, this is not true and one can formulate the statistical mechanics in a Lorentz covariant form.[29]

We consider a system in two space and one time dimension. Define the proper three-velocity u^μ of the heat bath. In the rest frame of the heat bath the three-velocity has the form $u^\mu=(1,0,0)$ and the inverse temperature β characterizes the thermal property of the heat bath.

Given the three-velocity vector u^μ one can decompose any three-vector into parallel and orthogonal components with respect to the proper velocity of the heat bath, the velocity u^μ. In particular, the parallel and transverse components of the three-momentum q^μ with respect to u^μ read

$$q_{\parallel}^{\mu}=(q\cdot u)u^{\mu}, \tag{A1}$$

$$\tilde{q}^{\mu}=q^{\mu}-q_{\parallel}^{\mu}. \tag{A2}$$

Similarly, one can decompose any vector and tensor into components that are parallel and transverse to a given momentum vector q^μ,

$$\bar{u}_\mu=u_\mu-\frac{(q\cdot u)}{q^2}q_\mu, \tag{A3}$$

$$\bar{\eta}_{\mu\nu}=\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}. \tag{A4}$$

It is easy to define second-rank symmetric tensors constructed at finite temperature from q^μ, u^μ, and $\delta_{\mu\nu}$ which are orthogonal to q^μ:

$$A_{\mu\nu}=\delta_{\mu\nu}-u^{\mu}u^{\nu}-\frac{\tilde{q}_\mu\tilde{q}_\nu}{\tilde{q}^2}, \tag{A5}$$

$$B_{\mu\nu}=\frac{q^2}{\tilde{q}^2}\bar{u}_\mu\bar{u}_\nu. \tag{A6}$$

Since one considers a spin system at finite temperature and "relativistic" covariance should be preserved, the polarization function may be put in the general form[29]

$$\Pi_{\mu\nu}=\Pi_A A_{\mu\nu}+\Pi_B B_{\mu\nu} \tag{A7}$$

and the Dyson equation (18) can now be expressed in a covariant form if one uses relation (A7).

*Electronic address: rdillen@lpt1.u-strasbg.fr
†Electronic address: richert@lpt1.u-strasbg.fr

[1] M. Franz, Z. Tesanovic, and O. Vafek, Phys. Rev. B **66**, 054535 (2002).
[2] P. Ghaemi and T. Senthil, Phys. Rev. B **73**, 054415 (2006).
[3] T. Morinari, cond-mat/0508251 (unpublished).
[4] P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006).
[5] R. Dillenschneider and J. Richert, Phys. Rev. B **73**, 224443 (2006).
[6] I. Affleck and J. B. Marston, Phys. Rev. B **37**, 3774 (1988).
[7] J. B. Marston and I. Affleck, Phys. Rev. B **39**, 11538 (1989).
[8] J. B. Marston, Phys. Rev. Lett. **61**, 1914 (1988).
[9] R. Dillenschneider and J. Richert, Phys. Rev. B **73**, 024409 (2006).
[10] V. N. Popov and S. A. Fedotov, Sov. Phys. JETP **67**, 535 (1988).
[11] A. Auerbach, *Interacting Electrons and Quantum Magnetism* (Springer-Verlag, Berlin, 1994).
[12] X. G. Wen, Phys. Rev. B **65**, 165113 (2002).
[13] D. P. Arovas, and A. Auerbach Phys. Rev. B **38**, 316 (1988).
[14] P. A. Lee and N. Nagaosa, Phys. Rev. B **46**, 5621 (1992).
[15] C. Itsykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1986).
[16] G. V. Dunne, in *Topological Aspects of Low Dimensional Systems*, edited by A. Comtet *et al.* (Springer-Verlag, Berlin, 2000).
[17] T. Itoh and H. Kato, Phys. Rev. Lett. **81**, 30 (1998).
[18] N. Nagaosa, *Quantum Field Theory in Strongly Correlated Electronic Systems* (Springer-Verlag, Berlin, 1998).
[19] T. W. Appelquist, M. Bowick, D. Karabali, and L. C. R. Wijewardhana, Phys. Rev. D **33**, 3704 (1986).
[20] N. Dorey and N. E. Mavromatos, Nucl. Phys. B **386**, 614 (1992).
[21] M. Rausch de Traubenberg, hep-th/0506011 (unpublished).
[22] T. Appelquist, D. Nash, and L. C. R. Wijewardhana, Phys. Rev. Lett. **60**, 2575 (1988).
[23] P. Maris, Phys. Rev. D **54**, 4049 (1996).
[24] D. K. Hong and S. H. Park, Phys. Rev. D **47**, 3651 (1993).
[25] S. Hands, J. B. Kogut, and B. Lucini, hep-lat/0601001 (unpublished).
[26] I. F. Herbut, Phys. Rev. B **66**, 094504 (2002).
[27] F. S. Nogueira and H. Kleinert, Phys. Rev. Lett. **95**, 176406 (2005).
[28] Z. Nazario and D. I. Santiago, cond-mat/0606386 (unpublished).
[29] A. Das, *Finite Temperature Field Theory* (World Scientific, Singapore, 1997).