Microwave-induced coupling of superconducting qubits

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We investigate the quantum dynamics of a system of two coupled superconducting qubits under microwave irradiation. We find that, with the qubits operated at the charge codegeneracy point, the quantum evolution of the system can be described by an effective Hamiltonian which has the form of two coupled qubits with tunable coupling between them. This Hamiltonian can be used for experimental tests on macroscopic entanglement and for implementing quantum gates.

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A significant interest in the quantum coherence properties of various superconducting devices has been manifest in the last years following the successful demonstration of superpositions of charge and flux based macroscopic quantum states.[1] It was immediately realized that, when supplemented with appropriate read-out components and protocols, these devices qualify as candidates for quantum bits in future quantum computing architectures.[2] Indeed, several types of superconducting qubits such as phase qubits,[3] charge qubits,[4] charge-phase qubits,[5] and flux qubits[6] have been operated since.

For systems of two superconducting qubits, a desirable feature to have in order to implement two-qubit gates is tunable coupling, and several schemes using for instance variable electrostatic transformers,[7] the dynamic inductance of a dc SQUID,[8] or resonator circuits[9] have been proposed. These ideas have not yet been fully tested experimentally: instead, fixed coupled qubits have been studied as a preliminary step, and a few significant results have already been reported, such as signatures of entanglement in coupled phase qubits,[10] and a protocol for implementing a controlled-NOT (CNOT) gate.[11] However, it has been recognized that these protocols will not be easily scalable because they manipulate the qubits far from the degeneracy points, where decoherence is strong (for single charge qubits, when operated off-degeneracy, the dephasing times are of the order of only a few hundred picoseconds,[12] due mostly to $1/f$ noise). The solution to this problem in the case of single qubits [5] is to keep the qubit at the degeneracy point during the quantum gate, and to move away from this point only during the measurements; this strategy has been successfully implemented for flux qubits as well.[6] But for two qubits the requirements are rather contradictory: on the one hand we would like to have the qubits operated at the special, low-decoherence degeneracy points; on the other hand, adding up the coupling will remove them from these points. More recent proposals have attempted to solve this problem by employing NMR-style strategies,[13] or by using a superconducting circuit that allows modulation of the coupling[14] or act as the vibration mode of two trapped ions.[15]

In this paper we show that it is possible to create entanglement and quantum gates in a system of two qubits with fixed coupling, irradiated with a monochromatic off-resonance microwave field and biased at the codegeneracy point. As a result, the proposed quantum circuit satisfies both

of the requirements above: it is insensitive to noise due to fluctuations of the external parameters (e.g., $1/f$ noise), and it can be mapped into a system of two qubits with tunable coupling. The scheme is therefore minimal from the point of view of the number of on-chip circuit elements required and can be realized immediately without any major change in the existing qubit experimental setups. Also, in contradistinction with the fast pulse method of Refs. 4 and 11 this technique does not rely on high-precision microwave electronic equipment, therefore, from an experimental point of view, could be regarded as more reliable and relatively inexpensive.

We do all the calculations for the case of coupled quantronium circuits,[5] with the observation that the results can be easily adapted to almost any other qubit species. Let us start with the Hamiltonian for two capacitively coupled charge qubits,[11]

$$H = E_{C1}(n_{g1} - n_1)^2 - E_{J1} \cos \varphi_1 + E_{C2}(n_{g2} - n_2)^2$$
$$- E_{J2} \cos \varphi_2 + E_m(n_{g1} - n_1)(n_{g2} - n_2), \qquad (1)$$

with $E_{C1,2} \approx 2e^2/C_{\Sigma1,2}$, $E_{J1,2}$ the standard charging and Josephson energies (Fig. 1) for each split Cooper pair box ($C_{\Sigma1,2}$ are predominantly given, for each qubit, by the sum of the corresponding island-to-lead capacitances) and $E_m \approx 4e^2 C_m/C_{\Sigma1}C_{\Sigma2}$ ($C_m \ll C_{\Sigma1,2}$ is the coupling capacitance).

The condition of insensitivity to fluctuations in the external parameters is satisfied automatically if the operating point of the qubits is fixed at $n_{g1} = n_{g2} = 1/2$—the so-called codegeneracy point—where the eigenvalues of the Hamiltonian are first-order independent of fluctuations in n_{g1} and n_{g2}. At the codegeneracy point, the 2-qubit Hamiltonian Eq. (1) written in the eigenbasis $|\uparrow\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$,
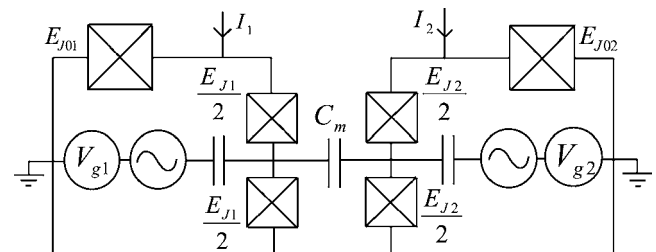

FIG. 1. Schematic of the circuit: two capacitively coupled quantronium circuits.

$|\downarrow\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$ of each qubit—considering the approximation of large charging energies in which we can restrict ourselves to the subspace spanned by $\{|n_1\rangle\} = \{|0\rangle, |1\rangle\}$, $\{|n_2\rangle\} = \{|0\rangle, |1\rangle\}$—has the form

$$H = \frac{1}{4}(E_{C1} + E_{C2}) + \frac{1}{4}E_m\sigma_1^x\sigma_2^x - \frac{1}{2}E_{J1}\sigma_1^z - \frac{1}{2}E_{J1}\sigma_2^z.$$

This Hamiltonian can be diagonalized by introducing the matrix

$$M_{\pm} = \begin{pmatrix} \cos\theta_{\pm}/2 & -\sin\theta_{\pm}/2 \\ \sin\theta_{\pm}/2 & \cos\theta_{\pm}/2 \end{pmatrix}, \tag{2}$$

where M_+ acts on the subspace spanned by $\{|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$ and M_- acts on the subspace $\{|\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle\}$. The angle $\theta_{\pm}$ is defined by

$$\tan\theta_{\pm} = -\frac{E_m}{2(E_{J2} \pm E_{J1})}. \tag{3}$$

The eigenbasis for the coupled qubits is

$$\begin{pmatrix} |uu\rangle \\ |dd\rangle \end{pmatrix} = M_+^{-1} \begin{pmatrix} |\uparrow\uparrow\rangle \\ |\downarrow\downarrow\rangle \end{pmatrix}, \tag{4}$$

$$\begin{pmatrix} |du\rangle \\ |ud\rangle \end{pmatrix} = M_-^{-1} \begin{pmatrix} |\downarrow\uparrow\rangle \\ |\uparrow\downarrow\rangle \end{pmatrix}. \tag{5}$$

The energy levels of the four-level system are defined by the quantities ξ and ϵ,

$$\xi = \frac{1}{2}\sqrt{(E_{J1} + E_{J2})^2 + (E_m/2)^2}, \tag{6}$$

$$\epsilon = \frac{1}{2}\sqrt{(E_{J1} - E_{J2})^2 + (E_m/2)^2}. \tag{7}$$

It is also useful to introduce the notation $\hbar\Omega = \xi + \epsilon$ and $\hbar\nu = \xi - \epsilon$. Since we will need to use Pauli operators also with respect to the new fixed-coupling eigenbasis Eqs. (4) and (5), we adopt the convention that $\sigma_{\{1,2\}}^{\{x,y,z\}}$ refer to the original qubits, and $\sigma_{x,y,z}$ together with the tensorial product $\otimes$ correspond to the basis Eqs. (4) and (5). For example, in the fixed-coupling eigenbasis the Hamiltonian describes two uncoupled qubits, $H = -\hbar(\nu\sigma_z \otimes I + \Omega I \otimes \sigma_z)/2$.

Consider now the case of an excitation produced by irradiating the qubits with a monochromatic microwave radiation of angular frequency ω, $n_{g1} = 1/2 + w_1\cos\omega t$ and $n_{g2} = 1/2 + w_2\cos\omega t$. The quantities w_1 and w_2 are the amplitudes of the radiation: experimentally, they can be adjusted relatively fast and independently for each qubit, by mixing the continuous microwave with a tunable signal from a pulse pattern generator or an arbitrary wave form generator. The Hamiltonian is then

$$H = \frac{1}{4}(E_{C1} + E_{C2}) + E_{C1}w_1\cos\omega t\sigma_1^x + E_{C2}w_2\cos\omega t\sigma_2^x$$
$$+ \frac{1}{4}E_m\sigma_1^x\sigma_2^x - \frac{1}{2}E_{J1}\sigma_1^z - \frac{1}{2}E_{J1}\sigma_2^z. \tag{8}$$

We expand at any time t the state vector by separating the eigenenergies $\pm\xi, \pm\epsilon$ of the four states $|uu\rangle, |dd\rangle$ and $|du\rangle, |ud\rangle$,

$$\Psi(t) = e^{(i/\hbar)\xi t}c_{uu}(t)|uu\rangle + e^{(i/\hbar)\epsilon t}c_{du}(t)|du\rangle + e^{-(i/\hbar)\epsilon t}c_{ud}(t)|ud\rangle$$
$$+ e^{-(i/\hbar)\xi t}c_{dd}(t)|dd\rangle. \tag{9}$$

Inserting this expansion into the Schrödinger equation with Hamiltonian Eq. (8), one can notice that it is possible to perform a rotating-wave approximation by neglecting fast-oscillating terms (containing the frequencies $\omega + \Omega$ and $\omega + \nu$; below we will neglect the Bloch–Siegert shifts[16] which become important only for higher values of the microwave amplitude).

If we define the detunings of the external microwave radiation from the two-qubit frequencies $\delta = \omega - \nu$ and $\Delta = \omega - \Omega$, we obtain the system of equations

$$i\hbar\frac{d}{dt}\tilde{c}_{uu} = \frac{\hbar}{2}(\delta + \Delta)\tilde{c}_{uu} + 2T_+^{(1)}\tilde{c}_{du} + 2T_+^{(2)}\tilde{c}_{ud}, \tag{10}$$

$$i\hbar\frac{d}{dt}\tilde{c}_{du} = 2T_+^{(1)}\tilde{c}_{uu} + \frac{\hbar}{2}(-\delta + \Delta)\tilde{c}_{du} + 2T_-^{(2)}\tilde{c}_{dd}, \tag{11}$$

$$i\hbar\frac{d}{dt}\tilde{c}_{ud} = 2T_+^{(2)}\tilde{c}_{uu} + \frac{\hbar}{2}(\delta - \Delta)\tilde{c}_{ud} + 2T_-^{(1)}\tilde{c}_{dd}, \tag{12}$$

$$i\hbar\frac{d}{dt}\tilde{c}_{dd} = 2T_-^{(2)}\tilde{c}_{du} + 2T_-^{(1)}\tilde{c}_{ud} + \frac{\hbar}{2}(-\delta - \Delta)\tilde{c}_{dd}, \tag{13}$$

where we introduced the notations

$$T_{\pm}^{(1)} = \frac{E_{C1}}{2}w_1\cos\frac{\theta_+ - \theta_-}{2} \pm \frac{E_{C2}}{2}w_2\sin\frac{\theta_+ + \theta_-}{2},$$

$$T_{\pm}^{(2)} = \frac{E_{C2}}{2}w_2\cos\frac{\theta_+ + \theta_-}{2} \pm \frac{E_{C1}}{2}w_1\sin\frac{\theta_+ - \theta_-}{2},$$

and the following substitutions have been used: $c_{uu}(t) = \exp[i(\delta + \Delta)t/2]\tilde{c}_{uu}(t)$, $c_{du}(t) = \exp[i(-\delta + \Delta)t/2]\tilde{c}_{du}(t)$, $c_{ud}(t) = \exp[i(\delta - \Delta)t/2]\tilde{c}_{ud}(t)$, $c_{dd}(t) = \exp[i(-\delta - \Delta)t/2]\tilde{c}_{dd}(t)$. This is a quantum evolution governed by a time-independent effective Hamiltonian,

$$H_{\text{eff}} = \left(\frac{\hbar\delta}{2}\sigma_z + \frac{E_{C1}w_1}{2}\cos\frac{\theta_+ - \theta_-}{2}\sigma_x\right) \otimes I$$
$$+ I \otimes \left(\frac{\hbar\Delta}{2}\sigma_z + \frac{E_{C2}w_2}{2}\cos\frac{\theta_+ + \theta_-}{2}\sigma_x\right)$$
$$+ \frac{E_{C1}w_1}{2}\sin\frac{\theta_+ - \theta_-}{2}\sigma_z \otimes \sigma_x$$
$$+ \frac{E_{C2}w_2}{2}\sin\frac{\theta_+ + \theta_-}{2}\sigma_x \otimes \sigma_z. \tag{14}$$

The Hamiltonian Eq. (14) is the central result of this paper; it describes a set of two coupled qubits of states $|u\rangle, |d\rangle$ with the coupling controlled by the radiation intensities w_1 and w_2. The elements of this Hamiltonian, namely $\sigma_z \otimes I$, $I \otimes \sigma_z$, $\sigma_x \otimes I$, $I \otimes \sigma_x$, $\sigma_z^1 \otimes \sigma_x^2$, and $\sigma_z^2 \otimes \sigma_x^1$ span the whole su(4) Lie algebra.[17] The entangling properties of this type of Hamiltonian have not been studied before in the quantum computing community and no ready-made analytical recipe

exists for the problem of generating a given two-qubit gate. We approach this problem numerically.

A typical experimental situation will be $E_{J1} \gg E_m$, $E_{J2} \gg E_m$, therefore $|\theta_+| \approx 0$. Also, we want to avoid an entangling dynamics between the qubits in the absence of quantum gate driving, therefore we impose the quasiseparability condition $E_m \ll |E_{J1} - E_{J2}|$ (a unitary transformation in the four-state basis $|uu\rangle, |du\rangle, |ud\rangle, |dd\rangle$ will be approximately the same when written in the original qubit basis $|\uparrow\uparrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\downarrow\rangle$). With these approximations, the Hamiltonian Eq. (14) reads

$$H_{\text{eff}} = \left(\frac{\hbar \delta}{2} \sigma_z + \frac{E_{C1}w_1}{2} \sigma_x \right) \otimes I + I \otimes \left(\frac{\hbar \Delta}{2} \sigma_z + \frac{E_{C2}w_2}{2} \sigma_x \right)$$
$$+ \frac{E_m}{8(E_{J2} - E_{J1})} (E_{C1}w_1 \sigma_z \otimes \sigma_x - E_{C2}w_2 \sigma_x \otimes \sigma_z).$$

We now see that in the presence of the driving microwave field, the relatively small value of E_m is compensated by the field intensity, and controlled entanglement becomes possible as the Larmor frequency of one qubit is modulated by the transversal part of the Rabi oscillations of the other one.

We have calculated numerically the evolution of the amplitudes $\tilde{c}$ for different values of the quantronium parameters. Two interesting particular cases emerge. Let us for simplicity neglect $\theta_+ \approx 0$ then take $\delta = -\Delta > 0$ $[\hbar\omega = \hbar(\nu + \Omega)/2 = \xi]$ and $E_{C1}w_1 = E_{C2}w_2 = \hbar W$. The effective Hamiltonian has then one eigenvalue zero and the other three given by the solutions of the third order equation $\lambda^3 - \lambda(\delta^2 + W^2) + \delta W^2 \sin\theta_- = 0$. Consider first the situation $\delta \gg W$. The solutions for the finite eigenvalues are found approximately $\lambda_{1,2} \approx \pm\delta - (W^2/2)\delta^{-1}\sin\theta_-$ and $\lambda_3 \approx W^2\delta^{-1}\sin\theta_-$. The evolution can be approximated by $\tilde{c}_{uu} = [1 + \exp(-i\lambda_3 t)]/2$, $\tilde{c}_{dd} = [-1 + \exp(-i\lambda_3 t)]/2$, $\tilde{c}_{du} = \tilde{c}_{ud} = 0$, and the concurrence assumes a remarkable simple form, $\mathcal{C} = 2|c_{uu}c_{dd} - c_{du}c_{ud}| = |\sin\lambda_3 t|$. In this case we note that the buildup of a probability amplitude on the state $|dd\rangle$ is due to a coherent effect similar to that which produces dark states for Λ atoms. There is no matrix element between the initial state $|uu\rangle$ and $|dd\rangle$; instead, the atoms are transferred coherently to $|uu\rangle$ through $|ud\rangle$ and $|du\rangle$, which have population zero due to destructive interference of the amplitude probabilities. This results at times $\lambda_3^{-1}(\pi/2 + n\pi)$ in the creation of a maximally entangled state of the form

$$\frac{1}{2}[1 + i(-1)^{n+1}]|uu\rangle + \frac{1}{2}[-1 + i(-1)^{n+1}]|dd\rangle.$$

The time required is of the order of λ_3^{-1}, and it gets larger for smaller and smaller microwave power.

The second case is $W \gg \delta$. In this case, the eigenvalues can be still determined approximately, $\lambda_{1,2} = \pm W - (\delta/2)\sin\theta_-$ and $\lambda_3 = \delta\sin\theta_-$, but the evolution of the coefficients $c_{uu,du,ud,dd}$ is not so simple anymore. Still, we have verified numerically that also in this limit the concurrence assumes a rather simple form (Fig. 2), but this time the oscillation period is not related only to λ_3. The state at $\mathcal{C} = 1$ has components on all four vectors $|uu\rangle, |du\rangle, |ud\rangle, |dd\rangle$, which tend to oscillate on a time scale of the order of W^{-1}; remarkably, this evolution conspires to give a concurrence of a simple form, as shown in the figure.
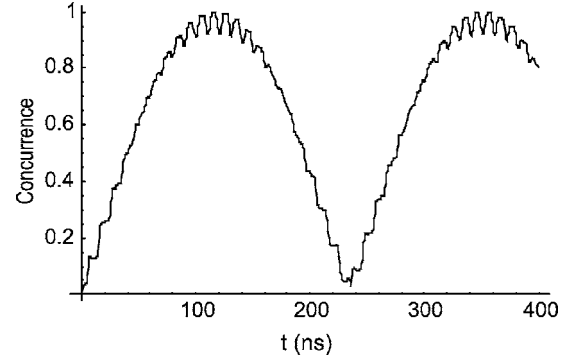

FIG. 2. Concurrence for the case $W \gg \delta$. The parameters for this figure are $W = 2\pi \times 96.3$ MHz, $\delta = -\Delta = 2\pi \times 15.2$ MHz, $\theta_+ \approx 0$, and $\theta_- = -0.29$.

In principle, one can measure these amplitudes (details at the end of the paper), and compare the results with the theory. These experiments are important because they are simple to realize (microwave amplitude is kept constant) and the theoretical prediction is an oscillation of the concurrence between the extreme values 0 and 1, therefore the comparison with experimental data (as well as the extraction of 2-qubit dephasing times) is straightforward.

We now address the problem of implementing quantum gates numerically. We first notice (and we also checked numerically) that local unitary transformations (single-qubit gates) can be realized simply by tuning the incoming microwave in resonance with either one of the two qubits and using relatively low power. For two-qubit gates we must find appropriate excursions in the parameter space $\{w_1, w_2\}$ such that the result of the evolution has the same Makhlin invariants G_1 and G_2 as CNOT;[18] we therefore must perform a numerical search for the minimum of the expression $|G_1(t)|^2 + |G_2(t) - 1|^2$. Our search method is based on simulated annealing;[19] for example, in the case of equal detunings $\delta = -\Delta$ one possible control parameter sequence $\{w_1, w_2\} = \{w_1(t), w_2(t)\}$ is presented in Fig. 3.

Measurements. We consider a measurement scheme (Fig. 1) in which each of the two qubits is shunted by large

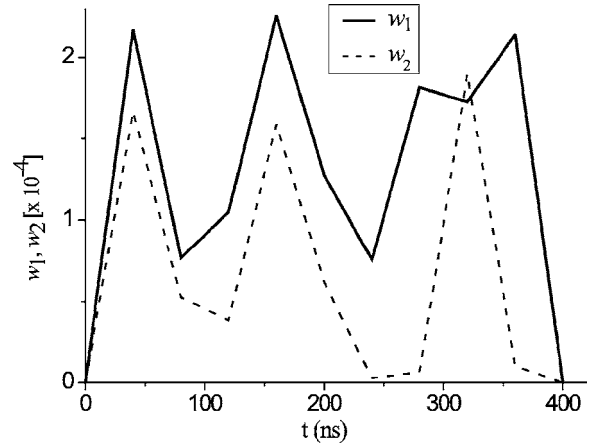FIG. 3. Parameters w_1 and w_2 for CNOT ($E_{C1} = 605$ GHz, $E_{C2} = 454$ GHz, $\delta = -\Delta = -19.2$ MHz, $\theta_+ \approx 0$, $\theta_- = -0.29$).

current-biased junctions. As in the single-qubit quantronium, the bias currents I_1 and I_2 will be kept to zero during the two-qubit gate. Next, currents are raised adiabatically to a value close to the critical current of the large junctions, and a switching event in a chosen time interval is recorded or not. Upon performing this experiment a large number of times, switching probabilities—as functions of the quantum state of the qubit—can be determined experimentally. In the approximation $E_m \ll |E_{J1} - E_{J2}|$ ($|uu\rangle \approx |\uparrow\uparrow\rangle$, $|du\rangle \approx |\downarrow\uparrow\rangle$, etc.), and the equations of motion for the macroscopic phase differences across the large read-out junctions separate. Therefore one can introduce independent switching rates $\Gamma_1^\uparrow$ and $\Gamma_2^\downarrow$ defined, as in the single-qubit case,[5,20] at two bias currents I_1 and I_2, where the sensitivity of the measurement is maximal. Let us now imagine a switching current experiment in which the large junctions are biased at I_1 and I_2 for times τ_1 and τ_2, respectively. The experimentalist can measure $P_{\text{yes,yes}}$, $P_{\text{yes,no}}$, $P_{\text{no,yes}}$, and $P_{\text{no,no}}$, the probabilities that both junctions have switched, that the first has switched while the second did not, etc. Using for instance the formalism described in Ref. 20 one can show that these probabilities are given by

$$P_{\text{no,no}} = |c_{\uparrow\uparrow}|^2 e^{-\Gamma_1^\uparrow \tau_1} e^{-\Gamma_2^\uparrow \tau_2} + |c_{\downarrow\uparrow}|^2 e^{-\Gamma_1^\downarrow \tau_1} e^{-\Gamma_2^\uparrow \tau_2}$$
$$+ |c_{\uparrow\downarrow}|^2 e^{-\Gamma_1^\uparrow \tau_1} e^{-\Gamma_2^\downarrow \tau_2} + |c_{\downarrow\downarrow}|^2 e^{-\Gamma_1^\downarrow \tau_1} e^{-\Gamma_2^\downarrow \tau_2},$$

$$P_{\text{yes,no}} = -P_{\text{no,no}} + (|c_{\uparrow\uparrow}|^2 + |c_{\downarrow\uparrow}|^2)e^{-\Gamma_2^\uparrow \tau_2} + (|c_{\uparrow\downarrow}|^2$$
$$+ |c_{\downarrow\downarrow}|^2)e^{-\Gamma_2^\downarrow \tau_2},$$

$$P_{\text{no,yes}} = -P_{\text{no,no}} + (|c_{\uparrow\uparrow}|^2 + |c_{\uparrow\downarrow}|^2)e^{-\Gamma_1^\uparrow \tau_1} + (|c_{\downarrow\uparrow}|^2$$
$$+ |c_{\downarrow\downarrow}|^2)e^{-\Gamma_1^\downarrow \tau_1},$$

$$P_{\text{yes,yes}} = 1 - P_{\text{no,no}} - P_{\text{yes,no}} - P_{\text{yes,yes}}.$$

These equations and the constraint $|c_{\uparrow\uparrow}|^2 + |c_{\uparrow\downarrow}|^2 + |c_{\downarrow\uparrow}|^2 + |c_{\downarrow\downarrow}|^2 = 1$ are sufficient to determine the amplitudes of the 2-qubit state.

In conclusion, we have investigated a problem of coupled two-level systems under the drive of a harmonic field. We have shown that the dynamics of this system is such that an effective tunable coupling, controlled by the intensity of the field, can be achieved, and as a result coherent controllable transfer of population resulting in entanglement is possible.

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