## Flux qubit on a mesoscopic nonsuperconducting ring

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The possibility of making a flux qubit on a nonsuperconducting mesoscopic ballistic quasi-one-dimensional ring is discussed. We showed that such a ring can be effectively reduced to a two-state system with two external control parameters. The two states carry opposite persistent currents and are coupled by tunneling, which leads to a quantum superposition of states. The qubit states can be manipulated by resonant microwave pulses. The flux state of the sample can be measured by a superconducting quantum interference device magnetometer. Two or more qubits can be coupled by the flux the circulating currents generate. The problem of decoherence is also discussed.

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## I. INTRODUCTION

Recently a number of systems which can be effectively reduced to two-level systems  $^{1-4}$  have been examined as candidates for quantum computing hardware. These include, e.g., ion traps, nuclear spins in molecules, charge, and flux states of superconducting circuits. Maintaining the coherence of quantum devices is a major challenge. The device should be maximally decoupled from the environment to avoid decoherence and thus, the loss of quantum information. In addition to further work on existing systems, new candidates for qubits can be investigated.

We are considering the mesoscopic ring made of a metal, semiconductor, or toroidal carbon nanotube. Persistent currents (PCs) in small, nonsuperconducting rings threaded by a magnetic flux are a manifestation of quantum coherence in a submicrometer system. If the ring circumference *L* is smaller than the phase coherence length  $L_{\phi}$ , the electron wave function may extend coherently over *L*, even in the presence of elastic scatterers.<sup>5,6</sup> In other words, a normal loop with *L*  $< L_{\phi}$  has a nontrivial ground state with a circulating PC.

The goal of this paper is to show that quantum tunneling between states with nearly equal energies and opposite persistent currents in a clean mesoscopic, nonsuperconducting quasi-one-dimensional (1D) ring with a barrier can lead to formation of a qubit. We argue that at low T such a ring can be effectively reduced to a two-state system with two external control parameters. Quantum tunneling between the states leads to the quantum superposition of two opposed current states. The problem of the qubit decoherence is also discussed. There are many ways in which one can make a two state system out of persistent current ring. It has been extensively discussed in Büttiker and Stafford (Ref. 7).

# II. A MESOSCOPIC NORMAL RING AS A TWO-STATE SYSTEM: FORMATION OF A QUBIT

Let us consider the mesoscopic metallic or semiconducting quasi-1D ring of radius  $R (2\pi R < L_{\phi})$  in the presence of static magnetic flux  $\phi_e$ ,  $\phi_e = B_e \pi R^2$ , where  $B_e$  is the applied magnetic field perpendicular to the plane of the ring. Mesoscopic systems are expected to behave according to the laws of quantum mechanics if they are separated well from the external degrees of freedom. Thus, we assume that the system is well insulated from the environment. We assume that a ring is made from a very clean material, i.e., we are in the ballistic regime. The energy levels in a quasi-1D ring are

$$E_n^0 = \frac{\hbar^2}{2mR^2} (n - \phi')^2, \qquad (1)$$

where  $\phi' = \frac{\phi}{\phi_0}$ ,  $\phi_0 = \frac{h}{e}$ ,  $n = 0, \pm 1, \dots$  is the orbital quantum number (winding number) for an electron going around the ring. With each energy level we can associate a current

$$I_n = -\frac{\partial E_n^0}{\partial \phi} = \frac{e\hbar}{2\pi m R^2} (n - \phi'), \quad n = 0, \pm 1, \dots.$$
 (2)

The current is persistent at  $kT \ll \Delta$ . In calculating the flux  $\phi$  for a very thin ring, the self-inductance effect can be neglected and  $\phi = \phi_e$ . The system has a set of quantum size energy gaps, the gap at the Fermi surface (FS) at  $\phi' = 0$  is  $\Delta = \frac{hv_F}{L}$ , where  $v_F$  is the electron velocity at the FS. Assuming the radius R = 400 Å, we get  $\Delta \sim 290$  K for a metallic ring and  $\Delta \sim 47$  K for a semiconducting ring.<sup>8</sup>

In the following we assume that the temperatures are close to zero and the system behaves coherently (the energy gap hampers the inelastic transitions). The energy spectrum as a function of  $\phi'$  is shown in Fig. 1. Neglecting the spin, each level  $E_n$  is occupied by a single electron. We can see from Fig. 1 that at  $\phi' = 0$  (and all integral  $\phi'$ ), if the number of electrons in the ring  $N=N_{even}=2n_F$ , then the level at the FS is doubly degenerate and occupied by a single electron only. The same situation happens at half integral  $\phi'$ , for N  $=N_{odd}=2n_F+1$ . Because the energy spectrum is periodic with period  $\phi' = 1$ , we can restrict our considerations to the neighborhood of the two degeneracy points  $\phi' = 0$  (for  $N_{even}$ ) and  $\phi' = \frac{1}{2}$  (for  $N_{odd}$ ). The electron at the FS behaves there as a particle in a double well potential, where the states in each well correspond to PC of opposite sign. It follows from (2) that with increasing magnetic field the ground state will change from angular momentum  $n_F$  to one with higher n.

We now introduce to the ring an energy barrier of finite length a < L and height V (positive or negative). In this case the tunneling occurs which mixes the states from both sides of the barrier. At the degeneracy points the eigenstates which are superpositions of states with different winding numbers can be formed. This causes the splitting of the initial energy



FIG. 1. The energy spectrum of a 1D ring as a function of the flux  $\phi'$ .

levels (Fig. 2). Quantum tunneling should thus lead to a qubit, i.e., a quantum superposition of the two opposed current states. The second quantized Hamiltonian in the presence of the barrier is

$$H = \sum_{m \neq n} \left[ E_n |n\rangle \langle n| - \frac{1}{2} \hbar(\omega_{m,n} |n\rangle \langle m| + \omega_{m,n} |m\rangle \langle n|) \right], \quad (3)$$

where  $E_n = E_n^0 + V_n$ ,  $V_n$  is the renormalization due to the barrier of the  $E_n^0$  at a given  $\phi'$ , and  $\omega_{m,n}$  is the phase slip rate between states  $|m\rangle$  and  $|n\rangle$ .

At  $\phi' = 0$  and at  $kT \ll \Delta$  the energy states with  $|n| < n_F$  are fully occupied and form the "Fermi sea." The energy states for  $|n| > n_F$  are separated by large energy gaps from the FS and are fully empty. The same separation takes place at  $\phi'$  $=\frac{1}{2}$ . Thus, the only states which can take part in the tunneling are the states in the immediate neighborhood of the FS, and



FIG. 2. The energy levels  $E_{\pm}$  as a function of the flux  $\phi'$  for R=400 Å and  $N_{even}$  with  $n_F=20$  (solid lines). Superposition states manifest themselves as an anticrossing of the initial energy levels (dashed lines).

we can consider a mesoscopic ring as a two-state quantum system.

In this case the summation in (3) can be restricted to the two states closest to the FS. If we assume that  $N=N_{even}$  and  $\phi'$  is close to 0, these states are  $|n_F\rangle = |\beta\rangle = {1 \choose 0}$  and  $|-n_F\rangle$  $=|\alpha\rangle = {\binom{0}{1}}$  and the Hamiltonian (3) becomes

$$H = \begin{bmatrix} E_{\alpha} & -\frac{1}{2}\hbar\omega_{\alpha,\beta} \\ -\frac{1}{2}\hbar\omega_{\alpha,\beta} & E_{\beta} \end{bmatrix}, \qquad (4)$$

where  $E_{\alpha} = E_{-n_F}$ ,  $E_{\beta} = E_{n_F}$ ,  $\omega_{\alpha,\beta} = \omega_{-n_F}$ ,  $h_{n_F}$ . For most values of  $\phi'$  the  $\omega_{\alpha,\beta}$  is small compared with the energy of orbital motion of an electron in the ring. However, close to the degeneracy points the  $\hbar \omega_{\alpha,\beta}$  term mixes the two states strongly. At  $\phi'=0$  the states  $|\beta\rangle$  and  $|\alpha\rangle$  have exactly the same energies  $E_{\alpha} = E_{\beta}$ , but opposite currents  $I_{\alpha} = -I_{\beta}$ . In this case the phase slip probability increases (it happens at all integers  $\phi'$ ).

In the case of  $\phi' = \frac{1}{2}$  we find  $\alpha = -n_F$  and  $\beta = n_F + 1$ , i.e., the energies involved are  $E_{\alpha} = E_{-n_F}$ ,  $E_{\beta} = E_{n_F+1}$ , and  $\omega_{\alpha,\beta}$  $=\omega_{-n_{E},n_{E}+1}$ . These states are degenerate and the respective currents are opposite. Notice that the case with  $N=N_{even}$  and  $\phi'=0$  does not require any external field and thus may be easier to decouple from the environment.

The amplitude of these currents is  $I_0 = \frac{ev_F}{L}$ . Assuming, e.g., R = 400 Å,  $I_0 \sim 1 \mu A$  for a metallic ring and  $I_0 \sim 0.16 \mu A$  for a semiconducting ring.

In a pseudospin notation Eq. (4) can be written as

$$H = -\frac{1}{2}B_z\hat{\sigma}_z - \frac{1}{2}B_x\hat{\sigma}_x,\tag{5}$$

where  $\hat{\sigma}_z, \hat{\sigma}_x$  denotes Pauli spin matrices. The term  $B_z$  can be tuned by the applied flux

$$B_{z} = E_{\beta} - E_{\alpha} = \begin{cases} \Delta \left[ 1 - 2\frac{\phi}{\phi_{0}} \right] & \text{for } N = N_{odd}, \\ -\Delta \cdot 2\frac{\phi}{\phi_{0}} & \text{for } N = N_{even}. \end{cases}$$
(6)

The x component of the effective magnetic field  $B_x$  describes the tunneling amplitude  $\hbar \omega_{\alpha,\beta}$  between the two potential wells and can be tuned by, e.g., electrical gating. With these two external control parameters the elementary single-bit operations, i.e., z and x rotations,<sup>2</sup> can be performed. The qubit can be driven by microwave pulses. The advantage of the proposed qubit is the large distance  $\Delta$  between the qubit energy levels and the next higher states.

By diagonalizing the Hamiltonian (4) we obtain two energy bands,

$$E_{\pm} = \pm \frac{1}{2} \sqrt{(E_{\beta} - E_{\alpha})^2 + \hbar^2 \omega_{\alpha,\beta}^2}.$$
 (7)

At the degeneracy point  $E_{\beta} = E_{\alpha}$ , the energy splitting can be estimated by making use of a transfer matrix method.<sup>9</sup> One obtains



FIG. 3. Qubit energy splitting  $\hbar \omega_{\alpha,\beta}$  at  $\phi'=0$  as a function of the barrier height V for two values of the barrier width a.

$$\hbar\omega_{\alpha,\beta} = \frac{\Delta}{\pi} \arccos \sqrt{T_F} \cos \phi' \left(1 - \frac{\delta_F}{\pi N}\right), \tag{8}$$

where  $T_F$  and  $\delta_F$  are the transmission probability and the phase shift of an electron at the FS, respectively. The energy eigenstates are then the symmetric and antisymmetric superpositions of states with opposite PC.

We performed numerical calculations of the energy levels  $E_{\pm}$  for different sets of the barrier parameters and extracted the magnitude of  $\hbar \omega_{\alpha,\beta}$  from it. The results are shown in Fig. 2 for a ring with  $N_{even}$ ,  $n_F=20$  near  $\phi'=0$  for V=0.005 eV,  $E_F \sim 0.01$  eV, and a=15 Å. The initial energy levels are splitted and shifted by the presence of the barrier. The qubit energy splitting depends on the width and height of the barrier, which can be raised or lowered by electrical gating. It is shown in Fig. 3.

The potential barrier in a mesoscopic ring can be realized in a number of ways. One of them is the ring in which the small fragment of a convex arc has been deformed to form a concave one.<sup>10</sup> One can produce such a mechanical deformation by pressing. Another possibility to form the barrier is the local point-like electric gate close to the ring. The electric potential of the gate forms the barrier, the height of which can be modified by the gate voltage. The electric potential barrier can be combined with the geometric one described above. In this case the barrier height and therefore the mixing term  $\omega_{\alpha,\beta}$  can be adjusted to the required value. To make the barrier more local we can use an atomic force microscope or a scanning tunneling microscope. Additionally, one can regulate the barrier height by the change of the potential of the tip. The other possibility is to place the single-wall carbon nanotube in the plane of the ring perpendicular to it and apply the voltage to it.

Another realization of such a system can be obtained by using a toroidal carbon nanotube (CN). It is known that the defect-free metallic CN behaves like a real 1D wire.<sup>11</sup> This is due to the small number of channels (two) available close to the FS. The FS consists of two points where the two bands cross. The properties of an ideal toroidal CN depend on the position with respect to the honeycomb lattice of two circles: circumference of CN and circumference of the torus, defining chiral and twist vectors  $(p_1, p_2; q_1, q_2)$  respectively.<sup>12</sup> In our model we assume that only one electron occupies the FS which is doubly degenerate. In CNs which are metallic in both vectors,  $p_1 - p_2 = 3k$  and  $q_1 - q_2 = 3l$ , where k and l are integers, there are four accessible states at the FS. If, however, a CN is metallic only in the chiral direction  $p_1 - p_2$ =3k and not metallic in twist direction  $q_1 - q_2 \neq 3l$ , we obtain the required structure by applying a flux  $\phi' = \pm \frac{1}{3} + integer$ , in the direction parallel to the torus symmetry axis. The dispersion relation of  $CN^{13,14}$  is different from that of a quasi-1D ring. However, in the neighborhood of the Fermi points both relations become similar, i.e., linear in  $\phi'$  and producing opposite currents.<sup>15</sup> Because the states close to the FS are well separated from the excited states, the system can also be treated as a two-level system. As an example, we consider the metallic armchair CN (10,10;-5000,5000) having the radius  $R \sim 1960$  Å and the width d=8 Å.<sup>16</sup> The energy gap between the first unoccupied state and the state at the FS is  $\Delta \sim 32$  K. The current of an electron at the FS can be roughly approximated by

$$I_{n_{r}}I_{0}(1-\phi'), \quad 0 \le \phi' \le 1.$$
 (9)

The current amplitude  $I_0$  is inversely proportional to R but independent of the toroid width. One obtains  $I_0 \sim 0.54 \ \mu\text{A}$  or equivalently, the magnetic flux  $\phi \sim 0.2 \times 10^{-3} \ \phi_0$  for the assumed parameters.

The mixing term  $\omega_{\alpha,\beta}$  is again due to the potential barrier along the toroidal CN. It can be obtained by a nonsmooth junction of both its ends by, e.g., a fullerene molecule<sup>17</sup> or by simply leaving a small gap between the ends. Alternatively, one can replace a fragment of a toroidal nanotube with one of similar circumference but different conductivity properties [e.g., (11,0) and (6,6)].<sup>18</sup> Finally, to obtain a system described by (5), one can apply a real magnetic field perpendicular to the torus symmetry axis close to the degeneracy point ( $B_z=0$ ).<sup>19</sup>

In our model calculations we have made simplifying assumptions that the ring is quasi-1D and its energy states (2)are single-particle states. However, the presented considerations are also valid with some modifications for the mesoscopic metallic or semiconducting ring with very small thickness d,  $d \ll R$ , i.e., with a few transverse channels.<sup>7,20</sup> Nanoscopic, semiconducting, defect-free quasi-1D rings in which the electronic states are in the true quantum limit have been already realized.<sup>21</sup> The change in the ground state angular momentum numbered by n has been observed in a magnetic field perpendicular to the plane of the ring. They also found that the single-particle states are a quite accurate basis for a description of the many particle states. Similar findings have been also obtained for CN tori.<sup>11,15</sup> It is also supported by persistent current measurements<sup>7</sup> on ballistic semiconducting rings with a few transverse channels. It was found that the measured current is of the same order as the current amplitude of noninteracting electrons  $I_0$ . This means that electron-electron interaction in real quantum rings is not so strong. For weakly interacting electrons we can consider the effect of the interaction<sup>22</sup> by a barrier renormalization.

The electron is scattered not only by the barrier but also by the potential induced by charge density fluctuations.

The renormalized transmission probability  $T_F^R$  at  $T \ll \Delta$  is

$$T_F^R = \frac{T_F \left(\frac{\Delta}{E_F}\right)^{2\xi}}{(1 - T_F) + T_F \left(\frac{\Delta}{E_F}\right)^{2\xi}},$$
(10)

where  $\xi = \frac{w}{hv_F}$  characterizes the strength of the electronelectron interaction, and *w* is the forward scattering amplitude of the interaction. The interaction parameter  $\xi$  can be expressed in terms of the Luttinger liquid stiffness constant  $\alpha = \frac{v_F}{s}$  (*s* is the plasmon velocity),

$$\xi = \frac{1}{2} \left( \frac{1}{\alpha^2} - 1 \right). \tag{11}$$

For weakly interacting electrons  $(\alpha \rightarrow 1) \xi \sim \alpha^{-1} - 1$  and the effect of the interactions is small.

Thus, the idea of the formation of a flux qubit on a mesoscopic ring is still valid for the interacting electrons.

Until now we neglected the electron spin. The orbital magnetic moments<sup>15,20</sup> in small ring structures are an order of magnitude larger than spin moments and usually the orbital states are successively populated with spin-up and spin-down electrons. By neglecting the small spin splitting in the magnetic field, our picture with spin included does not change qualitatively but more possibilities occur. If the number of electrons is  $N=2(2n_F+1)$ , the tunneling can take place at  $\phi' = \frac{1}{2}$ . For other values of N tunneling can take place at  $\phi' = 0$  and/or  $\phi' = \frac{1}{2}$ .

The flux qubit proposed by us is based on the similar idea as the flux qubit built on a superconducting ring.<sup>1</sup> Also the measurement of the flux state can be performed by a separate superconducting quantum interference device (SQUID) magnetometer inductively coupled to it in a similar way as in a superconducting case. Two or more qubits can be coupled by means of the flux that the circulating currents generate.

### **III. DECOHERENCE**

The quality of an effective information retrieval device depends on the conditions of its coherence. The question is to what extent the device behaves quantum mechanically when placed in a noisy environment generated by various fluctuations or measurements. Thus, the important constraints on the device are dephasing effects due to various decoherence sources. Below we discuss and estimate some of the main decoherence sources.

There is a natural question as to how the typical values of phase coherence length  $L_{\phi}$ , which is of the order of  $10^5 \text{ Å},^{23,24}$  translates into an applicability of mesoscopic rings as the qubit with relatively long decoherence times.

We have assumed that  $kT \ll \hbar \omega_{\alpha,\beta} \ll \Delta$  and that  $L < L_{\phi}$ . Under these conditions, the currents running in a state of thermodynamic equilibrium are genuinely persistent.<sup>6,24,25</sup> The finite decoherence time of the current is due to the interaction with the outside world and leads to the persistence of the currents on a time scale much longer than the coherence time  $\tau_{\phi} \sim L_{\phi}$ .

We also assume that a system is put in a shield that screens it from the unwanted radiation. Thin mesoscopic quantum rings are the systems with a relatively small number of degrees of freedom compared with other solid-state devices based on superconducting rings. However, they are still able to accommodate various intrinsic fluctuations.<sup>23–27</sup>

Thermal motion of any charge carriers is a source of thermal fluctuations related to the electronic Nyquist noise. At low T the weak electron-phonon coupling gives some decrease of the current amplitude but it does not lead to substantial level shifting and broadening. The effect of thermal noise on the equilibrium statistics of persistent currents has been studied in a semiclassical regime in Ref. 28.

The qubit can also decohere by spontaneous emission of photons. It follows from general considerations that this effect is small for the qubit size smaller than the radiated wavelength (the qubit is then an inefficient antenna).<sup>1</sup> Taking for our qubit, e.g., R = 400 Å we estimated  $t_m \sim 10^9$  s and for  $R = 2000 \text{ Å} t_m \sim 10^8 \text{ s}$ ; thus, the radiation is not a serious source of decoherence. The coupling between the magnetic moments of the current loops and those of nuclear spins can also be a reason of decoherence. However, it may be considerably reduced by aligning the spins or by applying the compensating pulse sequences.<sup>2</sup> We also estimated the dephasing from the unwanted dipole-dipole coupling.<sup>29</sup> It is of the order of 14 ms if the qubits are at the distance of 10  $\mu$ m. Because our basic states are flux states, the qubit will be sensitive to a flux noise but relatively insensitive to a charge noise. Thus, we expect that magnetic degrees of freedom in quantum rings should have longer decoherence times than charge degrees of freedom. The fluctuations in the barrier and in the magnetic field necessary to get the degeneracy of the states for  $N=N_{odd}$  can be also the source of decoherence. However, the system for  $N=N_{even}$  is naturally bistable, requiring no external bias.

The most important class of fluctuations comes from the inductive coupling of the qubit to the measuring apparatus, which is often a dc SQUID. It can be analyzed along the treatment developed for a superconducting flux qubit,<sup>30</sup> which can be generalized to related systems. We assume that the measurement is performed with the same device as in Ref. 30. The flux qubit coupled to the SQUID is an effective dissipative two-state or spin-boson system. There are two time scales related to the effect of environment. The first is the characteristic relaxation time of populations to approach an equilibrium Gibbs-like form. The second is the decoherence time after which coherences become negligible.<sup>31</sup> The relaxation  $\tau_r^{-1}$  and dephasing  $\tau_{\phi}^{-1}$  rates obtained using the spin-boson model for the system at temperature *T* are<sup>31</sup>

$$\tau_r^{-1} = \frac{1}{2} \left(\frac{B_x}{\nu}\right)^2 J(\nu/\hbar) \coth\left(\frac{\nu}{2k_B T}\right)$$
(12)

and

$$\tau_{\phi}^{-1} = \frac{\tau_r^{-1}}{2} + \left(\frac{B_z}{\nu}\right)^2 \alpha 2 \pi \frac{k_B T}{\hbar}, \qquad (13)$$

where  $J(\omega)$  is a spectral density function characterizing fully the environment and  $\alpha = \lim_{\omega \to 0} J(\omega)/(2\pi\omega)$  is the "ohmicity" parameter. The impedance  $Z(\omega)$  of the dc SQUID is a source of thermal voltage fluctuations,

$$\langle \delta V \delta V \rangle = \nu \Re\{Z(\omega)\} \operatorname{coth}\left(\frac{\hbar \omega}{2k_B T}\right),$$
 (14)

of the statistics governed by the Nyquist theorem. These fluctuations are related via current-voltage characteristics to the current fluctuations  $\langle \delta I_{sq} \delta I_{sq} \rangle_{\omega}$  and then inherited by the inductively coupled qubit,<sup>30</sup>

$$\delta B_z = -2I \delta \Phi_{sq}, \tag{15}$$

where  $\delta \Phi_{sq}$  is the fluctuation of the magnetic flux due to the measuring device.

It leads in our case to the following spectral density:

$$J(\omega) = \hbar^{-1} (2\pi)^2 \frac{1}{\omega} \left(\frac{MI}{\Phi_0}\right)^2 I_{sq}^2 \Re\{Z(\omega)\}, \qquad (16)$$

where *M* is the mutual inductance coefficient, and  $\nu = E_+$ -*E\_* is the qubit level spacing. *I* and *I<sub>sq</sub>* are the currents in the qubit and SQUID, respectively. The effect of fluctuations on the tunneling term  $\sigma_x$  can be included in a similar way. However, the effect of such fluctuations is relatively weak.

In these formulas a dimensionless factor

$$\gamma = \left(\frac{MI}{\Phi_0}\right) \tag{17}$$

is a measure of the coupling of the qubit and the measuring SQUID. Its magnitude  $\gamma \sim 2 \times 10^{-3}$  has been estimated for the superconducting qubit.<sup>30</sup> In our case, as the size of the nonsuperconducting ring is smaller, the coupling constant  $\gamma$  is also smaller. For the ring with  $R \sim 0.2 \ \mu m$  (toroidal nanotube) we estimated  $\gamma \sim 0.9 \times 10^{-3}$ . Assuming the same parameters for the measuring device as in Ref. 30 we obtain  $\tau_r \sim 75 \ \mu s$  and  $\tau_{\phi} \sim 10^2 \ \mu s$ .

In the nonsuperconducting flux qubit the number of degrees of freedom is relatively small in comparison to the superconducting devices. On one hand it limits significantly the number of decoherence mechanisms, but on the other hand as the system is normal and one cannot expect the suppression of some fluctuations as in the superconducting case. Therefore, because the state of the mesoscopic ring is not completely stable, some random fluctuations of the current are expected; actually, some modifications of the Aharonov-Bohm oscillations on a time scale of 10–40 h have been observed.<sup>7</sup> Finally, the small size of the qubit helps to reduce the influence of the environment,<sup>1</sup> which is significant for effective engineering. We can estimate the influence of various decoherence sources, but it is impossible to determine the real decoherence time with certainty, except by measurement.

The qubit level spacing sets<sup>2</sup> the fastest operation time to  $\tau_{op} \sim 10$  ns. Thus, the quality factor is of the order of  $10^4$ . Our model could in principle be tested on quantum rings investigated in Refs. 20 and 21 after application of the effective field  $B_x$ , i.e., when one introduces a controllable potential barrier.

The extended discussion of the effective engineering of the decoherence for the nonsuperconducting flux qubit is postponed to a further publication.

#### **IV. SUMMARY**

The advantage of microscopic quantum systems (atoms, spins) for qubit formation is that they can be easily isolated from the environment. The disadvantage is that the integration of many qubits into a complex circuit is a difficult task. From that point of view solid-state devices like charge and flux qubits built on superconducting rings are easier to integrate in a quantum computer using standard circuit technology. However, the large number of degrees of freedom makes it more difficult to maintain the coherence. The proposed flux qubit built on a normal quantum ring is on the border line between these two structures. The small number of degrees of freedom together with the small size of the qubit helps to decouple it from the environment. The proposed qubit can be addressed, manipulated, coupled to each other, and read out. The quality factor giving the number of quantum logic operations is of the order of 10<sup>4</sup>. The proposed qubit should be of considerable interest for fundamental studies of quantum coherence in mesoscopic systems and some aspects of quantum theory such as superposition of quantum states and entanglement.

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