

One-dimensional two-barrier quantum pump with harmonically oscillating barriers: Perturbative, strong-signal, and nonadiabatic regimes

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We study the stationary current in a one-dimensional quantum pump composed of two harmonically oscillating δ -functional barriers (wells). The harmonic signals are applied to any or both barriers. The low-signal and nonlinear regimes are considered. The former case is studied by the theory of perturbation, while the nonlinear regime is studied numerically. The numerical, perturbative, and adiabatic results are compared with each other. It is found that the current has peculiarities, caused by stationary and quasistationary electron states as well as threshold singularities. The direction and value of the current depend on the frequency, the Fermi momentum, the values of the stationary and alternating voltages, and the phase shift between them. In the case of a strong applied signal the stationary current exhibits multiphoton oscillations.

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INTRODUCTION

The quantum pump is a device that generates stationary current under the action of alternating voltage; it is a subject of numerous recent publications (for example, Refs. 1–22). The quantum pump is essentially analogous to various versions of the photovoltaic effect studied in details from the beginning of the 1980 (Refs. 23–27). The difference is that the photovoltaic effect is related to the emergence of a direct current in a homogeneous macroscopic medium (the only exception is the mesoscopic photovoltaic effect), while the pump is a microscopic object. From the phenomenological point of view, the emergence of a direct current in the pump is not surprising, since any asymmetric microcontact can rectify ac voltage. However, analysis of adiabatic transport in the quantum-mechanical object leads to new phenomena, such as quantization of charge transport.¹³

Just this, analytically solvable, adiabatic approach was utilized in most of the studies of quantum pumps.^{14–19}

In the present paper we consider a model of the quantum pump resembling a quantum wire with two narrow gates (see Fig. 1) to which alternating voltages are applied. The stationary bias between the source and drain is supposedly absent. Under these conditions the synchronized alternating voltage can induce a stationary current between the source and drain. We use a simplified model of this pump—namely, two δ -like harmonically oscillating barriers or wells. The system has a variety of regimes of the pump operation, depending on the system parameters: frequency, amplitudes, and the phase shift between the signals.

This model was studied in the adiabatic approach^{16–19} or by numerical solution of the equation for the Floquet scattering matrix.^{18–22} In Ref. 22 we considered the model beyond the adiabatic approach. We found the symmetry relations for the stationary current and analytical perturbative expressions and analyzed some multiphoton resonances both numerically and analytically.

Unlike Ref. 22, here we study both low-frequency (but not adiabatic) and nonlinear regimes of the electronic pump, including the phase shift, amplitude, and frequency dependence of the current.

BASIC EQUATIONS

The quantum pump considered is described by a potential of two oscillating δ -like barriers or wells:

$$U(x) = [u_1 + v_1(t)]\delta(x+d) + [u_2 + v_2(t)]\delta(x-d), \quad (1)$$

where $v_1(t) = v_1 \sin(\omega t)$, $v_2(t) = v_2 \sin(\omega t + \varphi)$, t is the time, and $2d$ is the distance between δ barriers (wells); the quantities u and v are measured in units of \hbar/md (m is the electron mass); p , E , and ω are the momentum, energy, and frequency measured in units of \hbar/d , $\hbar^2/2md^2$, and $\hbar/2md^2$, respectively. In the absence of an ac signal, the system has two barriers for positive values of u_1 and u_2 and two wells for negative values of these parameters. Asymmetry of the system is necessary for the appearance of stationary current. The specific direction of the asymmetry (and, hence, the current)

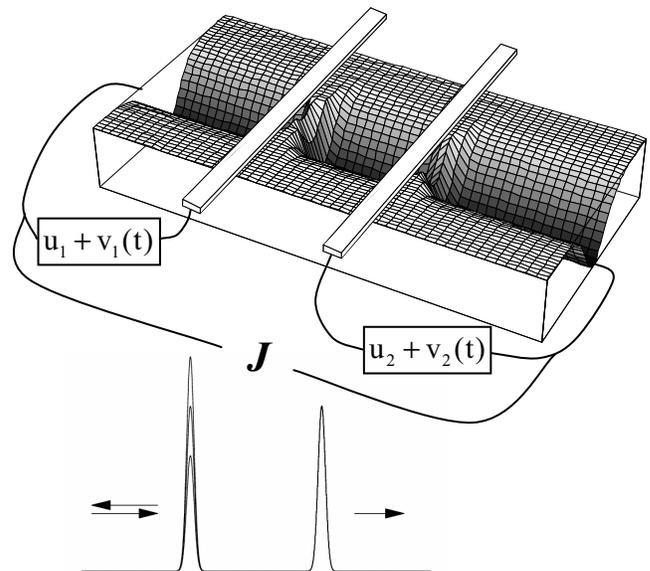


FIG. 1. Above: potential distribution in a one-dimensional quantum pump. The quantum wire is composed by the potential well. Narrow transversal gates produce oscillating barriers or wells across the wire. Below: one-dimensional model of the pump.

is conditioned by any of the factors: the difference of static voltages u_1 and u_2 , alternating voltages v_1 and v_2 , or the phase shift between alternating voltages.

We assume that the electron gas is in equilibrium and its distribution functions are identical in the regions $x < -d$ and $x > d$. The problem is to determine the direct current induced by the ac field.

The solution to the Schrödinger equation with the potential (1) can be written in the form

$$\psi = \sum_n \exp[-i(E+n\omega)t] \times \begin{cases} \delta_{n,0} \exp\left(\frac{ip_n x}{d}\right) + r_n \exp\left(-\frac{ip_n x}{d}\right), & x < -d, \\ a_n \exp\left(\frac{ip_n x}{d}\right) + b_n \exp\left(-\frac{ip_n x}{d}\right), & -d < x < d, \\ t_n \exp\left(\frac{ip_n x}{d}\right), & x > d. \end{cases} \quad (2)$$

Here, $p_n = \sqrt{p^2 + n\omega}$ and $p = \sqrt{E}$. The wave function (2) corresponds to the wave incident on the barrier from the left. (In the final formulas, we mark the directions of incident waves by the indices “ \rightarrow ” and “ \leftarrow ”). The form of solution (2) corresponds to absorption (for $n > 0$) or emission ($n < 0$) of n field quanta by an electron after interaction with the vibrating barriers; $n=0$ relates to the elastic process. The quantities t_n and r_n give the corresponding amplitudes of transmission (reflection). If the value of p_n becomes imaginary, the waves moving away from the barriers should be treated as damped waves, so that $\text{Im } p_n > 0$.

The transmission amplitudes obey the equations $t_n = e^{-i(p+p_n)} T_n$,

$$\begin{aligned} v_1 v_2 g_{n-1} e^{-i\varphi} T_{n-2}^{\rightarrow} - i[v_1 S_{n-1} + v_2 V_n e^{-i\varphi}] T_{n-1}^{\rightarrow} \\ - [2W_n + v_1 v_2 (g_{n-1} e^{i\varphi} + g_{n+1} e^{-i\varphi})] T_n^{\rightarrow} \\ + i[v_1 S_{n+1} + v_2 V_n e^{i\varphi}] T_{n+1}^{\rightarrow} + v_1 v_2 g_{n+1} e^{i\varphi} T_{n+2}^{\rightarrow} = 2ip \delta_{n,0}, \end{aligned} \quad (3)$$

and

$$\begin{aligned} v_1 v_2 g_{n-1} e^{-i\varphi} T_{n-2}^{\leftarrow} - i[v_1 S_n + v_2 V_{n-1} e^{-i\varphi}] T_{n-1}^{\leftarrow} \\ - [2W_n + v_1 v_2 (g_{n-1} e^{-i\varphi} + g_{n+1} e^{i\varphi})] T_n^{\leftarrow} \\ + i[v_1 S_n + v_2 V_{n+1} e^{i\varphi}] T_{n+1}^{\leftarrow} + v_1 v_2 g_{n+1} e^{i\varphi} T_{n+2}^{\leftarrow} = 2ip \delta_{n,0}. \end{aligned} \quad (4)$$

Here, $g_n = \sin 2p_n / p_n$,

$$S_n = 2u_2 g_n + e^{-2ip_n}, \quad V_n = 2u_1 g_n + e^{-2ip_n}, \quad (5)$$

$$W_n = 2u_1 u_2 g_n + (u_1 + u_2 - ip_n) e^{-2ip_n}. \quad (6)$$

Provided that electrons from the right and left of the pump are in equilibrium and that they have identical chemical potentials μ , the stationary current is

$$J = \frac{e}{\pi\hbar} \int dE \sum_n (|T_n^{\rightarrow}|^2 - |T_n^{\leftarrow}|^2) f(E) \theta(E + n\omega), \quad (7)$$

where $f(E)$ is the Fermi distribution function and $\theta(x)$ is the Heaviside step function. The current is determined by the transmission coefficients with real p_n only.

At a low temperature, it is convenient to differentiate the current with respect to the chemical potential:

$$\mathcal{G} = e \frac{\partial}{\partial \mu} J = G_0 \sum_n \theta(\mu + n\omega) (|T_n^{\rightarrow}|^2 - |T_n^{\leftarrow}|^2)_{p=p_F}. \quad (8)$$

Here $G_0 = e^2 / \pi\hbar$ is the conductance quantum and p_F is the Fermi momentum. The resultant quantity \mathcal{G} can be treated as a two-terminal photoconductance (the conductance for simultaneous change of chemical potentials of source and drain).

ASYMPTOTIC CASES

Let us consider the limit $v_1, v_2 \ll u_1, u_2$. The steady-state problem gives the transmission amplitude

$$T_0 = -\frac{ip}{W_0} = -\frac{ip^2}{2u_1 u_2 \sin 2p + (u_1 + u_2 - ip) p e^{-2ip}}, \quad T_{n \neq 0} = 0. \quad (9)$$

The scattering amplitude vanishes for $p \rightarrow 0$ and experiences oscillations with a period $\delta p = \pi/2$. For the large values of $u_{1,2}$, the quantity T_0 has poles in the vicinity of points $p = \pi n/2$.

In the zeroth order of perturbation theory, the direct and reverse transmission coefficients coincide; consequently, the current vanishes. The current appears only in the second order of perturbation theory. Second-order corrections to the current come only from the quantities T_0 , T_1 , and T_{-1} . Expanding in the ac signal, we obtain

$$\begin{aligned} \mathcal{G} = G_0 \frac{p^2}{4|W_0|^2} \left\{ v_1^2 \left(\frac{|S_0|^2 - |S_{-1}|^2}{|W_{-1}|^2} \theta(\mu - \omega) + \frac{|S_0|^2 - |S_1|^2}{|W_1|^2} \right) \right. \\ - v_2^2 \left(\frac{|V_0|^2 - |V_{-1}|^2}{|W_{-1}|^2} \theta(\mu - \omega) + \frac{|V_0|^2 - |V_1|^2}{|W_1|^2} \right) \\ + 2v_1 v_2 \text{Re} \left[\frac{S_0 V_{-1}^* - S_{-1} V_0^*}{|W_{-1}|^2} e^{-i\varphi} \theta(\mu - \omega) \right. \\ + \left. \frac{S_0 V_1^* - S_1 V_0^*}{|W_1|^2} e^{i\varphi} \right] + 4v_1 v_2 \sin \varphi \text{Im} \left[\frac{S_0 V_0 - S_{-1} V_{-1}}{W_0 W_{-1}} \right. \\ \left. - \frac{S_0 V_0 - S_1 V_1}{W_0 W_1} + 2 \frac{g_{-1} - g_1}{W_0} \right] \Bigg\}_{p=p_F}. \end{aligned} \quad (10)$$

In the particular case $u_1 = u_2$, the functions S_n and V_n coincide and expression (10) obtains the form

$$\mathcal{G} = G_0 \frac{p^2}{4|W_0|^2} \left\{ (v_1^2 - v_2^2) \left(\frac{|S_0|^2 - |S_{-1}|^2}{|W_{-1}|^2} \theta(\mu - \omega) + \frac{|S_0|^2 - |S_1|^2}{|W_1|^2} \right) + 4v_1 v_2 \sin \varphi \operatorname{Im} \left[\frac{S_0 S_{-1}^*}{|W_{-1}|^2} \theta(\mu - \omega) - \frac{S_0 S_1^*}{|W_1|^2} + \frac{S_0^2 - 1}{W_0} \left(\frac{1}{W_{-1}} - \frac{1}{W_1} \right) \right] \right\}_{p=p_F}. \quad (11)$$

The current is determined by the corrections $T_{\pm 1}$ associated with real emission (absorption) of a single photon. In addition, the correction to T_0 associated with the effect of a virtual single-photon process on the nonradiative channel also exists. Apart from the squares of ac signals v_1 and v_2 , the result for the regime $u_1 = u_2$ contains a bilinear combination; consequently, it is insufficient to consider the response only at one of the signals. The latter contribution is sensitive to the relative phase of the signals.

In the case of the large u_1 and u_2 compared with the Fermi momentum, expression (10) yields

$$\mathcal{G} = G_0 \frac{p^2 v_1^2 \sin \varphi}{8u_1^7 g_0} \left\{ \frac{(3p_{-1} - p)\theta(\mu - \omega)}{g_{-1}} - \frac{3p_1 - p}{g_1} \right\}_{p=p_F}. \quad (12)$$

If $u_1 = u_2 = 0$,

$$\mathcal{G} = G_0 v_1 v_2 \sin \varphi \left\{ \frac{\sin 2(p - p_{-1})}{p_{-1}^2} \theta(\mu - \omega) + \frac{\sin 2(p_1 - p)}{p_1^2} + \frac{2 \sin 2p}{p} \left(\frac{\cos 2p_{-1}}{p_{-1}} \theta(\mu - \omega) - \frac{\cos 2p_1}{p_1} \right) \right\}_{p=p_F}. \quad (13)$$

Expression (13) tends to infinity at the single-photon emission threshold. This singularity can be explained by the resonance with the state of an electron with zero energy: such an ‘‘immobile’’ state can be interpreted as a bound state.

In addition to the above-mentioned oscillations with period $\delta p = \pi/2$, the transmission amplitude experiences oscillations with periods $\delta p_{\pm 1} = \pi/2$. It can be seen from expression (10) that the extrema in the dependence of the current on p are located in the vicinity of the points corresponding to the minima of functions W_0 and $W_{\pm 1}$ and are connected with the elastic process as well as with the process involving the absorption or emission of a field quantum. For $v_2 = 0$ ($v_1 = 0$), the expression for the current contains only one term proportional to v_1^2 (v_2^2).

For $u_1, u_2 \gg p$ the oscillations are transformed into sharp peaks corresponding to the transmission resonances. For $p \sim 1$, the transmission amplitude has a characteristic scale of $p \sim u_1, u_2$. The corresponding structure for small values of u_1 and u_2 can be treated as a resonance at zero energy. For negative values of u_1 and u_2 , the resonance at bound states exists (at one or two such states depending on the distance between the wells).

Generally speaking, the solution of Eqs. (3) and (4) by a perturbative series in the n th order contains the amplitudes up to $\pm n$ th order. For a weak signal they exponentially decay with n . The growth of $v_{1,2}$ results in the growth of number of harmonics accompanied by appearance in the stationary cur-

rent of corresponding multiphoton resonances. For large $v_{1,2}$ the interference of these resonances determines complicated dependences of the current on all parameters.

ADIABATIC LIMIT

In the stationary limit Eq. (11) tends to zero, in accordance with the physical meaning. If the frequency is small, but finite, the current should be expanded in powers of ω . The lowest nonvanishing term is the first. Proportionality of the current to ω results also from the adiabatic perturbation theory which was subject of many papers.^{13–19} In the case of slow alternating voltage the current flowing in lead α can be expressed via stationary scattering matrix \hat{S}_0 (Ref. 18):

$$I_{ad,\alpha} = i \frac{e\omega}{4\pi^2} \int_0^{2\pi/\omega} dt \int_0^\infty dE \left(-\frac{\partial f_0(E)}{\partial E} \right) \times \left(\frac{\partial \hat{S}_0(E,t)}{\partial t} \hat{S}_0^\dagger(E,t) \right)_{\alpha,\alpha}, \quad (14)$$

where $f_0(E)$ is the Fermi distribution function. The scattering matrix in our case has the form

$$\hat{S}_0 = -\frac{1}{W_0} \begin{pmatrix} W_0 + iP S_0 & ip \\ ip & W_0 + iP V_0 \end{pmatrix}. \quad (15)$$

Collecting Eqs. (14) and (15) we find, at zero temperature,

$$\mathcal{G}_{ad} = G_0 \frac{i\omega}{8\pi p} \times \frac{\partial}{\partial p} \left(p^2 \int_0^{2\pi/\omega} dt \left[\frac{S_0^*}{W_0^*} \frac{\partial S_0}{\partial t} - \frac{V_0^*}{W_0^*} \frac{\partial V_0}{\partial t} \right] \right) \Big|_{p=p_F}. \quad (16)$$

Here u_1 and u_2 are treated as functions of instantaneous barrier amplitudes $u_1 \rightarrow u_1 + v_1(t)$ and $u_2 \rightarrow u_2 + v_2(t)$. Expanding Eq. (10) by the frequency we find, in the case of a slow and weak signal (where applicability regions of the ordinary perturbation theory and the adiabatic theory overlap),

$$\mathcal{G} = G_0 \frac{\omega p v_1 v_2 \sin \varphi}{|W_0|^2} \operatorname{Im} \left[\frac{(\partial S_0 / \partial p) V_0^* - S_0 (\partial V_0^* / \partial p)}{2|W_0|^2} + \frac{\partial(S_0 V_0) / \partial p}{W_0^2} - \frac{2 \partial g_0 / \partial p}{W_0} \right]_{p=p_F}. \quad (17)$$

Equation (17) follows also from the general adiabatic expression (16) if the signal is weak. If additionally $p_F \rightarrow 0$, Eq. (17) goes to

$$\mathcal{G} \approx -G_0 \omega p_F v_1 v_2 \sin \varphi \frac{1 + 2u_1 + 2u_2}{(u_1 + u_2 + 4u_1 u_2)^4}.$$

NUMERICAL RESULTS

The solution of Eqs. (3) and (4) was found numerically and substituted into Eqs. (7) and (8). Figure 2 shows the

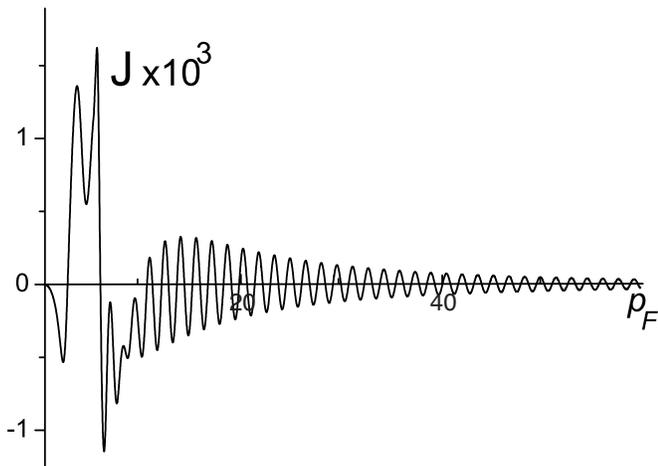


FIG. 2. The dependence of the stationary current J (in units of $e\hbar/2\pi md^2$) on the Fermi momentum in a symmetric structure with $u_1=u_2=-1$, $v_1=v_2=0.1$, $\omega=25$ and $\varphi=\pi/2$.

dependence of the stationary current J on the Fermi momentum in a symmetric structure with two δ wells ($|u_1|=|u_2|\gg v_1=v_2$, $\varphi=\pi/2$). The current exhibits $\pi/2$ oscillations with Fermi momentum. These oscillations are related to the resonance at quasistationary states between the wells. The threshold singularity at $p_F=5$ is associated with the zero-energy one-photon resonance.

Figure 3 demonstrates the dependence of the quantity \mathcal{G} on the Fermi momentum in the symmetric structure with two identical δ wells and δ barriers. These cases differ by the sign of \mathcal{G} and by the small relative shift of the position of the resonance singularities. Really, within the limits of large $u_1=u_2$ at $\omega\rightarrow 0$ the quantity $\mathcal{G}\propto u_1^{-7}$, Eq. (12), i.e., it is an odd function of the amplitude u_1 and, accordingly, changes sign together with u_1 . The shift of the position of the resonance singularities is connected with the difference of quasistationary energy levels in these cases.

Figure 4 depicts \mathcal{G} as a function of Fermi momentum for

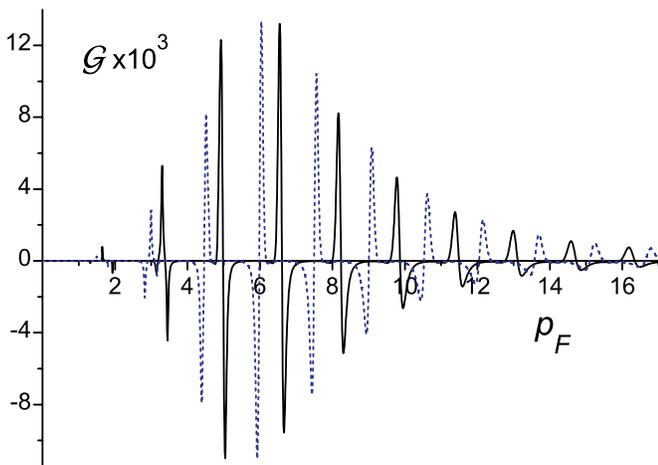


FIG. 3. (Color online) The dependence of \mathcal{G} on the Fermi momentum in a symmetric structure $u_1=u_2=\pm 1$, $v_1=v_2=0.1$, $\omega=1$, and $\varphi=\pi/2$. The solid and dashed curves correspond to $u_1=1$ and $u_1=-1$, accordingly.

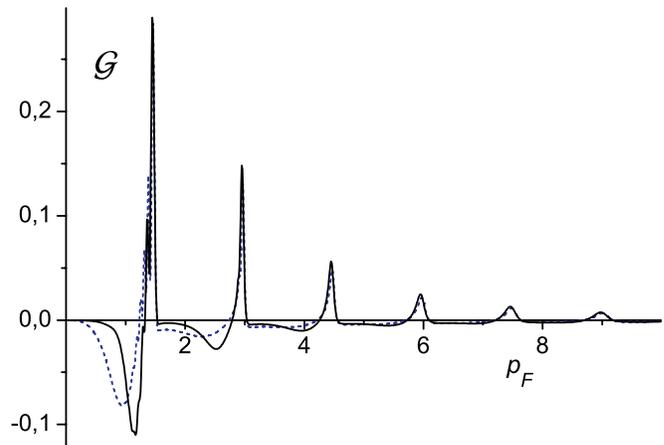


FIG. 4. (Color online) The dependence of \mathcal{G} on the Fermi momentum $u_1=u_2=v_1=v_2=5$, and $\omega=0.1$ for $\varphi=\pi/2$ (solid curve) and $\varphi=\pi/3$ (dashed curve).

two values of the phase φ in the symmetric device. It demonstrates that \mathcal{G} is phase sensitive for small p_F up to $p_F\sim 5$. The change of phase modifies the curve, in particular visibly shifts the first dip. For large $p_F>5$ the curves correspond to the perturbative expression (11).

We have compared the obtained results with the paper Ref. 17, which considered a similar problem in the adiabatic approximation. Figure 5 shows the frequency evolution of the current for weak alternating signals calculated according to Eq. (10). The dependence for small frequency ($\omega=0.1$) corresponds to Fig. 1 from Ref. 17. This demonstrates that the result of Ref. 17 is actually obtained for a weak alternating signal in the framework of ordinary perturbation theory. The perturbational expression (10) gives a harmonic phase dependence (sine like in the case of a symmetric system) of the current. At the same time Fig. 5(a) from Ref. 17 differs from a $\sin\varphi$ dependence while the parameters (small alternating signal on the base of large constant barrier) correspond to the applicability of the perturbation theory. In fact, in this case the perturbation parameter is $v_{1,2}/u_{1,2}\ll 1$ rather than $v_{1,2}/p_F\ll 1$. We have done calculations for the same

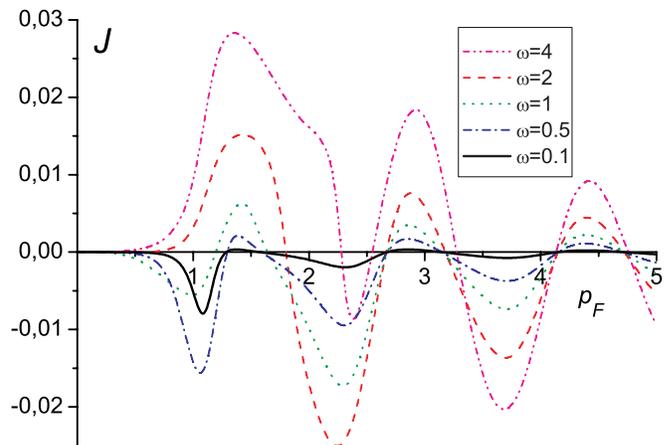


FIG. 5. (Color online) The dependence of the stationary current J on the Fermi momentum $u_1=u_2=1$, $v_1=v_2=0.4$, and $\varphi=\pi/2$ for different frequencies (specified in the box).

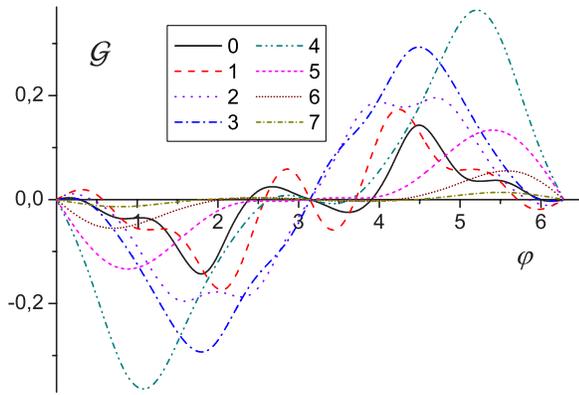


FIG. 6. (Color online) The dependence of \mathcal{G} on the phase shift φ in a symmetric structure $\omega=1$, $p_F=2$, $v_1=v_2=5$, and $u_1=u_2=0, 1, 2, 3, 4, 5, 6, 7$.

parameters as in Ref. 17 by both exact and approximate formulas and found that they give a sinelike result differing from Fig. 5(a) of Ref. 17.

Figures 6 and 7 show the evolution of the function $\mathcal{G}(\varphi)$ in the symmetric device with two barriers (Fig. 6) or two wells (Fig. 7) with the value of the alternating signal at a fixed $p_F=2$. The case of large $u_{1,2} \gg v_{1,2}$ corresponds to the perturbative expression (11). This explains the approximative sinusoidal dependence of \mathcal{G} on the phase for $|u_{1,2}| > 5$. For relatively small Fermi momenta $p_F < v_{1,2}$, and $u_{1,2} \leq v_{1,2}$, the harmonic (sinelike) dependence of $\mathcal{G}(\varphi)$ is superimposed on the short-period ($\pi/2$) oscillations conditioned by the resonance in fourth order of perturbation theory.

Figure 8 demonstrates the dependence of \mathcal{G} on the frequency of the alternating signal in the low-frequency limit. The linear dependence of \mathcal{G} in this limit agrees with Eq. (12). The threshold singularity at $\omega=0.5$ is related to zero-energy one-photon resonance.

Figure 9 depicts \mathcal{G} for strong low-frequency alternating

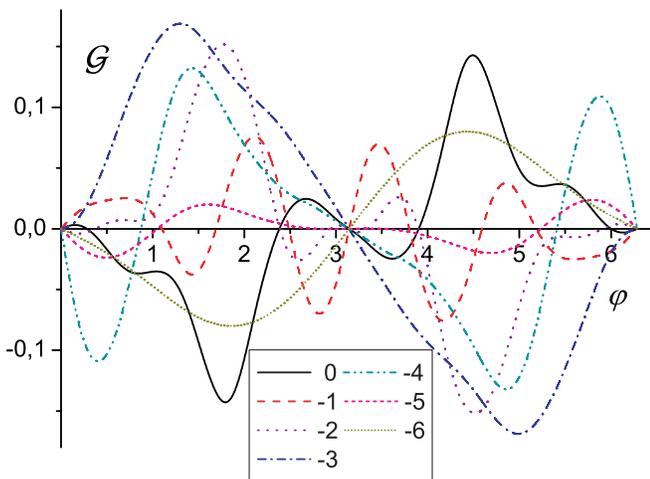


FIG. 7. (Color online) The dependence of \mathcal{G} on the phase shift φ in a symmetric structure $\omega=1$, $p_F=2$, $v_1=v_2=5$, and $u_1=u_2=0, -1, -2, -3, -4, -5, -6$.

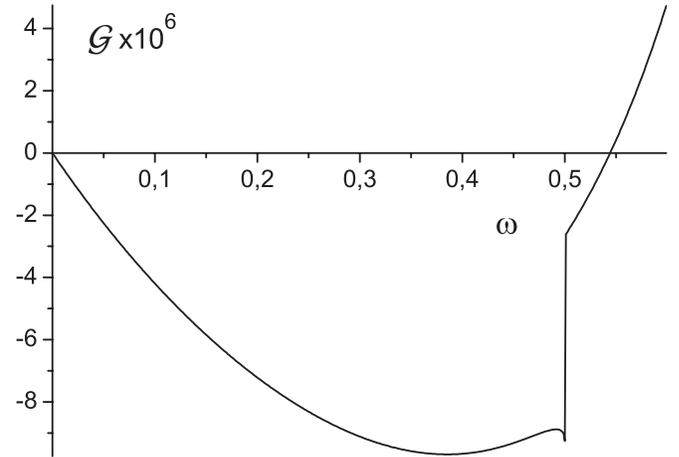


FIG. 8. The dependence of \mathcal{G} on the frequency $u_1=1$, $u_2=3$, $v_1=v_2=0.1$, $\varphi=\pi/2$, and $p_F=0.71$.

voltages. The resonance at $p_F=\pi/2$, which is present in the low-signal regime (see curve *a*), obtains photon repetitions. They overlap, composing damped (with the number of photons) oscillations. The oscillations rarefy with the increase of the frequency.

Figure 10 demonstrates a comparison of the results of exact calculations, according to Eqs. (3), (4), and (8) and an adiabatic approximation according to Eq. (16). The antisymmetric peak, appearing for small $v_{1,2}$ in the perturbative approach [see curve (a) in Fig. 9], is broadened to the wide dip and maximum if the signal grows. The adiabatic result is in good agreement with the smoothed exact dependence. The multiphoton oscillations are superimposed on this smooth dependence. The results of the adiabatic and exact approaches differ for small p_F , where the finiteness of the frequency is more essential. The maximal value of the exact \mathcal{G} drops as compared with the adiabatic limit due to the redistribution of transmitted electrons on energies and the consequent drop of the amplitude of the elastic channel.

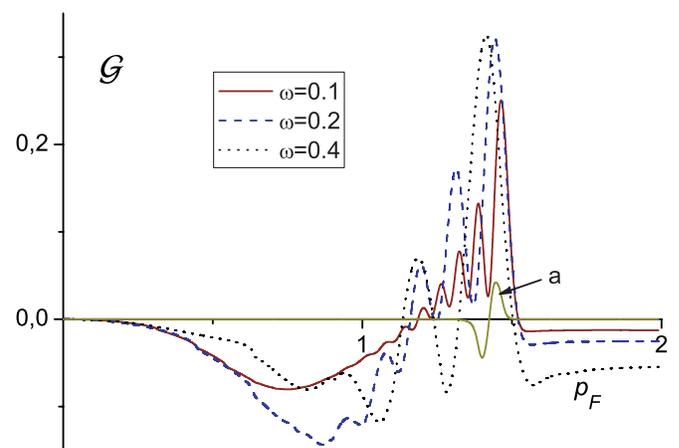


FIG. 9. (Color online) The dependence of \mathcal{G} on the Fermi momentum for different small frequencies (shown in the figure); $u_1=u_2=v_1=v_2=5$ and $\varphi=\pi/4$. Curve *a* represents the low-signal result for $u_1=u_2=5$, $v_1=v_2=1$, and $\varphi=\pi/4$, $\omega=0.1$.

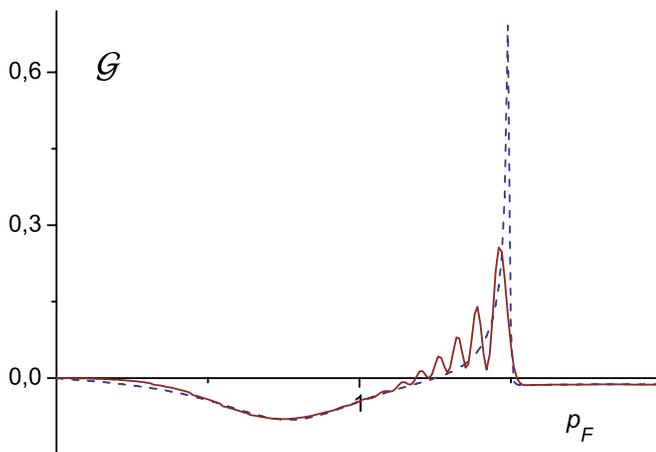


FIG. 10. (Color online) Comparison of the exact (solid curve) and adiabatic (dashed) results for \mathcal{G} . The parameters $\omega=0.1$, $\varphi=\pi/4$, and $u_1=u_2=v_1=v_2=5$ correspond to the solid curve in Fig. 9.

CONCLUSIONS

We have considered the one-dimensional quantum pump with two harmonically oscillating δ -functional barriers or wells. This system can be realized, e.g., as a quantum wire with two narrow gates across it. The system is characterized

by a variety of parameters: the static barriers heights, the amplitudes of the alternating signals and the phase shift between them, the Fermi momentum, and the signal frequency. Depending on the relation between parameters, the pump works in different modes. The system can operate in symmetric or nonsymmetric regimes. We have found an analytical perturbative expression for the current in the regime of weak signals and numerically studied the case of strong external alternating voltage. If the static barriers are high, then the conductance contains sharp resonances of different signs conversing to interference oscillations for large electron momentum. This dependence also exhibits the threshold singularities caused by the commensurability of the Fermi energy with the frequency.

The case of low frequency is studied in comparison with the extreme adiabatic case. It is found that the dependence of the current on the Fermi momentum experiences multiphoton oscillations for large alternating voltage.

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