Three electrons confined in multilayer quantum dots studied by the exact diagonalization method

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(Received 15 February 2006; revised manuscript received 15 June 2006; published 8 September 2006)

In this paper, we studied three electrons confined in one-, two-, and three-layer quantum dots, by the exact diagonalization method. A vertical magnetic field to the confinement plane is considered. The ground-state electronic structures and angular momentum transitions are investigated. These are connected to the exchange and rotational symmetries of the systems.

DOI: 10.1103/PhysRevB.74.115305 PACS number(s): 68.65.−k

I. INTRODUCTION

Small solid state devices known as quantum dots (QDs) can confine a few electrons in all spatial dimensions. In these systems, the electrons move in one or several planes with a lateral confinement potential. QDs are currently under intense study because they exhibit rich and elegant physics and have potential applications such as lasers and memory.[1,2] The problem of N electrons in QDs and magnetic fields has been widely considered in recent years.[3] The discrete energy spectra of few electrons depend on the number N of electrons and the details of the confinement potential, both of which can be controlled in experiment. While most of the theoretical calculations have been done for parabolic confinements, the interesting physics of a square QD have also studied by Creffield *et al.*[4] In general, the competition between quantum confinements and the pairwise Coulomb interactions determines most of the physical properties of QDs.

The theoretical study of few-electron QDs was the subject of many papers.[5–21] In 1983, Laughlin[5,6] first studied the states of a three-electron system in two dimensions in a strong magnetic field and confined by a parabolic potential. Laughlin explicitly constructed the spin-polarized correlated states and showed that they approximated the exact eigenstates well. Series of magic numbers of angular momentum $L=3k$ $(k=1,2,3,\ldots)$ which minimize the interaction energy were found. Then, Girvin and Jach[7] extended the analysis to systems containing more electrons. The magic numbers were seen to exist, but the rules explaining them seemed to increase in complexity as the number of particles increased. In 1995, Ruan and co-workers[10] theorized the origin of the magic numbers of three-electron QDs via an symmetry analysis. Next, Bao[14] explained the phase diagram and in the filling factors from an analysis of the inherent nodal structures of the internal wave functions of few-electron QDs.

The electron-electron interaction in a single QD has profound influence on the groundstate, which can occur, in a strong magnetic field, only at certain magic values of the total angular momentum L and total spin S.[9] This fact definitely implies a phase transition, i.e., a transition of structures. Thereby, when the magnetic field continuously increases, discontinuous changes of a number of physical properties (such as optical absorption spectrum,[22] electronic heat capacity,[9] and magnetization[23]) will occur.

When the device consists of two or more coupled dots, another controllable parameter comes into play: the interdot coupling. On the experimental side, recent advances in nanolithography and thin-film processing make it possible to fabricate vertically coupled QDs, where two-dimensional (2D) electrons are confined within an area smaller than 1 μm wide.[24] Recently, the successful growth of vertically aligned QDs of up to 10 layers of InAs islands separated by GaAs spacer layers were reported.[25] Furthermore, the number of electrons in each dot can be controlled at will. A coupled QD, which can be considered as an artificial molecule, has attracted much attention.[26–34] In contrast to the single QD, one must consider another degree of freedom along the growth direction in a vertically coupled QD. The main feature in this system is the effects of dot-dot and electron-electron interactions on the electronic structure. In 1993 Bryant[35] studied the energy spectra, charge densities, and correlation functions for interacting two-electron systems in coupled dots as functions of the applied bias. In 1996 Oh *et al.*[34] studied the electronic structure in coupled QDs with one or two electrons in magnetic fields. They were interested in the spin transitions of the ground state and the optical transitions between the energy levels. In 1998 Kaputkina and Lozovik[36] studied how the energy spectra for interacting two-electron system in horizontal and vertical coupled QDs depends on QD separation, lateral confinement, and magnetic field. They considered each dot as a strictly 2D system. Tokura *et al.*[37] next investigated the electron states in two vertically coupled QDs using an exact diagonalization method. From a theoretical point of view, it would be very interesting to further clarify the effect of interdot separation and external magnetic field on multilayer, vertically coupled QDs. Most of previous work focused on double-layer QD systems, and few works related to the multiple QD systems, e.g., Benjamin and Johnson[38] gave an analytical investigation of the coupled multidot systems using parabolic confinement and $1/r_{ij}^2$ type electron-electron interaction. In Ref. 39, we proposed a procedure for the exact diagonalization of the Hamiltonian of a N-layer QD molecule in the presence of a perpendicular magnetic field. To expose intuitively the effects of inter-dot correlation and quantum mechanical symmetries, in this paper, we will study the low-lying states of three electrons confined in one-, two-, and three-layer QDs.

This paper was organized as follows. In Sec. II, we presented the theoretical calculations of the low-lying states of three electrons in a one-layer QD. In Sec. III, we presented our results for low-lying states of a two-layer QD. The results of a three-layer single-electron QD are described in Sec. IV and in Sec. V, a discussion is given.

II. A ONE-LAYER QD

The standard theoretical model of a QD includes the following approximations. First, the motion of an electron is considered to be strictly two dimensional. Second, the confining potential is taken to be parabolic, and third the interaction between electrons is considered to be a pure Coulomb interaction. Let us first consider a system of three electrons moving in a single-layer QD subjected to a parabolic confinement $V(r)=m_e\omega_0^2 r^2/2$, where m_e is the effective mass of the electrons in the host semiconductor and ω_0 is the oscillator frequency of the confining potential. The external magnetic field is assumed to be along the z direction. With the symmetric gauge, the Hamiltonian takes the form

$$H=\sum_{i=1}^{3}\left(\frac{p_i^2}{2m_e}+\frac{1}{2}m_e\omega^2 r_i^2\right)+V(r_{ij})+\frac{1}{2}\omega_c L_z-g^*\mu_B BS_z, \tag{1}$$

with

$$V(r_{ij})=\sum_{i>j}\frac{e^2}{\epsilon r_{ij}}, \tag{2}$$

where $\vec{r}_i$ is the position of the ith particle, $r_{ij}=|\vec{r}_i-\vec{r}_j|$. The z component of the total orbital (spin) angular momentum is denoted by L_z (S_z). g^* stands for the effective Lande factor and μ_B the Bohr magneton. The frequency of the effective parabolic confinement is given by $\omega=\sqrt{\omega_0^2+\omega_c^2/4}$, where $\omega_c=eB/cm_e$ is the cyclotron frequency. Introducing the center-of-mass (c.m.) coordinates $R_{\text{c.m.}}=(\vec{r}_1+\vec{r}_2+\vec{r}_3)/3$ and the internal coordinates $\vec{\xi}_1=\vec{r}_1-\vec{r}_2$ and $\vec{\xi}_2=\vec{r}_3-(\vec{r}_1+\vec{r}_2)/2$, the Hamiltonian is then separated into

$$H=H_{\text{c.m.}}+H_I, \tag{3}$$

where

$$H_{\text{c.m.}}=\frac{P_{\text{cm}}^2}{2M}+\frac{1}{2}M\omega^2 R_{\text{c.m.}}^2+\frac{1}{2}\omega_c L_{\text{c.m.}}, \tag{4}$$

describes the c.m. motion, $M=3m_e$. The eigensolutions of $H_{\text{c.m.}}$ are obviously the ordinary 2D harmonic oscillator functions. H_I describes the internal motion

$$H_I=H_0+\sum_{i>j}u(r_{ij}), \tag{5}$$

with

$$H_0=\sum_{\nu=1}^{2}\left(\frac{p_\nu^2}{2\mu_\nu}+\frac{1}{2}\mu_\nu\omega^2\xi_\nu^2\right)+\frac{1}{2}\omega_c L-g^*\mu_B BS_z, \tag{6}$$

$$u(r_{ij})=\frac{1}{6}m_e\omega^2 r_{ij}^2+\frac{e^2}{\epsilon r_{ij}}, \tag{7}$$

where $\mu_1=m_e/2$ and $\mu_2=2m_e/3$. The term proportional to r_{ij}^2 arises from the confinement. A noteworthy point is that the equivalent particle-particle potential $u(r_{ij})$ is repulsive at small separation but attractive at large separation, with a minimum at $r_0=(3e^2/\epsilon m_e\omega^2)^{1/3}$. Hence, the landscape of the total potential energy surface $U=\Sigma_{i>j}u(r_{ij})$ in the multidimensional coordinate space is quite different from that without the Coulomb interaction. In this approach the c.m. motion is entirely separated from the internal motion and the multidimensional integrations can be reduced into one-dimensional integrations by means of the internal coordinates and Talmi-Moshinsky transformation coefficients.

Owing to the cylindrical symmetry of the problem, the eigenstates of H_I are classified by the total angular momentum L and the total spin S. To obtain the eigenfunctions and eigenenergies associated with internal motion, H_I is diagonalized in a model space spanned by the translationally invariant 2D harmonic product bases $\{\Phi_{[K]}\equiv\tilde{A}[\phi_{n_1 l_1}(\vec{\xi}_1)\phi_{n_2 l_2}(\vec{\xi}_2)\chi^S]_L\}$, where $[K]$ denotes the set of quantum numbers (n_1,l_1,n_2,l_2), $\chi^S=\{[\eta(1)\eta(2))]_{S_{12}}\eta(3)\}_S$ are the spin wave functions, $\phi_{nl}(\vec{\xi})$ are 2D oscillator harmonics with a frequency ω, and energies$(2n+|l|+1)\hbar\omega$. $\tilde{A}$ is an antisymmetrizer. $\eta(i)$ is the spinor of a single electron, S_{12} is the total spin of electrons 1 and 2. The matrix elements of H_I are then given by the following expressions:

$$\langle\Phi_{[K]}|H_0|\Phi_{[K']}\rangle=\left\{[2(n_1+n_2)+|l_1|+|l_2|+2]\hbar\omega+\frac{1}{2}\omega_c L_z-g^*\mu_B BS_z\right\}\delta_{[K],[K']} \tag{8}$$

and

$$\langle\Phi_{[K]}|U|\Phi_{[K']}\rangle=3U_{n_1,n_1'}\delta_{l_1,l_1'}\delta_{n_2,n_2'}\delta_{l_2,l_2'}, \tag{9}$$

with

$$U_{n,n'}=\int_0^\infty R_{nl}(\xi)\left(\frac{1}{6}m_e\omega^2\xi^2+\frac{e^2}{\epsilon\xi}\right)R_{n'l}(\xi)\xi d\xi, \tag{10}$$

where $R_{nl}(\xi)$ is the radial part of a 2D harmonic oscillator function. In our calculations, ω is treated as a variational parameter to minimize the ground-state energy. The accuracy of the solutions depends on how large the model space is. Since we are interested only in the low-lying states and in the qualitative aspect, the model space adopted is neither very large to facilitate numerical calculation, nor very small to assure the qualitative accuracy. This is achieved by extending the dimension of the model space step by step. In each step the new results are compared with previous results from a smaller space, until satisfactory convergence is achieved.

Our numerical computation is carried out for one of the typical semiconducting materials GaAs, as an example, with the material parameters shown below $m_e=0.067m_0$ (where m_0 is the single-electron bare mass), $\epsilon=12.58$, $g^*=0.44$, and $\hbar\omega_0=3.6$ meV (several experimental groups have fabricated such a dot). In what follows the energy unit is in meV and the length unit is in nm.

In Figs. 1, we calculate the energies of low-lying states of $L\leq 9$ as a function of the external magnetic field B for two different values of the total spin: (a) $S=1/2$, (b) $S=3/2$. It is the competition between the single particle energy and the interacting energy that finally determines the total energy. We know that the slope of the rising curve depends on L. A

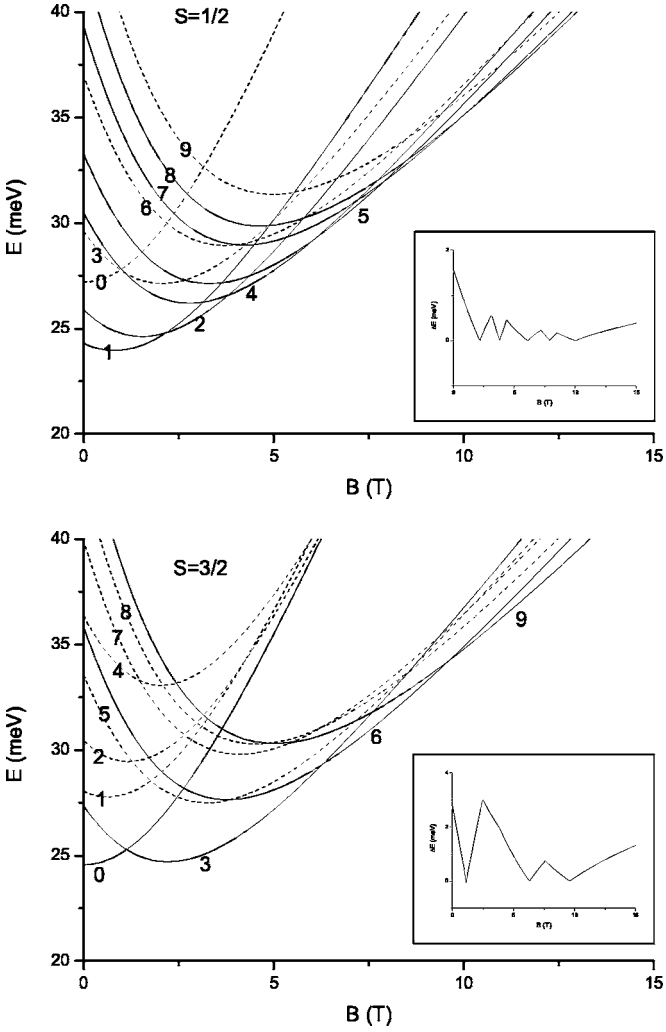

FIG. 1. Energy levels of low-lying states with $L \leq 9$ of a three-electron system in a single-layer QD versus external magnetic fields: (a) $S=1/2$, (b) $S=3/2$. The numbers in the figures label the total angular momentum L of the states. Parameters are taken appropriate for GaAs, $\hbar\omega_0=3.6$ meV. The inset gives the energy difference between ground and first excited states.

smaller L would lead to a larger slope because the negative term $\omega_c L_z/2$ is weaker. Therefore, when the magnetic field B increases, the curve with a small L cross the curve with a larger L because the former is rising faster. Obviously, the crossing would lead to a transition of L of the ground state from one to another. However, the transition is strictly limited to between two magic numbers of L. States with nonmagic L lie much higher and are excluded from becoming the lowest of the spin configuration due to the existence of the inherent nodal lines in the wave function and the failure to minimize the interaction energy.[15,40] It is readily seen that the transition of L is from a magic angular momentum to another which are $L \neq 3k$ $(k=0,1,2,\ldots)$ when $S=1/2$ and $L=3k$ when $S=3/2$, in a single QD. The origin of the magic numbers is the quantum constraint arising from the Pauli principle.

III. A DOUBLE-LAYER QD

The system we study is a vertically coupled two-layer QD containing three electrons. Without any loss of generality, we assume that electrons 1 and 2 are in the upper dot and the lower dot contains electron 3. In both dots the motion of electrons is strictly 2D, and the lateral confinement potential within each layer is assumed to be parabolic $V(r)=\frac{1}{2}m_e\omega_0^2 r^2$. The dots are separated by d in the vertical direction with their centers aligned on a common z axis. The electron tunneling between two dots are assumed to be negligible. The electrons experience both intralayer and interlayer Coulomb repulsions. The external magnetic field is assumed to be lying along the z direction. The interlayer separation d and the external magnetic field B are varied. The Hamiltonian is similar to that of three electrons in a single dot of except for the interaction potential $U=V_1+V_2$, where

$$V_1 = \frac{e^2}{\epsilon r_{12}}, \tag{11}$$

$$V_2 = \frac{e^2}{\epsilon}\left(\frac{1}{\sqrt{r_{23}^2+d^2}} + \frac{1}{\sqrt{r_{13}^2+d^2}}\right). \tag{12}$$

By using the same method as in Sec. II, the matrix elements of H_I are then given by the following expressions:

$$\langle \Phi_{[K]}|H_0|\Phi_{[K']}\rangle = \left\{[2(n_1+n_2)+|l_1|+|l_2|+2]\hbar\omega + \frac{1}{2}\omega_c L_z - g^*\mu_B B S_z\right\}\delta_{[K],[K']} \tag{13}$$

and

$$\langle \Phi_{[K]}|V_1|\Phi_{[K']}\rangle = U_{n_1,n_1'}\delta_{l_1,l_1'}\delta_{n_2,n_2'}\delta_{l_2,l_2'}, \tag{14}$$

$$\langle \Phi_{[K]}|V_2|\Phi_{[K']}\rangle = 2\sum_{[K''][K''']} B_{[K][K'']}B_{[K'][K''']} \times U^{II}_{n_1'',n_1'''}\delta_{l_1'',l_1'''}\delta_{n_2'',n_2'''}\delta_{l_2'',l_2'''}, \tag{15}$$

where

$$U^{\mathrm{I}}_{n,n'} = \int_0^\infty R_{nl}(\xi)\frac{e^2}{\epsilon}R_{n'l}(\xi)d\xi, \tag{16}$$

$$U^{\mathrm{II}}_{n,n'} = \int_0^\infty R_{nl}(\xi)\frac{e^2}{\epsilon\sqrt{\xi^2+d^2}}R_{n'l}(\xi)\xi d\xi, \tag{17}$$

$$B_{[K],[K']} = \int \Phi_{[K]}(\vec{\xi}_1,\vec{\xi}_2)\Phi_{[K']}(\vec{\xi}'_1,\vec{\xi}'_2)d\vec{\xi}_1 d\vec{\xi}_2. \tag{18}$$

$B_{[K],[K']}$ is the transformation bracket of 2D harmonic product states with two different sets of internal coordinates for three-body systems, which allows us to reduce the otherwise multiple integrals into a single integral. Nonvanishing $B_{[K],[K']}$ occurs only when both the states $\Phi_{[K]}(\vec{\xi}_1,\vec{\xi}_2)$ and $\Phi_{[K']}(\vec{\xi}'_1,\vec{\xi}'_2)$ have exact the same eigenenergy and eigenan-

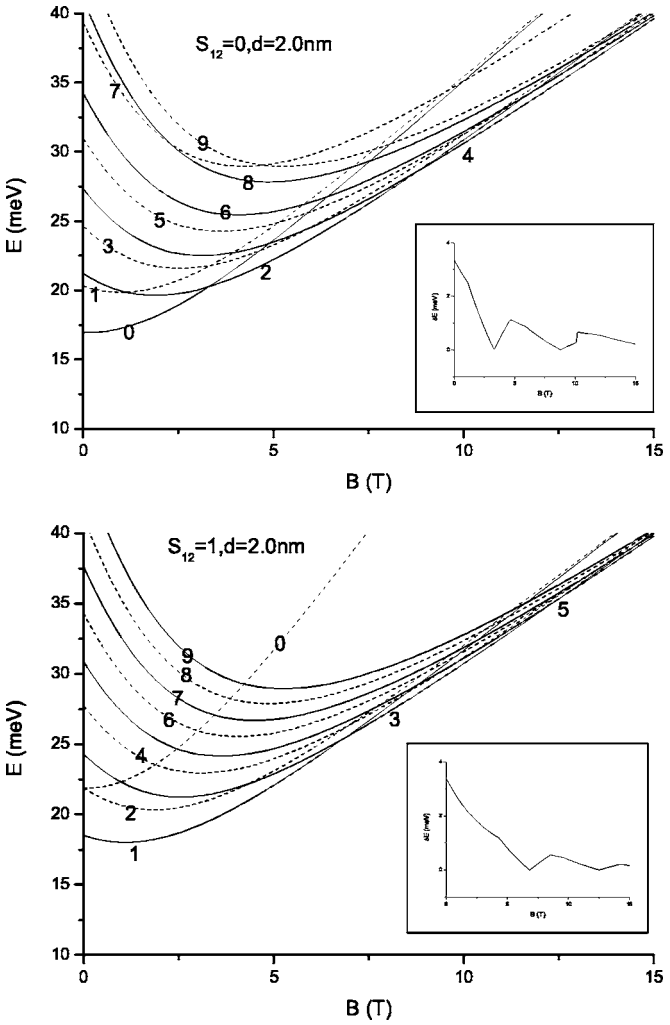

FIG. 2. Energy levels of low-lying states with $L \leq 9$ of a three-electron system in a double-layer QD versus external magnetic fields: (a) $S_{12}=0$, (b) $S_{12}=1$. The numbers in the figures label the total angular momentum L of the states. Parameters are taken the same as Fig. 1 except for $d=2.0$ nm. The inset gives the energy difference between ground and first excited states.

gular momentum. Analytical expression for $B_{[K],[k']}$ has already been derived in Ref. 41. The set of canonical coordinates $\{\vec{\xi}'_1, \vec{\xi}'_2\}$ are defined by $\vec{\xi}'_1 = \vec{r}_2 - \vec{r}_3$, $\vec{\xi}'_2 = \vec{r}_1 - (\vec{r}_2 + \vec{r}_3)/2$.

To see intuitively the effect of inter-dot correlation, we set $d=2.0$ nm and plotted in Figs. 2 the energies of the low-lying states of $L \leq 9$ as a function of the external magnetic field B for two different values of the total spin: (a) $S_{12}=0$, (b) $S_{12}=1$. From Figs. 2, it is readily seen that, in the double-layer coupled QD case, the ground state transitions are different from those of a three-electron single QD, i.e., the series for magic numbers are now $L=2k$ $(k=0,1,\ldots)$ when $S_{12}=0$ and $L=2k+1$ when $S_{12}=1$.

IV. A THREE-LAYER QD

Consider a coupled collinear three-layer QD containing three electrons, where the separations of the adjacent layers are d and the external magnetic field B can be varied. We assume that each QD contains only one electron and that the electron tunneling is negligible. Other configurations are also possible. But the configuration with one electron in a layer is more relevant to the recent experiment. The confinement potential and the separation between adjacent layers are experimentally adjustable by performing the same experiments on several different samples. The lateral potential $\frac{1}{2}m_e\omega_0^2 r^2$ is assumed to be identical for each layer. The external magnetic field is assumed to be along the z axis. The Hamiltonian is similar to that of three electrons in a single dot, except for the interaction potential $U=V_1+V_2$, where

$$V_1 = \frac{e^2}{\epsilon\sqrt{r_{12}^2 + d^2}}, \tag{19}$$

$$V_2 = \frac{e^2}{\epsilon}\left(\frac{1}{\sqrt{r_{23}^2 + d^2}} + \frac{1}{\sqrt{r_{13}^2 + 4d^2}}\right). \tag{20}$$

By using the same method as in Sec. II, the matrix elements of H_I are then given by the following expressions:

$$\langle \Phi_{[K]} | H_0 | \Phi_{[K']} \rangle = \left\{ [2(n_1+n_2) + |l_1| + |l_2| + 2]\hbar\,\omega + \frac{1}{2}\omega_c L_z - g^*\mu_B B S_z \right\} \delta_{[K],[K']} \tag{21}$$

and

$$\langle \Phi_{[K]} | V_1 | \Phi_{[K']} \rangle = U^I_{n_1,n_1'} \delta_{l_1,l_1'} \delta_{n_2,n_2'} \delta_{l_2,l_2'}, \tag{22}$$

$$\langle \Phi_{[K]} | V_2 | \Phi_{[K']} \rangle = \sum_{[K''][K''']} B_{[K][K'']} B_{[K'][K''']} U^{II}_{n_1'',n_1'''} \delta_{l_1'',l_1'''} \times \delta_{n_2'',n_2'''} \delta_{l_2'',l_2'''}, \tag{23}$$

where

$$U^{\mathrm{I}}_{n,n'} = \int_0^\infty R_{nl}(\xi) \frac{e^2}{\epsilon\sqrt{\xi^2 + d^2}} R_{n'l}(\xi)\xi d\xi, \tag{24}$$

$$U^{\mathrm{II}}_{n,n'} = \frac{e^2}{\epsilon}\int_0^\infty R_{nl}(\xi)\left(\frac{1}{\sqrt{\xi^2 + d^2}} + \frac{1}{\sqrt{\xi^2 + 4d^2}}\right) R_{n'l}(\xi)\xi d\xi, \tag{25}$$

$$B_{[K],[K']} = \int \Phi_{[K]}(\vec{\xi}_1, \vec{\xi}_2)\Phi_{[K']}(\vec{\xi}'_1, \vec{\xi}'_2) d\vec{\xi}_1 d\vec{\xi}_2. \tag{26}$$

To investigate further the effect of interdot correlation, we set $d=2.0$ nm and plotted in Fig. 3 the energies of the low-lying states of $L \leq 9$ as a function of the external magnetic field B. From Fig. 3, it is readily seen that, in the case of three layers, ground-state transitions are different from those of one and two layers. Now L takes all successive integers $0,1,2,\ldots$, as the ground-state transitions occur.

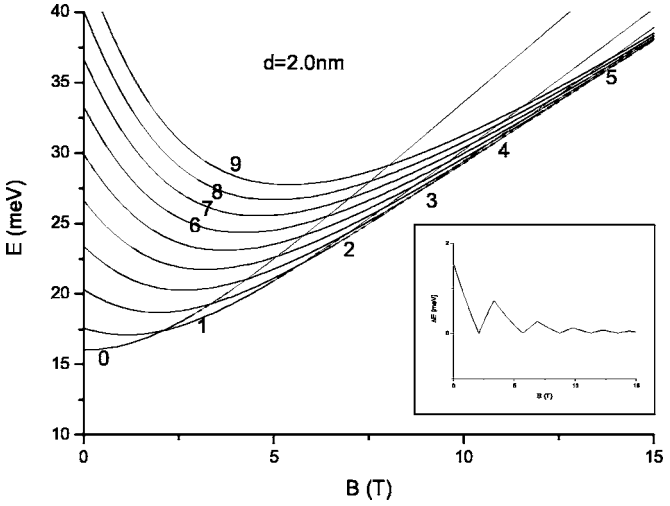

FIG. 3. Energy levels of low-lying states with $L \leq 9$ of a three-electron system in a three-layer QD versus external magnetic fields. The numbers in the figures label the total angular momentum L of the states. Parameters are taken the same as Fig. 2. The inset gives the energy difference between ground and first excited states.

V. DISCUSSION

From Figs. 1–3, we know that the ground states of three electrons confined in one-, two-, and three-layer QDs, are different from one another because they can have different magic numbers. This has a profound background from quantum mechanical symmetries,[11] such that rules for the determination of magic numbers can be set up by simple symmetry analysis.

First, in a single-layer QD case, it is obvious that if the particles form an equilateral triangle (ET) with all the side lengths being equal to r_0, then the potential energy is minimized. If the wave function is smoothly (without nodal lines) distributed around the above ET, the binding will be averagely strong and the internal motion will appear only as a gentle oscillation around the equilibrium configuration of ET. The total energy can then be minimized. Hence, the ET is the most important configuration of three-electron systems in a single-layer QD and should be pursued by low-lying states. However, we will see that in some cases this favorable configuration is prohibited by symmetry.

When $S=3/2$ (the polarized states), the spatial part of the wave function Φ is antisymmetric, thus it is invariant under a cyclic permutation $P_{(123)}$,

$$\Phi^L_{[K]}(\mathrm{ET}) = P_{(123)}\Phi^L_{[K]}(\mathrm{ET}). \tag{27}$$

Evidently, in an ET configuration a rotation about z axis by 120° is equivalent to a cyclic permutation. Thus, we have

$$R_z(120^\circ)\Phi^L_{[K]}(\mathrm{ET}) = P_{(123)}\Phi^L_{[K]}(\mathrm{ET}). \tag{28}$$

The former operator produces a factor of $\exp(i2\pi L/3)$, hence, we obtain

$$[1-\exp(i2\pi L/3)]\Phi^L_{[K]}(\mathrm{ET}) = 0. \tag{29}$$

This equation imposes a very strong constraint on $\Phi^L_{[K]}(\mathrm{ET})$: if the first factor is not equal to zero, $\Phi^L_{[K]}(\mathrm{ET})$ has to be zero. This is called an ET prohibition, it occurs when $L \neq 3k$, where k is an integer.

When $S=1/2$ (the unpolarized states), the wave function can be expanded as

$$\Psi = \Phi^a_L \chi^{1/2}_0 + \Phi^b_L \chi^{1/2}_1, \tag{30}$$

where χ^S_s is the spin wave function with the spins of electrons 1 and 2 coupled to form a two-electron spin state $s(=0,1)$, then it is coupled with the spin of electron 3 to form a three-electron spin state with total spin S. Since Ψ is antisymmetrized, from the representation theory of the symmetry group, we have

$$P_{(123)}\Phi^a_L = -\frac{1}{2}\Phi^a_L + \frac{\sqrt{3}}{2}\Phi^b_L, \tag{31}$$

$$P_{(123)}\Phi^b_L = -\frac{\sqrt{3}}{2}\Phi^a_L - \frac{1}{2}\Phi^b_L. \tag{32}$$

Combining Eqs. (31) and (32), when the particles form an ET, the following sets of homogeneous linear equations hold:

$$[1+\exp(i2\pi L/3)+\exp(i4\pi L/3)]\Phi^j_L(ET) = 0, (j=a,b). \tag{33}$$

If the first factor in Eq. (33) is not equal to zero, the ET prohibition occurs. In contrast with the $S=3/2$ states, now it occurs when $L=3k$.

Once the ET prohibition occurs, an inherent nodal line appears in the wave functions at the ET configuration, resulting in instability. This line is originated purely from symmetry, and it is named an inherent nodal line, which is found to be decisive to the structures of few-body systems.[42,43] Now the main features of the spectrum can be easily explained, where all the ET-accessible head(lowest) states are lower. In these head states, the wave function is smoothly distributed around an ET without nodal line. In this way the potential energy can be optimized and the internal excitation is avoided (the furiousness of an internal excitation is measured by the number of nodal lines contained in the wave function). Hence, the magic numbers in a single-layer QD containing three electrons are $L=3k$ when $S=3/2$ and $L \neq 3k$ when $S=1/2$.

Second, in a double-layer QD case, evidently, all the three electrons would prefer to be close to the common axis due to parabolic confinement. However, electrons 1 and 2 in the same layer would not be close to each other. Therefore, if the three electrons form an isosceles triangle (IST) with 1 and 2 at the base and a height d and with the height lying along the common axis, then this IST geometric configuration is favourable in binding. Let us call an arbitrary IST with its height paralled to the common axis an upstanding IST (or an UIST). Evidently, the UIST are the most important configuration. However, when the electrons form an UIST a rotation of 180° about the z axis is equivalent to an interchange of 1

and 2. Let $\Phi^L_{[K]}$ be the spatial part of the state with angular momentum L. Then we have

$$R_z(180^\circ)\Phi^L_{[K]}(\mathrm{UIST}) = P_{(12)}\Phi^L_{[K]}(\mathrm{UIST}). \quad (34)$$

The former operator produces a factor of $(-1)^L$, while the later produces a factor of $(-1)^{(s+1)}$. This implies that the UIST configuration would be completely prohibited [i.e., $\Phi^L_{[K]}(\mathrm{UIST})=0$] by symmetries unless the values of L are the magic numbers fulfilling $(-1)^L=-1$. Hence, the magic numbers in a single-layer QD containing three electrons are $L=2k$ when $s=0$ and $L=2k+1$ when $s=1$.

However, in our three-layer QD, such an ET or UIST prohibition does not exist because the tunneling of electrons is not allowed. Hence, instead of adopting the discrete magic numbers, the quantum number L can take all successive integers.

In conclusion, for three-electron QDs, it is found that the series of the magic numbers in one-, two-, and three-layer QDs, are different. We have shown that these magic numbers can be understood from symmetries. We noticed that the composite fermion model provides an alternative for the explanation of magic numbers in one-layer quantum dots,[44] which was found to be quite successful in the regime of weak interactions or for short-range interactions, but less so for the long-range Coulomb interaction and/or in the regime of strong interaction. Its applicability to multilayer quantum dots remains open.

ACKNOWLEDGMENTS

This work was financially supported by the National Natural Science Foundation under Grant No. 10475021.

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