Quantum left-handed metamaterial from superconducting quantum-interference devices

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A scheme for a kind of quantum left-handed metamaterial is proposed, which is composed of an array of superconducting quantum-interference devices and a dielectric background. Based on an analytical study, it is shown that (the real part of) the magnetic permeability μ can be negative with low loss and its frequency dependence is different from that of ordinary split-ring resonators. The structural requirements and frequency range for negative permeability are derived. Moreover, the permeability can be smoothly tuned over a large range by another (coupling) field via a quantum interference, which is similar to electromagnetically induced transparency of atomic systems. Negative refractivity with low loss can be achieved by tuning the permeability.

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Recently, left-handed metamaterials[1] (LHMs) have attracted much attention. This kind of metamaterial simultaneously has negative permittivity and negative permeability over some frequency band, which therefore leads to a negative refractive index. Many different phenomena can occur in LHMs, such as superprism, perfect flat lens, inverse light pressure, and reverse Doppler and Vavilov-Cherenkov effects. Large nonlinearity can also occur, causing, for example, bistable transition of the permeability from positive to negative.[2] Quantum phenomena in LHMs such as the modified spontaneous emission of atoms[3,4] have also attracted attention. In ordinary situations, however, LHMs [e.g., those composed of conductor lines and slit-ring resonators (SRRs)] are composed of "classical" systems where the negative ε and negative μ arise from classical plasma oscillations. In this sense they are quite different from quantum systems, e.g., atomic gases. How to make a quantum LHM (QLHM) is still an open question. For atomic gases, the magnetic response to a laser field is so weak that it is difficult to generate negative permeability. Some theoretical studies have been performed on negative refraction in atomic systems, e.g., Ref. 5, where the large atomic density and special restrictions on the frequencies of the driving fields are necessary. By contrast, an artificial quantum system with high magnetic sensitivity could be more advantageous for realizing negative permeability. However, two questions then arise: (i) How to design the unit cell of the metamaterial, so that it can be a quantum particle that behaves like an atom, and, at the same time play the role of an SRR, which is essential to left-handedness, and (ii) how to realize a negative refractive index, which, in general, requires overlapping of the frequency ranges of the negative Re(ε) and Re(μ). This may be difficult unless one of the frequency ranges can be continuously tuned over a considerably wide range.

In this paper, we propose a different kind of LHM composed of superconducting rings with Josephson junctions [superconducting quantum-interference devices (SQUIDs)] and a dielectric background. Here the cut of an ordinary SRR is replaced by a Josephson junction, which is essential to the quantum properties of the material. The purpose of our work is to show that the SQUID is a suitable candidate for the LHM unit cell. We will also show that due to quantum-interference effects,[6] the frequency of the negative permeability can be smoothly tuned over a large range by an external microwave field. It should be noted that superconducting metamaterials can be realized in other ways, such as with superconducting transmission lines[7] or superconducting SRRs and low-loss dielectric materials.[8]

A scheme for a QLHM is shown in Fig. 1, where the composite is composed of SQUIDs that are arrayed and placed in a dielectric background whose permittivity ε can be negative in a given frequency range. A schematic of the potential energy and energy levels of the first six eigenstates of the SQUID is shown at the left of the figure. For a microwave field, the dielectric can be, for example, an array of normal conducting wires or other structures with a single resonance model permittivity. For the sake of simplicity, we assume the structure of the composite to be two-dimensional, the geometry of the SQUID to be a cylindrical ring, and the dielectric to have no direct interaction with the SQUIDs. The radius of the ring is denoted by a, and the period of the array is denoted by d. We assume a microwave probe field is interacting with the composite and its wavelength (λ) satisfies the condition $a \ll d \ll \lambda$, in which case the SQUID array can be considered to be an effective medium with permeability μ, while the direct interaction among the SQUIDs can be neglected. In practice, we can use the parameters

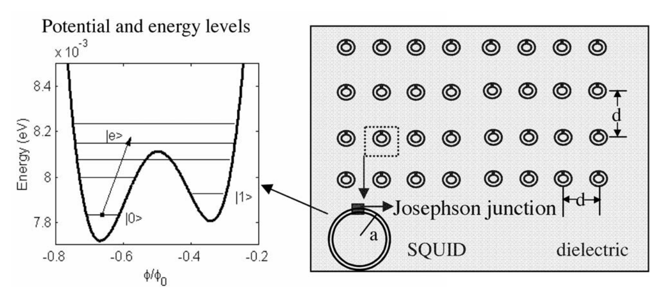

FIG. 1. Schematic of a composite metamaterial structure composed of superconducting rings with Josephson junctions (SQUIDs) arrayed and placed in a dielectric background (right), and the potential energy and the first six eigenstates of the SQUID (left). The energies of the ground state $|0\rangle$, metastable state $|1\rangle$, and excited state $|e\rangle$ are 7.81984, 7.90183, and 8.14057 meV, respectively.

$a \sim 40\ \mu$m (the typical SQUID size is 10–100 μm), $d \sim 400\ \mu$m, and $\lambda \sim 4$ mm.

For a classical SRR system, the permeability μ can be given (according to Ref. 2) by the relation $B(\omega)=H_x(\omega)+FH'(\omega)$, where $H_x(\omega)$ is the alternating external magnetic field and $H'(\omega)$ is the additional magnetic field induced by $H_x(\omega)$, which determines the magnetization of the composite, and $F=\pi a^2/d^2$ is the fraction of the structure. Therefore, the permeability can be given by

$$\mu(\omega) = 1 + F\frac{\phi(\omega)}{\phi_x(\omega)}, \tag{1}$$

where $\phi(\omega)$ is the flux induced by the external microwave field, i.e., $\phi(\omega)=H'(\omega)\pi a^2$. For the QLHM, here the SRRs are replaced by the SQUIDs, so $\phi(\omega)$ becomes an operator, although the external driven fields are assumed to be classical. In order to calculate μ, $\phi(\omega)$ in Eq. (1) should be replaced by the quantum average $\langle\phi(\omega)\rangle$.

The Hamiltonian of a SQUID, a superconducting ring with a Josephson junction, can be given by[9] $H_0=-\frac{\hbar}{2m}\frac{\partial}{\partial x^2}+V(x)$ with the potential of the SQUID being $V(x)=\frac{1}{2}m\omega_{LC}^2(x-x')-\frac{1}{4\pi^2}m\omega_{LC}^2\beta\cos(2\pi x)$, where $x=\phi/\phi_0$, $m=C\phi_0^2$, $\omega_{LC}^2=\frac{1}{LC}$, $\beta=2\pi LI_c/\phi_0$, and $x'=\phi_x/\phi_0$. Here, ϕ is the total flux in the ring, L is the ring inductance, ϕ_x the external magnetic flux applied to the SQUID, I_c the critical current of the junction, C the capacitance of the junction, and $\phi_0(=h/2e)$ the flux quantum. We consider a realistic SQUID system, which can be described by use of the parameters as in the work of Zhou *et al.*,[9] where $L=100pH$, $C=40fF$, and $I_c=3.95\mu A$, leading to $\omega_{LC}=5\times10^{11}$ rad/s, and $\beta=1.2$. The external dc magnetic field parameter x' is taken to be -0.501.

The interaction between the SQUID and microwave fields, which are assumed to be linearly polarized with their magnetic field perpendicular to the plane of the SQUID ring, is described by the time-dependent potential $V_{int}(x,t)=m\omega_{LC}^2(x-x')(\epsilon\cos\omega t+\epsilon_c\cos\omega_c t)$. In the interaction picture, the dynamics of the system is governed by the Schrödinger equation $i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle=V_{int}|\psi(t)\rangle$. Here, ϵ and ϵ_c are the microwave magnetic fluxes of the probe field and the coupling field, respectively, in units of ϕ_0. The frequencies of the probe and coupling fields are chosen to be nearly resonant with the transition $|0\rangle-|e\rangle$ and $|1\rangle-|e\rangle$, where $|0\rangle$, $|1\rangle$, and $|e\rangle$ are the eigenstates of H_0, which are the first and second ground states, and the excited state (with quantum number n_e), respectively. This is usually referred to as the three-level approximation. In this case, the wave function can be written as $|\psi(t)\rangle=c_0(t)|0\rangle+c_1(t)e^{-i(\delta-\delta_c)t}|1\rangle+c_e(t)e^{-i\delta t}|e\rangle$, and the interaction Hamiltonian in the rotating-wave approximation can be written as[9] $V_{int}=\hbar(\Omega e^{i\delta t}|0\rangle\langle n|+\Omega_c e^{-i\delta_c t}|1\rangle\langle n|)+\text{H.c.}$ Here, the Rabi frequency Ω and Ω_c are defined as $\Omega=-x_{0e}m\omega_{LC}^2\epsilon/\hbar$ and $\Omega_c=-x_{1e}m\omega_{LC}^2\epsilon_c/\hbar$, where x_{0e} and x_{1e} are defined as $\langle 0|x|e\rangle$and $\langle 1|x|e\rangle$, respectively, and $\delta=\omega-\omega_{0e}$, $\delta_c=\omega_c-\omega_{1e}$. According to the Schrödinger equation in the interaction picture and considering the decay of the excited state and the second ground state, the equations for amplitudes c_0, c_1, and c_e can be easily obtained as follows

$$i\frac{dc_0}{dt} = \frac{\Omega}{2}c_e,$$

$$i\frac{dc_e}{dt} = -\left(\delta + i\frac{\gamma}{2}\right)c_e + \frac{\Omega}{2}c_0 + \frac{\Omega_c}{2}c_1,$$

$$i\frac{dc_1}{dt} = -\left(\delta - \delta_c + i\frac{\gamma_1}{2}\right)c_1 + \frac{\Omega_c}{2}c_e, \tag{2}$$

where γ_1 and γ are the background decay rates of the states $|1\rangle$ and $|e\rangle$. The background decay rate of the ground state can be neglected because it is much smaller than that of the higher states. It should be noted that Eq. (2) can also be used in the case where the SQUID has more than one Josephson junction.[6] Therefore, the cell of the material discussed in this paper can also be replaced by a SQUID that has multijunctions.

It is easy to show that $(\langle\phi(\omega)\rangle)$ can be given by

$$\frac{\langle\phi(\omega)\rangle}{\phi_x(\omega)} = -\alpha\frac{c_0^*c_e}{\Omega}, \tag{3}$$

where $\alpha=\frac{m\omega_{LC}^2x_{0e}^2}{\hbar}=\frac{\phi_0^2}{\hbar L}x_{0e}^2$. Here, α is a parameter, in the dimension of frequency (Hertz), which measures the sensitivity of the effective medium to the probe magnetic field. For example, if we take $L=100pH$, $C=40fF$, $\beta=1.2$, and $x'=-0.501$ (typical values), for the $|0\rangle-|4\rangle$ transition, it can be obtained by numerically solving the time-independent Schrödinger equation that $x_{04}=5.39798\times10^{-3}$,[10,11] then $\alpha=11.815$ GHz.

Let us first consider the case of $\Omega_c=0$, i.e., a two-level system. If the SQUID is initially in the ground state $|0\rangle$ and coupled with a weak field $(\Omega \ll \gamma)$, according to Eqs. (1)–(3), the permeability can be given by

$$\mu = 1 - \frac{F}{2}\frac{\alpha}{\delta + i\gamma/2}. \tag{4}$$

This is clearly a Lorentz profile. When the probe field is nearly on resonance with the transition $|0\rangle-|e\rangle$, the absorption $\text{Im}(\mu)$ becomes large and the absorption peak becomes sharper with a decrease of γ. On the other hand, similar to an array of classical SRR, a negative $\text{Re}(\mu)$ can occur [see Fig. 2(a)]. It is worthy to note that the μ spectrum for the quantum composite here is similar to but not the same as that for a classical composite.[2] It is easy to derive from Eq. (4) that the negative $\text{Re}(\mu)$ frequency band can only occur in the situation when

$$g \equiv \frac{F\alpha}{2\gamma} > 1, \tag{5}$$

where g is a coupling constant. This simple inequality describes the structural requirement of the negative real part of permeability and is a necessary condition. For the two-level system there is only one frequency band for the negative $\text{Re}(\mu)$ if $g>1$, which is

$$x_- < \frac{\delta}{\gamma} < x_+, \tag{6}$$

where $x_\pm = \frac{1}{2}(g \pm \sqrt{g^2-1})$.

We shall now briefly discuss the condition for negative permeability (5). It is obvious that we should take smaller γ and larger F. However, if F is large (compact structure), the interactions among the SQUIDs cannot be neglected. For the sake of simplicity, we only deal with the case where F is small enough so that the interactions through local fields can be neglected. For example, in the case of $F=\pi\times 0.014$, if $\alpha=11.815$ GHz (as above), then $F\alpha/2=0.2598$ GHz. In this case, negative $\mathrm{Re}(\mu)$ can occur if the condition $\gamma < 0.2598$ GHz is met. In fact, the decay rate γ varies over a very large range and strongly depends on the quantum number (n_e) of the energy of the excited state ($|e\rangle$). In the case where the energy of $|e\rangle$ is near the top of the potential well, the quantum tunneling between the two wells (left and right wells) will strongly decrease γ, which can be ~ 1 GHz, whereas in the case where the energy of $|e\rangle$ is far from the top of the potential, it can be <1 MHz, which corresponds to intrawell transitions.[6,12] For $n_e=4$, a conservative estimate of γ based on experimental investigations[13] shows that γ can be 1 MHz or even much smaller. If we take $\gamma=1$ MHz, i.e., $\gamma/\alpha=8.4641\times 10^{-5}$, it can be obtained from Eq. (5) that $g=259.8(\gg 1)$, i.e., in this case negative permeability $[\mathrm{Re}(\mu)<0]$ is possible. In reality, one can choose the frequency ω very near the transparency window where $\mathrm{Re}[\mu(\omega)]\sim -1$ and $\mathrm{Im}[\mu(\omega)]\sim 0$.

If a coupling field is applied to the system ($\Omega_c \neq 0$), the magnetic response can be significantly modified. For a strong coupling field, i.e., $\Omega_c \gg \Omega$, γ, from Eqs. (1)–(3), the permeability can be given by

$$\mu = 1 - \frac{F}{2}\frac{\alpha\left(\delta-\delta_c+i\frac{\gamma_1}{2}\right)}{\left(\delta-\delta_c+i\frac{\gamma_1}{2}\right)\left(\delta+i\frac{\gamma}{2}\right)-\frac{\Omega_c^2}{4}}. \tag{7}$$

The spectrum of μ in the three-level case is shown in Fig. 2(b). It can be seen that the transparency occurs when $\delta=\delta_c=0$, which is similar to electromagnetically induced transparency (EIT) of atomic systems except that, here the coupling field is used to control the permeability μ instead of the permittivity ε of the probe field. The superconductive analog to EIT has also been investigated by Murali *et al.*[6] The width of the transparency window is about Ω_c due to Autler-Townes splitting. If g is very small, $\mathrm{Re}(\mu)$ is positive for all probe detunings (δ), which can be easily seen from Eq. (7). If g is large enough, however, negative $\mathrm{Re}(\mu)$ can occur near the transparency window. It is easy to prove that its frequency range only exists if inequality (5) is satisfied, i.e., the limitation on the structure of the material is the same as in the two-level case. However, here there are two frequency bands, which can be given by

$$f_-(x_-) < \delta/\gamma < f_-(x_+); \quad f_+(x_-) < \delta/\gamma < f_+(x_+), \tag{8}$$

where $f_\pm(x) \equiv \frac{x\pm\sqrt{x^2+\Omega_c^2/\gamma^2}}{2}$, and $x_\pm = \frac{1}{2}(g\pm\sqrt{g^2-1})$.

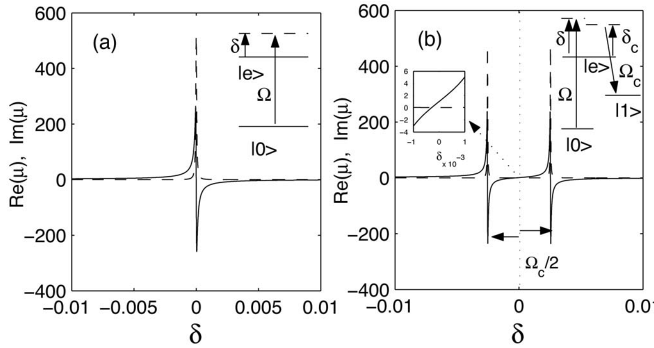

FIG. 2. Real part (solid curve) and imaginary part (dashed curve) of the permeability (μ) versus the probe detuning δ for $\delta_c=0$, where $F=\pi\times 0.014$, $\gamma/\alpha=8.4641\times 10^{-5}$, $\gamma_1=0.1\gamma$, $\beta=1.2$, and $x'=-0.501$. (a) is for a two-level system ($\Omega_c=0$) and (b) for a Λ-type three-level system with a coupling field of Rabi frequency $\Omega_c=0.5078$. All parameters are in units of α, where $\alpha=11.815$ GHz.

We assume that $\gamma_1 \ll \gamma$, in which case decoherence due to background decay of the metastable state $|1\rangle$ can be neglected, which is essential for EIT.

We assume that initially the system is prepared in the ground state $|0\rangle$, and for the sake of simplicity, we assume that the probe field is weak enough, i.e., $\Omega \ll \gamma$, $\Omega \ll \Omega_c$. In this case, the steady-state solution can be obtained as for ordinary EIT systems.

It should be noted that when $\mathrm{Re}(\mu)$ is negative, $\mathrm{Im}(\mu)$ is always nonzero, i.e., negative refractivity without loss can never occur. However, the absorption can be strongly suppressed near the condition of $\delta=\delta_c$. The permeability (μ) can be smoothly tuned over a large range by the Rabi frequency Ω_c (see Fig. 3) of the coupling field. Also, due to the smooth tuning, continuous transition from $\mathrm{Re}(\mu)=1$ to $\mathrm{Re}(\mu)=-1$ is possible.

We assume that the effective electric permittivity ε is generated by the dielectric[2,3] with the permittivity $\varepsilon=1+\frac{\omega_{pe}^2}{\omega_{Te}^2-\omega^2-i\omega\gamma_e}$, where ω_{pe} is the coupling strength, ω_{Te} the transverse resonance frequency and γ_e measures the loss of the medium. To achieve negative refraction $[\mathrm{Re}(n)<0]$ with low loss, we can choose the frequency range where $\mathrm{Re}(\varepsilon)<0$ and $\mathrm{Im}(\varepsilon)$ is small. For the sake of simplicity, we assume as in previous studies (e.g., Refs. 14, 3, and 2) that the dielectric is nonmagnetic and has contributes only to ϵ, not to μ, while the magnetic ring resonators have no contribution to ε. However, it should be noted that, in reality, we should consider undesirable excitation of the SQUIDs by the electric field. We can also eliminate or reduce this effect by use of SQUIDs with mirror symmetry with respect to the electric field,[15] as shown in Fig. 4(A). We can even redesign the unit cell, e.g., replace the one-junction SQUID with two-junction or four-junction systems, as shown in Figs. 4(B)–4(D). The SQUIDs with multijunctions have higher rotation symmetry and small total capacity, which is advantageous for high-frequency QLHMs.

In conclusion, we have proposed a quantum metamaterial composed of superconducting rings with Josephson junctions that are placed in a dielectric with negative permittivity in a

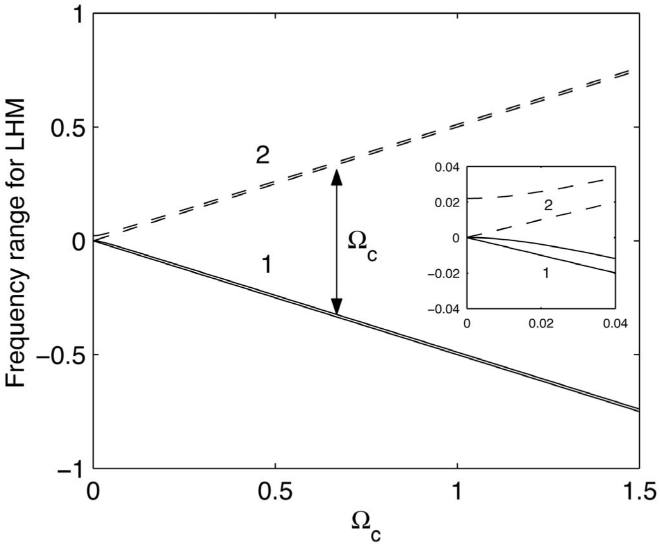

FIG. 3. Band edges of negative $\mathrm{Re}(\mu)$ vs Rabi frequency of the coupling field Ω_c, where negative $\mathrm{Re}(\mu)$ occurs when δ lies between the two solid curves, $f_-(x_-) < \delta < f_-(x_+)$ (band 1) or between the dashed curves, $f_+(x_-) < \delta < f_+(x_+)$ (band 2). The distance between the center frequencies of the two bands is about Ω_c. The inset shows the range of Ω_c from 0 to 0.04. The parameters are the same as in Fig. 2.

frequency band. The dielectric can be composed of normal conducting elements or superconducting elements, e.g., layers of $YBa_2Cu_3O_7$.[16] Negative permeability requires the structure of the SQUID array to satisfy the condition $g \equiv \frac{F\alpha}{2\gamma} > 1$. A coupling field can lead to an EIT effect in the case of Raman resonance $\delta = \delta_c$, and near resonance there are two passbands of negative refractivity, which are given by Eq. (8). Negative refractive index with low loss is easy to obtain due to the continuously tunable permeability. In the "quantum" environment, some different physics may be explored, such as that associated with the transient properties of left-handedness, and the large nonlinearity due to quantum interference effects or the strong magnetic response. Based on the principle of quantum interference, it should be possible to design high-frequency (close to the optical range) tunable metamaterials with mesoscopic particles, such as nanophotonic circuit elements.[17] The SQUID-based metamaterial may also have potential applications in quantum information processing[18] and high sensitive detection of electromagnetic fields.

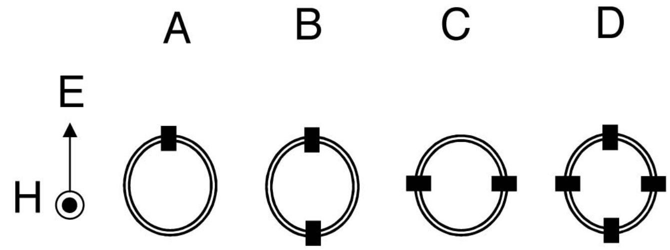

FIG. 4. Schematic of unit cells designed to have mirror symmetry, which avoid undesirable excitation by the electric field, (A) for the case of a single junction, (B) and (C) of two junctions, and (D) four junctions. Multijunction designs are desirable for isotropic LHMs.

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