Generation of multipartite entangled states in Josephson architectures

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We propose and analyze a scheme for the generation of multipartite entangled states in a system of inductively coupled Josephson flux qubits. The qubits have fixed eigenfrequencies during the whole process in order to minimize decoherence effects and their inductive coupling can be turned on and off at will by tuning an external control flux. Within this framework, we will show that a W state in a system of three or more qubits can be generated by exploiting the sequential one by one coupling of the qubits with one of them playing the role of an entanglement mediator.

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I. INTRODUCTION

Entanglement, "the striking feature of quantum mechanics" (as claimed by Schrödinger in 1935),[1] has been considered essential since the very beginning in order to investigate fundamental aspects of quantum theory. Quite recently, however, physicists have fully recognized that the generation of entangled states is an essential resource also in quantum communication and information processing. Entangled states have been generated in many experiments involving cavity QED and NMR systems, ion traps, and solid-state (superconducting) circuits, and their applications in the field of quantum computing have been demonstrated.[2–6] Among the previously mentioned physical systems, Josephson-junction based devices presently provide one of the best qubit candidates for the realization of a quantum computer due to the fact that a wide variety of potential designs for qubits and their couplings are available and that qubits can be easily scaled to large arrays and integrated in electronic circuits.[7,8] A series of successfully performed experiments for charge, flux, phase, and charge-flux qubits show indeed that they satisfy DiVincenzo's prescriptions[9] for quantum computing in terms of state preparation, state manipulation, and readout. Moreover, the nonlinearity characterizing Josephson junctions and the flexibility in circuit layout offer many possible options for coupling qubits together and for calibrating and adjusting the qubit parameters over a wide range of values.

In this field, remarkable achievements include the realization of complex single-qubit manipulation schemes,[10] the generation of entangled states[11,12] in systems of coupled flux[13] and phase[14,15] qubits, as well as the observation of quantum coherent oscillations and conditional gate operations using two coupled superconducting charge qubits.[11,16] The next major step toward building a Josephson-junction based quantum computer is therefore to experimentally realize simple quantum algorithms, such as the creation of an entangled state involving more than two coupled qubits.[3,6,17–19]

This goal may be achieved by selectively turning on and off the direct couplings between two qubits or their interaction with auxiliary systems (LC-oscillator modes,[20] inductances,[21] large-area current-biased Josephson junctions,[22,23] or the quantized modes of a resonant microwave cavity[24–26]) playing the role of a data bus. Typically, the coupling energy may be controlled by tuning the qubit level spacings in and out of resonance. However, in order to avoid introducing extra decoherence with respect to that characterizing single-qubit operations, other promising schemes for realizing a tunable coupling of superconducting (spatially separated) qubits have emerged: for instance, those wherein the interaction between two flux-based qubits is controlled by means of a superconducting transformer with variable flux transfer function,[27,28] or those wherein two qubits with an initial detuning can be made to (resonantly) interact by applying a time-dependent (microwave) magnetic flux to the qubits.[29]

Within these experimental frameworks, we propose a theoretical scheme by which it is possible to entangle *more than two* (spatially separated) flux qubits. It is based on the sequential inductive interaction of the qubits with one of them acting as an entanglement mediator.[18,30–32] More in detail, we will see that the scheme operates in such a way that it generates an entangled W state after a finite number of steps, where no conditional measurement is required. It should be noticed that, unlike in the ordinary cases so far considered,[18,30–32] the mediator will never be thrown away after use, but rather constitute an element of the entanglement in the present scheme. Furthermore, the proposed architectures are scalable, at least in principle, to an arbitrary number of qubits, and the detailed conditions on parameters are explicitly presented.

The paper is organized as follows. First, in Sec. II, we briefly describe the main features and the Hamiltonians characterizing two kinds of Josephson devices, namely the double rf superconducting quantum interference device(SQUID)[33–35] and the persistent (three-junction) qubit,[36] by which it is possible to build superconducting flux qubits. Then, we discuss the most common experimental procedures by which it is possible to initialize and to measure their quantum state. In Sec. III a scheme of successive interaction[18,30–32] is introduced in an $(N+1)$-qubit system, qubit M+qubit $1+\cdots+$qubit N, wherein entanglement me-

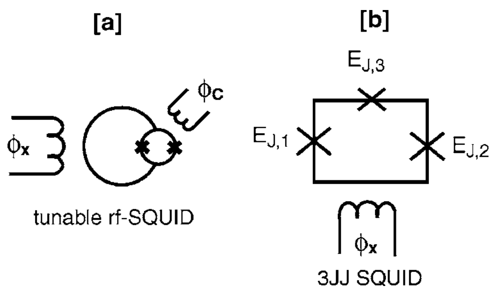

FIG. 1. Sketch (a) of the superconducting double rf-SQUID qubit and (b) of the three-junction SQUID by which it is possible to realize a persistent-current qubit.

diator M is coupled one by one with qubits $1,2,\ldots,N$. We discuss moreover the possibility of practically realizing this scheme by exploiting some of the different physical coupling elements currently available and how the coupling energy depends on the particular way in which the interaction between each qubit and the mediator is implemented. In Sec. IV we analyze the dynamics of the system showing that, by preparing the multiqubit system in a pure factorized state and by adjusting the coupling energies and/or the time of interaction between each of them and the mediator, an entangled W state can be generated. We demonstrate moreover that, under currently available conditions, the so-called rotating-wave approximation for the qubit-qubit interaction is indeed a good approximation and the counter-rotating terms bring about just a negligible effect for the generation of the W state. Finally, conclusions and discussions are given in Sec. V.

II. SUPERCONDUCTING FLUX QUBITS: MODELS AND HAMILTONIANS

In this section, we briefly describe the main features and the Hamiltonians of two devices, the tunable rf SQUID[33–35] and the three-junction SQUID,[36] by which it is possible to implement a two-state Josephson system, focusing our attention also on the experimental procedures to be considered in order to prepare their initial quantum state.

A. The double rf-SQUID qubit

We begin by considering a double rf-SQUID system,[33–35] that is a superconducting ring of self-inductance L interrupted by a dc SQUID, the smallest loop containing two identical Josephson junctions, each with critical current i_0 and capacitance c. This device [schematically illustrated in Fig. 1(a)] is biased by two magnetic fluxes ϕ_x and ϕ_c threading the greatest ring and the dc SQUID, respectively. The dc SQUID, if small enough (i.e., with inductance $l \ll i_0\phi_0/2\pi$, where $\phi_0=h/2e$ is the flux quantum), behaves like a single junction with a flux-dependent critical current $I_c=2i_0|\cos(\pi\phi_c/\phi_0)|$ and capacitance $C=2c$. This means that a double SQUID simulates a standard rf SQUID with tunable critical current $I_c\equiv I_c(\phi_c)$. Therefore, by taking into account both the charging energy of the "effective dc-SQUID junction" ($T=q^2/2C$) and the washboard potential, the Hamiltonian of the system is written down as

$$H=\frac{q^2}{2C}+\frac{(\phi-\phi_x)^2}{2L}-\frac{I_c\phi_0}{2\pi}\cos\left(\frac{2\pi\phi}{\phi_0}\right), \tag{1}$$

where the charge on the junction capacitance, q, and the flux through the SQUID loop, ϕ, are canonically conjugate operators satisfying the commutation relation $[\phi,q]=i\hbar$.

It is well known[35,37] that, by setting $\beta_L\equiv 2\pi LI_c/\phi_0>1$ and $\phi_x\approx\phi_0/2$, the circuit behaves as an artificial quantum two-level atom whose reduced Hamiltonian in the basis of the flux eigenstates $|L\rangle$ and $|R\rangle$ (which are localized in the two minima of the washboard potential and correspond to two different orientations of the current circulating in the large loop) assumes the form

$$H_{\mathrm{RF}}=-\frac{\hbar}{2}\Delta_{\mathrm{RF}}(\phi_c)\sigma_x-\frac{\hbar}{2}\epsilon_{\mathrm{RF}}(\phi_x)\sigma_z. \tag{2}$$

Here, $\hbar\epsilon_{\mathrm{RF}}(\phi_x)=2I_c\sqrt{6(\beta_L-1)}(\phi_x-\phi_0/2)$ is the energy difference between the two minima of the washboard potential,

$$\Delta_{\mathrm{RF}}(\phi_c)\simeq\frac{3}{2}\frac{\phi_0^2}{(2\pi)^2L}\left(1-\frac{\phi_0}{2\pi LI_c(\phi_c)}\right)^2 \tag{3}$$

(in the limit $0<\beta_L-1\ll 1$) the tunneling frequency between the left and the right wells that can be tuned by changing the junction critical current $I_c(\phi_c)$, and σ_x and σ_z the Pauli spin operators.

B. The persistent-current (3JJ) qubit

To minimize the susceptibility to external noise of a large-inductance rf SQUID, Mooij *et al.*[36] proposed to use a persistent-current qubit [schematically shown in Fig. 1(b)], namely a smaller superconducting loop containing three Josephson junctions, two of equal size (i.e., with $E_{J,1}=E_{J,2}=E_J$) and the third smaller by a factor α (i.e., with $E_{J,3}=\alpha E_J$, $\alpha<1$). By applying an external magnetic flux ϕ_x close to $\phi_0/2$ and choosing $\alpha\approx 0.8$, it has been proved that, in the low-inductance limit (in which the total flux coincides with the external flux and fluxoid quantization around the loop imposes the constraint $\varphi_1-\varphi_2+\varphi_3+2\pi f=0$ on the phase drops across the three junctions, $f=\phi_x/\phi_0$ being the reduced magnetic flux), the Josephson energy

$$U(\varphi_1,\varphi_2)=-E_J\cos\varphi_1-E_J\cos\varphi_2-\alpha E_J\cos(2\pi f+\varphi_1-\varphi_2) \tag{4}$$

forms a double well which permits two stable configurations of minimum energy corresponding to two persistent currents of opposite sign in the loop. This means that, also in this case, we may engineer a two-state quantum system whose effective Hamiltonian, in the basis of the two states that carry an average persistent current $\pm I_p\approx\pm 2\pi\alpha E_J/\phi_0$ (named $|L\rangle$ and $|R\rangle$ also in this case), reads

$$H_{3JJ} = -\frac{\hbar}{2}\Delta_{3JJ}\sigma_x - \frac{\hbar}{2}\epsilon_{3JJ}(\phi_x)\sigma_z. \quad (5)$$

Here, the tunneling matrix element between the two basis states, $\hbar\Delta_{3JJ}/2$, depends on the system parameters, and $\epsilon_{3JJ}(\phi_x)=2I_p(\phi_x-\phi_0/2)/\hbar$.

C. Initialization and readout

Using as a new basis that is spanned by the energy eigenstates $|0\rangle$ and $|1\rangle$ (which are the symmetric and the antisymmetric linear superpositions of $|L\rangle$ and $|R\rangle$, if ϕ_x is *exactly* equal to $\phi_0/2$), the Hamiltonians of both the rf-SQUID qubit and the 3JJ qubit take the diagonal spin-1/2–like form

$$H = \frac{\hbar}{2}\omega\sigma_z. \quad (6)$$

Here, $\hbar\omega \equiv E_1 - E_0 = \hbar\sqrt{\epsilon^2+\Delta^2}$ indicates the energy separation between their corresponding eigenstates (the analytic form of ϵ and Δ, as previously discussed, depends on the specific design of the qubit) and $\sigma_z = |1\rangle\langle 1| - |0\rangle\langle 0|$ is a Pauli operator.

It is well known that the unavoidable presence of noise in the magnetic flux ϕ_x induces fluctuations on the qubit frequency and therefore dephasing. For this reason, to minimize such an undesirable effect, we assume from the beginning that all the qubits are biased at their optimal points, that is at $\epsilon=0$, where $d\omega/d\epsilon=0$. As a consequence, the qubit quantum coherence is best preserved, since they are, to the first order, insensitive to noise in ϕ_x.[38] Before discussing the *modus operandi* of our scheme, we wish to underline that both the rf-SQUID qubit and the 3JJ qubit are easily addressed and measured. Usually, they are initialized to the ground state simply by allowing them to relax, so that the thermal population of their excited states can be neglected. The coherent control of the qubit state is instead achieved via NMR-like manipulation techniques, i.e., by applying resonant microwave pulses which, by opportunely choosing the interaction time and the microwave amplitude, can induce a transition between the two qubit energy levels.[10]

In addition, a flux state can be prepared producing a collapse of the wave function through a flux measurement or, as recently pointed out by Chiarello,[37] in the case of a double rf-SQUID qubit with an opportunely chosen sequence of variations of the washboard potential. A double SQUID can be indeed prepared in a particular flux state by strongly unbalancing the washboard potential in order to have just one minimum, then waiting a time sufficient for the relaxation to this minimum and finally sufficiently raising the barrier in order to freeze the qubit in this state. Finally, coherent rotation between the two flux states can be realized by lowering the barrier in order to induce fast free oscillations, waiting for fractions of the oscillation period to realize the desired rotation and opportunely raising the barrier in such a way to freeze the system in the desired target state.

Also for qubit readout, several detectors have been experimentally investigated. Most of them include a dc-SQUID magnetometer inductively coupled to the qubit to be measured by which it is possible to detect the magnetization signal produced by the persistent currents flowing through it, exploiting the fact that the dc-SQUID critical current is a periodic function of the magnetic flux threading its loop.[34] In addition, it is worth emphasizing that, besides these proposals, physicists have been focusing their efforts on the realization of nondemolition-measurement schemes (necessary for applications where low backaction is required) like that based on the dispersive measurement of the qubit state by coupling the qubit nonresonantly to a transmission-line resonator and probing the transmission spectrum.[39]

III. THE SCHEME FOR ENTANGLEMENT GENERATION: SEQUENTIAL INTERACTION OF N FLUX QUBITS WITH AN ENTANGLEMENT MEDIATOR

In this section, we propose a scheme for the generation of maximally entangled states in a multiqubit system, $M+1+2+\cdots+N$, where qubit M, playing the role of an entanglement mediator, is assumed to interact one by one with 1, 2,…, and N. Among the different forms of coupling theoretically proposed and experimentally realized, we consider those by which it is possible to realize an inductive interaction[27–29,40] between each qubit and the mediator, so that the free Hamiltonian of the whole system and the interaction Hamiltonians can be cast (in the basis of the energy eigenstates) in the following form:

$$H_0 = \sum_{i=M,1,2,\ldots,N} \frac{\hbar}{2}\omega_i\sigma_z^{(i)} \quad (7)$$

and

$$H'_{M1} = g_1\sigma_x^{(M)}\sigma_x^{(1)}, \quad H'_{M2} = g_2\sigma_x^{(M)}\sigma_x^{(2)}, \quad \ldots, \quad H'_{MN} = g_N\sigma_x^{(M)}\sigma_x^{(N)}. \quad (8)$$

We assume moreover that these qubits are properly initialized, by exploiting one of the previously mentioned experimental recipes, so that at $t=0$ the whole system is described by the pure factorized state $|1_M 0_1 0_2 \ldots 0_N\rangle \equiv |1\rangle_M \otimes |0\rangle_1 \otimes |0\rangle_2 \otimes \cdots \otimes |0\rangle_N$.

With this setup, if qubits 1, 2,…, and N are spatially separated (in order to strongly reduce their direct persistent coupling) and their interaction with mediator M can be turned on and off at will (by adjusting the coupling energies $g_1, g_2, \ldots, g_N$), we realize a step by step scheme which is sketched as follows:

(a) Mediator M prepared in the state $|1\rangle_M$ interacts inductively (by setting $g_1 \neq 0$) with qubit 1 during an opportunely chosen interval of time $0<t<\tau_1$, while qubits 2, 3,…, and N evolve freely (with $g_2=g_3=\cdots=g_N=0$).

(b) At $t=\tau_1$, we turn off the interaction between M and 1 and we adjust g_2 in order to couple the mediator and qubit 2 (by choosing $g_2 \neq 0$) for $\tau_1 < t < \tau_1+\tau_2$.

(c) In a similar manner, we put qubits 3, 4,…, and N in interaction with M one by one.

(d) Finally, at $t=\tau_1+\tau_2+\cdots+\tau_N$, we switch off the interaction between qubit N and mediator M, and the desired entangled state of the $(N+1)$-partite qubit M+qubit $1+\cdots$+qubit N system is generated, provided that the interaction

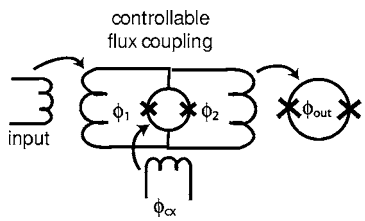

FIG. 2. Sketch of a controllable flux coupling (CFC) circuit. A signal flux is applied on the left side of the CFC and its response is read out/coupled to the SQUID on the right. The control of the transformer response flux $\phi_{\rm out}$ is achieved by modulating the flux ϕ_{cx}.[27]

times, τ_j (with $j=1,2,\ldots,N$), and/or the coupling constants, g_j, are accurately selected.

At this stage, it is important to consider a realistic experimental setup by which it is possible to implement our scheme. We begin by considering an inductive mediator-qubit coupling realized by means of a superconducting transformer with variable flux transfer function as in the paper of Castellano *et al.*[27] They show indeed that, by using a superconducting flux transformer modified with the insertion of a small dc SQUID, it is possible to control the flux transfer function and therefore the inductive coupling constant, by modulating via an externally applied magnetic flux ϕ_{cx} the critical current of the dc SQUID (see Fig. 2). More in detail, they prove that the transformer can operate between two states with very different behavior: the "off" state where the transfer ratio

$$\mathcal{R}(\phi_{cx}) = \left.\frac{d\phi_{\rm out}}{d\phi_{\rm in}}\right|_{\phi_{\rm in}=0} = \frac{1}{|1+\beta_0\cos(\pi\phi_{cx}/\phi_0)|} \qquad (9)$$

[$\phi_{\rm in}$ being the flux coupled to the left arm of the transformer, $\phi_{\rm out}$ the transformer response flux, and $\beta_0=2\pi(2I_0)L_0/\phi_0$ the so-called reduced inductance depending on the critical current I_0 of the junctions of the dc SQUID] is minimum ($\sim 0-0.1$) and the "on" state with a transfer function ratio which may be larger than 1. Under such conditions, it is possible to conceive an experimental scheme, like that depicted in Fig. 3, where the coupling between mediator M and the jth qubit [describable in terms of the interaction Hamiltonians given by Eq. (8)] may be effectively turned on by adjusting the control fluxes of the relative "switches"

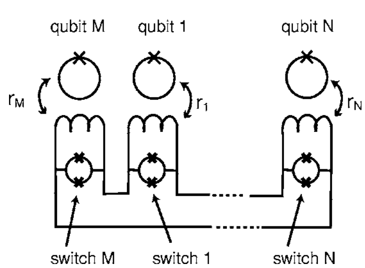

FIG. 3. Sketch of the scheme for the controlled coupling of many flux qubits proposed by Castellano *et al.*[27] The flux transfer function of the ith switch is controlled via a specific control flux $\phi_{cx}^{(i)}$ ($i=M,1,\ldots,N$) externally applied.

(namely, $\phi_{cx}^{(M)}$ and $\phi_{cx}^{(j)}$) in such a way that one obtains $g_j \neq 0$ with all the other "switches" kept in the off state.

We found indeed that the inductive coupling constants g_j may be expressed as a function of the applied magnetic fluxes $\phi_{cx}^{(M)}$ and $\phi_{cx}^{(j)}$ as follows. With reference to Fig. 3, the magnetic flux ϕ_M through the mediator couples to the arm of the corresponding switch, with a flux transforming ratio r_M (depending on the chip geometry). The flux $r_M\phi_M$ behaves as input for the switch M characterized by a flux transfer function $\mathcal{R}(\phi_{cx}^{(M)})$ given in Eq. (9). Under such conditions, indicating with $\mathcal{R}(\phi_{cx}^{(j)})$ the flux transfer function of the jth switch and with r_j the flux transforming ratio between the jth arm and the jth qubit, it is easy to convince oneself that the amount of the flux ϕ_M consequently appearing in the jth qubit is $[r_j\mathcal{R}(\phi_{cx}^{(j)})\mathcal{R}(\phi_{cx}^{(M)})r_M]\phi_M \equiv \xi_j\phi_M$. Similarly, the amount of the flux ϕ_j consequently appearing in the mediator turns out to be $\xi_j\phi_j$. These two extra fluxes through the qubits M and j, which shift their energy levels, imply that the total Hamiltonian of the system contains, besides the free Hamiltonians of the qubits [as expressed in Eq. (6)], the effective interaction term $H'_{Mj}=(1/L)(\xi_j\phi_M\phi_j+\phi_M\xi_j\phi_j) =(2\xi_j/L)(\phi_0/2)^2\sigma_x^{(M)}\sigma_x^{(j)}$, $\phi_i=(\phi_0/2)\sigma_x^{(i)}$ being the flux operator of the ith qubit ($i=\{M,j\}$) in the basis of the eigenstates of the free Hamiltonian, and $g_j\equiv(2\xi_j/L)(\phi_0/2)^2$ the effective tunable coupling constant between the mediator and the jth qubit. We finally emphasize that the effect of such switch control fluxes introduces an effective coupling between the two qubits involved in the process without changing their Hilbert space, that is without changing their optimal point of work.

Similarly, Plourde *et al.* propose to adjust the coupling strength characterizing the interaction between two 3JJ flux qubits by changing the critical current of a dc SQUID (which is coupled to each of these two qubits), finding that their coupling constant can be changed continuously from positive to negative values (and enables cancellation of the direct mutual inductive coupling if the two qubits are not spatially separated).[40]

Adopting one of these experimental coupling setups, it is therefore possible to realize the step by step scheme in the system of $N+1$ qubits whose Hamiltonian during each of the aforementioned steps takes the following form:

$$H_{Mj}=H_0+H'_{Mj}=H_M+H_1+H_2+\cdots+H_N+H'_{Mj} \quad (j=1,2,\ldots,N). \qquad (10)$$

We observe that the structure of the multipartite Hamiltonian during each step allows us to simplify the study of its dynamics by confining ourselves to the analysis of the dynamics of the bipartite mediator–SQUID-1 system during the time interval $0<t<\tau_1$, to that of the mediator–SQUID-2 system during the second period $\tau_1<t<\tau_1+\tau_2$, and so on (of course, provided that the free evolution of the other qubits is carefully taken into account).

Moreover, by assuming that all the qubits and entanglement mediator M have a common energy gap $\omega=\omega_i$, $\forall i \equiv\{M,1,2,\ldots,N\}$, due to the preponderance of rotating-wave terms of the interaction Hamiltonians with respect to the counter-rotating ones, it is not difficult to convince oneself that, during each step, the system dynamics is dominated by the bipartite Hamiltonian

$$H_{Mj}^{\mathrm{RWA}} = H_M + H_j + H_{Mj}^{\mathrm{RWA}\prime} = \frac{\hbar}{2}\omega_M \sigma_z^{(M)} + \frac{\hbar}{2}\omega_j \sigma_z^{(j)} + g_j(\sigma_+^{(M)}\sigma_-^{(j)} + \sigma_-^{(M)}\sigma_+^{(j)}), \quad \omega_M = \omega_j = \omega, \quad (11)$$

describing the rotating-wave coupling between the mediator and the jth qubit, $\sigma_+^{(i)}$ and $\sigma_-^{(i)}$ $(i=M,1,\ldots,N)$ being the raising and lowering operators for the ith qubit, respectively.

In the following section, we will demonstrate the validity of this assumption. It is interesting, however, to note that Hamiltonian (11) can be exactly implemented if the detuning $\Delta=\omega_M-\omega_j$ between the two flux (3JJ) qubits to be coupled is chosen sufficiently large so that initially each of them can be treated independently. As recently shown by Liu *et al.*,[29] in fact, this gap can be nullified by applying to one of the two SQUID loops a time dependent magnetic flux $\phi_x^{(\ell)}(t)=A_\ell \cos \omega_c^{(\ell)} t$ (with $\ell=M$ or j) satisfying the condition $\Delta \pm \omega_c^{(\ell)}=0$ when $\Delta \lessgtr 0$. This means that, by considering the reduced bias flux of each qubit close to $\phi_0/2$ and the aforementioned frequency-matching condition, it is possible to implement the interaction Hamiltonian $H_{Mj}^{\mathrm{int}} = g_j(\sigma_+^{(M)}\sigma_-^{(j)}+\sigma_-^{(M)}\sigma_+^{(j)})$, where $g_j A_j^2\langle 1_M 0_j | I^{(M)} \cos(\varphi_1^{(j)}+\varphi_2^{(j)} + 2\pi f^{(j)})|0_M 1_j\rangle/2$[29] can be controlled for instance by tuning the amplitude A_j of the time-dependent magnetic flux (TDMF) applied to the jth qubit, $I^{(M)}$ being the qubit loop current of the mediator without an applied TDMF, $f^{(j)}$ and $\varphi_1^{(j)}$, $\varphi_2^{(j)}$ the reduced magnetic flux, and the phase drops across the two identical junctions of the jth 3JJ qubit, respectively.

IV. *W*-STATE GENERATION

Now, we analyze the dynamics of the system and demonstrate that the scheme introduced in the previous section actually generates an entangled *W* state of a tripartite system. We show further that it can be extended to the case of a larger number of qubits.

We begin our analysis by looking at the eigensolutions of Hamiltonian (11), which is represented in the basis $\{|01\rangle_{Mj}, |10\rangle_{Mj}, |00\rangle_{Mj}, |11\rangle_{Mj}\}$ by the following simple diagonal block form:

$$H_{Mj}^{\mathrm{RWA}} = \begin{pmatrix} 0 & g_j & 0 & 0 \\ g_j & 0 & 0 & 0 \\ 0 & 0 & -\hbar\omega & 0 \\ 0 & 0 & 0 & \hbar\omega \end{pmatrix}. \quad (12)$$

These blocks describe three dynamically separate subspaces: the first with characteristic frequency $g_j/\hbar$ characterizing the appearance of the entanglement between the degenerate states $|01\rangle_{Mj}$ and $|10\rangle_{Mj}$, and the other ones describing the fact that the two states $|00\rangle_{Mj}$ and $|11\rangle_{Mj}$ evolve freely. We easily find that the eigenstates of H_{Mj}^{RWA} are

$$|u_{1j}\rangle = \frac{1}{\sqrt{2}}[|10\rangle_{Mj} - |01\rangle_{Mj}], \quad |u_{2j}\rangle = \frac{1}{\sqrt{2}}[|10\rangle_{Mj} + |01\rangle_{Mj}], \quad |u_{3j}\rangle = |00\rangle_{Mj}, \quad |u_{4j}\rangle = |11\rangle_{Mj}, \quad (13)$$

with eigenvalues

$$\lambda_{1j/2j} = \mp g_j, \quad \lambda_{3j/4j} = \mp \hbar\omega. \quad (14)$$

A. Generation of entangled *W* states of the tripartite "*M*+1+2" system

By exploiting the knowledge of the eigensolutions of Hamiltonian H_{Mj}^{RWA}, it is possible to follow step by step the dynamics of the three-qubit system characterized by the one by one interaction of mediator *M* with qubits 1 and 2. We choose as an initial condition the state

$$|1_M 0_1 0_2\rangle. \quad (15)$$

During the first step, we switch on the interaction between *M* and 1 and we let them interact for a time τ_1, while 2 evolves freely. This process generates the state

$$e^{-iH_{M1}\tau_1/\hbar}|1_M 0_1 0_2\rangle = e^{-i(H_0+H_{M1}^{\mathrm{RWA}\prime})\tau_1/\hbar}|1_M 0_1 0_2\rangle = e^{i\omega\tau_1/2}[\cos\theta_1 |1_M 0_1 0_2\rangle - i \sin\theta_1 |0_M 1_1 0_2\rangle], \quad (16)$$

where $\theta_1 \equiv g_1\tau_1/\hbar$. Next, by turning on the interaction between *M* and 2 at $t=\tau_1$ and by allowing qubit 1 evolves freely during the second step (i.e., during the interval of time $\tau_1 < t < \tau_1+\tau_2$), we obtain

$$|\varphi_2\rangle = e^{-iH_{M2}\tau_2/\hbar}e^{-iH_{M1}\tau_1/\hbar}|1_M 0_1 0_2\rangle = e^{i\omega(\tau_1+\tau_2)/2}[\cos\theta_1 \cos\theta_2 |1_M 0_1 0_2\rangle - i\sin\theta_1|0_M 1_1 0_2\rangle - i\cos\theta_1 \sin\theta_2 |0_M 0_1 1_2\rangle]. \quad (17)$$

Equation (17) clearly shows that, by adjusting $\theta_j \equiv g_j\tau_j/\hbar$ $(j=1,2)$ so that

$$|\sin\theta_1| = \frac{1}{\sqrt{3}} \text{ and } |\cos\theta_2| = \frac{1}{\sqrt{2}}, \quad (18)$$

a tripartite *W* state is generated. If we choose $\theta_2=\pi/4$ and θ_1 such that $\sin\theta_1 = 1/\sqrt{3}$ and $\cos\theta_1=\sqrt{2/3}$, for instance, we obtain

$$|W_2\rangle = \frac{1}{\sqrt{3}}e^{i\omega(\tau_1+\tau_2)/2}[|1_M 0_1 0_2\rangle - i|0_M 1_1 0_2\rangle - i|0_M 0_1 1_2\rangle]. \quad (19)$$

It is worth noting that we obtain this state by adjusting the coupling energies g_j during the aforementioned steps and/or by tuning the interaction times τ_j.

B. Generation of entangled *W* states of the multipartite "*M*+1+2+⋯+*N*" system

Within this framework, it is possible to look at the possibility of applying the same techniques in order to generate an entangled state of a multipartite "$M+1+2+\cdots+N$" system with $N>2$. As described in Sec. III we consider an entanglement mediator M in interaction one by one with N qubits. If the system is prepared at $t=0$ in the factorized state

$$|1_M 0_1 0_2 \ldots 0_N\rangle, \tag{20}$$

after a straightforward approach it is easy to show that at the end of the Nth step (namely at $t=t_N\equiv\Sigma_{j=1}^N \tau_j$) it can be described in terms of the state

$$\begin{aligned}|\varphi_N\rangle = e^{i(N-1)\omega t_N/2}[&\cos\theta_1\cos\theta_2\cdots\cos\theta_N|1_M 0_1 0_2\ldots 0_N\rangle \\ &- i\sin\theta_1|0_M 1_1 0_2\ldots 0_N\rangle - \cdots \\ &- i\cos\theta_1\cos\theta_2\cdots\cos\theta_{k-1}\sin\theta_k|0_M 0_1 0_2\ldots 1_k\ldots 0_N\rangle \\ &- \cdots \\ &- i\cos\theta_1\cos\theta_2\cdots\cos\theta_{N-1}\sin\theta_N|0_M 0_1 0_2\ldots 1_N\rangle].\end{aligned} \tag{21}$$

Assuming that it is possible to control the interaction time τ_j of each qubit with mediator M and/or their coupling constants g_j so that

$$\sin\theta_j = \frac{1}{\sqrt{N-j+2}} \text{ and } \cos\theta_j = \sqrt{\frac{N-j+1}{N-j+2}}, \tag{22}$$

we finally find that the generalized W entangled state

$$\begin{aligned}|W_N\rangle = \frac{e^{i(N-1)\omega t_N/2}}{\sqrt{N+1}}[&|1_M 0_1 0_2\ldots 0_N\rangle - i|0_M 1_1 0_2\ldots 0_N\rangle \\ &- i|0_M 0_1 1_2\ldots 0_N\rangle\cdots - i|0_M 0_1 0_2\ldots 1_N\rangle]\end{aligned} \tag{23}$$

of the $(N+1)$-partite system is created.

C. Estimation of the effect of counter-rotating terms

At this stage, we wish to test the validity of the rotating-wave approximation (RWA) performed at the end of Sec. III. To this end, we analyze the fidelity of the system state $|\psi_N\rangle$ calculated without performing the RWA on the Hamiltonian model with respect to the target state (23), i.e., $\mathcal{F}_N = |\langle W_N|\psi_N\rangle|$.

Let us first look at the fidelity for the tripartite W state. After a straightforward calculation, we find that, by following the aforementioned two-step procedure, the $\sigma_x^{(M)}\sigma_x^{(j)}$ couplings generate the state

$$\begin{aligned}|\psi_2\rangle = \frac{1}{\sqrt{3}}e^{i\omega(\tau_1+\tau_2)/2}\Bigg\{&|1_M 0_1 0_2\rangle - i|0_M 0_1 1_2\rangle - ie^{-i\omega\tau_2}\Bigg[\Bigg(\cos\chi_2 \\ &+ i\frac{\hbar\omega}{\sqrt{g_2^2+\hbar^2\omega^2}}\sin\chi_2\Bigg)|0_M 1_1 0_2\rangle \\ &- i\frac{g_2}{\sqrt{g_2^2+\hbar^2\omega^2}}\sin\chi_2|1_M 1_1 1_2\rangle\Bigg]\Bigg\}\end{aligned} \tag{24}$$

with the same θ_1 as that for Eq. (19), where $\chi_2 = \sqrt{g_2^2/\hbar^2+\omega^2}\tau_2$. Its fidelity $\mathcal{F}_2=0.999\,957$, calculated with $g_2/\hbar\approx 0.5$ GHz, $\omega\approx 10$ GHz (in agreement with the currently available experimental values), confirms that during each step the system dynamics is dominated by the bipartite Hamiltonian (11) describing the rotating-wave coupling between the mediator and the jth qubit.

For the $(N+1)$-partite system, the generated state reads

$$\begin{aligned}|\psi_N\rangle = \frac{e^{i(N-1)\omega t_N/2}}{\sqrt{N+1}}\Bigg[&|1_M 0_1 0_2\ldots 0_N\rangle \\ &- i|0_M 0_1 0_2\ldots 1_N\rangle - i\sum_{k=1}^{N-1}\Bigg\{\prod_{j=k+1}^{N} e^{-i\omega\tau_j}\Bigg(\cos\chi_j \\ &+ i\frac{\hbar\omega}{\sqrt{g_j^2+\hbar^2\omega^2}}\sin\chi_j\Bigg)\Bigg\}|0_M 0_1 0_2\ldots 1_k\ldots 0_N\rangle \\ &+ (\text{states orthogonal to}|W_N\rangle)\Bigg]\end{aligned} \tag{25}$$

with the tuning, Eq. (22), where $\chi_j=\sqrt{g_j^2/\hbar^2+\omega^2}\tau_j$, and the fidelity is given by

$$\begin{aligned}\mathcal{F}_N = \frac{1}{N+1}\Bigg|2 + \sum_{k=1}^{N-1}\prod_{j=k+1}^{N} e^{-i\omega\tau_j}\Bigg(&\cos\chi_j \\ &+ i\frac{\hbar\omega}{\sqrt{g_j^2+\hbar^2\omega^2}}\sin\chi_j\Bigg)\Bigg|,\end{aligned} \tag{26}$$

which results in

$$\begin{aligned}\mathcal{F}_3 = 0.999\,873,\quad \mathcal{F}_4 = 0.999\,601,\quad \ldots,\quad \mathcal{F}_{10} \\ = 0.997\,561,\quad \ldots,\quad \mathcal{F}_{20} = 0.993\,959,\text{ etc.},\end{aligned} \tag{27}$$

when $g_1/\hbar=\cdots=g_N/\hbar\approx 0.5$ GHz and $\omega\approx 10$ GHz. The physical meaning of such a list of values is that we might implement our scheme using up to 20 qubits maintaining a very good level of fidelity of the state given by Eq. (25) with respect to the target state expressed by Eq. (23).

V. CONCLUSIONS

In this paper, we have discussed a scheme for the generation of entangled W states among three Josephson (eventually spatially separated) flux qubits as well as its generalization to the case of $N+1$ qubits. The success of the scheme relies on the possibility of realizing controllable couplings between the qubits and the entanglement mediator M and on the possibility of preparing their initial quantum state. It should be stressed that no conditional measurement are required but we have to tune, for instance via the control fluxes $\phi_{cx}^{(i)}$, the coupling energy and/or the interaction time between each qubit and the mediator (see Fig. 3). The conditions on the parameters are explicitly presented in Eq. (22) for an arbitrary number of qubits. Furthermore, it is clarified that the counter-rotating terms give rise to just a negligible error in the generation of the W state.

The key is to find a method to precisely shape fast and simultaneously quasiadiabatic enough magnetic pulses, in such a way to avoid exciting upper noncomputational states of the qubits and further reducing the qubit relaxation and decoherence times, T_1 and T_2. Currently, rapid-single-flux-quantum (RSFQ) Josephson-junction based logic circuits[37,41] make it possible to produce flux pulses characterized by rise and/or fall times $t_{r/f}$ of the order of 10 ps. These devices seem to be very promising candidates for controlling a superconducting flux circuit like the one proposed here. Indeed, t_r and t_f are much smaller than the duration τ_j of any step in our scheme, typically less or of the order of $\hbar/g_j \approx 2$ ns. It is also of relevance that such RSFQ Josephson circuits take advantage of a technology which makes it possible to integrate a scalable large number of components on the same chip. Moreover, we wish to underline that there is a wide theoretical and experimental research activities focused on the development of the so-called unconventional RSFQ logic,[42] by which it might be possible to avoid reductions in T_1 and T_2 as well as the occurrence of transitions to excited noncomputational states of the qubits.

Nowadays, on the basis of both experimental estimations and theoretical analyses of the decoherence in superconducting flux qubits, we find that the operation duration (e.g., the time necessary to perform the desired quantum process) is small enough with respect to the decoherence time. On the one hand, in fact, the eigenfrequency of a Josephson qubit is of the order of 10 GHz and that correspondingly $g_j/\hbar \approx 0.5$ GHz.[8,29] Under such conditions, the length of each step (during which only a fraction of a Rabi oscillation takes place) is at most of the order of $\hbar/g_j \approx 2$ ns and consequently the whole process in the case, for instance, of a tripartite system lasts approximatively 4 ns. On the other hand, the relaxation and decoherence times, T_1 and T_2, of a superconducting flux qubit have been coarsely estimated and are in the range 1–10 μs.[8,37,43] Therefore, the number of operations required in our scheme to realize the step by step couplings of the mediator with the other two qubits can be performed before decoherence occurs. Extending this argument from a $(2+1)$-partite system to an $(N+1)$-partite system, the total duration for the preparation of a $|W_N\rangle$ state as in our scheme scales with N as $N\hbar/g_j \approx 2N$ ns. This time once more should be compared, with extreme caution, with the estimated decoherence time only with the scope of achieving a very rough idea of the maximum number of qubits which may be involved in the preparation of such W states. With T_1 and T_2 of the order of 1 μs, we feel that the realizability of $|W_N\rangle$ states with $N \leqslant 10$ is compatible with the currently available experimental setups.

Generally speaking, the ability to generate a multipartite entangled state raises, at least in principle, the question of how the prepared state can be measured as well as how the possibly developed entanglement may be quantified. These prominent issues of quantum mechanics and quantum information theory successfully dealt with the case of bipartite systems for which Wootters[44] introduced the powerful concept of the entanglement of formation. Unfortunately, this task becomes very difficult for many-particle systems. Some interesting attempts, yet under the *magnifying glass* of scientific community, include for instance applications of the relative entropy,[45] negativity,[46] Schmidt measure,[47] geometric measure of entanglement,[48] Mermin-Klyshko inequality,[49] and state preparation fidelity,[50] and some of them in particular have been proposed to quantify the entanglement of multipartite W states. A second issue is the measurement of the output quantum states. This goal can be achieved via a full tomographic reconstruction of the system states, as indicated both from theoretical[51] and experimental investigations on the motional quantum states of trapped atoms,[52] the internal states of trapped ions,[53] the states of photons,[54,55] and multiple spin-1/2 nuclei.[56] A method for the tomographic reconstruction of the states of qubits for a general class of solid-state systems has been quite recently proposed in Ref. 57 and successively applied to the case of superconducting qubits.[58] This last reference indicates that the tomography of a single superconducting (charge) qubit is in the grasp of experimentalists, while in the case of multiple qubits this process is not readily realizable. Nevertheless, there are other promising methods that can be exploited for estimating the fidelity of state preparation in multiqubit systems. It has been shown indeed that many useful entangled states (including W states) have certain symmetries allowing a more efficient fidelity determination via a reduced number of experimental measurements,[59] namely without performing full quantum state tomography.

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