Many electron theory of $1/f$ noise in hopping conductivity

A. L. Burin,[1] B. I. Shklovskii,[2] V. I. Kozub,[3,4] Y. M. Galperin,[5,3,4] and V. Vinokur[4]
[1]*Department of Chemistry, Tulane University, New Orleans, Louisiana 70118, USA*
[2]*William P. Fine Institute of Theoretical Physics, School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA*
[3]*A. F. Ioffe Physico-Technical Institute of Russian Academy of Sciences, 194021 St. Petersburg, Russia*
[4]*Argonne National Laboratory, 9700 South Cass Avenue, Argonne, Illinois 60439, USA*
[5]*Department of Physics, University of Oslo, P.O. Box 1048 Blindern, 0316 Oslo, Norway*
(Received 20 December 2005; revised manuscript received 19 May 2006; published 16 August 2006)

We show that $1/f$ noise in the variable-range hopping regime is related to transitions of many-electrons clusters (fluctuators) between two almost-degenerate states. Giant fluctuation times necessary for $1/f$ noise are provided by a slow rate of simultaneous tunneling of many localized electrons and by large activation barriers for their consecutive rearrangements. The Hooge constant steeply grows with decreasing temperature because it is easier to find a slow fluctuator at lower temperatures. Our conclusions qualitatively agree with the low-temperature observations of $1/f$ noise in *p*-type silicon and GaAs.

DOI: 10.1103/PhysRevB.74.075205 PACS number(s): 71.23.Cq, 72.70.+m, 72.20.Ee, 72.80.Sk

I. INTRODUCTION

At low temperatures the variable-range-hopping conductivity of doped semiconductors with strongly localized electrons obeys the Efros-Shklovskii (ES) law[1–3]

$$\sigma_{\mathrm{ES}} = \sigma_0 \exp\left[-\left(\frac{T_{\mathrm{ES}}}{T}\right)^{1/2}\right], \tag{1}$$

where the temperature T_{ES} is defined by the electron-electron interaction at the localization radius a of electronic states, σ denotes the conductivity of the system, and

$$T_{\mathrm{ES}} = \frac{Ce^2}{k_B \kappa a}. \tag{2}$$

Here $C \simeq 2.7$, e is the electron charge, k_B is the Boltzmann constant, and κ is the dielectric constant of the semiconductor. The conductivity behavior [Eq. (1)] is used, for example, in ion-implanted silicon (Si:P:B) bolometers working as detectors for high-resolution astronomical x-ray spectroscopy.[4,5] Absorption of an x ray increases the temperature of the semiconductor and this increase is detected by the change in its conductivity. The performance of some bolometers is limited by a $1/f$ noise, which obeys the Hooge's law[6,7]

$$\frac{\delta\sigma_\omega^2}{\sigma^2} = \frac{\alpha_H(\omega,T)}{\omega N_D}, \tag{3}$$

where N_D is the total number of donors,

$$\delta\sigma_\omega^2 \equiv \int_{-\infty}^{+\infty} dt e^{i\omega t}\langle \delta\sigma(t)\delta\sigma(0)\rangle, \quad \delta\sigma(t) \equiv \sigma(t) - \langle\sigma(t)\rangle,$$

and $\langle\cdots\rangle$ denotes ensemble average. The dimensionless Hooge factor $\alpha_H(\omega,T)$ measured for different doping levels grows by six orders of magnitude with the decreasing temperature following approximate power law[4,5,8]

$$\alpha_H \propto T^{-6}. \tag{4}$$

Reference 8 gives strong evidence that the noise is caused by the electron localization. This work investigates the bulk Si:P:B semiconductor in the range of dopant concentrations on both sides of the metal-insulator transition. The increase of the noise strength by several orders of magnitude, when crossing the metal-insulator transition from metal-like samples to insulating samples, was observed. The noise intensity continues to increase with decreasing electron localization radius a, which proves the primal significance of the electron localization in the noise formation. A similar behavior of the normalized $1/f$-noise power has been recently reported in the low-density hole system of a GaAs quantum well.[9] Thus, these and other experiments[10–13] show that $1/f$ noise is one of the manifestations of the complex correlated electronic state (Coulomb glass) formed by localized electrons coupled by the long-range Coulomb interaction.

The mechanism of $1/f$ noise in hopping conductivity has been investigated by several theoretical groups.[14–17] It was first suggested[14] that the $1/f$ noise in the nearest-neighbor-hopping transport is associated with electronic traps, in a way similar to McWorter's idea of $1/f$ noise in metal-oxide-semiconductor field-effect transistors (MOSFETs).[18] Each trap consists of an isolated donor within a spherical pore of the large radius r. Such rare configurations form *fluctuators*, which have two possible states (empty or occupied) switching back and forth with the very slow rate defined by the tunneling rate of electron out or into the pore

$$\nu(r) = \nu_0 \exp(-2r/a), \tag{5}$$

where $\nu_0 \sim 10^{12}$ s^{-1} is the hopping rate determined by the electron-phonon interaction. When a fluctuator is occupied the host electron cannot participate in the transport and thus the transitions of fluctuators change the effective number of "charge carriers" leading to a noise in a conductivity. The exponential sensitivity of the tunneling rate to the size r of the trap [Eq. (5)] makes the statistics of trap blinking close to the logarithmically uniform one leading to $\delta\sigma_\omega^2 \propto 1/\omega$.

The idea of a single-electron fluctuator has been extended to the regime of the variable-range hopping in later works.[16,17] These theories require that each fluctuator characterized by the slow relaxation rate ν has no neighboring

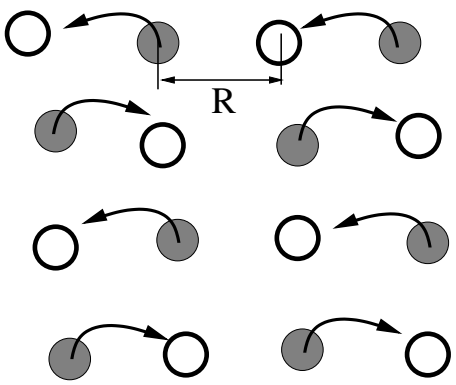

FIG. 1. A chessboard cluster of donors, responsible for a $1/f$ noise. The dark circles indicate occupied (neutral) donors, while the open circles indicate empty donors. All donors are in the energy band of the width $2e^2/\kappa R$ around the chemical potential ($E=0$). Arrows show the direction of the electron transition between two energy minima corresponding to two possible ways to occupy cluster sublattices. The length R stands for the size of the cluster cell.

donors, which belong to the energy band around the Fermi level of the width $E_\nu = k_B T \ln(\nu_0/\nu)$ in the sphere of the radius $R_\nu = (a/2)\ln(\nu_0/\nu)$ (here and everywhere below we choose the chemical potential as a reference point of energy). According to Ref. 17, the Hooge parameter is defined by the probability of such a "pore" in four-dimensional energy-coordinate space, which has the form

$$-\ln \alpha_H(\omega,T) \propto \left(\frac{T}{T_{\rm ES}}\right)^3 \ln^6 \frac{\nu_0}{\omega} = \left[\frac{\ln(\nu_0/\omega)}{\ln(\nu_0/\nu_{\rm ES})}\right]^6, \quad (6)$$

where $\nu_{\rm ES} = \nu_0 e^{-(T_{\rm ES}/T)^{1/2}}$ is the rate of typical hops contributing to Eq. (1). At small ω this Hooge parameter decreases very fast with decreasing ω. As a result, $1/f$ noise is limited to a relatively narrow frequency interval $(T_{\rm ES}/T)^{1/2} < \ln(\nu_0/\omega) < (T_{\rm ES}/T)^{3/5}$, where the dependence of $\alpha_H(\omega)$ is still weaker than $1/\omega$. Observation of $1/f$ noise in a wider range of frequencies $\omega \ll \nu_{\rm ES}$ still remains a challenge for the theory.

In this work we suggest the model of many-electron fluctuators, which possess small relaxation rates ν with much larger probability than single-electron traps. A new fluctuator is made of N occupied (neutral) and N empty (positively charged) donor sites quasiordered into the quasicubic lattice with the period R. They include only donors with energies within a band with the width

$$U_R \equiv \frac{e^2}{\kappa R}, \quad (7)$$

which are required to have somewhat diminished disorder energies (see below). We assume that the cluster (Fig. 1) has two energy minima with an energy difference of the order of $k_B T$ realized by two disordered chessboard configurations (Fig. 1). Optimizing the "lattice constant" R and the number of electrons N, we show below that a $1/f$ noise associated with such clusters behaves as

$$-\ln \alpha_H(\omega,T) \sim \left(\frac{T}{T_{\rm ES}}\right)^{3/5} \ln^{6/5} \frac{\nu_0}{\nu} = \left[\frac{\ln(\nu_0/\omega)}{\ln(\nu_0/\nu_{\rm ES})}\right]^{6/5}. \quad (8)$$

At small frequencies the absolute value of the exponent in Eq. (8) is apparently much smaller than that for the single-electron trap [Eq. (6)]. As a result, $1/f$ noise exists in the much broader range of frequencies,

$$(T_{\rm ES}/T)^{1/2} \ll \ln(\nu_0/\omega) \ll (T_{\rm ES}/T)^3.$$

At low temperatures, where $(T_{\rm ES}/T)^{1/2} \geqslant 6$ [see Eq. (26) and the discussion afterwards for details], this range is practically unlimited.

Our approach is similar to the previous analysis of clusters of many local two-level defects[19–21] suggested to interpret the universal behavior of amorphous solids. Long relaxation times have been achieved in Ref. 19 due to the exponential increase in the relaxation time with the number of defects belonging to a single cluster (note that the results of Ref. 19 cannot be directly applied to our system because they are obtained in the zero-temperature limit, i.e., ignoring thermally activated cluster transitions). The importance of strong electron-electron correlations and the many-electron nature of fluctuators determining $1/f$ noise was anticipated in Ref. 22.

The manuscript is organized as follows. In Sec. II we introduce the model of interacting localized electrons in a lightly doped semiconductor. In Sec. III we describe chessboard clusters and their statistics with respect to energies and relaxation rates. In Sec. IV we consider the noise in the variable-range-hopping conduction, induced by chessboard clusters. In Sec. V we report the generalization of our results to two-dimensional (2D) systems and amorphous semiconductors, where the variable-range-hopping conductivity may obey the Mott law.

II. MODEL OF A LIGHTLY DOPED SEMICONDUCTOR

Let us consider an n-type lightly doped semiconductor with the concentration n_d of donors and the concentration $n_a < n_d$ of acceptors. At low temperatures all acceptors are charged negatively, n_a donors are positively charged, while $n_d - n_a$ are occupied and neutral. The set of filling factors of donors $n_i = 0, 1$ is determined by the energy minimum of the Hamiltonian of the classical impurity band[2]

$$\hat{H} = \sum_i \left[\phi_i n_i + \frac{1}{2}\frac{e^2}{\kappa}\sum_{k\neq i}\frac{(1-n_i)(1-n_k)}{|\mathbf{r}_i - \mathbf{r}_k|}\right], \quad (9)$$

where $\phi_i = (e^2/\kappa)\Sigma_j |\mathbf{r}_i - \mathbf{r}_j|^{-1}$ is the random potential created on the donor i by all acceptors (enumerated by j). In the ground state, energies of donor states are

$$\epsilon_i = \phi_i - \frac{e^2}{\kappa}\sum_{k\neq i}\frac{1-n_k}{|\mathbf{r}_i - \mathbf{r}_k|}. \quad (10)$$

It is known[1,2] that the density of states $g(\epsilon)$ of such a system has a double-peak structure with the soft Coulomb gap

$$g(\epsilon) = \frac{3}{\pi}\frac{\epsilon^2\kappa^3}{e^6} \tag{11}$$

in the middle. At $n_a \sim n_d/2$ the width of the Coulomb gap in the density of states is comparable with the total width of the peak of the density of donor states. It is the Coulomb gap which leads to the ES law [Eq. (2)]. A characteristic length of hops leading to Eq. (1) is $R_{ES}=(a/2)(T_{ES}/T)^{1/2}$ and a characteristic width of the band of energies of states contributing to Eq. (1) is $E_{ES}=(T_{ES}T)^{1/2}$ (see Refs. 1–3).

III. LONG-LIVING CHESSBOARD CLUSTERS

A. Chessboard cluster and its probability

Chessboard clusters (see Fig. 1) are formed by the rare fluctuations in positions and energies of donors. We require the following conditions to be satisfied for the cluster. First, all occupied and empty donors i belonging to the cluster are placed approximately near the sites R_i of a cubic lattice with the period R. This requires

$$|\mathbf{r}_i - \mathbf{R}_i| < \eta_R R, \quad \eta_R \leqslant 1/2, \tag{12}$$

where $\eta_R \sim 1$ is the parameter restricting the deviations of electrons and holes belonging to the cluster from the cell centers.

In addition, all donor sites i of the cluster interact with the environment. This interaction includes the disorder energy ϕ_i produced by acceptors and the Coulomb interaction with "external" empty donors, which do not belong to the cluster because they either have energies larger than U_R or spatially are away from the cluster

$$\varphi_i = \phi_i - \frac{e^2}{\kappa}{\sum_k}' \frac{1-n_k}{r_{ik}}, \tag{13}$$

where Σ' indicates that the sum is taken over the "external" donors only. For all donors of the cluster disorder energy φ_i is required to be smaller than the characteristic energy U_R [Eq. (7)] of the Coulomb interaction within the cluster

$$\varphi_i < \eta_E U_R, \quad \eta_E < 1. \tag{14}$$

This second requirement involves the energy-constraint parameter η_E, which should be sufficiently small to keep the two minimal-energy states chessboardlike (Fig. 1).

First, we evaluate the probability p_s for each chessboard cell to have the donor located nearby its center with the diminished disorder energy. This probability is given by the product of the density of electronic states within the domain of the allowed energy $\int_0^{\eta_E U_R} g(E)dE$ [see Eq. (14)] and the allowed volume $4\pi\eta_R^3 R^3/3$ [see Eq. (12)]. Then, using the definition [Eq. (11)] of the density of states within the Coulomb gap we get

$$p_s = 4\eta_R^3\eta_E^3/3. \tag{15}$$

As discussed previously, $\eta_E < 1$ and $\eta_R < 1/2$ in order to have some ordering of the structure. These two constraints make the probability p_s substantially smaller than 1. It is convenient to express the probability p_s in the exponential form as $p_s=\exp(-\lambda/2)$. The probability to satisfy constraints [Eqs. (12) and (14)] for all $2N$ sites is given by the product of all $2N$ probabilities p_s,

$$p_{tot} = p_s^{2N} \approx \exp(-\lambda N). \tag{16}$$

Thus the probability to form the chessboard cluster with two energy minima separated by the large potential barrier decreases exponentially with the number of electrons within the cluster. It is important that within the Coulomb gap the parameter λ is the number of order of unity.

To characterize $1/f$-conductivity noise induced by clusters, one has to define the cluster unit-volume probability density $P(N,\Delta,R)$ of a transition energy Δ, a cell size R, and a number of donors N. By definition, the product $P(N,\Delta,R)dN\,dRd\Delta$ characterizes the number of clusters in the unit volume with the cell size in the domain $(R,R+dR)$, energy in the range $(\Delta,\Delta+d\Delta)$, and the number of donors in the range $(N,N+dN)$. The main dependence of the cluster density on the number of donors N is given by the exponent [Eq. (16)]. Next, we should define the preexponential factor. We will see that since the probability rapidly decreases with increasing N, the characteristic N is not very large and therefore, calculating the preexponential factor we can approximately ignore its sensitivity to N. We ignore the weak logarithmic dependence of cluster density on the energy Δ due to the dipole gap. Then, the probability density of clusters $P(\Delta,R,N)$ does not depend on Δ and can be expressed in the following way:

$$P(N,\Delta,R) \approx \frac{e^{-\lambda N}}{R^4 U_R} = \frac{\kappa e^{-\lambda N}}{e^2 R^3}. \tag{17}$$

Indeed, the density distribution of the parameters N, Δ, and R is given by the probability of finding a chessboard cluster having N donors, $p_{tot} \propto e^{-\lambda N}$, divided by the cluster's volume, NR^3, and the typical energy bandwidth of these donors, $U_R\sqrt{N}$. The probability density of clusters is then

$$P(N,\Delta,R) \propto \frac{\partial}{\partial R}\frac{p_{tot}}{N^{3/2}R^3 U_R}.$$

Below, we will keep track of the cluster size N only in the exponent because the power-law dependence on N in the preexponential factor is much weaker than the main exponential dependence. In this way we arrive at Eq. (17). This expression can be written starting just from the dimensionality requirement. The function $P(N,\Delta,R)$ cannot depend on other system parameters because of the universality of excitations within the Coulomb gap.[1,2]

B. Transition rate

Transition between two states A and C of a chessboard cluster (Fig. 2) can be made via a consecutive transition of single electrons. In the beginning this leads to a monotonous growth of energy until a proper saddle point is reached. Thus the rate of a thermally activated transition between the two states of fluctuator can be estimated using the minimum-activation energy barrier E_A separating two configurations. For the "chessboard" structure in Fig. 2 a minimum potential

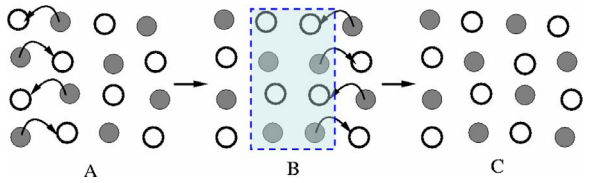

FIG. 2. (Color online) Thermally activated transition of the "chessboard" cluster between two energy minima A and C along the route $A \rightarrow B \rightarrow C$. The activation energy is proportional to the number of electrons $N^{2/3}$ in the domain wall shown in panel (B) by dashed lines.

barrier (saddle point) is defined by the energy of the "domain-wall" formation. One has to move all electrons in the one (leftmost) column of the cluster A (see Fig. 2) to the next positions B to the right side. The activation energy of that move reads $E_A \approx N^{2/3} U_R$ because the number of electrons belonging to a domain wall is $(N^{1/3})^2 = N^{2/3}$. Then, this domain wall can move in the system without acquiring or releasing a significant energy. When the domain wall reaches the right side the transition is completed. The rate of this process can be estimated using the standard Arrhenius law with the activation energy E_A;

$$\nu_A = \nu_0 e^{-N^{2/3} U_R / k_B T}. \tag{18}$$

An alternative process of transition between energy minima A and C (Fig. 2), is simultaneous tunneling of all electrons of the cluster. This transition of all N electrons takes place in the Nth order of the perturbation theory in the weak tunneling amplitude of an electron $t \propto e^{-R/a}$, so it can be approximately described by the exponential law

$$\nu_{\text{tun}} \approx \nu_0 e^{-2NR/a}. \tag{19}$$

The overall cluster-relaxation rate ν can be approximately expressed as the sum of the thermally activated and tunneling rates [Eqs. (18) and (19)]

$$\nu = \nu_0 \left[\exp\left(- N^{2/3} \frac{U_R}{k_B T} \right) + \exp\left(- 2N \frac{R}{a} \right) \right]. \tag{20}$$

C. Distribution of clusters over their relaxation rates

The distribution function of "chessboard" clusters over their relaxation rates ν (inverse relaxation times) can be calculated using Eqs. (17) and (20). To calculate this distribution function it is convenient to invert the function $\nu(N,R)$ given by Eq. (20) to obtain $N(\nu,R)$. The function $N(\nu,R)$ crosses over from $N_{\text{ac}}(\nu,R) = (k_B T / U_R)^{3/2} \ln^{3/2}(\nu_0/\nu)$ to $N_{\text{tun}}(\nu,R) = (a/2R) \ln(\nu_0/\nu)$ at

$$R_c \approx a \left(\frac{T_{\text{ES}}}{T} \right)^{3/5} \ln^{-1/5} \frac{\nu_0}{\nu},$$

$$N_c \approx \left(\frac{T_{\text{ES}}}{T} \right)^{-3/5} \ln^{6/5} \frac{\nu_0}{\nu}. \tag{21}$$

These values correspond to the case when the thermal-activation and tunneling-transition rates in Eq. (20) are equal to each other. Indeed, at small R tunneling is more frequent than a thermally activated motion. Then, according to Eq. (20), at a given ν we have $N \propto R^{-1}$. Since the cluster probability density $P(N,\Delta,R)$ decreases with N [Eq. (17)], it grows with R. This increase takes place until the crossover point R_c, where the thermal activation becomes comparable with the tunneling. A further increase of R will reduce the cluster probability, because for thermally activated processes the number of cluster sites grows as $N \sim R^{3/2}$. Therefore, the quantities R_c and N_c are the *optimum* parameters, which are defined by the crossover between thermal activation and tunneling regimes

$$\nu/\nu_0 = e^{-2N_c R_c/a} = e^{-N_c^{2/3} e^2/\kappa R_c k_B T}. \tag{22}$$

The cluster probability density $f(\nu,\Delta)$ can be found using the previously defined probability density $P(N,\Delta,R)$ [Eq. (17)] as

$$\begin{aligned} f(\nu,\Delta) &= \int_0^\infty dR \sum_{N=1}^{\infty} P(N,R,\Delta) \\ &\quad \times \delta[\nu - \nu_0 (e^{-N^{2/3} U_R/k_B T} + e^{-2NR/a})] \\ &\approx \frac{1}{\nu R_{\text{ES}}^3 k_B T_{\text{ES}}} e^{-\lambda N_c} \\ &= \frac{1}{\nu R_{\text{ES}}^3 k_B T_{\text{ES}}} \exp\left[- \lambda \left(\frac{T}{T_{\text{ES}}} \right)^{3/5} \ln^{6/5} \frac{\nu_0}{\nu} \right]. \end{aligned} \tag{23}$$

The above equation is valid only if $N_c \gtrsim 1$, which means $\nu \ll \nu_{\text{ES}}$ and $R < R_{\text{ES}}$. At $\nu \sim \nu_{\text{ES}}$ we arrive at $R = R_{\text{ES}}$ and $N_c = 1$, and clusters instead of slow modulating of ES hopping conductivity become a part of the conducting network of the ES variable-range hopping. As discussed previously, we ignore the dependence of the preexponential factor on $N \sim N_c$, which yields $R_c \approx R_{\text{ES}}$, where $R_{\text{ES}} = a(T_{\text{ES}}/T)^{1/2}$ is ES hopping length.

One can approximate Eq. (23) by the power law

$$f(\nu,\Delta) \propto \frac{1}{\nu} \left(\frac{\nu}{\nu_0} \right)^{\beta}, \tag{24}$$

with

$$\beta = - \frac{d \ln f}{d \ln \nu} + 1 = \frac{6\lambda}{5} \left(\frac{T}{T_{\text{ES}}} \right)^{3/5} \ln^{1/5} \frac{\nu_0}{\nu}. \tag{25}$$

Obviously, the statistics $1/\nu$ is applicable if the exponent $\beta \ll 1$. According to Eq. (24), this takes place at frequencies $\nu \gg \nu_{\text{min}}$, where

$$\nu_{\text{min}} \approx \nu_0 \exp[- (5/6\lambda)^5 (T_{\text{ES}}/T)^3]. \tag{26}$$

In this region the exponential term in Eq. (23) depends on ν only weakly, so the main dependence is defined by the $1/\nu$ distribution. As we mentioned in the Introduction, ν_{min} cannot be distinguished clearly from zero at low temperature. Our estimate of the unknown constant $\lambda \approx 2.2$ (see Sec. VI and Fig. 3) based on the comparison of the theory and the experiment leads to the approximate constraint $(T_{\text{ES}}/T)^{1/2} > 6$ required to have the small deviations from $1/f$-noise behavior [not to exceed 1/2 in accordance with Eq. (25)] in

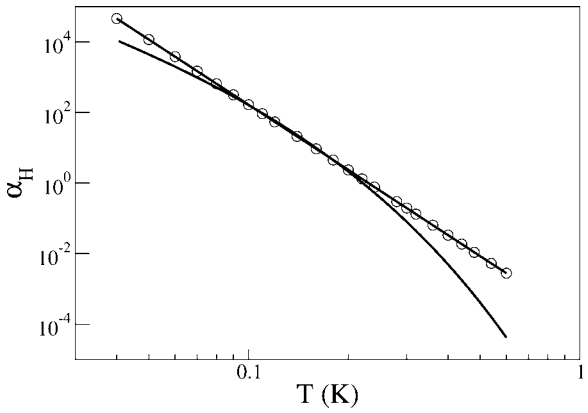

FIG. 3. Comparison of the theory [solid line, Eq. (30)] and the experiment [solid line with circles, Eq. (34)] for the case $\omega/2\pi = 1$ Hz and $T_{ES}=11$ K. We have used $\lambda=2.2$ and $n_d a^3=0.0084$ to make the optimum data fit in the range of temperatures (0.07–0.3 K) studied in the experiment.

the range of frequences $\nu \sim 1$ Hz. Measurements of $1/f$ noise in hopping conductivity[4,5] have been performed at temperatures satisfying the above constraint.

Note that the realistic variable-range-hopping measurements are done typically close to metal-insulator transition, where $n_d a^3 \simeq 1$. In this case, the inequality $\nu \gg \nu_{\min}$ always means that $R \gg n_d^{-1/3}$ and U_R does not exceed the width of the Coulomb gap. However, if in the lightly doped limit $n_d a^3 \ll 1$, strictly speaking, the distribution Eq. (23) is applicable within the Coulomb gap only, when $R > n_d^{-1/3}$. It can be shown that the case $a < R < n_d^{-1/3}$, where U_R exceeds the width of the Coulomb gap, only a minor revision of our theory is needed. Namely, the parameter λ should be increased by a logarithmically large factor.

IV. CONDUCTIVITY NOISE

Below we discuss how cluster transitions induce a $1/f$ noise of the electric current. The hopping conductivity [Eq. (1)] is provided by the critical network of resistances percolation cluster.[2] According to the Shklovskii–De Gennes model of infinite percolation cluster,[2] one can view this network as the random lattice formed by one-dimensional links of the approximate length $R_T \sim aT_{ES}/T$. Each link is a chain of $\sqrt{T_{ES}/T}$ donor sites separated by the hopping length $R_{ES} \sim a\sqrt{T_{ES}/T}$.

Following the ideas of Kozub,[15] we consider the conduction noise induced by the cluster transitions affecting adjacent links. A link has the critical hop with the largest resistance $r_h \propto e^{\sqrt{T_{ES}/T}}$, which is comparable with the resistance of the whole chain. The noise is defined by the clusters located in the vicinity of these critical sites. Assume that the critical hop is separated from the nearest-neighbor cluster by the distance R and this cluster makes slow transitions between its two energy minima. The transition of the cluster changes its characteristic dipole moment by the value $\mu \sim eRN^{1/2} \sim eR_{ES}$. The chessboard dipole potential leads to the energy fluctuation of the critical hop $\delta E \sim e^2 R_{ES}/R^2$. If this energy exceeds the thermal energy T, or

$$R < R_{\text{int}} = a(T_{ES}/T)^{3/4}, \tag{27}$$

the resistance of the critical hop changes substantially, leading to the addition or removal of the whole link. The fluctuation of the sample conductivity induced by a change of one link can be expressed as

$$\delta\sigma \sim \frac{\sigma_{ES}}{N_l} = \sigma_{ES}\frac{R_T^3}{V}, \tag{28}$$

where V is the system volume and the ratio $N_l \approx V/R_T^3$ estimates the total number of links.

Only clusters with the energy Δ comparable to the thermal energy are able to contribute to the noise, while the contribution of others is exponentially suppressed. Therefore, the relevant clusters density for the noise at frequency ω is $n_\omega \approx k_B T f(\omega, k_B T)$, where $f(\omega, k_B T)$ is given by Eq. (23). The probability p_l that the given link has the cluster located nearby [Eq. (27)] can be expressed through the cluster density as $p_l \approx f(\omega, k_B T) k_B T R_{\text{int}}^3$. Then the noise at the frequency ω is induced by $N_\omega \sim p_l N_l$ clusters making random contributions. Using Eq. (28) one can express the noise intensity as

$$\frac{\delta\sigma_\omega^2}{\sigma_{ES}^2} \sim \frac{N_\omega}{N_l^2} = \frac{p_l}{N_l}. \tag{29}$$

Using the latter equation one arrives at the Hooge parameter

$$\alpha_H = n_d a^3 \left(\frac{T_{ES}}{T}\right)^{11/4} \exp\left[-\lambda\left(\frac{T}{T_{ES}}\right)^{3/5} \ln^{6/5}\frac{\nu_0}{\nu}\right]. \tag{30}$$

The main exponential term of Eq. (30) apparently agrees with Eq. (8). According to this equation, at low temperature the prefactor of α_H is much larger than unity. This happens because standard normalization of α_H to the total number of donors (or electrons) is not natural for the variable-range hopping, where only a small fraction of all donors participates in transport.

It is the straightforward consequence of the cluster distribution given by Eqs. (23) and (30) that for ES hopping, the fluctuators, defined by the single electron pores[17] have much smaller density [see Eq. (6)] and they can always be neglected. Note that this is not the case for Mott variable-range hopping as discussed in Sec. V.

In the derivations of Eqs. (23) and (30) we assumed that $E_{ES}=k_B(TT_{ES})^{1/2} \ll e^2 n_d^{1/3}/\kappa$, $R_{ES}=(a/2)(T_{ES}/T)^{1/2} \gg n_d^{-1/3}$, and the variable-range hopping takes place. If these conditions are violated and $k_B T \gg e^2 n_d^{2/3} a/\kappa$, one deals with the nearest-neighbor hopping conductivity, where the Coulomb interaction plays a secondary role (see Ref. 2). In this case there is a range of not very small frequencies

$$\frac{1}{n_d^{1/3}a} < \ln\frac{\nu_0}{\omega} < \frac{1}{n_d^{1/3}a}\left(\frac{kk_BT}{e^2 n_d^{2/3}a}\right)^{1/3},$$

where $1/f$ noise is determined by single-electron pores.[14] At very small frequencies again the chessboard clusters take over.

Returning to the variable-range hopping we can generalize our theory to a two-dimensional system. In this case the density of states is proportional to $|\epsilon|$ rather than to ϵ^2.[3] In

TABLE I. Electronic density of states, parameters of the variable-range hopping in $2d$ and $3d$ systems, $1/f$-noise produced by chessboard clusters, and single-pore electrons and their comparison.

d	T	$g(\epsilon)$	$-\ln \sigma/\sigma_0$	$T_{\mathrm{M/ES}}$	$-(1/\lambda)\ln \alpha_H$	$-\ln \alpha_H^{\mathrm{pore}}$	$\alpha_H < \alpha_H^{\mathrm{pore}}$
2	$T < \frac{T_{\mathrm{ES}}^3}{T_{\mathrm{M}}^2}$	$\frac{2}{\pi}\frac{\epsilon\kappa^2}{e^4}$	$\left(\frac{T_{\mathrm{ES}}}{T}\right)^{1/2}$	$T_{\mathrm{ES}} \sim \frac{e^2}{k_B \kappa a}$	$\left(\frac{T}{T_{\mathrm{ES}}}\right)^{2/3} \ln^{4/3}\frac{\nu_0}{\omega}$	$\left(\frac{T}{T_{\mathrm{ES}}}\right)^{2} \ln^{4}\frac{\nu_0}{\omega}$	n/a
2	$\frac{T_{\mathrm{ES}}^3}{T_{\mathrm{M}}^2} < T$	g_0	$\left(\frac{T_{\mathrm{M}}}{T}\right)^{1/3}$	$T_{\mathrm{M}} \sim \frac{1}{k_B g_0 a^2}$	$\left(\frac{T}{T_{\mathrm{ES}}}\right)^{2/3} \ln^{4/3}\frac{\nu_0}{\omega}$	$\frac{T}{T_{\mathrm{M}}} \ln^{3}\frac{\nu_0}{\omega}$	$\ln\frac{\nu_0}{\omega} < \left(\frac{T_{\mathrm{M}}^3}{T T_{\mathrm{ES}}^2}\right)^{1/5}$
3	$T < \frac{T_{\mathrm{ES}}^2}{T_{\mathrm{M}}}$	$\frac{3}{\pi}\frac{\epsilon^2\kappa^2}{e^4}$	$\left(\frac{T_{\mathrm{ES}}}{T}\right)^{1/2}$	$T_{\mathrm{ES}} \sim \frac{e^2}{k_B \kappa a}$	$\left(\frac{T}{T_{\mathrm{ES}}}\right)^{3/5} \ln^{6/5}\frac{\nu_0}{\omega}$	$\left(\frac{T}{T_{\mathrm{ES}}}\right)^{3} \ln^{6}\frac{\nu_0}{\omega}$	n/a
3	$\frac{T_{\mathrm{ES}}^2}{T_{\mathrm{M}}} < T$	g_0	$\left(\frac{T_{\mathrm{M}}}{T}\right)^{1/4}$	$T_{\mathrm{M}} \sim \frac{1}{k_B g_0 a^3}$	$\left(\frac{T}{T_{\mathrm{ES}}}\right)^{3/5} \ln^{6/5}\frac{\nu_0}{\omega}$	$\frac{T}{T_{\mathrm{M}}} \ln^{4}\frac{\nu_0}{\omega}$	$\ln\frac{\nu_0}{\omega} < \left(\frac{T_{\mathrm{M}}^{5/2}}{T T_{\mathrm{ES}}^{3/2}}\right)^{1/7}$

addition, the "domain-wall" energy is proportional to $\sqrt{N}$, while the cluster volume is proportional to R^2. Taking these changes into account and following the previous derivation we arrive at the expression

$$\alpha_H^{(2d)}(\omega,T) \propto \exp\left[-\lambda\left(\frac{T}{T_{\mathrm{ES}}}\right)^{2/3} \ln^{4/3}\frac{\nu_0}{\omega}\right]. \tag{31}$$

We see that in 2D the $1/f$-noise amplitude α_H is smaller than in 3D. This happens because of the smaller activation energy in 2D systems compared to 3D systems (see Sec. III B). Similarly to a 3D case, in 2D a deviation from $1/f$ behavior becomes large at very small frequencies. The criterion for $1/f$ noise reads

$$\nu_{\mathrm{ES}} \gg \omega \gg \omega_{\min} \approx \nu_0 \exp\left[-\left(\frac{3}{4\lambda}\right)^3\left(\frac{T_{\mathrm{ES}}}{T}\right)^2\right]. \tag{32}$$

Although the low-frequency cutoff $\omega_{\min}$ exceeds that for a 3D system, it is still hardly distinguishable from zero at temperatures $T \lesssim T_{\mathrm{ES}}/30$.

Our theory cannot lead to an observable $1/f$ noise in the 1D case because in this case the domain wall contains one electron, its energy is small, and it is practically impossible to construct a slow enough trap at a finite temperature.

V. MOTT VARIABLE-RANGE HOPPING

In this section we briefly discuss the case of an amorphous semiconductor, where the bare density of states is diminished by a strong compositional disorder of non-Coulomb nature, so that the Coulomb gap is relatively narrow and the variable-range-hopping conductivity obeys the Mott law $\ln \sigma_{\mathrm{M}} \sim (T_{\mathrm{M}}/T)^{-1/(d+1)}$.[23]

For this purpose we depart from the original model [Eq. (33)] and consider the Efros lattice model[24] characterized by a Hamiltonian with a strong non-Coulomb disorder

$$\hat{H} = \sum_i \phi_i n_i + \frac{1}{2}\sum_{ij} \frac{e^2}{\kappa r_{ij}} n_i n_j. \tag{33}$$

Here $n_i = \pm 1/2$ stands for a hole and an electron, respectively. The density of bare states ϕ_i is defined as $g_0 = n/2W$, where n is the concentration of lattice sites i and W is the characteristic energy of disorder, i.e., random potentials ϕ_i are distributed uniformly within the domain $(-W, W)$. We assume that the dimensionless ratio $A = W\kappa/e^2 n^{1/d}$ is much larger than unity. In this case the Coulomb gap is much narrower than the width of the density of states W.

The results of our theory are summarized in Table I. The left half of this table characterizes the variable-range hopping at different temperatures and dimensionalities $d=2$ and 3 for both the classical impurity-band model and the Efros model. At low temperatures the variable-range-hopping conductance obeys the ES law [Eq. (1)], while at higher temperatures it obeys the Mott law.

The right half of Table I compares contributions from chessboard clusters α_H and one-electron pores α_H^{pore}. As we mentioned above, the pores are not important within the range of validity of the ES law. However, they become competitive in the case of the Mott law. Remarkably, in spite of the fact the Coulomb interaction is irrelevant to the Mott variable-range conductivity, the very-low-frequency $1/f$ noise is still determined by the chessboard clusters bound by the Coulomb interaction. Naturally, single-electron pores dominate at higher frequencies close to the frequency of Mott hops. They also become more important with decreasing dimensionality because in a low-dimensional system it is easier to create a pore, but it is more difficult to create a slow chessboard cluster. Therefore, the range of applicability of pores is broader in the 2D case.

VI. COMPARISON OF THEORY WITH EXPERIMENT

Our analysis predicts the strong temperature increase of the $1/f$ noise at low temperatures due to the exponential factor $e^{-F(\omega,T)}$. This expectation qualitatively agrees with the experiments,[4,5,8] where the Hooge parameter is approximately proportional to T^{-6}.

As an example, let us discuss the results of experiments on ion-implanted Si:P:B[4,5] showing the ES temperature dependence of the conductance. In this material the Hooge parameter α_H has been measured as a function of temperature for different levels of doping leading to different temperatures T_{ES} varying in the wide range from 1.4 to 44 K. According to Ref. 4, the behavior of the Hooge parameter can be described by the empirical law

$$\alpha_H = 0.034 T_{\mathrm{ES}}^{2.453} (T/0.153)^{-5.2-0.9 \log_{10} T_{\mathrm{ES}}}. \tag{34}$$

Here T and T_{ES} are measured in Kelvin. In particular, in the material with $T_{\mathrm{ES}} \approx 11$ K the relative resistance fluctuations

at $\omega/2\pi \sim 1$ Hz increased from 10^{-11} to 10^{-7} as the temperature decreased from 0.3 to 0.008 K.

Let us compare this empirical law with Eq. (30) derived in the previous section. Note that while deriving that equation we have neglected the powers of N_c in the preexponential factors since N_c cannot be much larger than unity because of the factor $e^{-\lambda N_c}$ [see Eq. (23)]. It is still important to take into account the temperature dependence of the minimum transition rate ν_0 because the cluster transitions are induced by their interaction with phonons. For this mechanism, $\nu_0=(k_B T^3)/\hbar E_c^2$ where E_c is some characteristic energy (see Ref. 25, and references therein). This energy can be estimated as $E_c \approx \sqrt{\rho \hbar^3 s^5/D}$, where ρ is the mass density of the semiconductor while s is the sound velocity. Assuming $D \approx 2$ eV we find $E_c/k_B \approx 10$ K. Using this value we obtain $\nu_0=10^9 T^3$, where T is measured in Kelvin. Then for the sample with $T_{\mathrm{ES}}=11$ K one can obtain a quite reasonable fit of the experimental data setting $\lambda=2.2$ and $n_d a^3=8.4\times 10^{-3}$ in the domain of measurements $0.07<T(\mathrm{K})<0.3$, as shown in Fig. 3.

The estimate for the dimensionless constant λ agrees with the expectation $\lambda/2 \sim 1$. The value of $n_d a^3$ seems to be too small for the material used in the experiments[4,5] performed in the vicinity of the metal-insulator transition. However, our fit is not sufficiently accurate in the interpretation of preexponential factors. In particular, the fitted factor $n_d a^3$ can be significantly underestimated due to the neglect of the N_c-dependent preexponential factor. What can be even much more important for a greater understanding of $1/f$ noise in noisy samples is the fact that the density of dopants in them is not uniform,[5] but depends on the distance from the surface.

We also show the frequency dependence of the Hooge parameter in the frequency domain 1–1000 Hz and at three different temperatures, $T=0.05$, 0.1, and 0.5 K, using Eq. (30) with its parameters defined above (see Fig. 4). The Hooge parameter increases with the increase in frequency. In agreement with our expectations [Eq. (25)], this increase is negligibly weak at lowest temperatures 0.05 or 0.1 K while it is more pronounced at the highest temperature $T=0.5$ K. We do not perform a comparison of theoretical and experimental frequency dependences because the theory is derived ignoring the possible frequency dependence of the preexponential factor. Also, there are no experimental data in the most interesting case of $T=0.5$ K or higher where the deviations from the $1/f$ noise spectrum are significant. Hopefully, future experiments will be extended to this high-temperature domain. Then they can be used to verify the theory.

VII. CONCLUSIONS

In this manuscript we have suggested a distinct mechanism of the $1/f$ noise in doped semiconductors in the hopping regime. This mechanism is associated with many-electron transitions of the chessboard clusters, the rate of which decreases exponentially with the cluster size. This exponential dependence results in the close to $1/\nu$ distribution of clusters over their relaxation rates ν. The slow fluctuations of cluster states modulate the critical resistors forming the backbone cluster leading to the $1/f$ noise in the electronic conductivity.

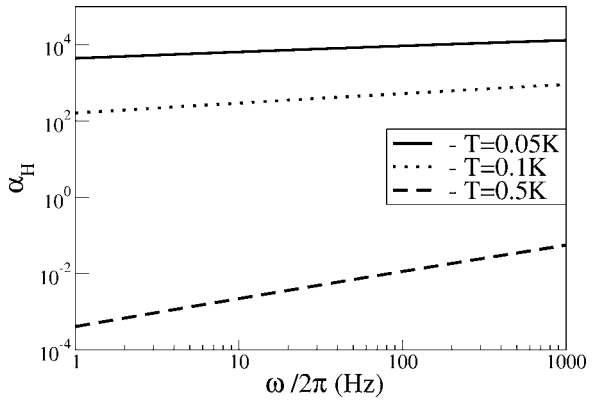

FIG. 4. Theoretical predictions for the frequency dependence of the Hooge parameter [Eq. (30)] at different temperatures for samples with $T_{ES}=11$ K. All parameters are as in Fig. 3.

Our predictions regarding the magnitude and temperature dependence of the Hooge parameter α_H are at least in qualitative agreement with experiments in ion-implanted silicon (Si:P:B).[4,5] In particular, both specific features—dramatic decrease of the noise with the temperature increase and its increase with the increase of T_{ES}—are explained. Moreover, the experimental results can be fitted quantitatively using reasonable values of the adjustable parameters.

The low-temperature ($T<0.2$ K) experimental values of the Hooge parameter exceed unity. This fact also agrees with Eq. (30), since at low temperatures the main temperature dependence is given by the prefactor. Experimental results obtained in Refs. 8 and 9 are also in qualitative agreement with our theory.

ACKNOWLEDGMENTS

We are grateful to A. L. Efros, D. McCammon, M. Moore, M. Mueller, B. Spivak, and C. C. Yu for useful discussions. A.B. greatly appreciates the hospitality of William P. Fine Theoretical Physics Institute of the University of Minnesota after the hurricane disaster in the city of New Orleans. The work of A.B. is supported by the Louisiana Board of Regents [Contract No. LEQSF (2005-08)-RD-A-29] and NSFEpsCor LINK Program through the Louisiana Board of Regents. The work of Y.G., V.K., and V.V. was supported by the U.S. Department of Energy Office of Science through Contract No. W-31-109-ENG-38.

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