Numerical simulations of thermomagnetic instability in high-T_c superconductors: Dependence on sweep rate and ambient temperature

You-He Zhou* and Xiaobin Yang
Department of Mechanics, College of Physical Science and Technology, Lanzhou University, Lanzhou, Gansu 730000, People's Republic of China

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We report the results of numerical simulations of flux jumps on the basis of dynamic process of thermomagnetic interaction to the nonisothermal and nonadiabatic high-T_c superconductors in the regime of thermally activated flux creep when an applied magnetic field is parallel to a slab of the high-T_c superconductors. The simulations for the samples of BiSrCaCuO show that the flux jumps may occur only in the region of low ambient temperature, which is dependent upon the heat contact, and the sweep rate is greater than a lower critical value of about 20 G/s and lesser than a large one up to the order of 1–10 T/s. It is found that the predictions of the first flux-jump field B_{fj1} are quantitatively in good agreement with the existing experimental data, and the temperature jumps are observed in the superconductors, corresponding to each flux jump in the magnetization loop. When the field sweep rate exceeds the large critical value for the case of the superconductor at 4.2 K, the phenomenon of experimental observations without flux jump is successfully predicted by the theoretical simulation, where the thermomagnetic interaction is smoothly circulated at a new dynamic equilibrium state in the temperature region of about 10.6–16.4 K higher than the ambient one, which is mainly dependent on the tradeoff of speeds of the dissipation energy in the slab and the heat removed into the coolant. After that, the sensitivity of the thermomagnetic instability to the parameters, such as critical current density, heat conductivity, heat transfer coefficient, critical geometrical scale, etc. is also discussed.

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I. INTRODUCTION

Since the 1960's, the peculiar phenomenon of thermomagnetic instability (or flux jumps) has been experimentally observed both in conventional hard type II superconductors[1–3] and in high-temperature superconductors (HTS).[4–16] Such studies are motivated by the interest from a basic point of view to understand the underlying mechanism of the instability and also in light of their potential applications (see the review papers[11,17]). It is known that the critical state of a superconductor may become unstable when the system under appropriate conditions undergoes a small fluctuation of either the external magnetic field and/or the temperature, which is associated with a sudden puncture of magnetic flux into the volume of the superconductor, an increase of temperature in the superconductor, and the decrease or quenching of the screening current. Thus, the thermomagnetic instability is problematic in applications as it drives the superconductors into a normal or resistive state.

To reveal the mechanism of magnetothermal instability, both experimental and theoretical researches focus on the critical condition or the first flux-jump field B_{fj1} of magnetothermal instability. The experimental measurements indicated that the first flux-jump field is mainly dependent on both the ambient temperature of the coolant and the field-sweep rate,[10,11,17,18] and the phenomenon of flux jumps of the high-T_c superconductors are detected only in the region where the ambient temperature decreases below about 10 K[5,11,17] when the sweep rate is greater than a threshold of about 20 G/s, where the flux-jump field increases with the ambient temperature and decreases with the sweep rate. When the sweep rate exceeds a certain limit that is of the order of 1–10 T/s for the samples of LaSrCuO and BiSrCaCuO at 4.2 K, however, the measurements demonstrated that no flux jump was observed and its magnetization was stabilized on a lower level after the first flux jump.[10] For the samples of $Bi_2Sr_2CaCu_2O_{8+\delta}$, it is found that the first flux-jump field approaches a saturation value of about 0.6–1 T.[10,13]

On the basis of an assumption of $J_c=J_c(T)$, or dependence of critical current density on temperature with negative gradient, the thermomagnetic instability was recently described as a positive feedback process or chain.[11] In such a case, two subprocesses of thermal and magnetic diffusions mutually interact by the subprocess of the heat dissipation from the change of electromagnetic fields and the subprocess of the change of critical current density generated by the resulting temperature rising. Therefore, Swartz *et al.*[11,19,20] proposed a basic theory appropriate to flux jumps of the conventional type II superconductors on the basis of Bean's critical state model of superconductivity[21] and some assumptions of local adiabatic condition. After the prerequisite criterion of temperature disturbance $\Delta T_1=\Delta T_2$ was employed; here, ΔT_1 and ΔT_2, respectively, represent the original and further fluctuations of temperature, an analytical formula for predicting the flux-jump field was obtained by the form[11,13]

$$B_{fj1}=\sqrt{3\mu_0 c J_c/(-dJ_c/dT)}, \qquad (1)$$

in which μ_0 is the magnetic permeability of vacuum, c stands for the specific heat of the superconductor material, and J_c represents the critical current density. The investigations pointed out that the fulfillment of the adiabatic condition depends upon the relative value between the thermal (D_t) and the magnetic (D_m) diffusivity, or the thermal diffusion time $\tau_t(\sim 1/D_t)$ and the magnetic diffusion time

$\tau_m(\sim 1/D_m)$ of the material.[13] Whether $D_t \ll D_m$ or $\tau_t \gg \tau_m$, the adiabatic condition for the occurrence of the flux jump is thought to be satisfied. Recently, Eq. (1) was directly applied to predict the flux-jump field for the case of high-T_c superconductors.[11] According to Eq. (1), we know that the flux-jump field is independent of the sweep rate, and the flux-jump phenomenon always occurs in the whole range of $0 \le T_0 \le T_c$ of ambient temperature.[11] Here, T_0 and T_c are the ambient temperature and the critical temperature of a superconductor, respectively. Usually, the value of T_c for high-T_c superconductors is greater than the boiling point of liquid nitrogen ($\approx$77.3 K). The quantitative results of Eq. (1) tell us that B_{fj1} increases first up to a peak, then decreases to zero as T_0 increases from zero to T_c. Corresponding to the peak, the ambient temperatures all are over 50 K.[11] Meanwhile, the predictions of Eq. (1) are roughly lower than the experimental data by about one order of magnitude.[13] When a criterion of disturbance of the magnetic field was employed on the same assumptions of the above theory, Muller and Andrikidis[6] deduced an analytical formula that is similar to Eq. (1) except for a different factor. Once we consider the heat propagation in the sample and the heat part flowing into the coolant, we know that the practical value of ΔT_2 should be smaller than that calculated by the theory with the adiabatic conditions, which should yield lower predictions of Eq. (1) than those in practice. In addition, the assumption of $J_c = J_c(T)$ may also result in some deviation. From the knowledge of superconductivity, we know that the critical current density is dependent on both local temperature and the magnetic field,[22] i.e., $J_c = J_c(T,B)$. In this case, we have

$$\dot{J}_c = \frac{\partial J_c}{\partial T}\dot{T} + \frac{\partial J_c}{\partial B}\dot{B}. \tag{2}$$

It is obvious that $\dot{B} \sim \dot{B}_e$ where B_e represents the external magnetic field, and $\dot{B}_e$ stands for the sweep rate, or time change rate of B_e. It is obvious that the last term in Eq. (2) is neglected in the theoretical framework of Eq. (1). Thus Eq. (1) is feasible only when the sweep rate is very small.

To reflect the effect of sweep rate on the flux-jump field in the theoretical analysis, on the other hand, Mints[18] proposed a theoretical model to study the thermomagnetic instability based on either the Bean's critical state model or the Kim-Anderson model in the flux-creep regime of type II superconductors and the approximation of spatial distribution of flux remaining fixed during the stage of rapid heating, or a condition of $\tau_1 \ll \tau_m$. In this theory, the temperature rise in the material is neglected, and the prerequisite criterion is employed when the additional heat release in the slab is fully removed into the coolant. After that, another formula for predicting the flux-jump field, for example, on the basis the Kim-Anderson model [i.e., $J_c(B) = \alpha(T)/B$], was obtained by[18]

$$B_{fj1} = \left(\frac{2\mu_0^2 \alpha(T_0) h (T_c - T_0)}{n\dot{B}_e}\right)^{1/3}. \tag{3}$$

Here, h is the heat transfer coefficient to the coolant with ambient temperature T_0, $n(=J_c/J_1 \gg 1)$ stands for the exponential factor in the relation $J = J_c(E/E_0)^{1/n}$, and $\alpha(T_0) = J_c(T_0)B_{0k}$, where B_{0k} is a phenomenological parameter and it is selected as about $B_{0k} = 0.3$ T in Ref. 23. From Eq. (3), one sees that the predicted flux-jump field decreases with the sweep rate and approaches zero when $\dot{B}_e \to \infty$. This result tells us that the thermomagnetic instability should always occur when the sweep rate is very high, which is inconsistent with the experimental observation of no flux jump when the sweep rate is about 6 T/s.[10] Since the rise of temperature generated by the heat dissipation in the superconductor is not taken into account, it is natural that the predictions based on Eq. (3) should be higher than their measured data. Meanwhile, Eq. (3) gives also a prediction of flux-jump field in the whole region of $0 \le T_0 \le T_c$, no matter what sweep rate is specified.

Here, we propose an approach to analyze the thermomagnetic dynamic interaction on the basis of the coupling of magnetic and heat diffusion equations associated with the intrinsic nonlinear behaviors of $J_c(T,B)$, $c(T)$, $h(T)$, etc., from which the experimental phenomena mentioned above are fully described.

II. ESSENTIAL EQUATIONS

Here, we focus our attention on a high-T_c superconducting slab with thickness $2d$ subjected to an external magnetic field $\mathbf{B}_e$ parallel to the sample surface (yz plane), at which the distributions of the z component of magnetic induction $B(x,t)$, the y component of electric intensity $E(x,t)$, the current density $J(x,t)$, and the temperature $T(x,t)$, are governed by the Maxwell equations coupled to the heat diffusion[24]

$$c(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\kappa \frac{\partial T}{\partial x}\right) + JE, \quad -d < x < d \tag{4}$$

$$\frac{\partial B}{\partial t} = -\frac{\partial E}{\partial x}, \tag{5a}$$

$$\frac{\partial B}{\partial x} = -\mu_0 J, \quad -d < x < d. \tag{5b}$$

After a $E \sim J$ relation of the superconductor is taken into account, i.e.,

$$E = \rho(B,J,T)J, \tag{6}$$

the Maxwell equations in Eqs. (5a) and (5b) can be reduced into the form

$$\mu_0 \frac{\partial B}{\partial t} = \frac{\partial}{\partial x}\left(\rho(B,T,J)\frac{\partial B}{\partial x}\right), \quad -d < x < d. \tag{7}$$

Here, $c(T)$ is the heat capacity, $\kappa(T)$ indicates the thermal conductivity, and $\rho = \rho(B,T,J)$ stands for the effective electric resistance. After the superconductivity state is considered by the zero field cooling, the boundary and initial conditions of the magnetic field and heat diffusion can be written as

$$x = \pm d: \quad B(x,t) = B_e(t), \tag{8}$$

$$x = \pm d: \quad -\kappa\frac{\partial T}{\partial x} = \pm h(T)(T - T_0), \tag{9}$$

$$t = 0: \quad B(x,t) = 0, \quad T(x,t) = T_0. \tag{10}$$

The experimental and theoretical investigations of superconductivity exhibit a nonlinear relation of $E \sim J$ dependent on the local magnetic and temperature fields, which yields some peculiar effects, e.g., magnetic relaxation,[25] and that the time scales of short- and long-magnetic relaxation are relevant to some of the directly measured parameters, such as $\dot{B}_e$ and E_c, which is a voltage criterion at which J_c is defined.[25] Due to that the $E \sim J$ relation is dependent on a phenomenological theory of superconductivity recently, it has been found to still be an open problem. For the high-T_c superconductors considered here, the pinning effect plays an important role for the flux creep which can be characterized by a phenomenological theory of the thermally activated flux movement of vortex current. In the regime of flux creep, the drift velocity of flux lines can be formulated in the form[22,26]

$$\nu = 2\nu_0 e^{-U_0/KT}\sinh(JU_0/J_ckT), \tag{11}$$

where ν_0 is a velocity prefactor related to the attempt frequency of flux line hopping, U_0 denotes the activation energy, k is the Boltzmann constant, and JU_0/J_c represents the energy change of a flux line associated with the Lorentz force acting on a vortex. The movement of the flux-line results in an induced electric field of $\mathbf{E}=\mathbf{v}\times\mathbf{B}$. For the superconducting slab that we consider here, we get the $E \sim J$ relation of the form

$$E = 2B\nu_0 e^{-U_0/kT}\sinh(JU_0/J_ckT). \tag{12}$$

The present research on the activation energy tells us that the energy is dependent on the local temperature and magnetic fields, or $U_0 = U_0(T,B)$, and it is explicitly expressed by[26]

$$U_0 = U_{00}[1-(T/T_c)^4][1-B/B_{c2}(T)], \tag{13}$$

in which U_{00} is the barrier height when $T=B=J=0$, and $B_{c2}(T)=B_{c2}(0)[1-(T/T_c)^2]$ stands for the upper critical magnetic field. Here, $B_{c2}(0)=B_{c2}(T)|_{T=0}$.

The temperature and magnetic-field dependence of critical current density is here employed as the Kim-Anderson model,[27–29] i.e.,

$$J_c = J_c(T,B) = J_c(T,0)\frac{B^*}{|B|+B^*}, \tag{14}$$

where B^* is a phenomenological parameter. For the samples of $Bi_2Sr_2CaCu_2O_{8+\delta}$, the dependence of the critical current density on temperature $J_c(T,0)$ is given by fitting the experimental data with relation[13]

$$J_c(T,0) = J_{c0}e^{-T/[T_e(1-(T/T_c)^2)]} \tag{15}$$

in which $J_{c0}=3\times10^{10}$ A/m^2 and $T_e=8.4$ K.

Substituting Eqs. (13)–(15) into Eq. (12), we obtain

$$\begin{aligned}E = {}& 2B\nu_0 e^{-U_{00}[1-(T/T_c)^4]\{1-B/\{B_{c2}(0)[1-(T/T_c)^2]\}\}/kT}\\ &\times\sinh[JU_{00}[1-(T/T_c)^4](1-B/\{B_{c2}(0)[1-(T/T_c)^2]\})\\ &\times(|B|+B^*)/\{J_{c_0}B^*kTe^{-T/[T_e(1-(T/T_c)^2)]}\}].\end{aligned} \tag{16}$$

It is obvious that the $E \sim J$ relation in Eq. (16) is dependent on the local temperature and magnetic fields. Consequently, the system with thermomagnetic interaction is nonlinearly coupled by the initial-boundary problem of Eqs. (4), (5a), (7), and (8)–(10) associated with Eq. (16). To quantitatively solve the nonlinear coupling problem, we propose a numerical code by means of the finite difference method and iteration, where to search for a critical value of magnetic field at each flux jump, the technique of variable time steps is utilized.

After the magnetic flux in the superconductor is numerically obtained at each instant, the magnetization is calculated by the following formula:

$$\mu_0M(t) = \frac{1}{d}\int_0^d B(x,t)dx - B_{ex}(t). \tag{17}$$

III. SIMULATION RESULTS TO THE EXPERIMENTAL MEASUREMENTS

In this section, we display the simulation results to the experimental measurements of the thermomagnetic interaction system based on the theory proposed in the previous section. After that, the sensitivity of the flux jump to the system parameters will be discussed in the following section. In the simulations, all external magnetic fields are swept from 0 to 9 T, back to −9 T, and again back to zero with a specified sweep rate of $\dot{B}_e=\mathrm{v}_{ex}$. We have noted that some parameters, e.g., U_{00}, ν_0, and h etc., that appeared in the theoretical model cannot be directly measured by the present experiments with a determinant value. In this case, we select the parameters in their possible regions given in the literature. For the system with the materials of BiSrCaCuO and/or $Bi_2Sr_2CaCu_2O_{8+\delta}$, for example, the parameters are taken as $J_{c0}=3.0\times10^{10}$ A/m^2, $2d=42\times10^{-3}$ m, $T_c=92$ K, $B_{c2}(0)=110$ T, $\nu_0=10$ m/s, $U_{00}/kT_0=20$, $\kappa=1.0$ J/(msK). For the temperature dependence of heat capacity $c(T)$, the experimental measurement shows that it is formulated by $c(T)=\gamma(0)T+B_3T^3$ to fit the experimental data of the Bi and Tl samples,[30] where $\gamma(0)$ and B_3 are the fitting coefficients. For the Bi samples, it is found that $\gamma(0)=0$ (see Ref. 30) and $c(4.2\text{ K})=11\times10^2$ J/Km^3 (see Ref. 13). Then, we get $B_3\approx14.8$ J/K^{4}m^3. In this section, we pay attention to the case of ideal heat contact between the slab surface and the coolant, in which the heat transfer coefficient can be expressed by the empirical formula[31]

$$h(T) = 0.05(T^4-T_0^4)/(T-T_0) = 0.05(T+T_0)(T^2+T_0^2). \tag{18}$$

Here, h is in units of J/(m^{2}s K).

Figure 1 illustrates the hysteresis magnetization loops and flux jumps of the simulation when the ambient temperature changes from 2.0 to 7.5 K and the sweep rate is specified by 50 G/s. From this figure, one sees that when the coolant

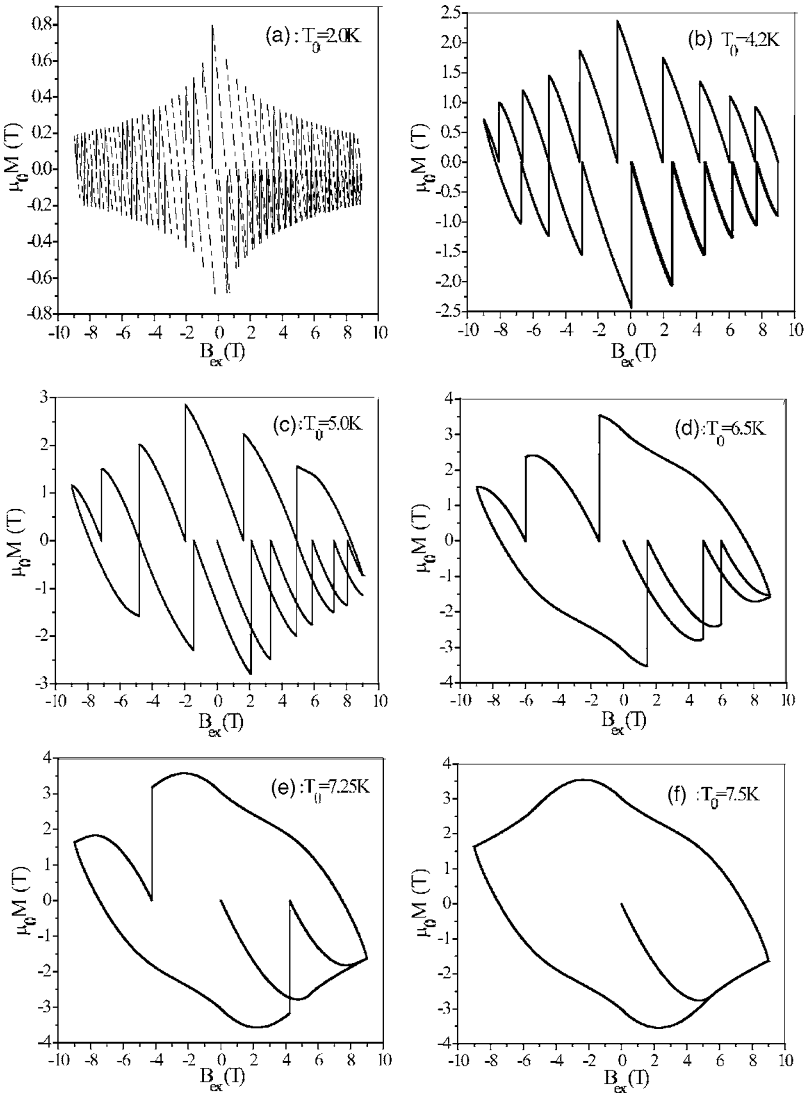

FIG. 1. The simulated magnetization loops of the thermomagnetic interaction system versus the coolant temperature when the sweep rate is 50 G/s.

temperature is higher than 7.5 K, no flux jump occurs in the simulation system, and the number of jumps increases with the decrease of temperature. These features are in good agreement with those observed in the experiment.[13] When the ambient temperature is 7.25 K, the simulation shows that the first flux jump occurs at the third quadrant of the magnetization loop [see Fig. 1(e)], which was also observed in the experiments[5,6] and attributed to the history of the magnetization of the sample.

According to the results shown in Fig. 1, we get the dependence of first flux-jump field on the ambient temperature, which is plotted in Fig. 2, where a comparison of the predictions with the experimental data given in Ref. 13 and the theoretical predictions of Eqs. (1) and (3) are also displayed. In the calculations of Eqs. (1) and (3), we use the approximate relations of $c=14.8T_0^3$, $\alpha(T_0)\approx J_{c0}B_{0k}e^{-T_0/T_e}$, $J_c(T_0)\approx J_{c0}e^{-T_0/T_e}$, and $h(T_0)\approx 0.2T_0^3$ under consideration of $T_0 \leq 7\text{ K} \ll 92\text{ K}=T_c$, $T\approx T_0$, and $B_{0k}=0.3$ T, and other parameters that are employed in this simulation. From Fig. 2, one finds that the predictions of this simulation are quantitatively in good agreement with the experimental data, and those of

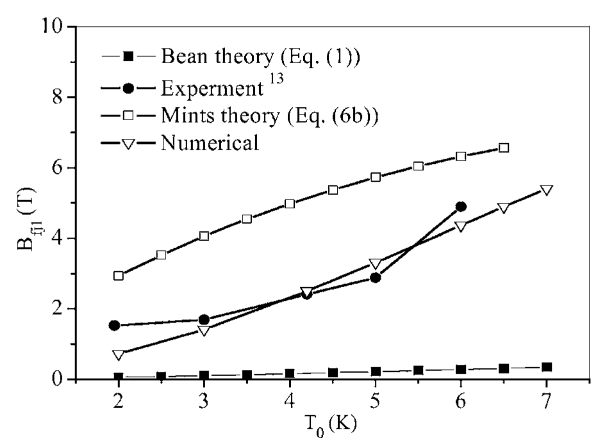

FIG. 2. The prediction curves of the flux-jump field versus the ambient temperature compared with the experimental measurements ($v_{ex}=50$ G/s). The prediction values of B_{fj1} shown here are gained from Fig. 1, while the predictions of Eqs. (1) and (3) are conducted on the basis of $h(T_0)=0.2T_0^3$, $c(T_0)=14.8T_0^3$, $J_c(T_0)\approx J_{c0}e^{-T_0/T_e}$, and $\alpha(T_0)\approx J_{c0}B_{0k}e^{-T_0/T_e}$, with $J_{c0}=3.0\times 10^{10}$ A/m^2, $B_{0k}=0.3$ T, and $T_e=8.4$ K ($n=20$).

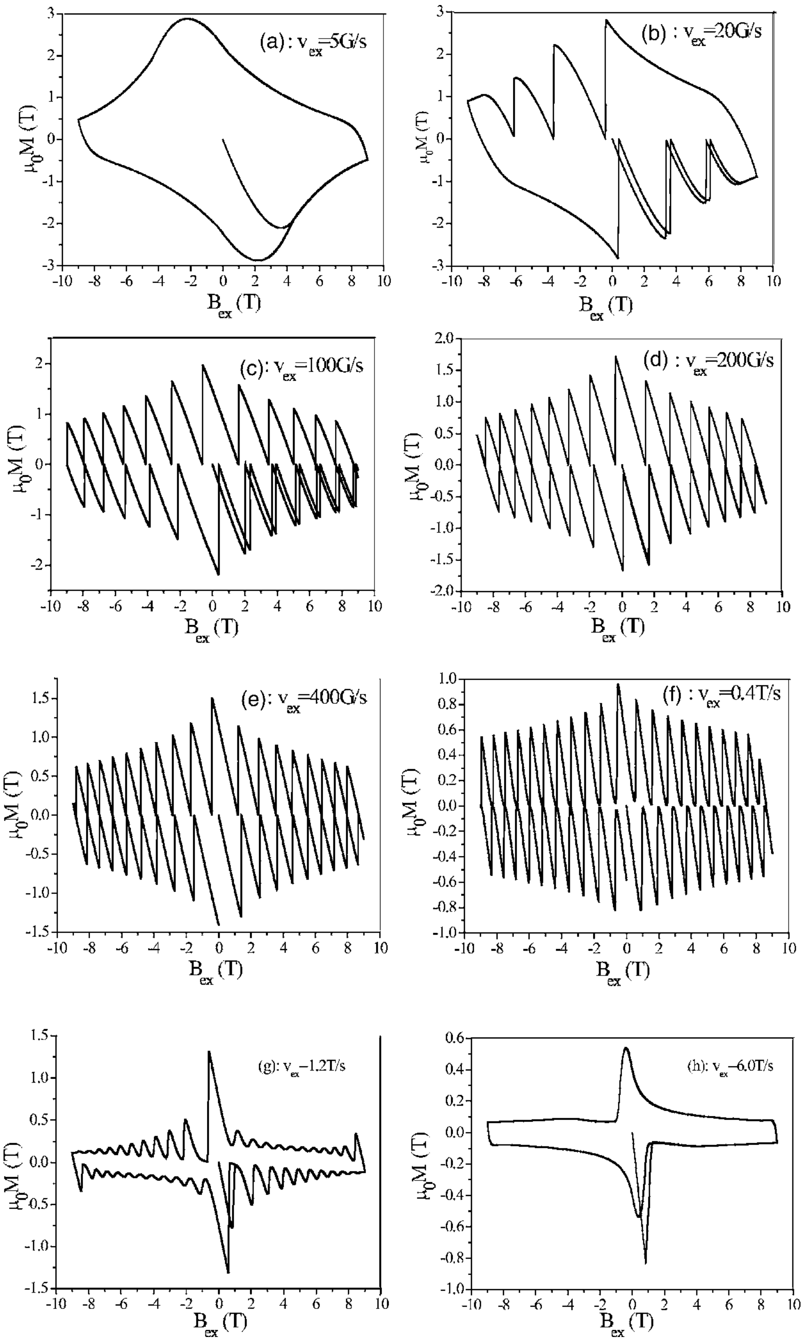

FIG. 3. The magnetization loops of simulating the thermomagnetic interaction system versus the sweep rate when the coolant temperature is 4.2 K.

Eq. (1) are lower than the experimental data by about one order of magnitude, while those of Eq. (3) are higher than the experimental data by about a factor of 2 when $T_0 \leq 5$ K. When the coolant temperature is 4.2 K, for example, the experimental value of the flux-jump field is about 2.1 T, but the predictions of Eqs. (1) and (3) are about 0.15 T (which is the same as one given in Ref. 13) and 5.1 T, respectively. In our simulations, the prediction is 2.44 T.

Once we specify the coolant temperature of 4.2 K and change the sweep rate of external magnetic field, we can obtain the magnetization loops versus the sweep rate, which are drawn in Fig. 3. When the sweep rate is very small, e.g., 5 G/s, Fig. 3(a) shows that the magnetization loop is smooth without a flux jump on the path. As the sweep rate increases up to a certain value of about 20 G/s and lower than a certain value of about 1 T/s, the flux jumps take place, where the number of jumps increases with the sweep rate [see Figs. 3(b)–3(f)]. According to the simulation results shown in Fig. 3, the sweep rate dependence of the flux-jump field is shown in Fig. 4 to compare with their experimental results in Ref. 13. From Fig. 4, we find that the numerical predictions are also in good agreement with the measurement data, while the predictions of the Mints' theory all are higher than the experimental data by a factor of about 2. As the sweep rate increases, the simulation results illustrate that the flux-jump field decreases and approaches a saturation value [about 0.85 T, see Fig. 3(h)], which is fairly close to the estimated value of about 1 T in Ref. 13. When the sweep rate exceeds 1 T/s, the magnetization loop oscillates during the application of the external magnetic field. After the sweep rate in-

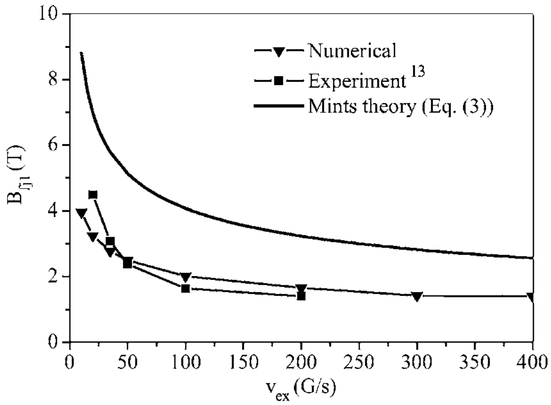

FIG. 4. The dependence of B_{fj1} on the sweep rate at ambient temperature of 4.2 K. Here, the results of the numerical prediction are determined on the basis of the data shown in Fig. 3, and the triangle dashed line, the square dashed line, and the solid line without a symbol represent the numerical predictions, the experimental data,[13] and the predictions of the Mints' theory or Eq. (3), respectively.

creases up to 6 T/s, the magnetization shown in Fig. 3(h) becomes stabilized on a lower level after the first flux jump, which is the same as what was observed in the experiment.[10]

In order to reveal the reason why it is, let us see the change of temperature in the superconductor. Corresponding to the flux jumps in the magnetization loops shown in Fig. 3, the time response curves of temperature at the center of slab ($x=0$) are plotted in Fig. 5 for different sweep rates. Here, T^c indicates the period of applied magnetic field. From the results of this figure, we know that a temperature jump occurs at each flux jump, which was measured in the experiment.[8] When the sweep rate is so small, the heat dissipation may be removed sooner into the coolant so that the temperature in the superconductor changes very little. In this case, no flux jump occurs, and the temperature continuously changes from the ambient temperature (4.2 K) to the highest peak of 4.45 K [see Fig. 5(a)]. Here, the inset figure displays that the increase of temperature oscillates. When the sweep rate is higher than a lower critical value of about 20 G/s and is less than an upper critical one of about 1 T/s, the flux jump and the temperature jump are concomitant. Before a jump, the heat released in the slab is greater than that removed into the coolant and the remaining heat in the slab results in a temperature rise, which leads to a decrease of current density, and further, more heat dissipation again and again till the jump happens rapidly. The numerical results indicate that the current density decreases about 5–6 orders of magnitude at each jump, which yields a fast decrease of heat dissipation after a jump such that the released heat in the slab is soon removed out and the temperature in the slab returns to the ambient one in a short time. Then, the current density gets back the state of superconductivity and the system enters a new circulation of thermomagnetic interaction. From Fig. 5(b) when the sweep rate is 20 G/s, one sees that the temperature may sometimes jump from the ambient temperature of 4.2 K up to about 65 K. As the sweep rate increase up 400 G/s, the peaks decrease to about 12–18 K [see Fig. 5(c)]. According to Fig. 5(c), it is also found that the temperature peaks decrease with time in the first and the third quarters of the period, or subprocesses of $0\rightarrow B_{e0}(=9\text{ T})$ and $0\rightarrow -B_{e0}$, while the peaks increase with time in the second and fourth quarters, or subprocesses of $B_{e0}\rightarrow 0$ and $-B_{e0}\rightarrow 0$. For different sweep rate in the region of $0.04\text{ T/s}\leq v_{ex}<1\text{ T/s}$, the maximum and minimum peaks of temperature change little, but the number of jumps increases with the sweep rate. When the sweep rate exceeds 1 T/s as indicated in Figs. 5(d) and 5(e), the temperature oscillates around a new equilibrium temperature of 8–10 K. and the lower valley of temperature is higher than the ambient temperature; sometimes the temperature peak may be up to 19 K. The same as the oscillation of magnetization shown in Fig. 3, the temperature jumps oscillate too. When the sweep rate is very high, e.g., 6 T/s or higher, the numerical results shown in Fig. 5(f) tell us that the temperature continuously changes from the ambient one (4.2 K) to 16.4 K, then continuously varies in the region of 10.6–16.4 K. That is, the thermomagnetic interaction dynamically persists around a new equilibrium temperature of about 14 K at which the acceleration of remaining heat is equal to zero. Corresponding to this case, the magnetization is continuously changed too [see Fig. 3(h)]. This continuous change of temperature and magnetization without jumps is mainly relevant to the tradeoff of the dissipation heat and the heat removed into the coolant during the application process of the external magnetic field. This dynamic mechanism is similar to the movement of a simple pendulum, where the remainder of the heat in the superconductor is similar to the displacement of the pendulum. Due to the dependence of the current density, the dissipated heat, and the transfer coefficient on the local temperature, the difference of the speed of the dissipated heat in the slab and the heat removed into the coolant determines the speed of the remainder heat, which yields either an increase or decrease of temperature in the slab. When the difference is positive, the temperature in the slab increases, which yields a decrease of the current density and an increase of the heat transfer coefficient. This subprocess persists till the temperature approaches a peak of temperature where the speed of the remainder heat is equal to zero. After the peak, the difference becomes negative so that the temperature in the slab decreases till the temperature approaches a valley point. During the latter subprocess, the current density increases and the speed of the transfer heat decreases. If the speed of the dissipated heat in this subperiod is much smaller than the speed of the heat transfer into the coolant, the temperature in the slab will be returned back to an ambient one, which is the case of flux jumps. Otherwise, some remaining heat can be collected in the slab even when the speed of the remaining heat is equal to zero when the temperature in the slab approaches a valley point higher than the coolant temperature.

IV. SENSITIVITY OF A FLUX-JUMP FIELD TO PARAMETERS

In this section, we briefly display the results of sensitivity of the flux jumps to some parameters appearing in the theoretical model of thermomagnetic interaction proposed in this paper. For the case of ideal contact Figs. 6 and 7 illustrate the simulation results for the dependences of the thermomag-

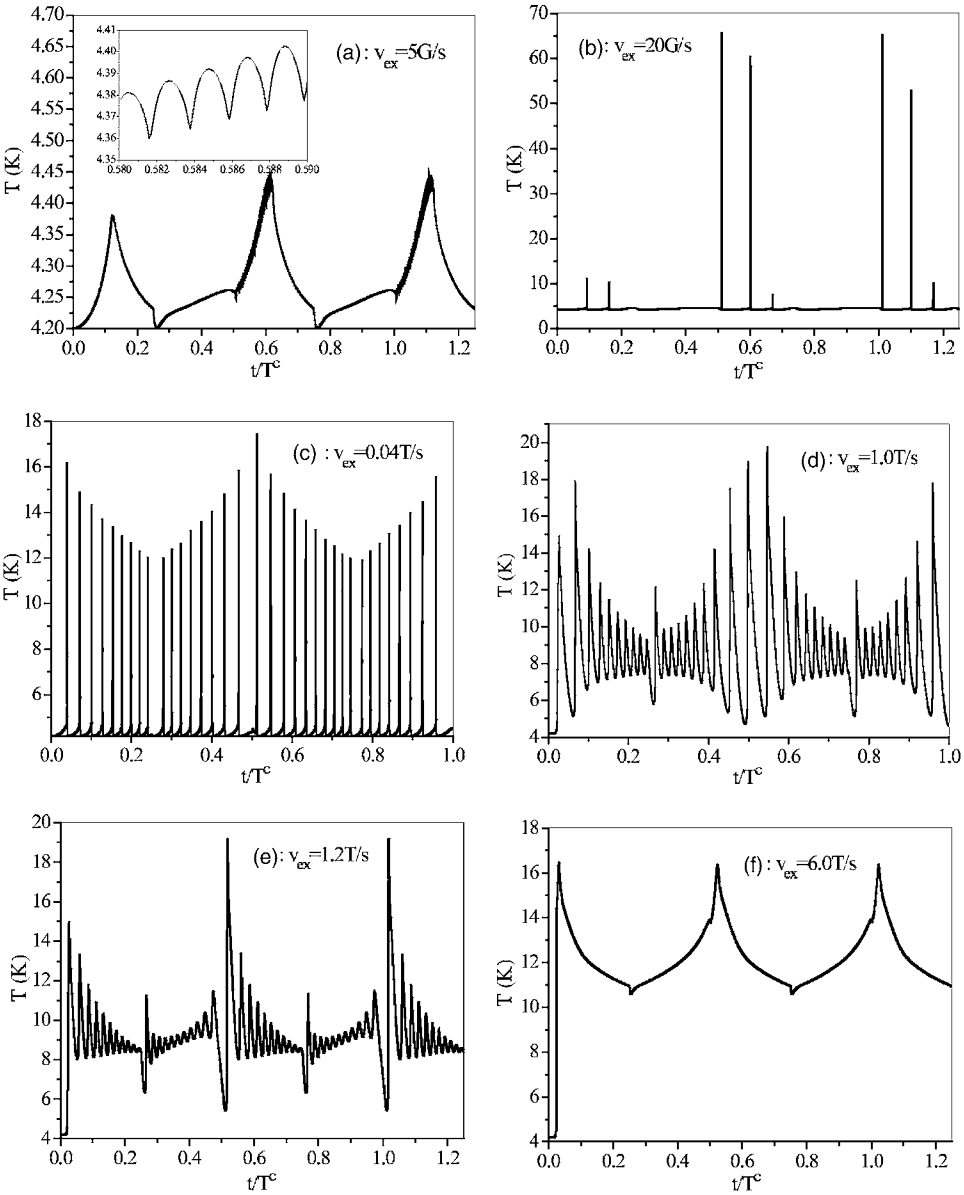

FIG. 5. Time responses of temperature at the center of the slab ($x=0$) varying with an application of external magnetic fields of different sweep rate. Here, $T^c(=4B_{e0}/v_{ex})$ means the period of one loop of the external magnetic field ($B_{e0}=9.0$ T).

netic instability on the parameters of thickness, thermal conductivity, and critical current density, respectively.

Figure 6 illustrates the critical curves of thickness versus sweep rate, in which a curve divides the stability and instability regions of the parameters in the figure. From this figure, one finds that the critical thickness for the flux jumps decreases as the ambient temperature decreases and the sweep rate increases. That is to say, the flux-jump phenomenon of thermomagnetic interaction is sensitive to the thickness of slabs. Comparing the minimum thickness obtained here with one given by the adiabatic theory,[11,13] we know that the latter one is lower than the former by about one order of magnitude. Thus, the result of critical thickness given by the adiabatic theory is conservative. When we change the thermal conductivity and the critical current density, respectively, the numerical results indicate that the flux-jump field is insensitive to the heat conductivity [see Fig. 7(a)], but is sensitive to the critical current density [see Fig. 7(b)]. That is, the flux-jump field increases with the critical current density.

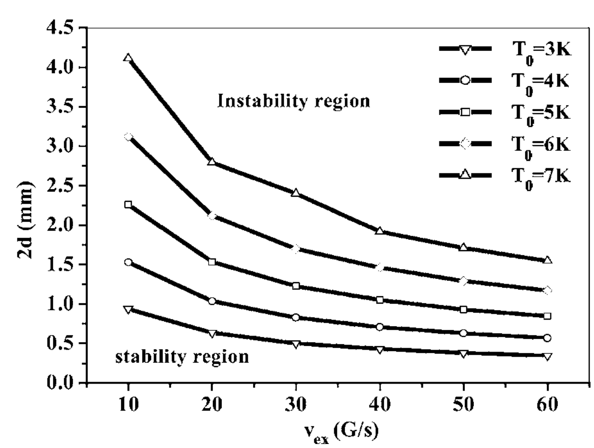

FIG. 6. Stability and instability regions divided by one curve of the critical thickness versus the sweep rate of the thermomagnetic interaction for the different ambient temperature. Here, $J_{c0}=3.0\times10^{10}$ A/m^2, $\kappa=1.0$ J/(m s K).

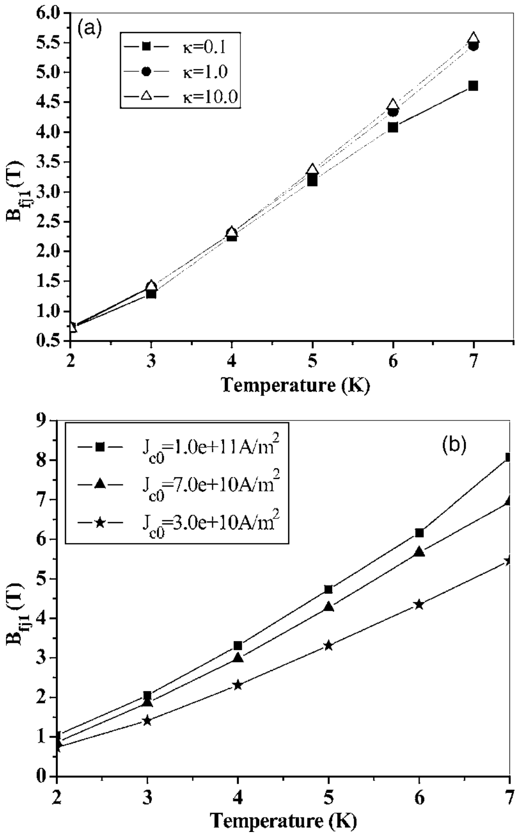

FIG. 7. Curves of the flux-jump field varies with the ambient temperature (a) for different thermal conductivity $[J_{c0}=3.0\times10^{10}\ \mathrm{A/m^2}]$, and (b) for different critical current density $[\kappa=1.0\ \mathrm{J/(m\ s\ K)}]$. Here, $v_{ex}=50$ G/s, and $2d=4.2$ mm.

It is obvious that the speed of heat transfer passing though the surface of slab is another important parameter to influence the flux-jump field, which is characterized by the heat transfer coefficient. For the heat contact between the superconductor slab and the coolant, either nonideal or ideal, we know that the heat transfer coefficient can be uniformly formulated by[31]

$$h(T) = 0.05(T^{\sigma_A} - T_0^{\sigma_A})/(T - T_0). \tag{19}$$

Here, the exponential factor σ_A is usually taken in the region of [3,4], and its experimental date is about 3.3. It is obvious that when $\sigma_A=4.0$, Eq. (19) becomes Eq. (18) for the ideal heat contact. If $\sigma_A=0$, we have $h(T)=0$, which corresponds to the case of adiabatic condition. Figure 8 illustrates the curves of the flux-jump field versus the sweep rate for a different exponential factor or the heat transfer coefficient, which tells us that the flux-jump field B_{fj1}, is sensitive to the exponential factor or the heat transfer coefficient, i.e., the flux-jump field increases with the exponential factor, wherein the ambient temperature region for the flux jumps has some extending as the factor decreases (see Fig. 8). These results are in accord with the practical system. If the system is adiabatic, for example, the collected heat in the slab cannot be removed and it should lead to a magnetic flux jump for any temperature of ambient environment as long as the magnetic application persists for enough time. In practice, the heat contact is usually nonideal and nonadiabatic. To quantitatively get a prediction of the flux-jump field in agreement with its experimental measurement for such a case, the critical current density should be higher than $J_{c0}=3.0\times10^{10}\ \mathrm{A/m^2}$ that is employed in the simulation of Sec. III for the ideal heat contact.

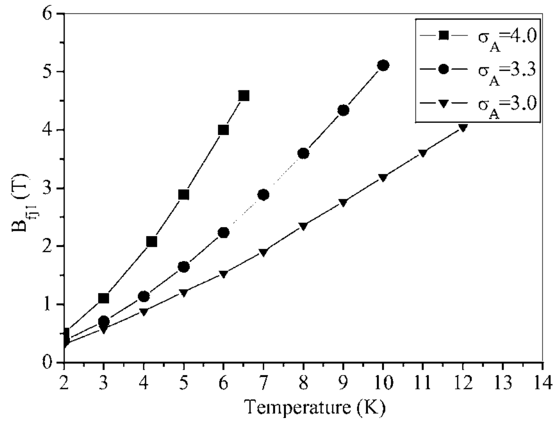

FIG. 8. Curves of the flux-jump field changes with the ambient temperature when $v_{ex}=50$ G/s for the different exponential factor σ_A in the relation of heat transfer coefficient varying with the ambient temperature, or Eq. (19). Here, $2d=4.2$ mm, $\kappa=1.0$ J/(m s K), $J_{c0}=3.0\times10^{10}$ A/m^2.

V. CONCLUSIONS

Based on the coupling of magnetic and heat diffusions associated with a nonlinear $E-J$ relation and the intrinsic nonlinear relations of $c=c(T)$ and $h=h(T)$, a theoretical investigation is presented in this paper for insight into the comprehensive mechanism of the thermomagnetic instability or flux jumps of practical nonisothermal and nonadiabatic high-T_c superconductors. The phenomena of both the flux jump and no-flux jump, observed in experiments, are described by the numerical simulations, in which the predictions of the first flux-jump field are quantitatively in good agreement with their measurements dependent on both the low temperature of the ambient environment and the sweep rate in the region of [20 G/s, 1 T/s]. Corresponding to each magnetic flux jump, the temperature in the slab experiences also a jump from the coolant one to a peak, then back down to the coolant one. When the sweep rate increases over 1 T/s and up to 6 T/s, the numerical results reveal the experimental observation that the system of thermomagnetic interaction converts to a smooth change of the magnetization loop after the first flux jump, while the temperature in the slab continuously changes without jumps in the region of about 10.6–16.4 K, which gives us an understanding of the mechanism of thermomagnetic interaction to this distinct phenomenon, i.e., the thermomagnetic interaction is smoothly circulated at a new dynamic equilibrium state in the temperature region of about 10.6–16.4 K higher than the ambient one, which is dependent on the tradeoff of dissipation heat and removed heat into the coolant. After that, the influence of some system parameters on the flux-jump field are discussed by the simulation model, which indicates that the flux-jump field is sensitive to the thickness of the slab, critical current density, and the heat transfer coefficient, but it is insensitive to the thermal conductivity.

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*Corresponding author. FAX: (086) 0931-8625576. Electronic address: zhouyh@lzu.edu.cn

[1] Y. B. Kim, C. F. Hempstead, and A. R. Strand, Phys. Rev. **129**, 528 (1963).
[2] S. L. Wipf and M. S Lubell, Phys. Lett. **16**, 103 (1965).
[3] P. W. Goedemoed and C. V. Kolmeschate, Meteslaar Physica **31**, 573 (1965).
[4] J. L. Tholence, H. Noel, J. C. Levet, M. Potel, and P. Gougeon, Solid State Commun. **65**, 1131 (1988).
[5] K. Chen, S. W. Hsu, T. L. Chen, S. D. Lan, W. H. Lee, and P. T. Wu, Appl. Phys. Lett. **56**, 2675 (1990).
[6] K.-H. Muller and C. Andrikidis, Phys. Rev. B **49**, 1294 (1994).
[7] G. C. Han, K. Watanabe, S. Awaji, N. Kobayashi, and K. Kimura, Physica C **274**, 33 (1997).
[8] L. Legrand, I. Rosenman, Ch. Simon, and G. Collin, Physica C **211**, 239 (1993).
[9] M. E. McHenry, H. S. Lessure, M. P. Maley, J. Y. Coulter, I. Tanaka, and H. Kojima, Physica C **190**, 403 (1992).
[10] A. Gerber, Z. Tarnawski, and J. J. M. Franse, Physica C **209**, 147 (1993).
[11] S. L. Wipf, Cryogenics **31**, 936 (1991).
[12] A. Milner, Physica B **294-295**, 388 (2001).
[13] A. Nabialek, M. Niewczas, H. Dabkowska, A. Dabkowski, J. P. Castellan, and B. D. Gaulin, Phys. Rev. B **67**, 024518 (2003).
[14] V. Chabanenko, R. Puzniak, A. Nabialek, S. Vasiliev, V. Rusakov, L. Huanqian, R. Szymczak, H. Szymczak, J. Jun, J. Karpinski, and V. Finkel, J. Low Temp. Phys. **130**, 175 (2003).
[15] Z. W. Zhao. S. L. Li, Y. M. Ni, H. P. Yang, Z. Y. Liu, H. H. Wen, W. N. Kang, H. J. Kim, E. M. Choi, and S. I. Lee, Phys. Rev. B **65**, 064512 (2002).
[16] D. Monler and L. Fruchter, Eur. Phys. J. B **3**, 143 (1998).
[17] R. G. Mints and A. L. Rakhmanov, Rev. Mod. Phys. **53**, 551 (1981).
[18] R. G. Mints, Phys. Rev. B **53**, 12311 (1996).
[19] S. L. Wipf, Phys. Rev. **161**, 404 (1967).
[20] P. S. Swartz and C. P. Bean, J. Appl. Phys. **39**, 4991 (1968).
[21] C. P. Bean, Phys. Rev. Lett. **8**, 250 (1962).
[22] M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975).
[23] V. V. Chabonenko, A. I. D'yachenko, M. V. Zalutskii, V. F. Rusakov, H. Szymozak, S. Piechota, and A. Nabielek, J. Appl. Phys. **88**, 5875 (2000).
[24] A. Igor, G. Alex, and V. Valerii, Phys. Rev. Lett. **87**, 067003 (2001).
[25] A. Gurevich and H. K. Küpfer, Phys. Rev. B **48**, 6477 (1993).
[26] R. Griessen, Phys. Rev. Lett. **64**, 1674 (1990).
[27] Y. B. Kim, C. F. Hempstead, and A. R. Strand, Phys. Rev. Lett. **9**, 306 (1962).
[28] P. W. Anderson and Y. B. Kim, Rev. Mod. Phys. **36**, 39 (1964).
[29] M. J. Qin and X. X. Yao, Phys. Rev. B **54**, 7536 (1996).
[30] R. A. Fisher, S. Kim, S. E. Lacy, N. E. Phillips, D. E. Morris, A. G. Markelz, J. Y. T. Wei, and D. S. Ginley, Phys. Rev. B **38**, 1942 (1988).
[31] E. T. Swartz and R. O. Pohl, Rev. Mod. Phys. **61**, 605 (1989).