Current-induced motion of a domain wall in a magnetic nanowire

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The current-induced motion of a magnetic domain wall in a quasi-one-dimensional ferromagnet with both easy-axis and easy-plane anisotropies is studied theoretically. We analyze the spin-transfer-induced torque on a sharp domain wall upon the flow of a dc electric current in the wire. The torque is shown to have two components; one of them acts as a driving force on the domain wall. The other torque component leads to changes to the domain-wall shape in that it forces the magnetic moments to diverge from the easy plane.

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I. INTRODUCTION

The increased interest in the dynamics of domain walls (DWs) in magnetic nanostructures is fueled by possible applications in spintronic devices. Several recent experiments have demonstrated the controllability of the DW position by means of either external magnetic fields or electric currents.[1–5] On the theoretical side, several theoretical treatments have been put forward to deal with the driven DW dynamics.[6–8] The magnetic dynamics within these models is usually viewed as classical and a separation of the magnetic and electronic degrees of freedom is then assumed.

Current research efforts are devoted to the study of the current-induced DW motion in nanowires and nanoconstrictions. In the presence of an electric current, the DW can be set in motion by the spin torque exerted on the magnetic system by the spin-polarized electron gas. In addition (linear) momentum can be transferred directly to the DW by the scattering of the charge carriers. Few recent papers[9–14] address the spin torque and the DW motion.

The problem of the DW motion is described by the Landau-Lifshitz equations, which have well-known static solutions. Deriving dynamic solutions is a nontrivial task, particularly in the presence of an external force. This problem is usually circumvented pragmatically by assuming physically reasonable approximations which are not strictly justified mathematically. The simplest approach is to consider the moving wall to be described by the static shape solution. The validity range of this approximation is unclear and one has to resort to numerical simulations to assess its reliability.[15]

As well known, a DW moving in response to a steady magnetic field can be described by the Walker solution.[16,17] This solution is exact and is based on the assumption of a constant deviation of the magnetization from the plane perpendicular to the field. However, it describes correctly the DW motion only for magnetic fields smaller than some critical value, H_c.[18] It is not clear whether the Walker solution is appropriate for a DW dragged by an electric current, for the current-induced torque tends to push the moments out of the plane.

Several recent approaches addressing the torque calculations as well as the solution of the dynamical equations of motion for the DW were developed recently[9,10,19] and are partially revised in this work. We consider the spin torque and the wall dynamics in a magnetic nanowire with a DW which is sharp on the scale set by the wave length of the relevant charge carriers. Our approach is particularly appropriate for magnetic semiconductors with low charge carrier (electrons or holes) concentrations (small Fermi momentum).[20]

In Sec. II we consider the torque due to spin transfer. The DW motion is discussed in Sec. III, whereas Sec. IV contains final conclusions and discussions.

II. CURRENT-INDUCED SPIN TORQUE

We consider the spin torque transferred to the DW in the presence of a steady current of spin-polarized charge carriers. Our main objective is to demonstrate the existence of two components of the torque that rotate a magnetic moment in different directions.

We adopt a one-dimensional model for the charge carriers, with a pointlike interaction between the electron spin $\boldsymbol{\sigma}$ and the magnetic moment $\mathbf{M}(x)$ located at a point x along the wire,

$$H_{int} = g\boldsymbol{\sigma}\cdot\mathbf{M}(x), \tag{1}$$

where g is the coupling constant. Here the one dimensionality of the electronic system means that we consider the electrons within a wire with transversal dimensions smaller than the electron wavelength λ, so that only the lowest electron subband is relevant. Strictly speaking, the coupling of the localized moment to the carriers' spin depends on the coordinates y and z that characterize the location of the moment within the wire, $g(y,z)\sim|\psi_0(y,z)|^2$, where $\psi_0(y,z)$ is the wave function of the transverse motion of electrons in the lowest subband. For simplicity, in the following we neglect this dependence, assuming an average coupling, $g(y,z)\to g = A^{-1}\int g(y,z)dy\,dz$, where A is the cross section of the wire.

When considering scattering of electrons from a magnetic moment $\mathbf{M}(x)$ we assume that the magnetic moment is frozen at the point x on the scale of the characteristic times of electron motion. This assumption renders possible the calculation of the torque as in the case of a static DW. The calculated torque is then used to investigate the DW dynamics. Our assumption relies on an adiabatic approximation insofar as we require the time scale for the motion of the magnetic subsystem to be slow as compared to that of the electrons.

To calculate the torque in the case of a sharp DW, we start from a model describing electron scattering from a localized moment in nonmagnetic and magnetic wires. The results obtained for the simplified models are then used to calculate the torque acting locally on the moments within the DW.

A. Single magnetic moment in a nonmagnetic wire

Let us consider first the scattering of electrons in a nonmagnetic one-dimensional system (nonmagnetic nanowire). The electrons are scattered from a single frozen magnetic moment $\mathbf{M}_0$ situated at the point $x=0$, i.e., $\mathbf{M}(x)=\mathbf{M}_0\delta(x)$. Here we denote the coordinate along the wire as x. In the absence of spin-orbit interaction, it is convenient to use a separate coordinate system (x',y',z') for the spin space. We calculate the total scattering amplitude (not just in the first Born approximation) of an electron with arbitrary spin polarization, coming from $x=-\infty$ and elastically scattered into another spin polarization.

Assuming the quantization axis z' along the moment $\mathbf{M}_0$, we can write the spinor wave function of the electrons as

$$\psi(x)=\begin{cases} e^{ikx}\begin{pmatrix} a \\ b\end{pmatrix}+e^{-ikx}\begin{pmatrix} ra \\ r^*b\end{pmatrix}, & x<0 \\ e^{ikx}\begin{pmatrix} ta \\ t^*b\end{pmatrix}, & x>0, \end{cases} \tag{2}$$

where the coefficients a and b correspond to an arbitrary spin polarization of the incident electron wave, r and t are the reflection and the transmission coefficients in the spin-up channel, respectively,

$$r=-\frac{i\alpha}{1+i\alpha},\quad t=\frac{1}{1+i\alpha}, \tag{3}$$

$\alpha=gM_0m/k\hbar^2$, and M_0 is the magnitude of the localized magnetic moment.

Using Eqs. (2) and (3) we can calculate the corresponding spin density in the wire,

$$S_\mu(x)=\psi^\dagger(x)\sigma_\mu\psi(x). \tag{4}$$

The relevant spin density components are

$$S_{x'}(x<0)=s_{x'}\frac{1+\alpha^2+2\alpha^4}{(1+\alpha^2)^2}+s_{y'}\frac{2\alpha^3}{(1+\alpha^2)^2} - 2\cos(2kx)\left(s_{x'}\frac{\alpha^2}{1+\alpha^2}+s_{y'}\frac{\alpha}{1+\alpha^2}\right), \tag{5}$$

$$S_{y'}(x<0)=s_{y'}\frac{1+\alpha^2+2\alpha^4}{(1+\alpha^2)^2}-s_{x'}\frac{2\alpha^3}{(1+\alpha^2)^2} + 2\cos(2kx)\left(-s_{y'}\frac{\alpha^2}{1+\alpha^2}+s_{x'}\frac{\alpha}{1+\alpha^2}\right), \tag{6}$$

$$S_{z'}(x<0)=s_{z'}\left(\frac{1+2\alpha^2}{1+\alpha^2}-\cos(2kx)\frac{2\alpha^2}{1+\alpha^2}\right) - \sin(2kx)\frac{2\alpha}{1+\alpha^2}, \tag{7}$$

where s_μ is the unit vector along the spin polarization of the incident wave. Similarly, for $x\geqslant 0$ we obtain

$$S_{x'}(x\geqslant 0)=s_{x'}\frac{1-\alpha^2}{(1+\alpha^2)^2}-s_{y'}\frac{2\alpha}{(1+\alpha^2)^2}, \tag{8}$$

$$S_{y'}(x\geqslant 0)=s_{y'}\frac{1-\alpha^2}{(1+\alpha^2)^2}+s_{x'}\frac{2\alpha}{(1+\alpha^2)^2}, \tag{9}$$

$$S_{z'}(x\geqslant 0)=s_{z'}\frac{1}{1+\alpha^2}. \tag{10}$$

From Eqs. (5)–(10) follows that the spin density induced by the spin-polarized wave incoming from $x=-\infty$, oscillates for $x<0$ with the period π/k, and is constant for $x>0$.

The spin current is defined as

$$j^s_\mu(x)=\frac{i\hbar}{2m}\{[\nabla_x\psi^\dagger(x)]\sigma_\mu\psi(x)-\psi^\dagger(x)\sigma_\mu\nabla_x\psi(x)\} \tag{11}$$

$(\mu=x',y',z')$, and can also be calculated using Eqs. (2) and (3). We find that the spin current is constant for $x<0$ and $x>0$, with a jump of its x' and y' components at $x=0$.

The spin torque acting on the moment $\mathbf{M}_0$ can be calculated as the transferred spin current at the point $x=0$

$$T_\mu=j^s_\mu(-\delta)-j^s_\mu(+\delta). \tag{12}$$

Using Eqs. (2), (3), (11), and (12) we obtain

$$T_{x'}=\frac{j_0}{e}\left[s_{x'}\frac{4\alpha^2}{1+\alpha^2}+s_{y'}\frac{2\alpha(1-\alpha^2)}{1+\alpha^2}\right], \tag{13}$$

$$T_{y'}=\frac{j_0}{e}\left[s_{y'}\frac{4\alpha^2}{1+\alpha^2}-s_{x'}\frac{2\alpha(1-\alpha^2)}{1+\alpha^2}\right], \tag{14}$$

and $T_{z'}=0$, where e is the electron charge $(e<0)$, j_0 is the electric current associated with the scattering state

$$j_0=\frac{ie\hbar}{2m}\{[\nabla_x\psi^\dagger(x)]\psi(x)-\psi^\dagger(x)\nabla_x\psi(x)\}=\frac{ev}{1+\alpha^2}, \tag{15}$$

and $v=\hbar k/m$ is the velocity. Note that the x' and y' axes are perpendicular to $\mathbf{M}_0$, so the torque components (13) and (14) are not related in any way to the direction of the current j_0.

Using Eqs. (5)–(10), the results (13) and (14) can also be obtained from the equation of motion for the magnetic moment $\mathbf{M}_0$,

$$T_\mu = -\frac{gM_0}{\hbar}\epsilon_{\mu\nu\lambda}n_\nu S_\lambda(0), \tag{16}$$

where $\mathbf{n}$ is the unit vector along $\mathbf{M}_0$, and $\epsilon_{\mu\nu\lambda}$ is the unit antisymmetric tensor.

In a general coordinate system for the electron spin (not necessarily with the axis z' along $\mathbf{M}_0$), Eqs. (13) and (14) become

$$T_\mu = \frac{j_0}{e}[\eta(\delta_{\mu\nu} - n_\mu n_\nu)s_\nu + \zeta\epsilon_{\mu\nu\lambda}n_\nu s_\lambda], \tag{17}$$

where

$$\eta = \frac{4\alpha^2}{1+\alpha^2}, \quad \zeta = -\frac{2\alpha(1-\alpha^2)}{1+\alpha^2}. \tag{18}$$

In Eqs. (17) and (18), there are two components of the torque—both transverse to the localized moment. One tends to align the moment along the direction of the spin polarization of the incoming electrons, whereas the other is perpendicular to the spin polarization of the incident wave. In the first Born approximation for the scattering amplitude, valid for $\alpha \ll 1$, only the second term in Eq. (17) survives, which tends to rotate the moment $\mathbf{M}_0$ to the direction perpendicular to the vector $\mathbf{s}$ (and also to $\mathbf{n}$).

The spin torque acting on a single magnetic moment can be found either as a change of the spin current due to scattering from the localized moment or from the calculation of the interaction of accumulated spin with the localized moment. In the following, to calculate the torque in the domain wall, we will use both methods but the second one (coupling to the accumulated spin) is more convenient in the case of a sharp DW.

In the case of a smooth DW, it can be more convenient to consider the propagation of a spin-polarized wave with subsequent scattering from the magnetic moments. Alternatively, one can calculate the torque as the divergence of the spin current.

B. Scattering from a single magnetic moment in a magnetic wire

Now we calculate the torque in the case of a magnetic wire with magnetization $\mathbf{M}$ oriented along the axis x for $x<0$ (left of the wall) and in the opposite direction for $x>0$ (right of the wall). We assume now that the (x',y',z') coordinate system coincides with the (x,y,z) one. As before, we introduce an additional frozen magnetic moment $\mathbf{M}_0 = M_0(n_x,n_y,0)$ located at the point $x=0$. For definiteness, let the vector $\mathbf{M}_0$ lie in the x-y plane.

The corresponding Hamiltonian can be written as

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + gM\sigma_x\,\mathrm{sgn}(x) + gM_0\mathbf{n}\cdot\boldsymbol{\sigma}\delta(x). \tag{19}$$

We consider the torque created by spin-polarized electron waves coming from the left (with the spin polarization along the axis x labeled as "↑"). We choose the quantization axis along the axis z. Then, the wave function containing the reflected and the transmitted waves of opposite polarization is

$$\psi_\uparrow(x) = \begin{cases} \dfrac{e^{ik_\uparrow x} + r_\uparrow e^{-ik_\uparrow x}}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} + \dfrac{r_{\uparrow f}e^{-ik_\downarrow x}}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} & \text{for } x<0, \\ \dfrac{t_\uparrow e^{ik_\downarrow x}}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} + \dfrac{t_{\uparrow f}e^{ik_\uparrow x}}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} & \text{for } x>0, \end{cases} \tag{20}$$

where $k_{\uparrow,\downarrow} = [2m(\varepsilon \mp gM)]^{1/2}/\hbar$, and ε is the energy. Note that the spin-up electrons are the spin-minority, while the spin-down electrons are the spin-majority ones.

Using the continuity of the wave function at $x=0$ and the discontinuity in slope of the function at $x=0$, resulting from

$$-\frac{\hbar^2}{2m}\left(\left.\frac{d\psi}{dx}\right|_{+\delta} - \left.\frac{d\psi}{dx}\right|_{-\delta}\right) + gM_0(n_x\sigma_x + n_y\sigma_y)\psi(0) = 0, \tag{21}$$

we find the transmission coefficients for the spin-up polarized wave

$$t_\uparrow = \frac{2k_\uparrow(k_\uparrow + k_\downarrow - ig_o n_x)}{(k_\uparrow + k_\downarrow)^2 + g_0^2}, \tag{22}$$

$$t_{\uparrow f} = -\frac{2g_o n_y k_\uparrow}{(k_\uparrow + k_\downarrow)^2 + g_0^2}, \tag{23}$$

and the reflection factors $r_\uparrow = t_\uparrow - 1$, $r_{\uparrow f} = t_{\uparrow f}$, where $g_0 = 2gmM_0/\hbar^2$.

Using Eqs. (11) and (20), we find that the spin current associated with the incoming spin-up wave is

$$j^s_{\uparrow x}(x) = \begin{cases} v_\uparrow(1-|r_\uparrow|^2) + v_\downarrow|r_{\uparrow f}|^2, & x<0, \\ v_\downarrow|t_\uparrow|^2 - v_\uparrow|t_{\uparrow f}|^2, & x>0, \end{cases} \tag{24}$$

$$j^s_{\uparrow y}(x) = \begin{cases} t_{\uparrow f}\,\mathrm{Im}[v_\uparrow(e^{ik_+x} - r_\uparrow e^{-ik_-x}) + v_\downarrow(e^{-ik_+x} + r^*_\uparrow e^{ik_-x})], & x<0, \\ t_{\uparrow f}\,\mathrm{Im}[-v_\uparrow t^*_\uparrow e^{ik_-x} + v_\downarrow t_\uparrow e^{-ik_-x}], & x>0, \end{cases} \tag{25}$$

$$j^s_{\uparrow z}(x) = \begin{cases} t_{\uparrow f}\,\mathrm{Re}[v_\uparrow(e^{ik_+x} - r_\uparrow e^{-ik_-x}) - v_\downarrow(e^{-ik_+x} + r^*_\uparrow e^{ik_-x})], & x<0, \\ t_{\uparrow f}\,\mathrm{Re}[v_\uparrow t^*_\uparrow e^{ik_-x} + v_\downarrow t_\uparrow e^{-ik_-x}], & x>0, \end{cases} \tag{26}$$

where $k_\pm = k_\uparrow \pm k_\downarrow$ and $v_{\uparrow,\downarrow} = \hbar k_{\uparrow,\downarrow}/m$.

Hence, the transverse components of the spin currents, $j^s_{\uparrow y}(x)$ and $j^s_{\uparrow z}(x)$, are nonzero for $x<0$ and for $x>0$. As we see from Eqs. (24)–(26), the transverse components of the spin current are oscillating functions of x. The nonconservation of spin currents in the magnetic wire is related to indirect magnetic interactions accompanying the inhomogeneous distribution of the spin density. In the nonmagnetic case, corresponding to the limit of $k_- \to 0$, it reduces to the conservation of spin current at $x<0$ and $x>0$, as in the previous section.

The spin transfer (12) gives now the torque

$$T_{\uparrow x} = 2v_\uparrow \,\mathrm{Re}\, t_\uparrow + (v_\uparrow + v_\downarrow)(|t_{\uparrow f}|^2 - |t_\uparrow|^2), \tag{27}$$

$$T_{\uparrow y} = -2t_{\uparrow f}(v_\uparrow + v_\downarrow)\mathrm{Im}\, t_\uparrow, \tag{28}$$

$$T_{\uparrow z} = 2t_{\uparrow f}[v_\uparrow - (v_\uparrow + v_\downarrow)\mathrm{Re}\, t_\uparrow]. \tag{29}$$

For the coordinate system assumed, when the vector $\mathbf{M}_0$ lies in the x-y plane, the transverse components of the torque acting on the moment $\mathbf{M}_0$ are

$$T_{\uparrow\perp} = -n_y T_{\uparrow x} + n_x T_{\uparrow y}, \tag{30}$$

which tends to rotate the moment in the x-y plane, and $T_{\uparrow z}$ which tends to rotate it out of the plane. The label $\perp$ in Eq. (30) means the torque projection on the direction perpendicular to the moment $\mathbf{M}_0$ in the x-y plane.

Scattering of incident electrons with the spin polarization opposite to the axis x (labeled as $\downarrow$) can be considered in a similar way. The corresponding scattering state has the form of Eq. (20) with $k_\uparrow \leftrightarrow k_\downarrow$ and interchanged spin states. The relevant transmission coefficients are

$$t_\downarrow = \frac{2k_\downarrow(k_\uparrow + k_\downarrow + ig_o n_x)}{(k_\uparrow + k_\downarrow)^2 + g_0^2}, \tag{31}$$

$$t_{\downarrow f} = \frac{2g_o n_y k_\downarrow}{(k_\uparrow + k_\downarrow)^2 + g_0^2}, \tag{32}$$

where the change $g_0 \leftrightarrow -g_0$ is equivalent to the flip of moment $\mathbf{M}_0$. The components of the associated spin current $j^s_{\downarrow\mu}$ have the following form:

$$j^s_{\downarrow x}(x) = \begin{cases} -v_\downarrow(1 - |r_\downarrow|^2) - v_\downarrow |r_{\downarrow f}|^2, & x<0, \\ -v_\uparrow |t_\downarrow|^2 + v_\downarrow |t_{\downarrow f}|^2, & x>0, \end{cases} \tag{33}$$

$$j^s_{\downarrow y}(x) = \begin{cases} -t_{\downarrow f}\,\mathrm{Im}[v_\downarrow(e^{ik_+x} - r_\downarrow e^{-ik_-x}) + v_\uparrow(e^{-ik_+x} + r^*_\downarrow e^{ik_-x})], & x<0, \\ t_{\downarrow f}\,\mathrm{Im}[v_\downarrow t^*_\downarrow e^{ik_-x} - v_\uparrow t_\downarrow e^{-ik_-x}], & x>0, \end{cases} \tag{34}$$

$$j^s_{\downarrow z}(x) = \begin{cases} t_{\downarrow f}\mathrm{Re}[v_\downarrow(e^{ik_+x} - r_\downarrow e^{-ik_-x}) - v_\uparrow(e^{-ik_+x} + r^*_\downarrow e^{ik_-x})], & x<0, \\ t_{\downarrow f}\,\mathrm{Im}[v_\downarrow t^*_\downarrow e^{ik_-x} + v_\uparrow t_\downarrow e^{-ik_-x}], & x>0. \end{cases} \tag{35}$$

As in the case of the $\uparrow$-incident wave, Eq. (12) can be used to calculate $\mathbf{T}_\downarrow$. The relevant formula is similar to Eqs. (27)–(29). We note, that the components of $\mathbf{T}_\uparrow$ and $\mathbf{T}_\downarrow$ can also be calculated from Eq. (16), taking into account the net spin at $x=0$,

$$S_{\uparrow,\downarrow x}(0) = \frac{4k^2_{\uparrow,\downarrow}[(k_\uparrow + k_\downarrow)^2 + g_0^2(n_x^2 - n_y^2)]}{[(k_\uparrow + k_\downarrow)^2 + g_0^2]^2}, \tag{36}$$

$$S_{\uparrow,\downarrow y}(0) = \frac{8g_0^2 k^2_{\uparrow,\downarrow} n_x n_y}{[(k_\uparrow + k_\downarrow)^2 + g_0^2]^2}, \tag{37}$$

$$S_{\uparrow,\downarrow z}(0) = \mp \frac{8g_0 k^2_{\uparrow,\downarrow}(k_\uparrow + k_\downarrow)n_y}{[(k_\uparrow + k_\downarrow)^2 + g_0^2]^2}. \tag{38}$$

The above formulas will be used later to calculate the torque exerted on a DW.

In the case of a fully spin-polarized electron gas, only the spin current components Eqs. (33)–(35) are relevant (corresponding to the majority electrons). Accordingly, in these equations we should substitute $k_\uparrow \to i\kappa_\uparrow$, where $\kappa_\uparrow$ is real.

The eigenfunctions of the Hamiltonian (19) correspond to the spin-polarized incoming electron waves (spin-up and spin-down). An arbitrary-polarized incoming wave is not the eigenfunction of the Hamiltonian. Nevertheless, we can still consider the scattering of electron waves with different spin polarizations. For example, such a state can be created by means of an injection from the tip, and its lifetime τ can be long enough on the scale of the characteristic time of the DW motion. In this case, a superposition of states with different incoming spin-polarized waves can be used to calculate the torque.

C. Magnetic wire with a thin domain wall

In the case of a thin metallic wire, when $k_{F\uparrow,\downarrow}d \ll 1$ (d is the wire diameter and $k_{F\uparrow,\downarrow}$ are the Fermi momenta of minority and majority electrons), we can assume that only one quantization level is filled with electrons.

Consider a magnetic wire with a single DW corresponding to the magnetization $\mathbf{M}$ along the axis x for $x<-w$ and opposite to the axis x for $x>w$. Here $2w$ is the DW width, and we chose the spin coordinate system like in the previous section, with the x axis along the wire.

Upon applying a small voltage, an electric current can flow in the wire. We assume the current in the negative x axis direction (i.e., the electron flux is along x). If the only imperfection in the wire is the DW, one can assume a jump $\Delta\phi$ in the electrostatic potential at the wall, and both the charge and the spin currents can be calculated as integrals over the energies in the interval between ε_{FR} and $\varepsilon_{FL}=\varepsilon_{FR}+e\Delta\phi$, where ε_{FL} and ε_{FR} are the Fermi levels on the left and right sides. In the limit of a small voltage, $|e\Delta\phi| \ll \varepsilon_F$, the transport is linear and is associated with electrons at the Fermi level.

We assume the electrons approaching the domain wall from the left are spin-polarized according to the magnetization direction in the left part of the wire. The incoming electrons are scattered from a large number of magnetic moments in the wall. We consider this scattering using the point interaction of an electron with each of the localized moments. This corresponds to the picture with an array of well-separated magnetic moments as in a magnetic semiconductor doped with magnetic impurities. Accordingly, an electron transmitted through the wall is multiply scattered by many magnetic moments.

To calculate the transmission of electrons through the DW, we take the perturbation created by the total magnetic moment $\widetilde{\mathbf{M}}(x)=\Sigma_i \mathbf{M}_i \delta(x-x_i)$, where $\mathbf{M}_i$ is the localized moment at the point $x=x_i$, and all of the moments $\mathbf{M}_i$ are located within a region of the wall width, $|x_i|<w$, which in turn is assumed to be small as compared to the wavelength of electrons, $k_{F\uparrow,\downarrow}w \ll 1$.

Electron scattering from the total moment $\widetilde{\mathbf{M}}(x)$ located within a region much smaller than the electron wavelength can be described using the spin-dependent delta-function potential model.[12] Then, in the limit of small voltage, the current takes the form

$$j_0 \simeq \frac{e^2\Delta\phi}{2\pi\hbar}\left(|\widetilde{t}_{\uparrow f}|^2 + \frac{v_\downarrow}{v_\uparrow}|\widetilde{t}_\uparrow|^2 + |\widetilde{t}_{\downarrow f}|^2 + \frac{v_\uparrow}{v_\downarrow}|\widetilde{t}_\downarrow|^2\right), \qquad (39)$$

where the tilde means the transmission coefficients for the scattering of electrons from the effective moment[12] $\mathbf{M}_{eff}$

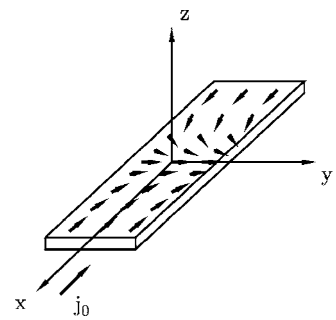

FIG. 1. Schematic picture of the domain wall.

$\simeq \int_{-w}^{+w}\widetilde{\mathbf{M}}(x)dx$. This is the Büttiker-Landauer formula for conductivity, which can be obtained in the linear response approximation using the basis of scattering states. There are two contributions in Eq. (39) related to the incoming waves with spin-up and spin-down polarizations, and with the corresponding Fermi momenta $k_{\uparrow,\downarrow} \equiv k_{F\uparrow,\downarrow}$.

In the DW with the magnetization profile of Fig. 1, the effective moment $\mathbf{M}_{eff}$ is along the y axis. The transmission coefficients $\widetilde{t}_\uparrow$, $\widetilde{t}_{\uparrow f}$ and $\widetilde{t}_\downarrow$, $\widetilde{t}_{\downarrow f}$ have the form of Eqs. (22), (23), (31), and (32), respectively, with $n_x=0$, $n_y=1$, and with substitution $g_0 \to \widetilde{g}_0 \equiv 2mgM_{eff}/\hbar^2$. The magnitude of M_{eff} is $M_{eff} \simeq \int_{-w}^{w} M_y(x)dx$.

The spin current can be also calculated in the linear response approximation using the scattering states.[12] It includes the sum of partial spin currents

$$\mathbf{j}^s(x) = \frac{e\Delta\phi}{2\pi\hbar}\left(\frac{\widetilde{\mathbf{j}}^s_\uparrow(x)}{v_\uparrow} + \frac{\widetilde{\mathbf{j}}^s_\downarrow(x)}{v_\downarrow}\right), \qquad (40)$$

where the components of $\widetilde{\mathbf{j}}^s_{\uparrow,\downarrow}$ can be found using Eqs. (24)–(26) and (33)–(35) with the substitution $t_{\uparrow,\downarrow}, t_{\uparrow,\downarrow f} \to \widetilde{t}_{\uparrow,\downarrow}, \widetilde{t}_{\uparrow,\downarrow f}$. The appearance of $v_\uparrow$ and $v_\downarrow$ in the denominators of Eq. (40) is related to the one-dimensional (1D) density of states for spin-up and spin-down electrons. The spin current components perpendicular to the axis x are oscillating functions. As we see from Eqs. (24)–(26) and (33)–(35), the wavelength of the oscillations is determined by the inverse momentum at the Fermi level. Hence, the oscillation wavelength of the transverse component of the spin current is much larger than the DW width.

It is worth noting that in three-dimensional systems, the transverse component of the spin current decays due to the integration over momentum in the DW plane. In metallic ferromagnets, the decay is very fast due to the large electron Fermi momentum. However, there is an additional nonvanishing spin transfer for the transverse component in the 3D case.

We can also calculate the net spin density induced by the external current j_0. It can be found as the expectation value of the spin σ_μ in the scattering state of the incoming electrons, integrated over all energies between ε_F and $\varepsilon_F+e\Delta\phi$, as in the calculation of the charge and spin currents. We obtain

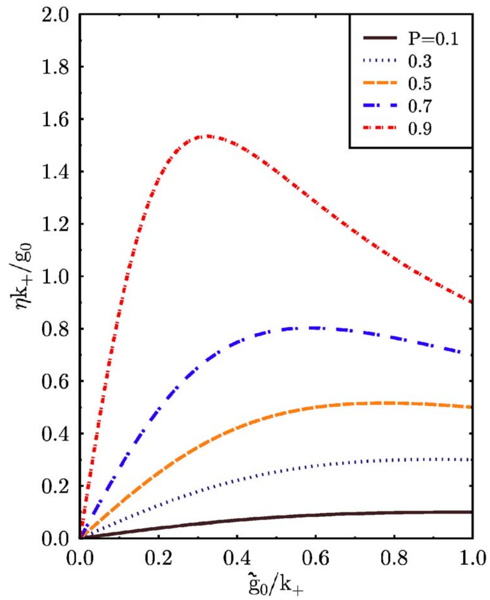

FIG. 2. (Color online) Dependence of the factor η on the effective coupling $\tilde{g}_0$ for different values of the electron polarization P.

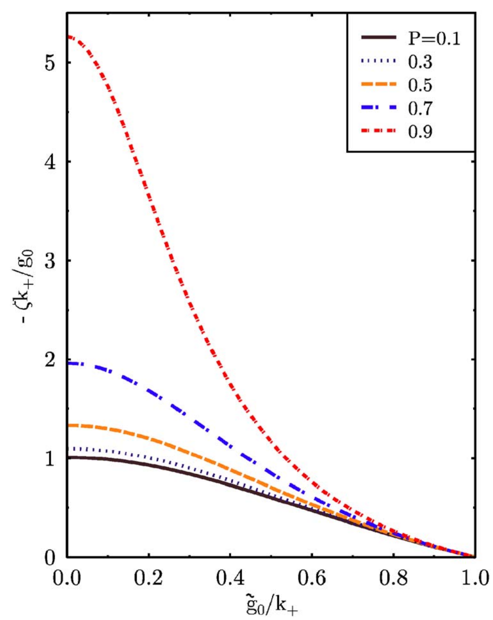

FIG. 3. (Color online) Coefficient ζ vs coupling constant $\tilde{g}_0$ for different values of P.

$$\mathbf{S}(0) = \frac{e\Delta\phi}{2\pi\hbar}\left(\frac{\widetilde{\mathbf{S}}_\uparrow(0)}{v_\uparrow} + \frac{\widetilde{\mathbf{S}}_\downarrow(0)}{v_\downarrow}\right), \tag{41}$$

where $\widetilde{\mathbf{S}}_{\uparrow,\downarrow}(0)$ can be found using Eqs. (36)–(38) with $n_x=0$ and the substitution $g_0 \to \tilde{g}_0$, which corresponds to the scattering from the effective magnetic moment $\mathbf{M}_{eff}$.

Finally, we find the torque acting on a single localized moment in the domain wall. For this purpose we use Eq. (16) with $\mathbf{S}(0)$ from (41), describing the spin accumulation created by scattering from the domain wall as a whole. In its turn, $\widetilde{\mathbf{S}}_{\uparrow,\downarrow}(0)$ is calculated as explained after Eq. (41) using (36)–(38). The result can be presented in the general form

$$\mathbf{T} = \frac{j_0}{e}[\eta\mathbf{n}\times(\mathbf{n}\times\mathbf{s}) + \zeta\mathbf{n}\times\mathbf{s}]. \tag{42}$$

where

$$\eta = \frac{g_0\tilde{g}_0(k_\downarrow^2 - k_\uparrow^2)}{2k_\uparrow k_\downarrow(k_\uparrow + k_\downarrow)^2 + \tilde{g}_0^2(k_\uparrow^2 + k_\downarrow^2)}, \tag{43}$$

$$\zeta = -\frac{g_0(k_\uparrow + k_\downarrow)^2[(k_\uparrow + k_\downarrow)^2 - \tilde{g}_0^2]}{2[2k_\uparrow k_\downarrow(k_\uparrow + k_\downarrow)^2 + \tilde{g}_0^2(k_\uparrow^2 + k_\downarrow^2)]}, \tag{44}$$

and $\mathbf{s}$ is the unit vector along the spin polarization corresponding to magnetization $\mathbf{M}$ at $x<-w$. The dependence of the coefficients η and ζ on the parameters $\tilde{g}_0$ and on the electron gas polarization $P=(k_\downarrow - k_\uparrow)/(k_\uparrow + k_\downarrow)$ is presented in Figs. 2 and 3. As we see, both coefficients strongly depend on the parameters describing the ferromagnet and on the parameters of the wall. In the case of small coupling $\tilde{g}_0$, we obtain $\zeta \gg \eta$, i.e., the torque is mostly related to the second component in Eq. (42). In contrast, if $\tilde{g}_0$ is larger, the first term in Eq. (42) dominates.

D. Spin torque in *p*-type magnetic semiconductors

Electrical conductivity of magnetic semiconductors like $Ga_xMn_{1-x}As$ is usually of the p type. The valence band of these semiconductors can be described by a matrix Hamiltonian, which includes the spin-orbit interaction.[21] Thus, a calculation of the hole transmission through the DW requires a model, which takes into account complex band structure of such compounds.

In this paper we use the Luttinger model for the energy spectrum of holes with the angular momentum $J=\frac{3}{2}$,[22] and neglect anisotropy of the energy spectrum. To simplify calculations, we assume the quantization axis along the wire (axis z). In the quasi-one-dimensional case, with the domain wall in the y-z plane, the Hamiltonian of holes takes then the form

$$H = \frac{\hbar^2}{2m_0}\left(\gamma_1 + \frac{5\gamma_2}{2}\right)\frac{d^2}{dz^2} - \frac{\hbar^2\gamma_2}{m_0}J_z^2\frac{d^2}{dz^2} - g[J_yM_y(z) + J_zM_z(z)], \tag{45}$$

where m_0 is the free electron mass, γ_1 and γ_2 are the Luttinger parameters, and J_μ are the matrices of the total angular momentum $\frac{3}{2}$. Note that we are using (45) to describe the holes as unfilled electron states in the valence band. The correct statistics of holes corresponds to the negative energy as compared to that of electrons.

As in the previous section, we take the magnetization $\mathbf{M}$ along the axis z for $z<-w$ and in the opposite direction for $z>w$, while in the region $-w<z<w$ the moment changes its orientation rotating in the y-z plane. For $z<-w$, the holes can be described by the energy spectrum consisting of four parabolic bands labeled by the angular momentum projection J_z,

$$E_{\pm 3/2}(k) = -\frac{\hbar^2 k^2}{2m_t} \mp \frac{3gM}{2}, \tag{46}$$

$$E_{\pm 1/2}(k) = -\frac{\hbar^2 k^2}{2m_l} \mp \frac{gM}{2}, \tag{47}$$

where $m_t = m_0/(\gamma_1 - 2\gamma_2)$ and $m_l = m_0/(\gamma_1 + 2\gamma_2)$ are the masses of heavy and light holes, respectively. In accordance with Eqs. (46) and (47), the energy band of the heavy holes with the moment projection $J_z = -\frac{3}{2}$ is above all the other bands. In the region $z > 0$, the spectrum is the same but with the opposite signs of J_z.

We assume that the holes are fully polarized so that the hole density is rather small. Correspondingly, we assume that the chemical potential μ is located between $gM/2$ and $3gM/2$, i.e., $gM/2 < \mu < 3gM/2$.

The scattering state of $J_z = -\frac{3}{2}$ holes, corresponding to the wave incoming from $z = -\infty$, is

$$\psi^\dagger(z) = (r_3^* e^{\kappa_3 z}, r_2^* e^{\kappa_2 z}, r_1^* e^{\kappa_1 z}, e^{-ikz} + r^* e^{ikz}), \quad z < -w, \tag{48}$$

$$\psi^\dagger(z) = (t^* e^{-ikz}, t_1^* e^{-\kappa_1 z}, t_2^* e^{-\kappa_2 z}, t_3^* e^{-\kappa_3 z}), \quad z > +w, \tag{49}$$

where $r, r_1, \ldots, r_3$ and $t, t_1, \ldots, t_3$ are the reflection and transmission coefficients, respectively. The momentum k of the heavy hole is taken at the Fermi surface, $-\hbar^2 k^2/2m_t + 3gM/2 = \varepsilon$. The other momenta κ_i correspond to the decaying components of the wave function, $\kappa_1 = [2m_l(\varepsilon - gM/2)/\hbar^2]^{1/2}$, $\kappa_2 = [2m_l(\varepsilon + gM/2)/\hbar^2]^{1/2}$, and $\kappa_3 = [2m_t(\varepsilon + 3gM/2)/\hbar^2]^{1/2}$. Note that the transmission coefficient t in this notation corresponds to the transmission from the state with moment $J_z = -\frac{3}{2}$ to the state $J_z = 3/2$.

In the limit of $w \to 0$, the matching condition can be presented in the matrix form

$$\mathrm{diag}(m_t^{-1}, m_l^{-1}, m_l^{-1}, m_t^{-1}) \left(\left. \frac{d\psi}{dz} \right|_{\delta} - \left. \frac{d\psi}{dz} \right|_{-\delta} \right) - \lambda_0 J_y \psi(0) = 0, \tag{50}$$

where $\lambda_0 = 2gM_{eff}/\hbar^2$.

Using Eq. (50) and the continuity of the wave function at $z=0$, we can calculate eight reflection and transmission coefficients. The accumulated spin density $\mathbf{S}(0)$ induced by the current flowing along the axis z can be calculated as in the previous section, but with the opposite sign because the accumulation of polarized holes means a loss of real particles (electrons). We find

$$S_x(0) = -\frac{e\Delta\phi}{2\pi\hbar v_t} \,\mathrm{Im}(\sqrt{3} t_1^* t + 2 t_2^* t_1 + \sqrt{3} t_3^* t_2), \tag{51}$$

$$S_y(0) = -\frac{e\Delta\phi}{2\pi\hbar v_t} \,\mathrm{Re}(\sqrt{3} t_1^* t + 2 t_2^* t_1 + \sqrt{3} t_3^* t_2), \tag{52}$$

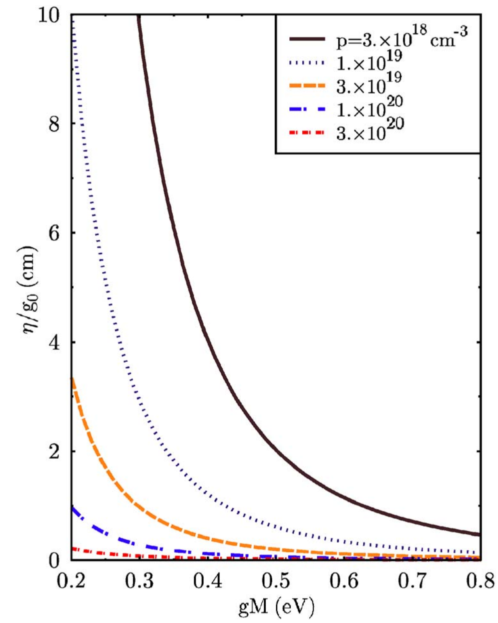

FIG. 4. (Color online) Dependence of the factor η on the magnetic splitting gM in the valence band of magnetic semiconductors for different values of the bulk hole density p.

$$S_z(0) = -\frac{e\Delta\phi}{4\pi\hbar v_t}(3|t|^2 + |t_1|^2 - |t_2|^2 - 3|t_3|^2), \tag{53}$$

where $v_t = \hbar k/m_t$ is the velocity of heavy holes at the Fermi level, $e\Delta\phi = \varepsilon_{FR} - \varepsilon_{FL} > 0$, and ε_{FL} and ε_{FR} are the Fermi levels at $z < -w$ and $z > w$, respectively.

Using Eqs. (16) and (42) we find the parameters η and ζ determining the torque acting on a single magnetic moment M_0,

$$\eta = \frac{egM_0}{j_0 \hbar} S_z(0), \tag{54}$$

$$\zeta = -\frac{egM_0}{j_0 \hbar} S_x(0), \tag{55}$$

where $j_0 = -e^2 \Delta\phi |t|^2 / 2\pi\hbar$, and the "$-$" sign in the current is due to the positive charge of the holes.

The dependence of η and ζ on the magnitude of magnetic splitting gM for different bulk hole densities p is presented in Figs. 4 and 5. We take the cross section $A = 1$ nm^2, and the momentum of heavy holes $k = \pi p_{1D}$, where p_{1D} is the linear density of holes.

As we can see from Figs. 4 and 5 the factor ζ is negligibly small as compared to η. In our model, the density of holes and the spin splitting are independent parameters. Thus, the magnitude of the torque η increases with the decreasing hole density p at a fixed value of gM. However, in real magnetic semiconductors these values are not independent, and the magnetic splitting increases with the increasing hole density.[23]

It should be noted that the approximation $w \to 0$ implies that not only the wavelength of holes with $J_z = \frac{3}{2}$ is large as compared to the DW width, $kw \ll 1$, but also the conditions

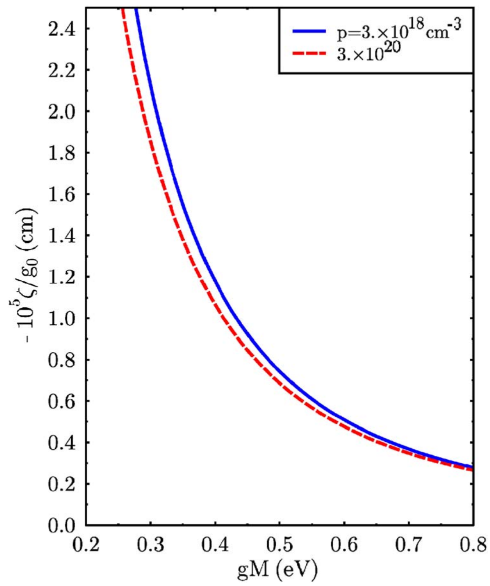

FIG. 5. (Color online) Coefficient ζ vs magnetic splitting gM for different values of p.

$\kappa_i w \ll 1$ for all κ_i should be fulfilled. This condition restricts the magnitude of the magnetic splitting, $(gMm_t)^{1/2} w/\hbar \ll 1$.

III. MOTION OF THE DOMAIN WALL

A. Hamiltonian and equations of motion

Now we consider the Hamiltonian $\mathcal{H}_0$ describing a quasi-one-dimensional magnetic system with a DW. We adopt a model including the magnetic exchange interaction and two different anisotropy constants λ_1 and λ_2 in the z and y directions, respectively (see Fig. 1),

$$\mathcal{H}_0 = \frac{a}{2}\left(\frac{\partial \mathbf{n}}{\partial x}\right)^2 + \frac{\lambda_1}{2} n_z^2 + \frac{\lambda_2}{2} n_y^2, \tag{56}$$

where a is the exchange constant, and $\mathbf{n}(x)$ is the unit vector along the magnetization. This Hamiltonian will be used to describe the magnetic nanowire like that presented in Fig. 1. We assume that the vector $\mathbf{n}$ depends only on the coordinate x and time t. The Hamiltonian $\mathcal{H}_0$ describes the magnetic system in the absence of the spin torque.

In our work we concentrate on the domain-wall motion due to spin transfer, so in the Hamiltonian (56) we neglected the magnetostatic contribution due to a stray field produced by the wire and domain wall. Such a contribution is expected to be small. However, it could be taken into account within the micromagnetic simulations,[24] which is however beyond the scope of this paper.

As we have already pointed out in the Introduction, our approach differs from the Walker solution of the magnetic dynamics equations. The main reason of the difference originates from the fact that the spin torque giving rise to current-induced DW motion [see Eq. (42)] explicitly contains a term rotating magnetic moments out of the plane. Our method is approximative but justified for the strong easy-plane anisotropy.

In the following we consider the spin torque as a driving force on the DW. Hence, we neglect the direct transfer of momentum from electrons reflected from the DW. In the Appendix we show that this effect is smaller than that due to spin torque.

Using spherical coordinates $\theta(x,t)$ and $\varphi(x,t)$, we can rewrite the Hamiltonian $\mathcal{H}_0$ as

$$\mathcal{H}_0 = \frac{a}{2}\left(\frac{\partial \theta}{\partial x}\right)^2 + \frac{a}{2}\left(\frac{\partial \varphi}{\partial x}\right)^2 \sin^2\theta + \frac{\lambda_1}{2}\cos^2\theta + \frac{\lambda_2}{2}\sin^2\theta \sin^2\varphi. \tag{57}$$

The Landau-Lifshitz-Gilbert equation of motion includes a damping term and two possible components of the current-induced torque, as discussed in the previous section,

$$\frac{1}{\gamma}\frac{\partial \mathbf{n}}{\partial t} = -\mathbf{n}\times\left(\frac{\partial \mathcal{H}_0}{\partial \mathbf{n}} - \frac{\partial}{\partial x}\frac{\partial \mathcal{H}_0}{\partial(\partial \mathbf{n}/\partial x)}\right) - \alpha \mathbf{n}\times\frac{\partial \mathbf{n}}{\partial t} + J_0\zeta \mathbf{n}\times\mathbf{s} + J_0\eta \mathbf{n}\times(\mathbf{n}\times\mathbf{s}), \tag{58}$$

where α is the damping constant, $\gamma = g\mu_B/\hbar M$ is the gyromagnetic ratio divided by M, $J_0 = j_0\hbar/eg\Omega_0$, and Ω_0 is a volume per magnetic moment. In Eq. (58) the spin torque is expressed in terms of the transferred moment per unit volume, and enters directly into the equation of motion. The corresponding spin-torque terms in the magnetic Hamiltonian can be represented as

$$\mathcal{H}_{int} = J_0\zeta \mathbf{n}\cdot\mathbf{s} + J_0\eta \int_0^1 d\tau\, \mathbf{n}\cdot\left(\frac{\partial \mathbf{n}}{\partial \tau}\times\mathbf{s}\right), \tag{59}$$

where $\mathbf{n}(\tau=0)=0$ and $\mathbf{n}(\tau=1)=\mathbf{n}$.

The Lagrangian of the magnetic system contains a term with a time derivative as follows[9,25]

$$\mathcal{L} = A\int dx\left[\frac{1}{\gamma}\frac{\partial \varphi}{\partial t}(\cos\theta - 1) - \mathcal{H}\right]. \tag{60}$$

Neglecting the damping term, equation of motion for the magnetization leads to the following equations for the spherical coordinates:

$$\frac{1}{\gamma}\frac{\partial \theta}{\partial t} = -a\frac{\partial^2 \varphi}{\partial x^2}\sin\theta + \lambda_2 \sin\theta\sin\varphi\cos\varphi - J_0\eta\cos\theta\cos\varphi - J_0\zeta\sin\varphi, \tag{61}$$

$$\frac{\sin\theta}{\gamma}\frac{\partial \varphi}{\partial t} = a\frac{\partial^2 \theta}{\partial x^2} - a\left(\frac{\partial \varphi}{\partial x}\right)^2 \sin\theta\cos\theta + \lambda_1\cos\theta\sin\theta - \lambda_2\sin\theta\cos\theta\sin^2\varphi + J_0\eta\sin\varphi + J_0\zeta\cos\theta\cos\varphi. \tag{62}$$

In the absence of current, $j_0=0$, they have the well-known[26] kinklike static solution $\varphi_0(x) = \arccos[\tanh(\beta_0 x)]$ and $\theta_0 = \pi/2$, where $\beta_0 = (\lambda_2/a)^{1/2}$ is the inverse width of the static DW. From now on we assume for definiteness that $\lambda_1 > \lambda_2$, so that the static DW with the magnetization in the x-y plane is energetically more favorable.

In a general case, the solution of the nonlinear equations (61) and (62) for a moving DW is a difficult problem. Therefore, we assume in the following that one of the anisotropy constants is large, $\lambda_1 \gg \lambda_2$.

B. Strong easy-plane anisotropy

We consider the case of a large easy-plane anisotropy, and, accordingly, assume that for the moving DW (subjected to the torque) the deviation of magnetization $\mathbf{M}$ from the x-y plane is small. Then, we can write $\theta(x,t)=\pi/2+\chi(x,t)$ and take $|\chi(x,t)|\ll 1$. To second order in the field $\chi(x,t)$, the Lagrangian $\mathcal{L}$ is

$$\mathcal{L}=A\int dx\left[-\frac{1}{\gamma}\frac{\partial\varphi}{\partial t}(\chi+1)-\frac{a}{2}\left(\frac{\partial\chi}{\partial x}\right)^2-\frac{a}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2(1-\chi^2)-\frac{\lambda_1}{2}\chi^2-\frac{\lambda_2}{2}\sin^2\varphi(1-\chi^2)+J_0\eta\chi\sin\varphi+J_0\zeta\cos\varphi\right], \tag{63}$$

Since we restrict our considerations to quadratic terms in χ, the integral over χ is Gaussian, and we can integrate out[27] the χ fields to obtain

$$\mathcal{L}=A\int dx\left[\frac{1}{2}\int dx' G(x,x')\left(\frac{1}{\gamma}\frac{\partial\varphi(x)}{\partial t}-J_0\eta\sin\varphi(x)\right)\times\left(\frac{1}{\gamma}\frac{\partial\varphi(x)}{\partial t}-J_0\eta\sin\varphi(x)\right)-\frac{a}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2-\frac{\lambda_2}{2}\sin^2\varphi+J_0\zeta\cos\varphi\right], \tag{64}$$

where the Green function $G(x,x)$ obeys the equation

$$\left[-a\frac{\partial^2}{\partial x^2}-a\left(\frac{\partial\varphi}{\partial x}\right)^2+\lambda_1-\lambda_2\sin^2\varphi\right]G(x,x')=\delta(x-x'). \tag{65}$$

Note that the φ fields are taken at the same time t in Eq. (64). This follows from the equation for the Green function describing propagation in time, $G(x,t;x',t')\sim\delta(t-t')$. Equation (64) contains $\varphi(x,t)$, which should be the saddle point solution of the Lagrangian, i.e., the self-consistency should be preserved.

Neglecting the first term in Eq. (65) we can find an approximate formula for the Green function proportional to $\delta(x-x')$

$$G(x,x')=\delta(x-x')\left[-a\left(\frac{\partial\varphi}{\partial x}\right)^2+\lambda_1-\lambda_2\sin^2\varphi\right]^{-1}. \tag{66}$$

This form of $G(x,x')$ leads to the point interaction of the φ fields in the first term of Eq. (64). Physically, by neglecting the first term with derivatives in Eq. (65) we substitute the finite-range interaction by the δ-like one.

One can estimate the conditions for which the use of Green function (66) is justified. The exact solution of Eq. (65) is

$$G(x,x')=\sum_n\frac{\phi_n(x)\phi_n^*(x')}{\varepsilon_n+\lambda_1}, \tag{67}$$

where $\phi_n(x)$ and ε_n are the eigenfunctions and the corresponding eigenvalues of the equation

$$\left[-a\frac{\partial^2}{\partial x^2}-a\left(\frac{\partial\varphi}{\partial x}\right)^2-\lambda_2\sin^2\varphi-\varepsilon_n\right]\phi_n(x)=0. \tag{68}$$

We expect that the function $\varphi(x)$ in Eqs. (65) and (68) is similar to the form of the static solution $\varphi_0(x)$. Thus, Eq. (68) corresponds to the Schrödinger equation for a particle of mass $m=\hbar^2/2a$ in the potential well $V(x)$ of width $L_0\sim(a/\lambda_2)^{1/2}$. The energy spectrum of this problem consists of a level in the well, $\varepsilon_0\simeq-\lambda_2$, and a continuous spectrum for all positive energies.

Equation (68) determines the eigenmodes (local spin excitations) of the static DW. This is because we use the harmonic expansion (63) of the Lagrangian in small deviations χ from the static solution with $\theta=\pi/2$. Note that the current-induced torque does not affect terms of the order of χ^2 in (63). The excitation modes of the DW are also known as *Winter modes*.[28] By integrating out the χ field from Eq. (63) we take into account the effective interaction of the φ fields via the Winter modes.

For $\varphi(x)=\varphi_0(x)$, the potential has the form $V(x)=-2\lambda_2/\cosh^2(\beta_0 x)$, and the discrete energy spectrum[29] has one level, $\varepsilon_0=-4\lambda_2$. The eigenfunctions $\phi_n(x)$ corresponding to the continuous spectrum are oscillatory functions, so that their contribution to Eq. (67) can be estimated as $G^{(cont)}\times(x,x')\simeq(a\lambda_1)^{-1/2}e^{-\kappa_1|x-x'|}$, where $\kappa_1=(\lambda_1/a)^{1/2}$. Since $\lambda_1\gg\lambda_2$, it is a strongly localized function on the scale of the distance L_0 (static domain-wall width). On the other hand, the contribution of the localized state gives $G^{(0)}(x,x')\simeq(1/L_0\lambda_1)e^{-\kappa_0|x-x'|}$, where $\kappa_0=1/L_0$. Thus, in the case of $\lambda_1\gg\lambda_2$ (i.e., strong in-plane anisotropy), the contribution of $G^{(0)}$ can be neglected as compared to the short-range interaction. Using the condition of strong easy-plane anisotropy, $\lambda_1\gg\lambda_2$, we can simplify Eq. (66) essentially and obtain the expression

$$G(x,x')\simeq\frac{\delta(x-x')}{\lambda_1}. \tag{69}$$

In this approximation, the Lagrangian (64) acquires the following form:

$$\mathcal{L}=A\int dx\left[\frac{1}{2\lambda_1}\left(\frac{1}{\gamma}\frac{\partial\varphi}{\partial t}-J_0\eta\sin\varphi\right)^2-\frac{a}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2-\frac{\lambda_2}{2}\sin^2\varphi+J_0\zeta\cos\varphi\right]. \tag{70}$$

The corresponding saddle-point equation is

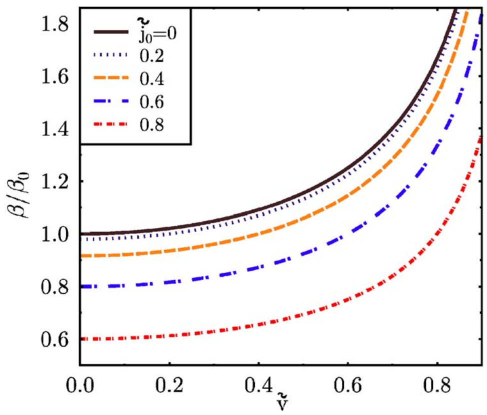

FIG. 6. (Color online) Dependence of the parameter β on the domain-wall velocity $\tilde{v}$ for different values of the current.

$$-\frac{1}{\gamma^2\lambda_1}\frac{\partial^2\varphi}{\partial t^2}+\frac{J_0^2\eta^2}{\lambda_1}\sin\varphi\cos\varphi+a\frac{\partial^2\varphi}{\partial x^2}-\lambda_2\sin\varphi\cos\varphi - J_0\zeta\sin\varphi=0. \tag{71}$$

The problem of the DW dynamics is reduced to a single $\varphi(x,t)$ field.

C. Solution for $\zeta=0$

Let us consider the possibility of the kinklike solution, moving with an arbitrary constant velocity v, $\varphi(x,t)\equiv\varphi(x-vt)$. We can find such solutions in the case of $\zeta=0$, trying a function which obeys the equality $\partial\varphi(x)/\partial x=\beta\sin\varphi(x)$. This function differs from the static solution only by a different choice of β instead of $\beta_0=(\lambda_2/a)^{1/2}$. Substituting it into Eq. (71), we obtain the equation that relates the values of β and v,

$$\beta^2\left(a-\frac{v^2}{\gamma^2\lambda_1}\right)-\lambda_2+\frac{J_0^2\eta^2}{\lambda_1}=0, \tag{72}$$

as in the Walker solution. The dependence $\beta(\tilde{v})$ is presented in Fig. 6, where we denoted $\tilde{v}=v/\gamma\sqrt{\lambda_1 a}$ and $\tilde{j}_0=J_0\eta/\sqrt{\lambda_1\lambda_2}$.

When $j_0=0$, we find from Eq. (72) that $\beta^2=\beta_0^2/(1-v^2/\gamma^2\lambda_1 a)$. This means that in the absence of current, the solution for a moving DW is more sharp than for a static wall. For $j_0\neq 0$ and $v\to 0$, the value of β depends on the current as $\beta^2=\beta_0^2(1-J_0^2\eta^2 a/\lambda_1\lambda_2)$, i.e., the current makes the thickness of the static DW larger.

Now we can use the velocity v as a variational parameter to minimize the Lagrangian

$$\mathcal{L}=\frac{AF(v)}{2\beta(v)}\int \sin^2\varphi(x)d(\beta x), \tag{73}$$

where

$$F(v)=\frac{1}{\lambda_1}\left[\frac{v\beta(v)}{\gamma}+J_0\eta\right]^2-a\beta^2-\lambda_2, \tag{74}$$

the function $\beta(v)$ is defined by Eq. (72), and the integral in Eq. (73) does not depend on v.

Using Eqs. (72)–(74) we find that for $j_0=0$, the quantity $F(v)=-\lambda_2$ for any v. Thus, the minimum of $\mathcal{L}$ corresponds to $\beta=\beta_0$, which is the minimum value of the dependence $\beta(v)$ for $j_0=0$. In the limit of a small velocity, $v^2\ll\gamma^2\lambda_1 a$, and using the relation $\int\sin^2\varphi(\alpha)d\alpha=2$, we find the kinetic energy of the moving DW in the form of $E_{kin}=m^*v^2/2$, where $m^*=A\sqrt{\lambda_2}/\gamma^2\lambda_1\sqrt{a}$ is the effective mass of the DW.[30] This is in agreement with the definition from Ref. 7 for $\lambda_1=2\pi M^2$.

For $j_0\neq 0$, we can present the dependence of the factor F on both parameters v and j_0 as

$$F(v)=-\lambda_2\left[1-2\tilde{j}_0^2-2\tilde{j}_0\tilde{v}\left(\frac{1-\tilde{j}_0^2}{1-\tilde{v}^2}\right)^{1/2}\right]. \tag{75}$$

In the limit of $v\to 0$, the factor F changes its sign for $j_0>j_{0cr}$, where

$$j_{0cr}=\frac{e\Omega_0\sqrt{\lambda_1\lambda_2}}{\sqrt{2}\hbar\eta} \tag{76}$$

is the critical current. Thus, if $j_0>j_{0cr}$, the solution with moving DW is energetically favorable. We can interpret the effect of the current as leading to an effective reduction of the effective mass of the DW. For $j_0>j_{0cr}$ the current induces an instability towards a spontaneous motion of the wall.

D. Case of $\zeta\neq 0$

In the case of $\zeta\neq 0$, there are no solutions of Eq. (71) corresponding to the motion of the DW with a constant velocity. This is because the last term in this equation acts as a force accelerating the DW. Indeed, if we assume a probe solution in the form of $\varphi(x,t)\equiv\varphi[x-x_0(t)]$, we find

$$m^*\ddot{x}_0(t)+J_0A\zeta=0, \tag{77}$$

where m is a constant in the limit of a small velocity. In other words, Eq. (77) describes the acceleration of the DW just after we apply some voltage. Hence, our model can describe the steady state if we include a viscosity (friction) into the equation of motion. We can use the damping term from Eq. (58). Writing the corresponding additional term in Eq. (71) as $F_d=-\alpha\partial\varphi/\partial t$, we find the following equation that determines the velocity of the moving DW:

$$v\beta(v)\simeq\frac{J_0\zeta}{\alpha}. \tag{78}$$

This equation indicates a linear dependence of the velocity on the current in the limit of a small velocity, when β is constant. As we see from Eq. (78), this corresponds to a large damping.

Effective friction may also stem from the pinning by impurities. This case can be described phenomenologically leading to another mechanism for the critical current.[9]

IV. CONCLUSIONS

We have calculated the spin-torque components, acting on a thin DW in a magnetic nanowire subject to an electric current. These components can induce rotation of magnetic moments in different directions. We have also considered the dynamics of a domain wall in the presence of a charge current. It has been demonstrated that a moving magnetic kink, similar to the static domain wall, can be a solution of the equations for the magnetic dynamics only at some special conditions characterized by a large ratio of the magnetic anisotropy constants. In the limit of small velocities, the solution is not a kink; its width decreases with increasing velocity. In the limit of a small velocity, the domain wall moves as a particle with a mass determined by the exchange interaction and anisotropies. The spin-torque component ζ dominates at small coupling and acts as a driving force on the DW, accelerating its motion (provided that there is no pinning to impurities).

Recent direct observations of the domain-wall configurations show that the spin structure of the wall changes with the current, and the structure depends on the velocity of the domain-wall motion.[5]

We have performed calculations of the torque in the limit of thin DW, $w \ll \lambda_F$. This simplifies the problem, so that the solution can be obtained analytically. Generally, the condition of a thin DW may not be well fulfilled. However, let us consider a wire with a cross section $A=1\ \text{nm}^2$ and a bulk carrier density $n_{3D}=10^{19}\ \text{cm}^{-3}$, corresponding to the linear density $n_{1D}=n_{3D}A=10^5\ \text{cm}^{-1}$ meaning that $k_F=\pi n_{1D}\simeq 3\times 10^5\ \text{cm}^{-1}$ and we obtain for the carrier wavelength $\lambda_F=2\pi/k_F\simeq 100$ nm. To estimate the DW width, we assume $M=100$ Oe, the demagnetizing factor along the y axis $n^{(y)}=0.3$, and calculate the anisotropy constant as $\lambda_2 \simeq 8\pi n^{(y)}M^2\simeq 10^5\ \text{erg/cm}^3$. For the energy of magnetic interaction $E_{int}\simeq 10$ meV at a distance between magnetic ions of $c_0=1$ nm, the exchange parameter $a=E_{int}c_0/A \simeq 10^{-8}$ erg/cm. Then, the DW width has a reasonable value of $w=(a/\lambda_2)^{1/2}\simeq 10$ nm. Comparing these estimations, we see that the main inequality of $w\ll\lambda_F$ is satisfied. At a larger carrier density, both w and λ can be of the same order of magnitude, or even the inequality is reversed as in magnetic metals. In this case, the constants ζ and η should be calculated numerically.

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APPENDIX A: SPIN TORQUE DUE TO THE MOMENTUM TRANSFER

The reflection of electrons from a DW is accompanied by the transfer of momentum from the electron system to the DW. In the presence of the electric current flowing through the magnetic wire, this creates an additional force acting on the wall.[8] Here we estimate the magnitude of this effect in the case of a thin DW, $k_F w \ll 1$.

The force F is determined by the total transferred momentum in a unit time. Taking into account the contributions of spin-up and spin-down scattering states, corresponding to the waves incoming from $-\infty$ in the energy range between ε_F and $\varepsilon+e\Delta\phi$, we find

$$\mathcal{F}=\frac{e\Delta\phi}{2\pi}\left[k_\uparrow\left(1+|r_\uparrow|^2-|t_{\uparrow f}|^2+\frac{v_\uparrow}{v_\downarrow}|r_{\downarrow f}|^2-\frac{v_\uparrow}{v_\downarrow}|t_\downarrow|^2\right)+k_\downarrow\left(1+|r_\downarrow|^2-|t_{\downarrow f}|^2+\frac{v_\downarrow}{v_\uparrow}|r_{\uparrow f}|^2-\frac{v_\downarrow}{v_\uparrow}|t_\uparrow|^2\right)\right]. \tag{A1}$$

This force tends to shift the DW along the x direction. For a local moment within the wall it is equivalent to the presence of a torque. To estimate the magnitude of this mechanical torque acting on a single moment we use a simplified model.

We describe the DW by the field $\varphi(x)$, which is the angle in the x-y plane determining orientation of moment $\mathbf{M}(x)$, as shown in Fig. 1. We assume that the shift along the axis x is related to the following interaction:

$$\mathcal{H}_{int}=\lambda\varphi(x)v(x), \tag{A2}$$

where λ is a constant, $v(x)=-d\varphi_0/dx$, and $\varphi_0(x)$ is the static solution for the domain wall. The potential $v(x)$ has the form of a potential well in the vicinity of the DW, and it forces (makes energetically favorable) a correction to the $\varphi(x)$ field of the same form, $\delta\varphi(x)\sim d\varphi_0/dx$. On the other hand, the correction $\delta\varphi(x)=(d\varphi/dx)\delta x_0$ is the shift along the axis x by δx_0. Thus, the interaction term in the form of (A2) in the equation of motion for the $\varphi(x)$ acts as a shifting force.

The constant λ should be determined by the condition that the energy δE associated with the shift gives the force $\mathcal{F}$

$$\mathcal{F}=-\frac{\delta E}{\delta x_0}=\lambda A\int\left(\frac{d\varphi_0}{dx}\right)^2dx. \tag{A3}$$

Using the known solution, $d\varphi_0/dx=\beta\sin\varphi_0(x)$, we find $\lambda=\mathcal{F}/2\beta A$.

The equation of motion for $\varphi(x)$ [Eq. (64)] includes the additional torque term as $\lambda v(x)=\mathcal{F}v(x)/2\beta A$. Using (A1) we estimate the torque acting on the localized moment $M_0=M\Omega$

$$T_{mt}\simeq\frac{j_0}{e}\frac{k_F\Omega}{A}, \tag{A4}$$

where Ω is the volume of an elementary cell. We find that the relative contribution of the momentum-induced torque with respect to the spin transfer is

$$T_{mt}/T_{st}\simeq k_F\Omega/A\ll 1. \tag{A5}$$

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