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Surface state peculiarities in one-dimensional photonic crystal interfaces

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We study the optical properties of a photonic crystal interfaced with a uniform medium with the negative dielectric constant or with another photonic crystal. We show that, at such an interface, nonpropagating surface states may arise. These states result in a sharp feature in the transmission and reflection spectra of the system. We also show that interfacing magnetic and nonmagnetic photonic crystals gives rise to giant Faraday and Kerr effects.

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I. INTRODUCTION

Within the last two decades, optical properties of photonic crystals (PCs)—structures with periodically modulated dielectric properties, have attracted a great deal of attention (see Refs. 1–3, and references therein). In unbounded PCs, the solutions of Maxwell's equations have the form of Bloch functions,[1–4] which results in the appearance of band gaps in the electromagnetic spectrum as well as new effects that do not exist in uniform media.[1]

In bounded PCs the translational invariance is broken and as a result new solutions of Maxwell's equations can arise within the band gap. Some of these solutions may have the form of surface modes that decay exponentially from the boundary into the crystal.[1–3] In this paper we analyze surface states in PCs as well as peculiarities of optical and magneto-optical properties of PCs related to these states.

In order to emphasize the difference between surface states and evanescent modes, we will not consider solutions of the scattering problem. In particular, we do not consider Bloch modes with complex wave numbers that are excited by a plane wave incident on the PC with a frequency within the band gap. We are only interested in modes that do not have sources outside of a PC.

Surface modes at the interface between uniform media have been known in electrodynamics for more than a century.[5] They can be observed on a surface between two media with dielectric constants with opposite signs. More precisely, these modes appear when the following condition is satisfied:[6]

$$k_x^2 = k_0^2 \frac{|\epsilon_{<0}| \cdot \epsilon_{>0}}{|\epsilon_{<0}| - \epsilon_{>0}} > k_0^2 \epsilon_{>0}, \tag{1}$$

where $k_0=\omega/c$, k_x is a wave vector component along the surface between media with positive, $\epsilon_{>0}$, and negative, $\epsilon_{<0}$, dielectric constants, with the z axis perpendicular to the surface. Such a wave is *P*-polarized (TM wave), i.e., its magnetic field is parallel to the interface between the media. The wave does not propagate along the z axis in both media. Indeed, in the medium with the negative dielectric constant, the respective wave vector component, $k_z=\sqrt{\epsilon_{<0}k_0^2-k_x^2}$, is purely imaginary; in the medium with the positive dielectric constant, the condition of total internal reflection, Eq. (1), is satisfied (see Fig. 1).

The requirement of the sign change of the dielectric constant at the boundary follows from the condition of continuity of the tangential components of the magnetic and electric fields at the boundary, H_y and $E_x \sim (1/\epsilon)(\partial H_y/\partial z)$, respectively. In other words, the impedances of the conjugate semispaces must match for incoming waves. Note, that at the interface between two media with different signs of the magnetic permeabilities, *S*-polarized surface waves (TE waves) can propagate.

A peculiar characteristic of a PC interface is that not only TM, but also TE, surface modes can exist even if a PC is made of nonmagnetic materials.[7–13] In this sense, a PC possesses properties of a medium in which both the dielectric permittivity ϵ and the magnetic permeability μ are negative. Therefore, depending on the properties of the medium ad-

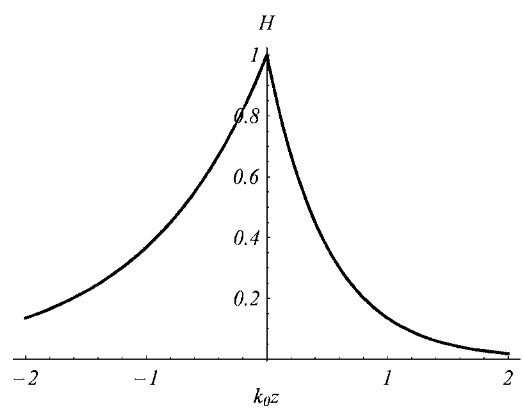

FIG. 1. The coordinate dependence of the magnetic field for the surface wave on the interface between two uniform media with dielectric constants $\epsilon_1=1(z<0)$ and $\epsilon_2=-2(z>0)$. The wave parameters satisfy $k_x/k_0=1.414$.

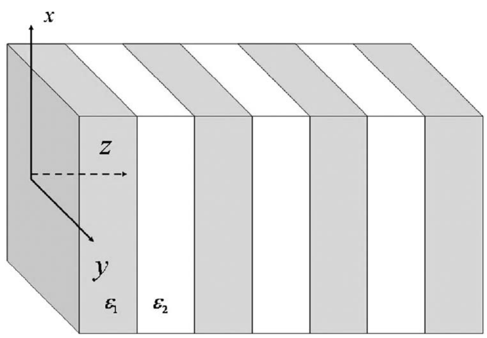

FIG. 2. One-dimensional photonic crystal. The elementary cell consists of one shaded slab and one clear slab.

joining the PC (e.g., ideally conducting metals, dielectrics with $\epsilon > 0$ or $\epsilon < 0$, another PC, etc.), both propagating surface modes (surface waves) and nonpropagating surface modes (surface states) can be excited on the interface. The situation becomes even more complicated when two PCs are joined via a defect.

In this paper we demonstrate that one of the possible types of surface states in PCs is an exact analog of the Tamm electron state on a crystal surface.[14] Therefore, following Ref. 15, we suggest referring to all localized surface states in PCs as Tamm states. This emphasizes the difference between surface states and surface waves or defect modes.[16] For the sake of simplicity, we consider one-dimensional structures.

The paper is organized as follows. In Sec. II we systematize the results of Refs. 7–13, 15, and 17–19 and analyze conditions for surface mode propagation in a PC. As mentioned above, TE and TM modes can coexist on a PC surface. Thus, a PC adjoining a medium with positive values of both ϵ and μ can be treated as a uniform medium characterized by a negative value of either ϵ or μ. In Sec. III we consider Tamm states for the most interesting cases of a PC interfaced to a uniform material or another PC. In Sec. IV we show that when nonmagnetic and magnetic PCs are juxtaposed, Tamm states may result in substantial Faraday and Kerr effects.

II. SURFACE WAVES IN PHOTONIC CRYSTALS

The reason for the variety of surface modes in PCs is that Bloch waves differ from plane waves. More precisely, even when the wave numbers of Bloch and plane waves coincide, the difference in field values of these waves is considerable on the length scale of an elementary cell. As an example, let us consider a one-dimensional PC in which the elementary cell of the total thickness d consists of two uniform slabs (see Fig. 2) of thicknesses d_1 and d_2, respectively. In such a crystal the Bloch wave field in the nth slab ($n=1,2$) is a sum of fields of two plane waves. The electric and magnetic fields of this Bloch wave have the form

$$E_{yn} = A_n e^{ik_{zn}(z-z_n)+ik_x x} + B_n e^{-ik_{zn}(z-z_n)+ik_x x},$$

$$H_{xn} = \frac{i}{\mu k_0}\frac{\partial}{\partial z}E_y = -\frac{k_{zn}}{\mu k_0}[A_n e^{ik_{zn}(z-z_n)+ik_x x} - B_n e^{-ik_{zn}(z-z_n)+ik_x x}],$$

$$H_{zn} = -\frac{i}{\mu k_0}\frac{\partial}{\partial x}E_y = \frac{k_{xn}}{\mu k_0}[A_n e^{ik_{zn}(z-z_n)+ik_x x} + B_n e^{-ik_{zn}(z-z_n)+ik_x x}], \qquad (2)$$

for the TE polarization, and

$$H_{yn} = A_n e^{ik_{zn}(z-z_n)+ik_x x} + B_n e^{-ik_{zn}(z-z_n)+ik_x x},$$

$$E_{xn} = -\frac{i}{\epsilon k_0}\frac{\partial}{\partial z}H_y = \frac{k_{zn}}{\epsilon k_0}[A_n e^{ik_{zn}(z-z_n)+ik_x x} - B_n e^{-ik_{zn}(z-z_n)+ik_x x}],$$

$$E_{zn} = \frac{i}{\epsilon k_0}\frac{\partial}{\partial x}H_y = -\frac{k_{xn}}{\epsilon k_0}[A_n e^{ik_{zn}(z-z_n)+ik_x x} + B_n e^{-ik_{zn}(z-z_n)+ik_x x}] \qquad (3)$$

for the TM polarization. Using the continuity of the tangential components E_t, H_t of the fields on the boundary between two slabs and the Floquet's theorem,

$$E_t(z+d) = \exp(ik_B d)E_t(z),$$

$$H_t(z+d) = \exp(ik_B d)H_t(z),$$

one can obtain a system of linear uniform equations for coefficients A_n and B_n.[7–13,15,20] The determinant of the corresponding matrix must be equal to zero. This results in a dispersion relation that determines the Bloch wave number (see Refs. 15 and 21). In our case, this equation has the form

$$\cos[k_B(d_1+d_2)] = \cos(k_{z1}d_1)\cos(k_{z2}d_2) - \frac{1}{2}\left(\frac{\zeta_1}{\zeta_2}+\frac{\zeta_2}{\zeta_1}\right)\sin(k_{z1}d_1)\sin(k_{z2}d_2). \qquad (4)$$

Here ζ_n is equal to $-E_y/H_x$ for TE waves and H_y/E_x for TM waves. Note that even within the band gap where the Bloch factor falls exponentially, $\exp(ik_B z) \sim \exp(-\kappa_B z)$, fields may propagate in each slab including the one adjacent to the boundary of the PC. Thus, on the boundary, the surface wave field, which decays exponentially into the uniform medium, must match with the field of waves propagating along the first slab of the PC. This does not require a sign change of ϵ or μ. In the other words, the surface waves may propagate along the surface of a PC contacting with a double positive medium ($\epsilon > 0, \mu > 0$).

In the general case of a wave on the interface of the PC and a uniform medium, the dispersion relation follows from the equality of tangential components E_t and H_t, or from the equality of their ratios in the medium, ζ_m, and a PC, Z:

$$Z = \zeta_m. \qquad (5)$$

For a crystal in which the elementary cell consists of two uniform slabs, it follows from Eqs. (2)–(4) that

$$Z = -\zeta_1 \frac{(\zeta_2 \cos k_{z1}d_1 + i\zeta_1 \sin k_{z1}d_1)\exp(ik_{z2}d_2) - \zeta_2 \exp[ik_B(d_1+d_2)]}{(\zeta_1 \cos k_{z1}d_1 + i\zeta_2 \sin k_{z1}d_1)\exp(ik_{z2}d_2) - \zeta_1 \exp[ik_B(d_1+d_2)]}, \quad (6)$$

where

$$\zeta_j = k_{zj}/\mu_j k_0, \quad \zeta_m = -k_z/\mu k_0, \quad (7)$$

for the TE polarization and

$$\zeta_j = k_{zj}/\epsilon_j k_0, \quad \zeta_m = -k_z/\epsilon k_0 \quad (8)$$

for the TM polarization. $k_{zj}=\sqrt{\epsilon_j\mu_j k_0^2 - k_x^2}$ is the normal component of the wave vector in the jth slab. In the expressions for ζ_m the negative sign appears because in the uniform space we deal with the "reflected" wave. Using Eqs. (5)–(8) one can obtain the existence condition for the TE- and TM-polarized surface waves:[7–13]

$$\zeta_m = -\zeta_1 \cdot \frac{(\zeta_2 \cos k_{z1}d_1 + i\zeta_1 \sin k_{z1}d_1)\exp(ik_{z2}d_2) - \zeta_2 \exp[ik_B(d_1+d_2)]}{(\zeta_1 \cos k_{z1}d_1 + i\zeta_2 \sin k_{z1}d_1)\exp(ik_{z2}d_2) - \zeta_1 \exp[ik_B(d_1+d_2)]}. \quad (9)$$

If ϵ and μ of the uniform medium are positive, then the solutions of Eq. (9) correspond to waves that transfer energy along the PC interface. Indeed, to satisfy the condition for the total internal reflection from the uniform slab, the wave vector must have a tangential component. By combining solutions corresponding to the waves propagating in the opposite directions one can obtain a nonuniform state corresponding to a standing wave that does not transfer energy.

III. TAMM STATES

It follows from Sec. II that a PC can play the role of a uniform medium with negative ϵ or μ that supports surface TE and TM waves along the interface with a medium with positive ϵ and μ. However, if the PC is interfaced with a material characterized by a negative permittivity $\epsilon<0$ or a negative permeability $\mu<0$, then in contrast to the case considered in Sec. II, solutions (states) may exist near the boundary, which are uniform along the surface (k_x=0), and do not transfer energy (Fig. 3).[20] This state is twice degenerated because at $k_x\neq 0$ we have to distinguish TE and TM waves. The k_x dependence of the wavelength corresponding to the surface solutions is presented in Fig. 4.

In fact, here we deal with a plane resonator. One of its mirrors is made of a material with negative dielectric constant, the other one is a dielectric mirror well known in laser fabrication. The peculiarity of such a resonator is the absence of a resonant cavity: reflectors are in direct contact with each other and the field is concentrated inside the mirrors.

The case k_x=0, $\epsilon<0$ is analogous to the Tamm surface

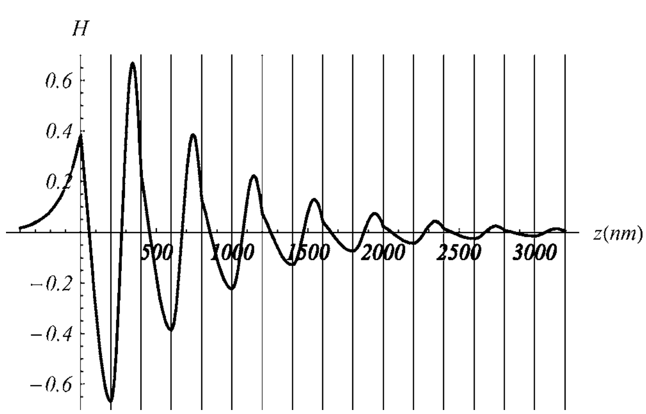

FIG. 3. The Tamm state at the boundary of a medium with ϵ=−0.5 and PC comprised of slabs having dielectric constants ϵ_1=4 and ϵ_2=1 and thicknesses d_1=d_2=200 nm. The wavelength of the normally incident wave is λ=575 nm. The dashed line shows the amplitude of an enveloping plane wave $f(0)\exp(ik_B z)$ [the Bloch wave differs from this plane wave by the periodic amplitude $f(z)$].

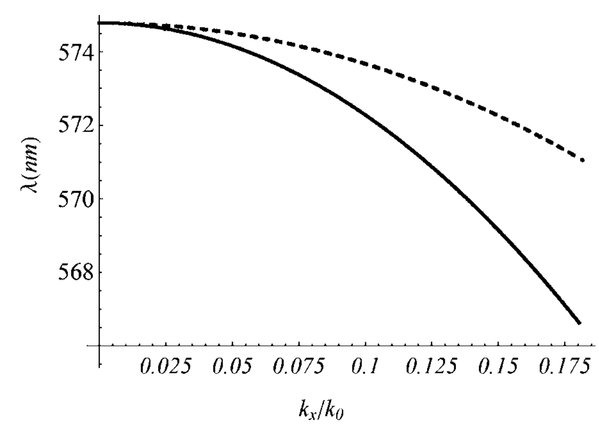

FIG. 4. The dependence of the wavelength on k_x/k_0 (k_0=$2\pi/\lambda$), corresponding to the surface solutions. Solid and dashed lines correspond to TE- and TM- polarized surface waves, respectively. At k_x=0 there is no difference between both polarizations and the nontraveling state is twice degenerated. The parameters of the system are the same as in Fig. 3.

state[14] well known in solid state physics. Indeed, in this case, Maxwell's equations can be reduced to the Helmholtz equation

$$\frac{\partial}{\partial z}\left(\frac{1}{\mu}\frac{\partial E_y}{\partial z}\right) + k_0^2 \epsilon E_y = 0,$$

and boundary conditions require continuity of the field E_y and its derivative $H_x \sim 1/\mu\, \partial E_y/\partial z$. If all materials are assumed to be nonmagnetic, $\mu=1$, this corresponds exactly to the quantum-mechanical problem. On the other hand, the same problem formulated in terms of magnetic field equations has physical solutions with a noncontinuous first derivative (Fig. 3). This distinguishes this solution from the ones known in quantum mechanics for the wave function. It is necessary to note that solutions with a noncontinuous first derivative are used in the theory of semiconductor superlattices for the envelope function but not for the wave function itself.[22] On the boundary, one must match H_y and $E_x \sim 1/\epsilon\, \partial H_y/\partial z$, which resembles the situation for uniform media (Fig. 1). In this case, the PC works as a medium with a positive dielectric constant. However, since the wave attenuation inside the crystal is due to the band gap, the requirement of total internal reflection is not necessary, and instead of a state corresponding to a surface wave, there may appear a surface state, which does not propagate along the surface.

The situation is reversed in the case of a uniform medium with $\mu<0$ and constant ϵ. Now, there is an exact analogy between the problem formulated in terms of the magnetic field with the quantum-mechanical problem, while in the problem formulated in terms of the electric field we find solutions with a noncontinuous first derivative.

In the general case, the dispersion equation for the surface state considered has the form given by Eq. (9) with $k_x=0$, i.e., $\zeta_j=\sqrt{\epsilon_j/\mu_j}$, $\zeta_m=-\sqrt{\epsilon/\mu}$, $k_{zj}=\sqrt{\epsilon_j\mu_j}$:

$$\sqrt{\epsilon/\mu} = \sqrt{\epsilon_1/\mu_1} \times \frac{[\sqrt{\epsilon_2/\mu_2}\cos(k_0\sqrt{\epsilon_1\mu_1}d_1) + i\sqrt{\epsilon_1/\mu_1}\sin(k_0\sqrt{\epsilon_1\mu_1}d_1)]\exp(ik_0\sqrt{\epsilon_2\mu_2}d_2) - \sqrt{\epsilon_2/\mu_2}\exp[ik_B(d_1+d_2)]}{[\sqrt{\epsilon_1/\mu_1}\cos(k_0\sqrt{\epsilon_1\mu_1}d_1) + i\sqrt{\epsilon_2/\mu_2}\sin(k_0\sqrt{\epsilon_1\mu_1}d_1)]\exp(ik_0\sqrt{\epsilon_2\mu_2}d_2) - \sqrt{\epsilon_1/\mu_1}\exp[ik_B(d_1+d_2)]}. \quad (10)$$

The Tamm state can be detected experimentally by measuring the transmission coefficient of waves through a PC of a finite thickness joined to a material with negative permittivity. At the frequency corresponding to the Tamm state, a narrow peak must be observed (Fig. 5). At the same time, the transmission coefficient either through the uniform slab or the PC, in the band gap region, when measured separately will be much smaller than through the interfaced media.

The case of the interfacing of two PCs with overlapping band gaps is of special interest (Fig. 6). The equation defining parameters of the wave at such an interface is given by the equality of the values of Z in the crystals, Eq. (6):

$$\zeta_1^I \frac{(\zeta_2^I \cos k_{z1}^I d_1^I + i\zeta_1^I \sin k_{z1}^I d_1^I)\exp(ik_{z2}^I d_2^I) - \zeta_2^I \exp[ik_B^I(d_1^I+d_2^I)]}{(\zeta_1^I \cos k_{z1}^I d_1^I + i\zeta_2^I \sin k_{z1}^I d_1^I)\exp(ik_{z2}^I d_2^I) - \zeta_1^I \exp[ik_B^I(d_1^I+d_2^I)]}$$
$$= \zeta_1^{II} \frac{(\zeta_2^{II} \cos k_{z1}^{II} d_1^{II} + i\zeta_1^{II} \sin k_{z1}^{II} d_1^{II})\exp(ik_{z2}^{II} d_2^{II}) - \zeta_2^{II} \exp[ik_B^{II}(d_1^{II}+d_2^{II})]}{(\zeta_1^{II} \cos k_{z1}^{II} d_1^{II} + i\zeta_2^{II} \sin k_{z1}^{II} d_1^{II})\exp(ik_{z2}^{II} d_2^{II}) - \zeta_1^{II} \exp[ik_B^{II}(d_1^{II}+d_2^{II})]}. \quad (11)$$

Here superscripts I and II designate the crystals. The Bloch wave numbers in both crystals must be complex and waves attenuate away from the boundary.

In the system of two interfaced PCs, the Tamm state can also be revealed by the peak of the transmission coefficient in the regions in which band gaps of both crystals overlap (Fig. 7).

IV. MAGNETO-OPTIC FARADAY AND KERR EFFECTS CAUSED BY TAMM STATES

In Refs. 23–27 it was shown that if a defect in a PC is made of electrogyrotropic material, the magneto-optic properties of such a system are enhanced nearly two orders of magnitude over the magneto-optic properties of the defect layer itself. For the Faraday effect enhancement, a microresonator scheme should be used. This is equivalent to using a Fabry-Perot resonator in which finite-size PCs (Fig. 8) having band gaps at a working frequency serve as mirrors.[24–27] For the Kerr effect enhancement, the asymmetric scheme should be used.[25]

Magneto-optic effects are also enhanced at certain frequencies when the magneto-photonic crystal is constructed of alternating magnetic and nonmagnetic layers, e.g., $Bi:DyIG/SiO_2$. In this case, however, the enhancement of magneto-optic effects is somewhat smaller.[27,28] Below we

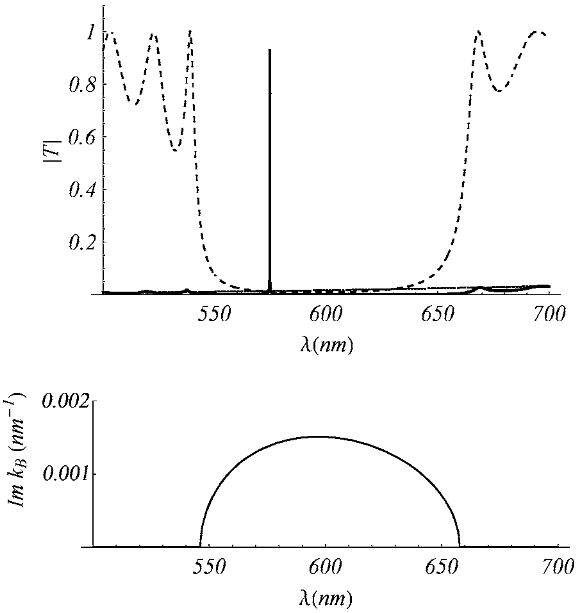

FIG. 5. The Tamm state at the boundary of the nine-layer-PC and 640-nm-slab with the same parameters as in Fig. 3. The upper graph shows the transmission coefficients through the PC interfaced with the uniform medium (solid line), the vacuum conjugated PC (dashed line), and a slab of the uniform medium alone (dotted line). The lower graph shows the imaginary part of k_B that defines the positions of the band gaps. One can see that a transmission peak is observed at a wavelength λ within the band gap.

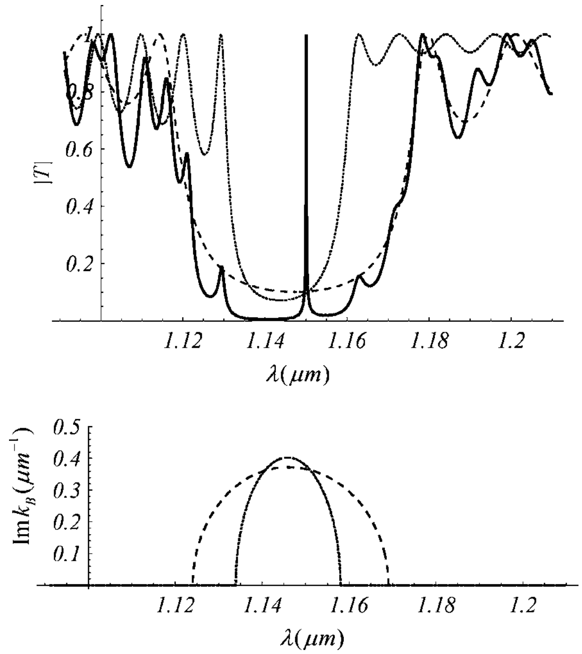

FIG. 7. The transmission coefficient of a system of two PCs (the PCs' parameters are the same as in Fig. 6). On the upper graph, the solid line shows the transmission coefficient of the joined crystals, the dashed and dotted lines correspond to the transmission coefficients of each crystal in vacuum. The lower graph shows the imaginary parts of k_B for each crystal. One can see that, for the wavelength of $\lambda = 1.15$ μm that belongs to the gaps of both crystals, there is a sharp peak of the transmission.

show that when magnetic and nonmagnetic PCs are interfaced, magneto-optic properties are also enhanced substantially. This enhancement is of the same order as in a Fabry-Perot resonator scheme.

Let us consider two interfaced PCs. The elementary cell of the first one consists of two layers of nonmagnetic dielectric materials, e.g., Ta_2O_5 ($n=2.1$) and SiO_2 ($n=1.44$). The elementary cell of the second crystal has one magnetic and one nonmagnetic layers, e.g., Bi:DyIG and SiO_2. At a frequency of 1.15 μm the dielectric permittivity tensor of the magnetic material has diagonal and nondiagonal elements, $\epsilon_d=5.58$ and $\epsilon_{nd}=-0.00\,198$, respectively.

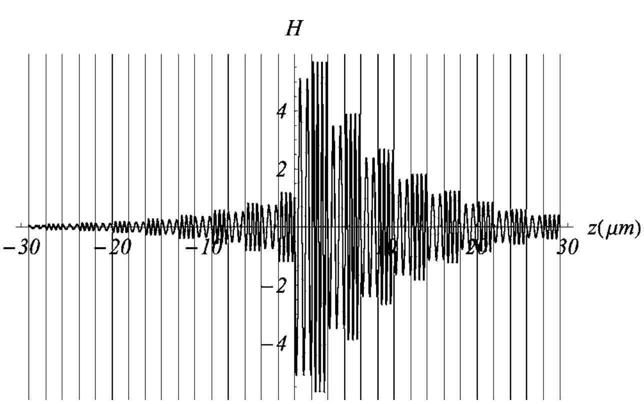

FIG. 6. The TM-polarized Tamm state at the boundary of two PCs. Parameters of the eight-period crystals correspond to the following materials: I-Ta_2O_5 ($\epsilon_1^I=4.41$, $d_1^I=0.696$ μm) and SiO_2 ($\epsilon_2^I=2.07$, $d_2^{II}=1.37$ μm); II-SiO_2 ($\epsilon_1^{II}=2.07$, $d_1^{II}=1.87$ μm) and Bi:DyIG ($\epsilon_2^{II}=5.58$, $d_2^{II}=1.77$ μm). The free wavelength is $\lambda=1.15$ μm; the tangential component of the wave vector is $k_x=0$. The dashed line shows the amplitude of the enveloping plane wave, $f(0)\exp(ik_B z)$.

In Fig. 7 we show the transmission coefficient for two PCs with eight periods each. One can clearly see the Tamm resonance inside the band gap. More precisely, there are two resonances corresponding to the left- and right-polarized modes. These resonances are so narrow and positioned so closely to each other that they merge into a single peak on the scale of Fig. 7. In Fig. 9, we enhance the scale to distinguish these resonances.

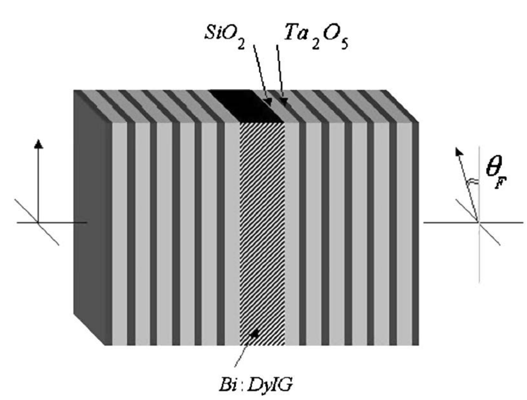

FIG. 8. The structure leading to the Faraday effect enhancement.

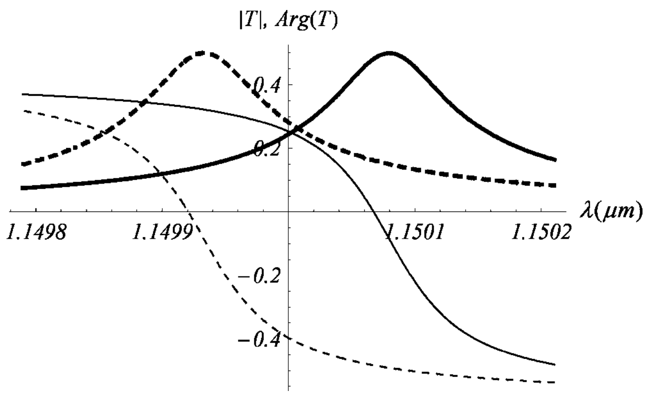

FIG. 9. The amplitude (thick lines) and the phase (thin lines) of the transmission coefficient for the same system as in Figs. 6 and 7. Solid and dashed lines correspond to the right and left polarizations, respectively.

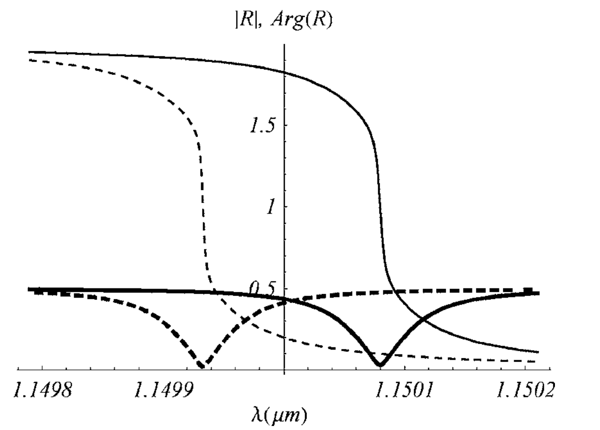

FIG. 10. The amplitude (thick lines) and the phase (thin lines) of the reflection coefficient for the same system as in Figs. 6 and 7. Solid and dashed lines correspond to right and left polarizations, respectively.

One can see from Fig. 9 that for $\lambda = 1.15\ \mu$m the transmission coefficients for both polarizations are equal. This means that the outgoing wave is linearly (not elliptically) polarized. Its plane of polarization is rotated by an angle equal to half of the phase difference of the transmission coefficients for each polarization. Numerical results for the transmission coefficient and the angle of rotation of the plane of polarization for different numbers of layers are shown in Table I.

A substantial Kerr effect, i.e., the rotation of the plane of the polarization in reflection, is also observed in the same scheme. The results for a normal incidence are shown in Fig. 10 and Table II.

We note that for a slab of the magneto-optic material discussed above with a thickness of eight magneto-optical layers of the PC, the polarization plane rotates by approximately 2° for both reflected and transmitted waves.

V. CONCLUSIONS

In this paper we show that when a PC is interfaced with a uniform medium characterized by a negative dielectric constant, a new type of surface state may arise. In contrast to the well-known surface waves, these states are nonpropagating and they can have both TM and TE polarizations. The physical reason for such states is the interference of surface waves propagating in opposite directions. As a result, a standing wave is formed. In the case of TE polarization, these states are analogous to electron states on a surface of a crystal, discovered theoretically by Tamm in 1932.[14] We therefore call them Tamm states.

Tamm states appear in the band gap region of the PC. They can be observed by studying transmission or reflection spectra of the system. Due to the resonance tunneling of an electromagnetic wave through the Tamm state, a narrow peak appears in the transmission spectrum together with a dip in the reflection spectrum.

Tamm states also appear at the boundary between two PCs, one of which is magnetic, in the region in which the band gaps overlap. Such a system is also characterized by giant Faraday and Kerr effects: the angle of rotation of the polarization plane of the transmitted and reflected waves is 20–30 times greater than that of the magnetic medium with the same thickness as the magnetic component of the PC.

The numerical data presented in the paper were obtained for the spectral range in which optical and magneto-optical losses are negligible (the loss tangent is less than 10^{-7} at $\lambda = 1.15\ \mu$m). However, our numerical calculations show that the effect is still observable even if the loss tangent increases up to 10^{-3}.

Of course, the effect disappears when the technological errors in layer's thickness are substantial. In accordance with Ref. 29 (see also Ref. 30), the Liapunov exponent of the finite periodic-on-average one-dimensional system decreases when relative fluctuations of layer thicknesses increase. This effect is responsible for the decrease of the Q factor of the Tamm state, and as a consequence, in the decrease of the

TABLE I. The Faraday effect for the PC slab containing different numbers of cells.

Number of layers in each PC	Transmittance $\|T\|^2$	The angle of the polarization plane rotation in degrees
6	0.88	20.1
7	0.63	37.7
8	0.28	58.6
9	0.079	73.9

TABLE II. The Kerr effect for the PC slab containing different numbers of cells.

Number of layers in each PC	Reference $\|R\|^2$	The angle of the polarization plane rotation in degrees
6	0.12	74.3
7	0.37	55.4
8	0.72	33.4
9	0.92	17.3

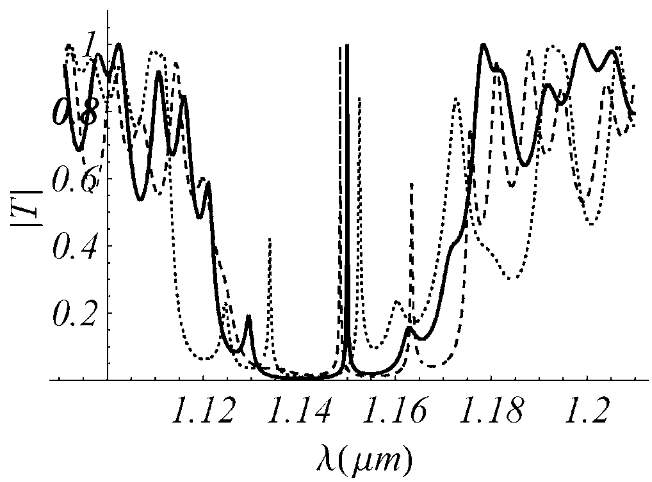

FIG. 11. The amplitude of the transmission coefficient for a system of two PCs without disorder, as in Fig. 7 (solid line), and two random realizations with relative thickness deviation of layer thickness of 2% (dashed and dotted lines).

resonant transmittance and the angle of rotation of the polarization plane. Indeed, thickness fluctuations should lead to a violation of the Bragg condition and degradation of the band gap. Fortunately, this happens gradually. First of all, while a decrease of the Liapunov exponent is observed near the band edges, in the middle of the band gap it does not change until the relative fluctuations in the layer thickness exceed 5%.[29] These effects become more pronounced with an increase of the number of layers the system consists of. It is well known that in an infinite one-dimensional system arbitrarily small disorder leads to the localization of waves and an observation of magneto-optical effects becomes impossible.

The results of our computer simulation are in a good agreement with the results of Ref. 29. For our system containing 32 layers, in the middle of the band gap, the Liapunov exponent is almost constant when the relative standard deviation of the layer thickness, σ/d, is less than 2%. In this region, in each realization the random shifting of eigenfrequency of the Tamm state is observed without the line broadening (Fig. 11). The value of this shift is distributed with the relative standard deviation being of the order of σ/d. If σ/d is larger than 2%, the band gap shrinks and the effects of the magneto-optical enhancement are not observed.

An experimental observation of Tamm states for electrons in regular crystals is extremely difficult due to the complexity of their surfaces. Even though the electronic structure of crystal interfaces is one of the oldest problems of condensed matter physics, these states were first observed in superlattices only in 1990.[31] Since PC surfaces can be well controlled, they present an excellent opportunity for the experimental study of Tamm states.

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