

# Crystallization kinetics and glass-forming ability of bulk metallic glasses $\text{Pd}_{40}\text{Cu}_{30}\text{Ni}_{10}\text{P}_{20}$ and $\text{Zr}_{41.2}\text{Ti}_{13.8}\text{Cu}_{12.5}\text{Ni}_{10}\text{Be}_{22.5}$ from classical theory

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Due to their scientific significance and potential engineering applications, bulk metallic glasses are among the most intensively studied advanced materials. Understanding the glass-forming ability (GFA) of these metallic alloys is a long-standing subject. While a large number of empirical factors have been proposed to correlate with GFA of the alloys, a full understanding of GFA remains a goal to achieve. Since glass formation is a competing process against crystallization, we have performed a systematic analysis on the crystallization kinetics of two known best metallic glass-formers  $\text{Pd}_{40}\text{Cu}_{30}\text{Ni}_{10}\text{P}_{20}$  (in at. %) and  $\text{Zr}_{41.2}\text{Ti}_{13.8}\text{Cu}_{12.5}\text{Ni}_{10}\text{Be}_{22.5}$  based on classical nucleation and growth theory. Our results show that there is a dramatic difference between the two alloys in their nucleation behavior although they possess comparable GFA. Particularly, an extremely sharp nucleation peak ( $\sim 10^{18}/\text{m}^3 \text{ s}$ ) is found for  $\text{Pd}_{40}\text{Cu}_{30}\text{Ni}_{10}\text{P}_{20}$  around 632 K with a very small half maximum width of 42 K, implying that this alloy is an excellent candidate for nanocrystallization studies. Moreover, we have also found that the GFA of these alloys can be calculated to a high accuracy and precision based on the classical theory, suggesting that the classical theory may be sufficient to account for glass formation mechanism in these metallic alloys.

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## I. INTRODUCTION

Because of their unique structure and properties, bulk metallic glasses (BMGs), a new category of advanced materials, are receiving tremendous attention from both fundamental and practical research disciplines. Glass-forming ability (GFA) is one of the most important subjects in the study of BMGs, because it determines, to a large degree, the potential for this new category of materials to be utilized in various applications.<sup>1–8</sup> Upon continuous cooling from its molten state, an alloy can eventually form a glass if, and only if, the cooling rate exceeds a critical value. This critical cooling rate ( $R_c$ ) is a direct and universal measurement of the GFA of any substance including the conventional oxide glasses. Therefore, a complete understanding of GFA should ultimately be supported by an accurate and precise calculation of the  $R_c$ . While a large number of empirical factors (e.g., Refs. 9–14) have been proposed over the past decades to correlate with the GFA of various metallic glasses, they either were not verified against  $R_c$  or could not reproduce the experimentally measured  $R_c$  to a satisfactory accuracy and precision. The accurate and precise calculation of  $R_c$ , and accordingly, the full understanding of metallic glass formation mechanism remain unaccomplished.

Considering that glass formation is a competing process against crystallization, we perform a systematic study on the crystal nucleation and growth kinetics of two known best metallic glass formers  $\text{Pd}_{40}\text{Cu}_{30}\text{Ni}_{10}\text{P}_{20}$  (in at. %) and  $\text{Zr}_{41.2}\text{Ti}_{13.8}\text{Cu}_{12.5}\text{Ni}_{10}\text{Be}_{22.5}$  based on classical theory. We also calculate the  $R_c$  values for the two alloys by virtue of a continuous integral method (first introduced by Weinberg *et al.*<sup>15</sup> for oxide glasses) in the framework of the classical theory, and analyze the accuracy and precision of the  $R_c$  calculation.

## II. CRYSTAL NUCLEATION AND GROWTH KINETICS

### A. $\text{Pd}_{40}\text{Cu}_{30}\text{Ni}_{10}\text{P}_{20}$

$\text{Pd}_{40}\text{Cu}_{30}\text{Ni}_{10}\text{P}_{20}$  and  $\text{Zr}_{41.2}\text{Ti}_{13.8}\text{Cu}_{12.5}\text{Ni}_{10}\text{Be}_{22.5}$  were selected for this study primarily because of the availability of their thermodynamic and kinetic data as well as reliable  $R_c$  values determined directly from continuous cooling experiments.<sup>16–19</sup>

In Ref. 16, the isothermal temperature-time-transformation (TTT) diagram of  $\text{Pd}_{40}\text{Cu}_{30}\text{Ni}_{10}\text{P}_{20}$  was measured and successfully fitted to the classical nucleation and growth theory using the following equations:

$$I_\nu = \frac{A_\nu}{\eta} \exp\left(-\frac{16\pi\sigma^3}{3k_B T \Delta G^2}\right), \quad (1)$$

$$u = \frac{k_B T}{3\pi l^2 \eta} \left[ 1 - \exp\left(-\frac{n\Delta G}{k_B T}\right) \right], \quad (2)$$

and

$$t = \left(\frac{3f_c}{\pi I_\nu \mu^3}\right)^{1/4}, \quad (3)$$

where  $I_\nu$  is the steady-state nucleation rate,  $u$  is the diffusion-controlled crystal growth rate,  $A_\nu$  is a fitting parameter,  $\eta$  is the temperature-dependent viscosity,  $k_B$  is the Boltzmann constant,  $\sigma$  and  $\Delta G$  are the interfacial energy per area and the Gibbs free energy difference per volume between the liquid and the crystalline phases, respectively,  $l$  is the average atomic diameter,  $n$  is the average atomic volume, and  $f_c$  is the critical crystallized volume fraction detectable by instruments ( $f_c$  is usually assigned to a somewhat arbitrary value for the fitting of a TTT diagram, and  $A_\nu$  and  $\sigma$  are the only two fitting parameters. The choice of  $f_c$  value, however, does not affect the fitting [Eq. (3)] or the calculation of  $R_c$

TABLE I. Thermodynamic and kinetic data for Pd<sub>40</sub>Cu<sub>30</sub>Ni<sub>10</sub>P<sub>20</sub> (taken from Ref. 16).

$\eta_0$ (Pa s)	$D$	$T_0$ (K)	$\Delta H_F$ (J/m <sup>3</sup> )	$T_l$ (K)	$\sigma$ (J/m <sup>2</sup> )	$A_v$ (Pa/m <sup>3</sup> )	$n$ (m <sup>3</sup> )	$l$ (m)	$f_c$	$T_g$ (K)	$T_n$ (K)	$t_n$ (s)
$9.34 \times 10^{-3}$	9.25	447	$7.69 \times 10^8$	823	0.061 <sup>a</sup>	$4.4 \times 10^{31}$	$1.52 \times 10^{-29}$	$3.1 \times 10^{-10}$	0.005	582	680	50

<sup>a</sup>The value of 0.067 J/m<sup>2</sup> for  $\sigma$  given in Ref. 16 is not accurate.

[Eq. (8) below] because the intrinsic fitting parameter besides  $\sigma$  is actually the ratio  $A_v/f_c$  rather than  $A_v$ . It does, on the other hand, affect the absolute value of  $I_v$  [Eq. (1) through  $A_v$ ]. In the fitting, a Vogel-Fulcher-Tamman (VFT) form was used to represent the temperature dependence of viscosity

$$\eta(T) = \eta_0 \exp\left(\frac{DT_0}{T - T_0}\right), \quad (4)$$

where  $\eta_0$ ,  $D$ , and  $T_0$  are three constants. In addition, a first order approximation was used for  $\Delta G$ , i.e.,  $\Delta G = \Delta H_F(1 - T/T_l)$ , where  $\Delta H_F$  is the enthalpy of fusion per unit volume, and  $T_l$  is the liquidus temperature.

Table I summarizes all the relevant thermodynamic and kinetic data for Pd<sub>40</sub>Cu<sub>30</sub>Ni<sub>10</sub>P<sub>20</sub> taken from Ref. 16. Using these data, one can calculate and plot the nucleation rate  $I_v$  and the growth rate  $u$  as functions of temperature. The results are shown in Fig. 1. Upon cooling from the liquidus temperature 823 K, the crystal growth rate increases immediately and quickly reaches its maximum value of  $3.2 \times 10^{-7}$  m/s at 794 K, and then drops rapidly afterwards. By contrast, the nucleation rate remains very low over a relatively large temperature interval of  $\sim 123$  K below the liquidus temperature. Starting from  $\sim 700$  K, however, the nucleation rate rises steeply as the temperature further decreases, and rapidly assumes its maximum value of  $10^{18}/\text{m}^3 \text{ s}$  at 632 K, followed by a steep decrease afterwards.

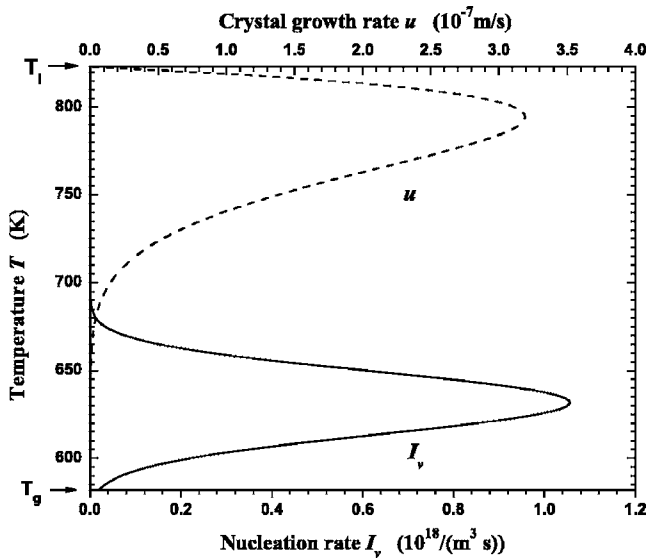


FIG. 1. Nucleation rate  $I_v$  and crystal growth rate  $u$  as functions of temperature for Pd<sub>40</sub>Cu<sub>30</sub>Ni<sub>10</sub>P<sub>20</sub>.

Both the nucleation rate and the growth rate are jointly controlled by thermodynamics and kinetics. In fact, one can separate the thermodynamic and kinetic factors by dividing each of the rates into two parts, with  $\eta$  representing kinetics and  $I_v\eta$  (or  $u\eta$ ) representing thermodynamics. Figure 2 shows the plots of the two thermodynamic factors, for the nucleation rate and the growth rate of Pd<sub>40</sub>Cu<sub>30</sub>Ni<sub>10</sub>P<sub>20</sub>, respectively. It can be seen that within the temperature range between the liquidus temperature and the glass transition temperature ( $T_g$ , 582 K for Pd<sub>40</sub>Cu<sub>30</sub>Ni<sub>10</sub>P<sub>20</sub>, refer to Table I), the two thermodynamic factors both increase monotonically as temperature decreases, which means that thermodynamics consistently enhances crystal nucleation and growth. On the other hand, kinetics consistently restrains crystal nucleation and growth, as represented by monotonically increasing  $\eta$  (plot not shown) during continuous cooling process. It is the competition between thermodynamics and kinetics that has resulted in the peaks of  $I_v$  and  $u$  that are shown in Fig. 1.

As we will discuss in next section, when compared with Zr<sub>41.2</sub>Ti<sub>13.8</sub>Cu<sub>12.5</sub>Ni<sub>10</sub>Be<sub>22.5</sub>, the nucleation behavior of Pd<sub>40</sub>Cu<sub>30</sub>Ni<sub>10</sub>P<sub>20</sub> appears very unique and has important implications for future nanocrystallization studies.

### B. Zr<sub>41.2</sub>Ti<sub>13.8</sub>Cu<sub>12.5</sub>Ni<sub>10</sub>Be<sub>22.5</sub>

In Ref. 17, the kinetic time scales for viscous flow, atomic transport, and crystallization (isothermal) in the liquid and

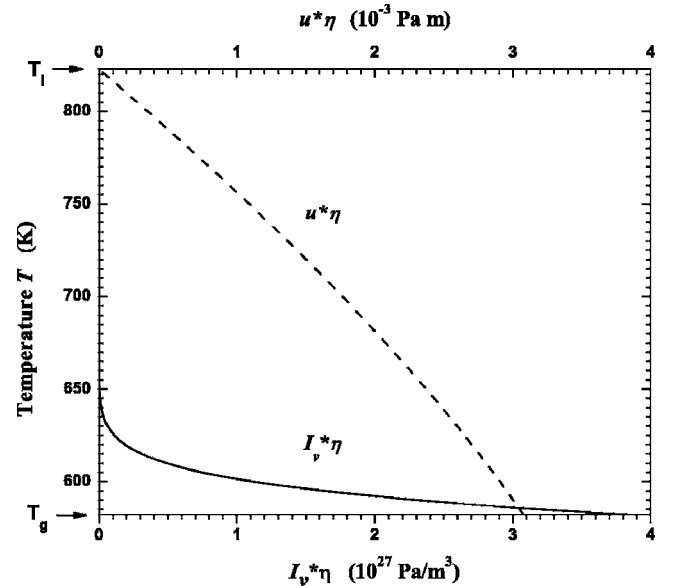


FIG. 2. Thermodynamic factors,  $I_v\eta$  and  $u\eta$ , for the nucleation rate  $I_v$  and the crystal growth rate  $u$ , respectively, as functions of temperature for Pd<sub>40</sub>Cu<sub>30</sub>Ni<sub>10</sub>P<sub>20</sub>.

TABLE II. Thermodynamic and kinetic data for  $Zr_{41.2}Ti_{13.8}Cu_{12.5}Ni_{10}Be_{22.5}$  (taken from Ref. 17 unless otherwise noted).

$\eta_0$ (Pa s)	$c_1$ (K)	$c_2$ (K)	$T_0$ (K)	$D_0$ (m <sup>2</sup> /s)	$Q_{eff}$ (eV)	$\Delta H_F$ (J/m <sup>3</sup> )	$T_l$ (K)	$\sigma$ (J/m <sup>2</sup> )	$A_v$ (m <sup>-5</sup> )	$n$ (m <sup>3</sup> )	$l$ (m)	$f_c$	$T_g$ (K)	$T_n$ (K)	$t_n$ (s)
$4 \times 10^{-5}$	9866	162	672	$6.85 \times 10^{-8}$	1.2	$7.9 \times 10^{8a}$	1026	0.04	$3 \times 10^{20}$	$1.72 \times 10^{-29}$	$3.2 \times 10^{-10}$	$10^{-4}$	620 <sup>a</sup>	895	60

<sup>a</sup>Taken from Ref. 19.

supercooled liquid states of  $Zr_{41.2}Ti_{13.8}Cu_{12.5}Ni_{10}Be_{22.5}$  (Vitreloy-1) were reported. It was found that the measured viscosity of Vitreloy-1 cannot be fitted well by a VFT equation in the supercooled liquid range. Rather, the Cohen-Grest (CG) free volume model<sup>20</sup> provides a better description for the temperature dependence of viscosity, i.e.

$$\eta = \eta_0 \exp(c_1/[T - T_0 + \sqrt{(T - T_0)^2 + c_2 T}]). \quad (5)$$

The measured isothermal TTT diagram was fitted well using Eq. (3) and the following expressions for the nucleation rate  $I_v$  and the crystal growth rate  $u$ :

$$I_v = A_v D_{eff} \exp\left(-\frac{16\pi\sigma^3}{3k_B T \Delta G^2}\right) \quad (6)$$

and

$$u = \frac{D_{eff}}{l} \left[ 1 - \exp\left(-\frac{n\Delta G}{k_B T}\right) \right], \quad (7)$$

where  $D_{eff}$  is a piecewise-defined effective diffusivity: above 850 K, the effective diffusivity obeys a Stokes-Einstein relationship with viscosity, i.e.,  $D_{eff} = k_B T / (3\pi l \eta)$ ; below 850 K, however, the effective diffusivity is better described by an Arrhenius law, i.e.,  $D_{eff} = D_0 \exp(-Q_{eff}/k_B T)$  with an activation energy  $Q_{eff} = 1.2$  eV.

Table II provides a list of the relevant data for Vitreloy-1 taken from Refs. 17 and 19. The nucleation rate  $I_v$  and the crystal growth rate  $u$  are plotted versus temperature in Fig. 3. The general profiles of  $I_v$  and  $u$  for Vitreloy-1 are very similar to those for  $Pd_{40}Cu_{30}Ni_{10}P_{20}$ , both increasing initially as temperature decreases, reaching a peak value and then decreasing subsequently. The peak positions of  $I_v$  and  $u$  for Vitreloy-1 are 793 K and 989 K, respectively.

More careful comparison of Vitreloy-1 with  $Pd_{40}Cu_{30}Ni_{10}P_{20}$ , however, reveals that while the maximum crystal growth rates for the two alloys are different by only two orders of magnitude, the maximum nucleation rates differ by almost fourteen orders of magnitude (Note that there is a factor of 50 difference between the  $f_c$  values used for the two alloys, which may affect  $I_v$  by the same factor through  $A_v$ ). However, this cannot at all account for the  $\sim 14$  orders of

magnitude difference in the two maximum nucleation rates.). Nevertheless, the two alloys show very similar shortest crystallization time, i.e., “nose time”  $t_n$ , on their measured isothermal TTT diagrams, being 50 s and 60 s for  $Pd_{40}Cu_{30}Ni_{10}P_{20}$  and for Vitreloy-1, respectively.<sup>16,17</sup> This has resulted from the different degree of overlapping and the different sharpness (with respect to temperature) of  $I_v$  and  $u$  for the two alloys. Particularly worth noting is the distinct sharpness of the  $I_v$  peak for  $Pd_{40}Cu_{30}Ni_{10}P_{20}$ , being characterized by its very small half maximum width of 42 K and extremely high magnitude of  $\sim 10^{18}/m^3$  s, as compared to 131 K and  $2.3 \times 10^4/m^3$  s, respectively, for Vitreloy-1 (refer to Fig. 1, Fig. 3, and also Table III). From these results, one can expect that nanocrystallization can be obtained more easily in  $Pd_{40}Cu_{30}Ni_{10}P_{20}$  than in Vitreloy-1. An extremely large number of nuclei will form when  $Pd_{40}Cu_{30}Ni_{10}P_{20}$  is maintained at around 632 K, and yet they cannot easily grow into large grains due to the very low growth rate corresponding to that temperature. The characteristic data listed in Table III together with the information contained in Figs. 1–3 will provide important guidelines for future nanocrystallization experiments.

### III. CALCULATION OF CRITICAL COOLING RATE

#### A. Continuous integral method

In Ref. 15, Weinberg *et al.* first utilized a continuous integral method to evaluate the relative effects of different factors (such as liquid-crystal interfacial energy  $\sigma$ ) on the calculated critical cooling rate  $R_c$  for certain oxide glasses, on the basis of the Johnson-Mehl-Avrami (JMA) phase transformation model<sup>21,22</sup> and the classical theory for crystal nucleation and growth. In the method, assuming a spherical mode of crystal growth, the crystallized volume fraction  $f$  of a glass-forming liquid upon continuous cooling, if  $f \ll 1$ , depends on the cooling rate  $R$  as follows:

$$f = \frac{4\pi}{3R^4} \int_{T_l}^{T_g} I_v(T') dT' \left[ \int_{T'}^{T_g} u(T'') dT'' \right]^3.$$

Therefore, the critical cooling rate required for glass formation is determined as

TABLE III. Characteristics of the nucleation and growth peaks for  $Pd_{40}Cu_{30}Ni_{10}P_{20}$  and  $Zr_{41.2}Ti_{13.8}Cu_{12.5}Ni_{10}Be_{22.5}$ . Notation:  $T_p$ —peak temperature,  $HMW$ —half maximum width.

Alloy Composition (at. %)	$u_{max}$ (m/s)	$T_p^u$ (K)	$HMW^u$ (K)	$I_{v,max}$ (m <sup>-3</sup> s <sup>-1</sup> )	$T_p^I$ (K)	$HMW^I$ (K)
$Pd_{40}Cu_{30}Ni_{10}P_{20}$	$3.2 \times 10^{-7}$	794	60	$1.1 \times 10^{18}$	632	42
$Zr_{41.2}Ti_{13.8}Cu_{12.5}Ni_{10}Be_{22.5}$	$8.3 \times 10^{-5}$	989	80	$2.3 \times 10^4$	793	131

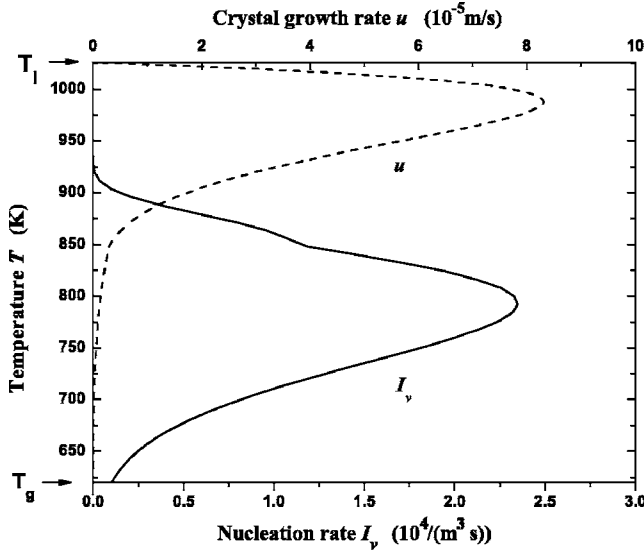


FIG. 3. Nucleation rate  $I_v$  and crystal growth rate  $u$  as functions of temperature for  $Zr_{41.2}Ti_{13.8}Cu_{12.5}Ni_{10}Be_{22.5}$ .

$$R_c = \left\{ \frac{4\pi}{3f_c} \int_{T_l}^{T_g} I_v(T') dT' \left[ \int_{T'}^{T_g} u(T'') dT'' \right]^3 \right\}^{1/4}. \quad (8)$$

By substituting Eqs. (1), (2), and (4) into Eq. (8) and using the data listed in Table I, the critical cooling rate  $R_c$  for  $Pd_{40}Cu_{30}Ni_{10}P_{20}$  is calculated to be 0.46 K/s. This calculated value agrees remarkably well with the experimentally measured 0.33 K/s,<sup>16</sup> particularly considering the large variation, usually by several orders of magnitude, in the critical cooling rates of different metallic alloys.

Similarly, the critical cooling rate for Vitreloy-1 can be calculated using Eqs. (5)–(8), the piecewise-defined effective diffusivity  $D_{eff}$ , and the thermodynamic and kinetic data listed in Table II. The calculation yields  $R_c=0.91$  K/s. As for the case of  $Pd_{40}Cu_{30}Ni_{10}P_{20}$ , this calculated value for Vitreloy-1 agrees strikingly well with the experimentally measured 1.4 K/s.<sup>18</sup>

### B. Accuracy and precision of the calculations

It was warned in the original paper by Weinberg *et al.*<sup>15</sup> that small error in the interfacial energy  $\sigma$  may cause large uncertainty in the calculated  $R_c$ . For example, the authors demonstrated (see Fig. 3 in Ref. 15) when  $\sigma$  [proportional to a Turnbull's constant  $\alpha=(N_A V_m^2)^{1/3} \sigma / \Delta H_F$  (Ref. 9)] increases by  $\sim 15\%$ , the calculated  $R_c$  for  $SiO_2$  drops by one order of magnitude. As a matter of fact, the use of the absolute value of the calculated  $R_c$  was not recommended by the authors.

However, the error (or, “sensitivity”) analysis in Ref. 15 was performed in a way that errors from different factors were isolated and were not allowed to interfere with each other. Nevertheless, this is not the case when a measured isothermal TTT diagram (or part of it, particularly the part around its “nose”) is available, as in the present study. In the present calculation of  $R_c$ , the measured TTT diagrams place constraints on the errors in  $\sigma$  and  $A_v$ . Take  $Pd_{40}Cu_{30}Ni_{10}P_{20}$  as an example. As an illustration, Fig. 4 displays the experi-

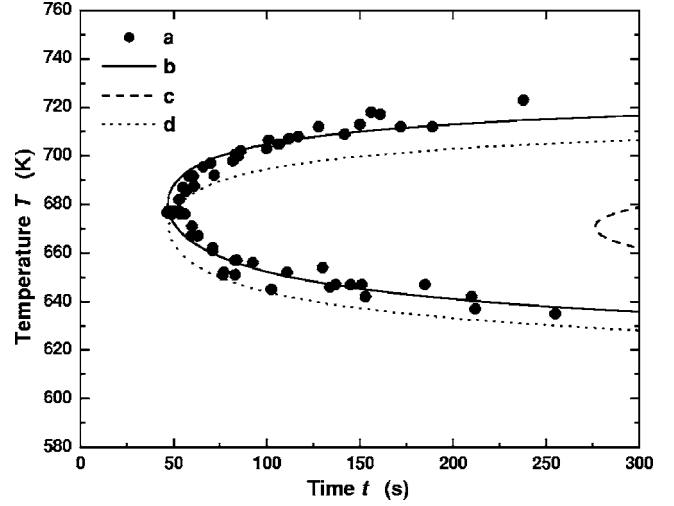


FIG. 4. Measured and calculated isothermal TTT diagrams of  $Pd_{40}Cu_{30}Ni_{10}P_{20}$ : (a) experimental data (Ref. 16); (b) calculated using  $A_v=4.4 \times 10^{31}$  Pa/m<sup>3</sup> and  $\sigma=0.061$  J/m<sup>2</sup>; (c) calculated using  $A_v=4.4 \times 10^{31}$  Pa/m<sup>3</sup> and  $\sigma=0.0671$  J/m<sup>2</sup>; (d) calculated using  $A_v=5 \times 10^{34}$  Pa/m<sup>3</sup> and  $\sigma=0.0671$  J/m<sup>2</sup>.

mental data together with three calculated TTT diagrams for  $Pd_{40}Cu_{30}Ni_{10}P_{20}$ . In Fig. 4, curve (b) is the best fit, with  $A_v=4.4 \times 10^{31}$  Pa/m<sup>3</sup> and  $\sigma=0.061$  J/m<sup>2</sup>, curve (c) is calculated by fixing  $A_v=4.4 \times 10^{31}$  Pa/m<sup>3</sup> and introducing 10% error to  $\sigma$  (and thus making  $\sigma=0.0671$  J/m<sup>2</sup>), and curve (d) is calculated using  $A_v=5 \times 10^{34}$  Pa/m<sup>3</sup> and  $\sigma=0.0671$  J/m<sup>2</sup> such that the nose time  $t_n$  is restored to the measured 50 s while keeping the 10% error in  $\sigma$ . Apparently, when introduced isolatedly, the 10% error can change the position of the calculated TTT diagram dramatically. In order to keep reasonable fit to the measured data, accordingly, the 10% error in  $\sigma$  has to be accommodated with a three orders of magnitude change in  $A_v$ . The mutually constrained errors in  $\sigma$  and  $A_v$  result in a fairly stabilized (precise)  $R_c$ , e.g.,  $R_c=0.42$  K/s for  $A_v=5 \times 10^{34}$  Pa/m<sup>3</sup> and  $\sigma=0.0671$  J/m<sup>2</sup>, in comparison with  $R_c=0.46$  K/s obtained using the best fit of  $\sigma$  and  $A_v$ .

A further extension to the above analysis is to allow  $\sim \pm 10$  s uncertainties in the nose time  $t_n$  while keeping the 10% error in  $\sigma$  (i.e., fixing  $\sigma=0.0671$  J/m<sup>2</sup>). The calculated TTT diagrams are shown in Fig. 5. Correspondingly, the critical cooling rate is evaluated to be:  $R_c=0.34$  K/s for  $A_v=2 \times 10^{34}$  Pa/m<sup>3</sup>, and  $R_c=0.60$  K/s for  $A_v=2 \times 10^{35}$  Pa/m<sup>3</sup>.

Similar analysis has also been performed on Vitreloy-1. It is concluded that the error bars to the calculated  $R_c=0.46$  K/s for  $Pd_{40}Cu_{30}Ni_{10}P_{20}$  and  $R_c=0.91$  K/s for Vitreloy-1, caused by the uncertainties in their measured TTT diagrams, are both within  $\pm 0.2$  K/s. Therefore, the calculation is very precise. On the other hand, comparing the calculated values with the measured 0.33 K/s and 1.4 K/s for  $Pd_{40}Cu_{30}Ni_{10}P_{20}$  and Vitreloy-1, respectively, indicates the high accuracy of the calculation.

Certain empirical methods, such as the Turnbull's  $T_{rg}$  rule<sup>9</sup> and the one based on the  $\gamma$  parameter,<sup>10</sup> have been used frequently to obtain a quick and rough estimation of GFA since they do not require all the data input as needed for the



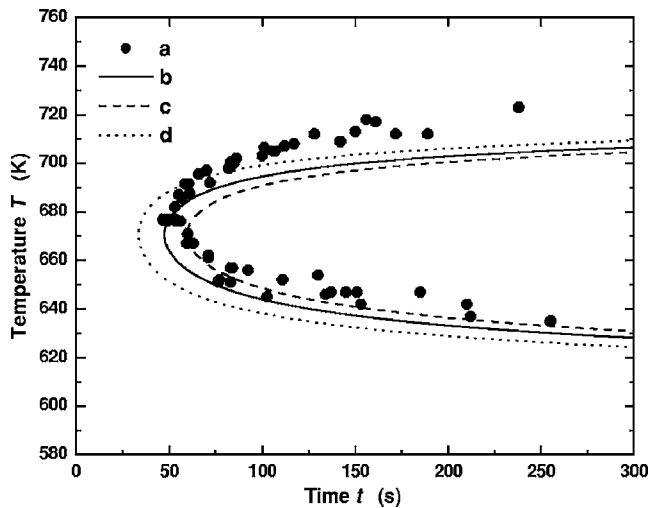


FIG. 5. Measured and calculated (with fixed  $\sigma=0.0671 \text{ J/m}^2$ ) isothermal TTT diagrams of  $\text{Pd}_{40}\text{Cu}_{30}\text{Ni}_{10}\text{P}_{20}$ : (a) experimental data (Ref. 16); (b) calculated using  $A_v=5 \times 10^{34} \text{ Pa/m}^3$ ; (c) calculated using  $A_v=2 \times 10^{34} \text{ Pa/m}^3$ ; (d) calculated using  $A_v=2 \times 10^{35} \text{ Pa/m}^3$ .

integral method and accordingly much simpler to apply. While many alloy systems were reported to support the “predictive power” of the empirical methods, there were also a comparable number of exceptions with distinct deviations. It should be noted that the main concern here in this present paper is to obtain a more complete and accurate fundamental understanding of why a metallic glass can form from thermodynamic and kinetic point of view, rather than to establish

another quick and rough criterion that will almost certainly encounter many exceptions.

#### IV. CONCLUSIONS

Within the framework of classical nucleation and growth theory, we have performed a systematic study on the crystallization kinetics of two known best metallic glass formers  $\text{Pd}_{40}\text{Cu}_{30}\text{Ni}_{10}\text{P}_{20}$  and  $\text{Zr}_{41.2}\text{Ti}_{13.8}\text{Cu}_{12.5}\text{Ni}_{10}\text{Be}_{22.5}$ , and calculated the critical cooling rates ( $R_c$ ) for the two alloys using a continuous integral method. We have found the existence of an extremely sharp nucleation peak ( $\sim 10^{18}/\text{m}^3 \text{ s}$ ) around 632 K for  $\text{Pd}_{40}\text{Cu}_{30}\text{Ni}_{10}\text{P}_{20}$  with a very small half maximum width of 42 K, which implies that this alloy is an excellent candidate for nanocrystallization studies. The calculated  $R_c$  values agree remarkably well with experimental measurements, with only very small deviation of 0.13 K/s and 0.5 K/s for  $\text{Pd}_{40}\text{Cu}_{30}\text{Ni}_{10}\text{P}_{20}$  and  $\text{Zr}_{41.2}\text{Ti}_{13.8}\text{Cu}_{12.5}\text{Ni}_{10}\text{Be}_{22.5}$ , respectively. The error bars for the calculations, caused by the uncertainties in the measured TTT diagrams, are evaluated to be within  $\pm 0.2 \text{ K/s}$  for both alloys. The high accuracy and precision of the calculations suggest that the classical theory may be sufficient to account for glass formation mechanism in these metallic alloys.

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