

Lifetime of a quasiparticle in an electron liquid. II

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Inelastic lifetime of an electron quasiparticle in an electron liquid due to electron-electron interaction evaluated in previous work is calculated in an alternative way. Both the contributions of the “direct” and “exchange” processes are included. The results turn out to be exactly the same as those obtained previously, and hence confirm the latter and consequently fully resolve the theoretical discrepancies existing in the literature. Derivation in the two-dimensional case is presented in detail due to its intricacies. The effects of local field and finite well width on the effective electron interaction in the two-dimensional case are also investigated in a quantitative comparison of the electron relaxation rate between theory and experiment. These effects are shown to make a rather small contribution to the quasiparticle lifetime.

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I. INTRODUCTION

Low-energy electron excitations in solids can be successfully described in terms of quasiparticles in the Landau theory of Fermi liquids.^{1,2} An excited quasiparticle with definite momentum is not stable due to scattering by phonons, disorders, and other electrons. Hence a quasiparticle has finite lifetime. Among this, the intrinsic inelastic scattering lifetime τ_e , i.e., the lifetime that arises purely from the electron-electron scattering processes, is a central quantity in the Landau theory of the electron liquid. It plays a key role in our understanding of a broad variety of phenomena in solids such as electron dephasing,³ tunneling,^{4,5} and localization,⁶ etc. It might also have an effect on electron transport.⁷

In fact, the electron tunneling techniques in semiconductor quantum wells^{4,5} have enabled experimentalists to directly determine τ_e in two-dimensional (2D) electron liquids. For weakly coupled wells, the lifetime principally arises from electron-electron scattering processes. On the other side, huge progress has also been made, by the use of the techniques of ultrafast laser, in measuring the lifetime of photon-excited electrons in metals such as copper.⁸ These advances have made it possible to carry out quantitative comparisons between theories and experiments. The theory of the inelastic lifetime in three dimensions (3D) is rather well established within random phase approximation.^{9,10} It was later extended to include the exchange contribution (see Ref. 11 for a detailed discussion).^{12,13} Several theoretical investigations had also been carried out in 2D, but with quantitative disagreement.^{14–20} In an earlier paper,¹¹ hereafter referred to as I, we have managed to clarify the origin of the disagreement that exists among these previous investigations. The results in I are summarized as follows. The inverse lifetime of a quasiparticle with low energy ξ_p (relative to the chemical potential μ) at temperature T in a 3D electron liquid is

$$\frac{1}{\tau_e} = \frac{m^3 e^4 \pi^2 k_B^2 T^2 + \xi_p^2}{\pi \rho k_s^3} \left\{ \frac{\lambda}{\lambda^2 + 1} + \tan^{-1} \lambda - \frac{1}{\sqrt{\lambda^2 + 2}} \right. \\ \left. \times \left[\frac{\pi}{2} - \tan^{-1} \left(\frac{1}{\lambda} \sqrt{\frac{1}{\lambda^2 + 2}} \right) \right] \right\}, \quad (1)$$

where $\lambda = 2k_F/k_s$, and $k_s = \sqrt{\frac{4k_F}{\pi a_0}}$ is the 3D Thomas-Fermi

screening wave vector. k_B , k_F , and a_0 are the Boltzmann constant, the Fermi wave vector, and the Bohr radius, respectively. For a 2D electron liquid, we found

$$\frac{1}{\tau_e} = - \frac{m^2 \xi_p^2}{16\pi^3 E_F} [3W^2(0) + 2W^2(2k_F) - 2W(0)W(2k_F)] \ln \frac{\xi_p}{2E_F} \quad (2)$$

for $k_B T \ll \xi_p$, and

$$\frac{1}{\tau_e} = - \frac{(mk_B T)^2}{32\pi E_F} [3W^2(0) + 2W^2(2k_F) - 2W(0)W(2k_F)] \ln \frac{k_B T}{2E_F} \quad (3)$$

for $\xi_p \ll k_B T$. Here $E_F = \hbar^2 k_F^2 / 2m$, and $W(q)$ is the effective interaction between quasiparticles.

The calculation of τ_e is quite a nontrivial task in many-body theory, which helps explain the disagreement among various previous theoretical results in 2D. Evidently, correctness of the results in Eq. (1) and Eqs. (2) and (3) is crucial in any meaningful comparisons with experiments. In fact, big discrepancies remain between experiments and theories, and call for explanations.^{4,5,8,11,14–20} In this paper, we calculate $1/\tau_e$ in an alternative way and confirm the above results. Various theoretical discrepancies in the literature, as mentioned above, are fully resolved.

The present calculation appears very different to the previous calculation in I. In I, we calculated $1/\tau_e$ by expressing it as the frequency convolution of the imaginary part of the density-density response function. The present calculation is technically straightforward, and somewhat in a textbook fashion. But it is by no means much simpler than that in I. In fact, the present approach seems rather clumsy for the case of $\xi_p \ll k_B T$. Hence we shall restrict ourselves to the case of zero temperature.

After giving the general formulas for $1/\tau_e$ in the next section, we present our calculation for the 3D and 2D cases separately in Secs. III and IV. In Sec. V, we shall discuss the contribution to the inverse quasiparticle lifetime in 2D arising from the effects of local field and finite well width on the effective electron interaction, and then briefly summarize the paper.

II. GENERAL FORMULAS

We start by rewriting Eqs. (4) and (5) in I as follows:

$$\frac{1}{\tau_{\sigma}^{(D)}} = 2\pi \sum_{\mathbf{k}, \mathbf{q}} \sum_{\sigma'} W_{\sigma\sigma'}^2(\mathbf{k} - \mathbf{p}) \bar{n}_{\mathbf{k}\sigma} n_{\mathbf{k}+\mathbf{q}-\mathbf{p}\sigma'} \bar{n}_{\mathbf{q}\sigma'} \times \delta(\xi_{\mathbf{p}} + \xi_{\mathbf{k}+\mathbf{q}-\mathbf{p}\sigma'} - \xi_{\mathbf{k}\sigma} - \xi_{\mathbf{q}\sigma'}) \quad (4)$$

and

$$\frac{1}{\tau_{\sigma}^{(ex)}} = -2\pi \sum_{\mathbf{k}, \mathbf{q}} W_{\sigma\sigma}(\mathbf{p} - \mathbf{q}) W_{\sigma\sigma}(\mathbf{k} - \mathbf{p}) \bar{n}_{\mathbf{k}\sigma} n_{\mathbf{k}+\mathbf{q}-\mathbf{p}\sigma} \bar{n}_{\mathbf{q}\sigma} \times \delta(\xi_{\mathbf{p}} + \xi_{\mathbf{k}+\mathbf{q}-\mathbf{p}\sigma} - \xi_{\mathbf{k}\sigma} - \xi_{\mathbf{q}\sigma}), \quad (5)$$

where $W_{\sigma\sigma'}(\mathbf{q})$ is the effective interaction between quasiparticles of spin σ and spin σ' . We have set $\hbar=1$. We consider only the paramagnetic electron liquid, and hence the index σ of the $1/\tau_{\sigma}^{(D), (ex)}$ is unnecessary and will be dropped hereafter. The sum of $1/\tau_{\sigma}^{(D)}$ and $1/\tau_{\sigma}^{(ex)}$ yields the inverse inelastic lifetime $1/\tau_e$,

$$\frac{1}{\tau_e} = 2\pi \sum_{\mathbf{k}, \mathbf{q}} \sum_{\sigma'} \left(1 - \frac{1}{2} \delta_{\sigma\sigma'}\right) [W_{\sigma\sigma'}(\mathbf{k} - \mathbf{p}) - \delta_{\sigma\sigma'} W_{\sigma\sigma'}(\mathbf{p} - \mathbf{q})]^2 \bar{n}_{\mathbf{k}\sigma} n_{\mathbf{k}+\mathbf{q}-\mathbf{p}\sigma'} \bar{n}_{\mathbf{q}\sigma'} \times \delta(\xi_{\mathbf{p}} + \xi_{\mathbf{k}+\mathbf{q}-\mathbf{p}\sigma'} - \xi_{\mathbf{k}\sigma} - \xi_{\mathbf{q}\sigma'}). \quad (6)$$

We are only interested in the case of low excited energy ($\xi_p \ll E_F$). In this case the contribution to the summations over momenta in the above expression arises only from the region of $\xi_{\mathbf{k}+\mathbf{q}-\mathbf{p}\sigma}, \xi_{\mathbf{k}\sigma}, \xi_{\mathbf{q}\sigma} \ll E_F$. Therefore

$$\frac{1}{\tau_e} = 2\pi \sum_{\mathbf{k}, \mathbf{q}} \sum_{\sigma'} I^{\sigma\sigma'}(\mu_{\mathbf{k}}, \mu_{\mathbf{q}}) \bar{n}_{\mathbf{k}\sigma} n_{\mathbf{k}+\mathbf{q}-\mathbf{p}\sigma'} \bar{n}_{\mathbf{q}\sigma'} \times \delta(\xi_{\mathbf{p}\sigma} + \xi_{\mathbf{k}+\mathbf{q}-\mathbf{p}\sigma'} - \xi_{\mathbf{k}\sigma} - \xi_{\mathbf{q}\sigma'}), \quad (7)$$

where $\mu_{\mathbf{k}} = \hat{p} \cdot \hat{k}$ and $\mu_{\mathbf{q}} = \hat{p} \cdot \hat{q}$, respectively, and the hats mean unit vectors. We have defined formally

$$I^{\sigma\sigma'}(x, y) = \left(1 - \frac{1}{2} \delta_{\sigma\sigma'}\right) [W_{\sigma\sigma'}(\sqrt{2k_F}\sqrt{1-x}) - \delta_{\sigma\sigma'} W_{\sigma\sigma'}(\sqrt{2k_F}\sqrt{1-y})]^2. \quad (8)$$

Below we present our calculation for the 3D and 2D cases separately in Secs. III and IV. We set the volume of the 3D system and the area of the 2D system, respectively, to be units in this paper.

III. THE INVERSE LIFETIME IN 3D

The integrations over the azimuthal angles of \mathbf{k} and \mathbf{q} with respect to \mathbf{p} in Eq. (7) can be straightforwardly carried out. After that, it becomes

$$\frac{1}{\tau_e} = 2\pi \left(\frac{1}{(2\pi)^3}\right)^2 \frac{\sqrt{2}\pi}{E_p} \sum_{\sigma'} \int_{k_F}^{\infty} dk k^2 \int_{k_F}^{\infty} dq q^2 \times \theta(p^2 + k_F^2 - k^2 - q^2) \int_{-1}^1 d\mu_{\mathbf{k}} \int_{-1}^1 d\mu_{\mathbf{q}} \times I^{\sigma\sigma'}(\mu_{\mathbf{k}}, \mu_{\mathbf{q}}) \frac{1}{\sqrt{(1-\mu_{\mathbf{k}})(1-\mu_{\mathbf{q}})}} \frac{\theta(\mu_{\mathbf{k}} + \mu_{\mathbf{q}})}{\sqrt{\mu_{\mathbf{k}} + \mu_{\mathbf{q}}}}, \quad (9)$$

where $E_p = p^2/2m$. We have defined $\theta(x)=1$ for $x>0$, $\theta(x)=0$ for $x\leq 0$. Notice that Eq. (9) is essentially the same as Eq. (4) in Ref. 13 except typos of $\left(\frac{\Omega}{2\pi^3}\right)^3$ and a missing factor of $1 - \frac{1}{2} \delta_{\sigma\sigma'}$ in Eq. (3b) in Ref. 13. The integrations over k and q in Eq. (9) can be carried out analytically to the accuracy of the leading order of $O(\xi_p^2)$ and yield

$$\frac{1}{\tau_e} = A \frac{\xi_p^2}{E_F E_p} \sum_{\sigma'} \int_{-1}^1 d\mu_{\mathbf{k}} \int_{-1}^1 d\mu_{\mathbf{q}} I^{\sigma\sigma'}(\mu_{\mathbf{k}}, \mu_{\mathbf{q}}) \times \frac{1}{\sqrt{(1-\mu_{\mathbf{k}})(1-\mu_{\mathbf{q}})}} \frac{\theta(\mu_{\mathbf{k}} + \mu_{\mathbf{q}})}{\sqrt{\mu_{\mathbf{k}} + \mu_{\mathbf{q}}}}, \quad (10)$$

where

$$A = \frac{e^2}{a_0} \left(\frac{1}{2^{5/2}\pi^2}\right) \left(\frac{k_F^2}{4\pi e^2}\right)^2. \quad (11)$$

Notice that there seems to be an error of a factor of 1/2 in Eq. (6b) in Ref. 13. However, this does not effect the results in Table II in Ref. 13 and the subsequent conclusions, since the coefficient A cancels in the ratio $p^{\sigma\sigma}/p^{\sigma\bar{\sigma}}$ in the table.

The contributions from the ‘‘direct’’ and ‘‘exchange’’ processes are, separately, given as

$$\frac{1}{\tau^{(D)}} = A \frac{\xi_p^2}{E_F E_p} \int_{-1}^1 d\mu_{\mathbf{k}} \int_{-\mu_{\mathbf{k}}}^1 d\mu_{\mathbf{q}} \times [W_{\sigma\sigma}^2(\sqrt{2k_F}\sqrt{1-\mu_{\mathbf{k}}}) + W_{\sigma\bar{\sigma}}^2(\sqrt{2k_F}\sqrt{1-\mu_{\mathbf{k}}})] \times \frac{1}{\sqrt{(1-\mu_{\mathbf{k}})(1-\mu_{\mathbf{q}})}} \frac{1}{\sqrt{\mu_{\mathbf{k}} + \mu_{\mathbf{q}}}}, \quad (12)$$

where $\bar{\sigma} = -\sigma$ and

$$\frac{1}{\tau^{(ex)}} = -A \frac{\xi_p^2}{E_F E_p} \int_{-1}^1 d\mu_{\mathbf{k}} \int_{-\mu_{\mathbf{k}}}^1 d\mu_{\mathbf{q}} W_{\sigma\sigma}(\sqrt{2k_F}\sqrt{1-\mu_{\mathbf{k}}}) \times W_{\sigma\sigma}(\sqrt{2k_F}\sqrt{1-\mu_{\mathbf{q}}}) \frac{1}{\sqrt{(1-\mu_{\mathbf{k}})(1-\mu_{\mathbf{q}})}} \frac{1}{\sqrt{\mu_{\mathbf{k}} + \mu_{\mathbf{q}}}}. \quad (13)$$

The spin dependence of the effective interaction is not crucial in determining the total inelastic lifetime.¹³ We shall ignore this dependence and follow the usual practice of characterizing the screening effects by the screening wave vector k_s :

$$W(q) = \frac{4\pi e^2}{q^2 + k_s^2}. \quad (14)$$

The integrations over $\mu_{\mathbf{q}}$ and $\mu_{\mathbf{k}}$ can be analytically carried out. After that, one finally has

$$\frac{1}{\tau^{(D)}} = \frac{m^3 e^4 k_F}{\pi p^2 k_s^3} \xi_p^2 \left(\frac{\lambda}{\lambda^2 + 1} + \tan^{-1} \lambda \right), \quad (15)$$

and

$$\frac{1}{\tau^{(ex)}} = -\frac{m^3 e^4 k_F}{\pi p^2 k_s^3} \xi_p^2 \frac{1}{\sqrt{\lambda^2 + 2}} \left[\frac{\pi}{2} - \tan^{-1} \left(\frac{1}{\lambda} \sqrt{\frac{1}{\lambda^2 + 2}} \right) \right]. \quad (16)$$

Equations (15) and (16) are, to the leading order of $O(\xi_p^2)$, exactly the same as the results for $1/\tau^{(D)}$ and $1/\tau^{(ex)}$ obtained in I for the 3D case.

IV. THE INVERSE LIFETIME IN 2D

The derivation in the 2D case is relatively intricate. We shall present it in detail. We first rewrite Eq. (7) as

$$\begin{aligned} \frac{1}{\tau_e} &= 2\pi m^2 \sum_{\sigma'} \int_{k_F}^{\infty} dq q \\ &\times \int_{k_F}^{\infty} dk k \theta(p^2 + k_F^2 - k^2 - q^2) \int_{-\pi}^{\pi} d\phi_{\mathbf{k}} \int_{-\pi}^{\pi} d\phi_{\mathbf{q}} \\ &\times I^{\sigma\sigma'}(\mu_{\mathbf{k}}, \mu_{\mathbf{q}}) \delta[p^2 + kq \\ &\times \cos(\phi_{\mathbf{k}} - \phi_{\mathbf{q}}) - pk \cos \phi_{\mathbf{k}} - pq \cos \phi_{\mathbf{q}}], \quad (17) \end{aligned}$$

where $\mu_{\mathbf{k}} = \cos \phi_{\mathbf{k}}$, $\mu_{\mathbf{q}} = \cos \phi_{\mathbf{q}}$, and $\phi_{\mathbf{k}}$ and $\phi_{\mathbf{q}}$ are the angles of \mathbf{k} and \mathbf{q} relative to \mathbf{p} , respectively. After some algebraic manipulations, the preceding expression can be written as

$$\begin{aligned} \frac{1}{\tau_e} &= 8\pi m \left(\frac{1}{(2\pi)^2} \right)^2 \sum_{\sigma'} \int_{k_F}^p dq q \int_{k_F}^{\sqrt{p^2 + k_F^2 - q^2}} dk k \\ &\times \int_{-1}^1 d\mu_{\mathbf{k}} \int_{-1}^1 d\mu_{\mathbf{q}} I^{\sigma\sigma'}(\mu_{\mathbf{k}}, \mu_{\mathbf{q}}) \\ &\times \delta[(p - k\mu_{\mathbf{k}})^2 (p - q\mu_{\mathbf{q}})^2 - k^2 q^2 (1 - \mu_{\mathbf{k}}^2)(1 - \mu_{\mathbf{q}}^2)]. \quad (18) \end{aligned}$$

We now define the following variables,

$$\lambda = \frac{p}{\sqrt{z}}, \quad \lambda' = \frac{p}{\sqrt{z'}}, \quad (19)$$

and function,

$$f(x, y) = (\lambda - x)^2 (\lambda' - y)^2 - 1 + x^2 + y^2 - x^2 y^2, \quad (20)$$

and write Eq. (18) in the following form:

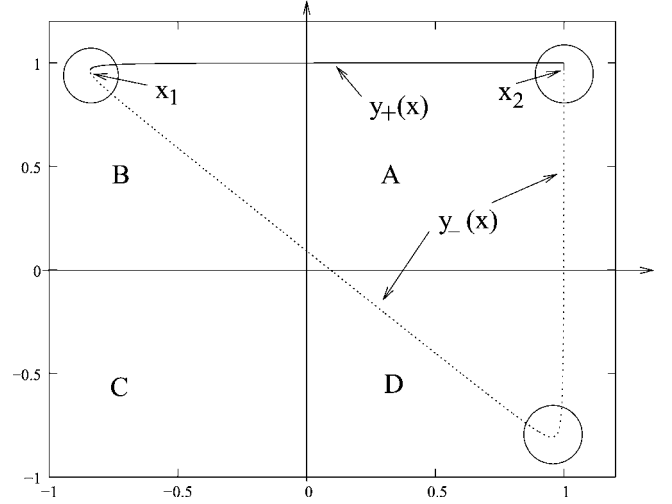


FIG. 1. The contour of $f(x, y) = 0$, defined in Eq. (20), and its two parts: $y_+(x)$ (solid line) and $y_-(x)$ (dotted line), defined in Eq. (28).

$$\begin{aligned} \frac{1}{\tau_e} &= \frac{2\pi m}{k_F^2} \left(\frac{1}{(2\pi)^2} \right)^2 \sum_{\sigma'} \int_{k_F}^p dz \int_{k_F}^{p^2 + k_F^2 - z} dz' \\ &\times \int_{-1}^1 dx \int_{-1}^1 dy \delta[f(x, y)] I^{\sigma\sigma'}(x, y). \quad (21) \end{aligned}$$

The contour of $f(x, y) = 0$ is illustrated in Fig. 1.

We further define

$$G_{\sigma\sigma'}^{A, B, D} = \int_{k_F}^p dz \int_{k_F}^{p^2 + k_F^2 - z} dz' J_{\sigma\sigma'}^{A, B, D}, \quad (22)$$

where

$$J_{\sigma\sigma'}^A = \int_0^1 dx \int_0^1 dy \delta[f(x, y)] I^{\sigma\sigma'}(x, y), \quad (23)$$

$$J_{\sigma\sigma'}^B = \int_{-1}^0 dx \int_0^1 dy \delta[f(x, y)] I^{\sigma\sigma'}(x, y), \quad (24)$$

and

$$J_{\sigma\sigma'}^D = \int_0^1 dx \int_{-1}^0 dy \delta[f(x, y)] I^{\sigma\sigma'}(x, y), \quad (25)$$

and express $1/\tau_e$ as follows:

$$\frac{1}{\tau_e} = \frac{2\pi m}{k_F^2} \left(\frac{1}{(2\pi)^2} \right)^2 \sum_{\sigma'} (G_{\sigma\sigma'}^A + G_{\sigma\sigma'}^B + G_{\sigma\sigma'}^D). \quad (26)$$

Evidently, A , B , and D denote the contributions arising from the first, second, and fourth quadrants, respectively, as shown in Fig. 1. Notice that $f(x, y) \neq 0$ in the third quadrant. In fact, the leading order contributions only arise from the circled regions in Fig. 1. By interchanging the integral variables x and y in Eq. (25), it is easy to see that the evaluation of $G_{\sigma\sigma'}^D$ is totally analogous to that of $G_{\sigma\sigma'}^B$. Hence we only present the latter. The evaluation of $G_{\sigma\sigma'}^B$ turns out to be relatively

simpler than that of $G_{\sigma\sigma'}^A$, and it is in the meanwhile instructive for the latter. We hence start with the former below.

We first evaluate the integrations over the variables x and y in Eq. (24). To this end, we rewrite Eq. (24) as

$$J_{\sigma\sigma'}^B = \int_{-1}^0 dx \int_0^1 dy \frac{1}{(\lambda-x)^2 + 1 - x^2} \times \delta[y - y_+(x)][y - y_-(x)] I^{\sigma\sigma'}(x, y), \quad (27)$$

where

$$y_{\pm}(x) = \frac{1}{(\lambda-x)^2 + 1 - x^2} [\lambda'(\lambda-x)^2 \pm \sqrt{1-x^2}\sqrt{(1-\lambda'^2)(\lambda-x)^2 + 1 - x^2}]. \quad (28)$$

The functions $y_+(x)$ and $y_-(x)$ are two components of the contour of $f(x, y)=0$, and they are illustrated in Fig. 1. The integration over the variable y in Eq. (27) is now straightforward, which yields

$$J_{\sigma\sigma'}^B = \frac{1}{2} \int_{-1}^0 dx \frac{\theta(x-x_1)\theta(x_2-x)}{\sqrt{1-x^2}\sqrt{(1-\lambda'^2)(\lambda-x)^2 + 1 - x^2}} \times [I^{\sigma\sigma'}(x, y_+(x)) + I^{\sigma\sigma'}(x, y_-(x))], \quad (29)$$

or, more explicitly,

$$J_{\sigma\sigma'}^B = \frac{1}{2\lambda'} \int_{x_1}^0 dx \frac{1}{\sqrt{(1-x^2)(x_2-x)(x-x_1)}} \times [I^{\sigma\sigma'}(x, y_+(x)) + I^{\sigma\sigma'}(x, y_-(x))]. \quad (30)$$

Here we have defined

$$x_{1,2} = -\frac{\lambda(1-\lambda'^2) \pm \sqrt{\lambda^2 + \lambda'^2 - \lambda^2\lambda'^2}}{\lambda'^2}, \quad (31)$$

which are also shown in Fig. 1. By using the fact that $x_1 = 4(\lambda' - 1) - 1$ and $y_+(x_1) = y_-(x_1) = 1$ for $\lambda' \rightarrow 1$, we have, to the leading order,

$$J_{\sigma\sigma'}^B = -\frac{1}{2} I^{\sigma\sigma'}(-1, 1) \ln(\lambda' - 1). \quad (32)$$

Substituting the preceding result into Eq. (22) and performing the integrations over z and z' , one obtains

$$G_{\sigma\sigma'}^B = -\frac{1}{4} I^{\sigma\sigma'}(-1, 1) (p^2 - k_F^2)^2 \ln[(p^2 - k_F^2)/2k_F^2]. \quad (33)$$

As pointed out previously, the evaluation of $G_{\sigma\sigma'}^D$ is similar to that of $G_{\sigma\sigma'}^B$. Here we only quote the final result,

$$G_{\sigma\sigma'}^D = -\frac{1}{4} I^{\sigma\sigma'}(1, -1) (p^2 - k_F^2)^2 \ln[(p^2 - k_F^2)/2k_F^2]. \quad (34)$$

Next we calculate $G_{\sigma\sigma'}^A$. First of all, from the experience in deriving $J_{\sigma\sigma'}^B$ in Eq. (32), it is not difficult to see that the leading order contribution to $J_{\sigma\sigma'}^A$ arises from the region of $x \rightarrow 1, y \rightarrow 1$. Therefore, we may directly rewrite Eq. (23) as

$$J_{\sigma\sigma'}^A = I^{\sigma\sigma'}(1, 1) \int_0^1 dx \int_0^1 dy \delta[f(x, y)]. \quad (35)$$

However, the following calculation is a little more delicate. Due to the fact that $y_-(x)$ becomes ill-defined (actually becomes $x=1$) in the first quadrant as $\lambda, \lambda' \rightarrow 1$, a straightforward calculation like the preceding one for $J_{\sigma\sigma'}^B$ does not work. To circumvent this difficulty, we make the following variable transform,

$$x = \frac{1}{\sqrt{2}}(x' - y'), \quad y = \frac{1}{\sqrt{2}} \frac{\lambda'}{\lambda}(x' + y'), \quad (36)$$

and rewrite Eq. (35) as

$$J_{\sigma\sigma'}^A = I^{\sigma\sigma'}(1, 1) \int_0^{(1/\sqrt{2})(1+\lambda/\lambda')} dx' \int dy' \times \delta\{a(x')[y' - y_1(x')][y' - y_2(x')]\}. \quad (37)$$

The Jacobian λ'/λ for the above integration variable transform can be set to be one in the limit of $\lambda, \lambda' \rightarrow 1$. The limits of the integration over y' are left unspecified because they are not really relevant simply due to the δ function in the integrand, while the integration region of x' and y' corresponds to the square of $0 \leq x', y' \leq 1$. In Eq. (37), we have defined

$$y_{1,2}(x) = \frac{-b(x) \pm \sqrt{b^2(x) - 4a(x)c(x)}}{2a(x)}, \quad (38)$$

where

$$a(x) = \frac{1}{2} (2\sqrt{2}x\lambda'^2/\lambda - 2\lambda'^2 + \lambda'^2/\lambda^2 + 1), \quad (39)$$

$$b(x) = -(1 - \lambda'^2/\lambda^2)x, \quad (40)$$

and

$$c(x) = \lambda^2\lambda'^2 - 1 - 2\sqrt{2}\lambda\lambda'^2x + \frac{1}{2}(\lambda'^2/\lambda^2 + 6\lambda'^2 + 1)x^2 - \sqrt{2}(\lambda'^2/\lambda)x^3. \quad (41)$$

The integration over y' yields

$$J_{\sigma\sigma'}^A = I^{\sigma\sigma'}(1, 1) \int_0^{(1/\sqrt{2})(1+\lambda/\lambda')} dx \frac{\theta[b^2(x) - 4a(x)c(x)]}{\sqrt{b^2(x) - 4a(x)c(x)}}, \quad (42)$$

which can be rewritten as

$$J_{\sigma\sigma'}^A = I^{\sigma\sigma'}(1, 1) \int_{-(1/\sqrt{2})(1+\lambda/\lambda')}^0 dx \frac{\theta(\alpha x^2 + \beta x + \gamma)}{\sqrt{\alpha x^2 + \beta x + \gamma}}, \quad (43)$$

where α, β , and γ , in the limit of $\lambda \rightarrow 1, \lambda' \rightarrow 1$, can be shown as

$$\alpha = 16, \quad (44)$$

$$\beta = -4\sqrt{2}(\lambda\lambda' + \lambda'^2 + \lambda + \lambda' + \lambda^2 + 3\lambda^2\lambda'^2 - 4\lambda\lambda'^2 - 4\lambda^2\lambda') \times (\lambda - 1)(\lambda' - 1), \quad (45)$$

$$\gamma = -8(\lambda - 1)^2(\lambda' - 1)^2. \quad (46)$$

The leading order contribution to $J_{\sigma\sigma'}^A$, in fact arises from the limiting region of $x \rightarrow 0$ in the integral of Eq. (43). Therefore the higher order terms of $O(x^3)$ and $O(x^4)$ have been ignored in the θ function and the square root denominator in Eq. (43). The term βx can be further neglected since it also is higher order smaller according to Eq. (45). Therefore, one has

$$J_{\sigma\sigma'}^A = I^{\sigma\sigma'}(1,1) \int_{-(1/\sqrt{2})(1+\lambda/\lambda')}^0 dx \frac{\theta\left[x^2 - \frac{1}{2}(\lambda-1)^2(\lambda'-1)^2\right]}{4\sqrt{x^2 - \frac{1}{2}(\lambda-1)^2(\lambda'-1)^2}}, \quad (47)$$

or

$$J_{\sigma\sigma'}^A = \frac{1}{4} I^{\sigma\sigma'}(1,1) \int_{-(1/\sqrt{2})(1+\lambda/\lambda')}^{-(1/\sqrt{2})(\lambda-1)(\lambda'-1)} dx \times \frac{1}{\sqrt{x^2 - \frac{1}{2}(\lambda-1)^2(\lambda'-1)^2}}. \quad (48)$$

Equation (48) can be evaluated as

$$J_{\sigma\sigma'}^A = -\frac{1}{4} I^{\sigma\sigma'}(1,1) \ln[(\lambda-1)(\lambda'-1)]. \quad (49)$$

Substituting the preceding result into Eq. (22) and carrying out the remaining integrations over z and z' , one finally has

$$G_{\sigma\sigma'}^A = -\frac{1}{4} I^{\sigma\sigma'}(1,1) (p^2 - k_F^2)^2 \ln[(p^2 - k_F^2)/2k_F^2]. \quad (50)$$

In view of the fact that $G_{\sigma\sigma'}^A$ in the preceding equation is totally similar to $G_{\sigma\sigma'}^{B,D}$ in Eqs. (33) and (34), it is curious that there seems no simpler way to derive it.

Substituting the results for $G_{\sigma\sigma'}^A$, $G_{\sigma\sigma'}^B$, and $G_{\sigma\sigma'}^D$ in Eqs. (50), (33), and (34) into Eq. (26) one finally obtains

$$\frac{1}{\tau_e} = \frac{\pi m}{2k_F^2} \left(\frac{1}{(2\pi)^2} \right)^2 (p^2 - k_F^2)^2 \ln[(p^2 - k_F^2)/2k_F^2] \times \sum_{\sigma'} [I^{\sigma\sigma'}(1,1) + I^{\sigma\sigma'}(1,-1) + I^{\sigma\sigma'}(-1,1)]. \quad (51)$$

The contributions from the ‘‘direct’’ and ‘‘exchange’’ processes can be separately written as

$$\frac{1}{\tau^{(D)}} = -\frac{m^2 \xi_p^2}{16\pi^3 E_F} \ln \frac{\xi_p}{2E_F} \sum_{\sigma'} \{2[W_{\sigma\sigma'}(0)]^2 + [W_{\sigma\sigma'}(2k_F)]^2\}, \quad (52)$$

and

$$\frac{1}{\tau^{(ex)}} = \frac{m^2 \xi_p^2}{16\pi^3 E_F} \ln \frac{\xi_p}{2E_F} \{[W_{\sigma\sigma}(0)]^2 + 2W_{\sigma\sigma}(0)W_{\sigma\sigma}(2k_F)\}. \quad (53)$$

We emphasize that the above results are accurate only to the leading order of $O(\xi_p^2 \ln \xi_p)$. In the case that the spin dependence of the effective interaction can be neglected, one has exactly the results shown in Eqs. (52) and (59) in I, respec-

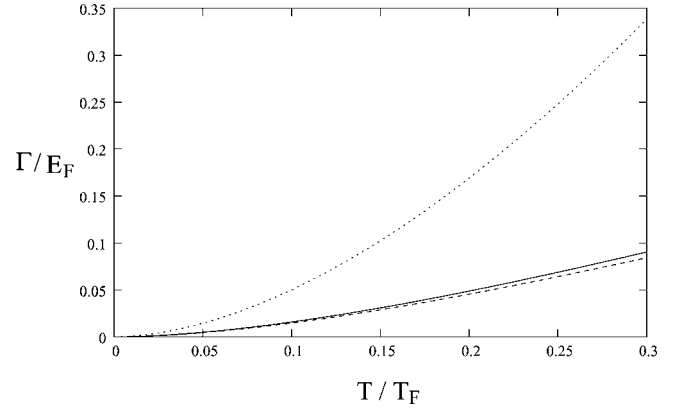


FIG. 2. Electron relaxation rate Γ in 2D (Refs.5, 11, and 18–20). Dotted line: experimental data from Ref. 5; dashed line: calculated one from Eq. (3) with RPA to the effective interaction $W(q)$ Ref. 11; solid line: calculated one from Eq. (3) with the effects of the local field and the finite well width on $W(q)$ included.

tively. The sum of $1/\tau^{(D)}$ and $1/\tau^{(ex)}$ yields $1/\tau_e$ as given in Eq. (2) in the Introduction.

V. DISCUSSION AND SUMMARY

The results in Eqs. (15) and (16) have been obtained with the approximation of the Thomas-Fermi screened Coulomb potential of Eq. (14) to the effective electron interaction. One can always resort to the more general expressions of Eqs. (12) and (13) if necessary. On the other side, in the 2D case, with which this paper is mainly concerned, no approximation has been made in the effective electron interaction $W(q)$ except that it is assumed to be static. The local field effects and the finite well width effects on the effective interaction can be readily taken into account. We now estimate their contribution to the inverse quasiparticle lifetime by using the form factor of $F(q) = \frac{2}{qb} [1 + \frac{1}{qb}(e^{-qb} - 1)]$ with b being the well width,^{19,21} and the local field factor evaluated in Ref. 22. These effects have been investigated earlier in Ref. 19. The clarification of the theoretical disagreement now enables us to definitely elucidate their contribution. Both of them are shown to yield in effect quite small corrections to the results calculated with random phase approximation (RPA) to $W(q)$. This conclusion should hold in a more general sense regardless of the particular choice of the form factor and the local field factor, since both of them mainly affect the short-range behavior of the effective interaction while the inverse lifetime of a low-energy quasiparticle is mainly determined by the long-range behavior of the effective interaction. The comparison with the experimental values of the electron relaxation rate Γ (Refs. 5, 11, and 18–20) from Ref. 5 is illustrated in Fig. 2. The result calculated with the RPA to the effective interaction is also plotted in Fig. 2 for comparison.¹¹

It seems that other factors must be taken into account in order to explain the difference between theory and experiment. One of the assumptions made in all previous work is that the couplings between electrons in different wells are weak and can be ignored. This assumption might require

further justification for a barrier width being about 250 Å, with the width of each well being about 200 Å. Furthermore, higher-order terms in electron interaction, usually not important at a density of $r_s \sim 1$ where r_s is the Wigner-Seitz radius, might not be simply ignored in this case, for they have been shown to contribute nontrivially higher-order logarithmic factors.²³ Other factors which might also play a role have been mentioned in Ref. 11. Evidently, further theoretical effort is needed in order to fully understand the discrepancy between theory and experiment.

In conclusion, we have calculated, in a rather different manner, the inelastic lifetime of an electron quasiparticle in

an electron liquid. The results confirm those in Eq. (1), and Eqs. (2) and (3) obtained in our previous work, and consequently finally resolve the theoretical discrepancies in the literature.

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