Theory of strong localization effects of light in disordered loss or gain media

Regine Frank,[1,2] Andreas Lubatsch,[1,3] and Johann Kroha[1]

[1]*Physikalisches Institut, Universität Bonn, 53115 Bonn, Germany*
[2]*Institut für Astronomie und Astrophysik, Universität Tübingen, 72076 Tübingen, Germany*
[3]*Laboratoire de Physique et Modélisation des Milieux Condensés/CNRS, 38042 Grenoble, France*

We present a systematical theory for the interplay of strong localization effects and absorption or gain of classical waves in three-dimensional, disordered dielectrics. The theory is based on the self-consistent Cooperon resummation, implementing the effects of energy conservation and its absorptive or emissive corrections by an exact, generalized Ward identity. Substantial renormalizations are found, depending on whether the absorption or gain occurs in the scatterers or in the background medium. We find a finite, gain-induced correlation volume which may be significantly smaller than the scale set by the scattering mean-free path, even if there are no truly localized modes. Possible consequences for coherent feedback in random lasers as well as the possibility of oscillatory in time behavior induced by sufficiently strong gain are discussed.

I. INTRODUCTION

The strong or Anderson localization (AL) (Ref. 1) of light in $d=$ three-dimensional, random media has remained a fascinating and controversial issue, despite its early theoretical anticipation[2,3] and despite the experimental observation of its precursor effect, the coherent backscattering light cone of weak localization.[4] Early experimental reports of an anomalously low diffusion constant in strongly scattering powders[5] were explained by a reduction of the transport velocity by Mie resonances.[6] However, recent works on the experimental proof of light localization[7] has lead to a controversial, still ongoing debate.[8,9] The reason resides in the difficulty to distinguish AL from absorption in the medium. The problem becomes even more pressing for localization in random media with stimulated emission (*random lasers*).[10] Here, the scattering mean-free path is large compared to the wave length, $\ell \gg \lambda$, so that localization effects are expected to be small. However, the experiments indicate unambiguously, by direct observation[11] and especially by measuring the photon statistics,[12] that the laser emission is due to coherent feedback and occurs from spatially confined spots in the sample. As an alternative to lasing from Anderson localized modes, the existence of preformed cavities in the random medium has been proposed in Ref. 13.

AL has been understood[14,15] as an effect of repeated self-interference (so-called Cooperon contributions) of diffusive modes. Since diffusion, as a hydrodynamic phenomenon, relies on particle number or energy conservation, this raises the fundamental question for the fate of AL in active media altogether. Intuitively one expects coherent amplification to enhance the transmission, but at the same time also the Cooperon interference. Since the latter tends to localize a wave and hence to reduce the transmission, the overall effect is left an open issue. Paasschens *et al.*,[16] treating the transmission in a linearized way, prove analytically in $d=1$, that amplification diminishes the transmission just like absorption does. In contrast, Jiang *et al.*,[17] going beyond the linear approximation, find numerically an enhanced transmission in the long-time limit. Although important insight has been ob-

tained by further numerical studies in $d=1$ (Ref. 18) and $d=2$,[19] this controversial situation calls for a semianalytical theory to systematically analyze transport quantities like the diffusion coefficient D and the intensity correlation length ξ_a. This is especially so in $d=3$, where the realization of AL modes is unclear even in the passive system.

In this paper we present such a theory, based on the self-consistent Cooperon resummation pioneered by Vollhardt and Wölfle.[15] It allows us to distinguish AL properties from absorption or gain-induced decay or growth. The evaluations will be done for $d=3$, although the theory is valid for $d=1,2,3$. As the central results we (1) recover, for small gain, the duality of AL with respect to absorption or gain, together with a spatially homogeneous, exponential decay or growth in the long-time limit, thus reconciling the results of Refs. 16 and 17. (2) In addition, we find strong loss- or gain-dependent renormalizations of D and of ξ_a, asymmetrical in loss or gain, originating not from wave interference but from the violation of the conservation laws. These renormalizations are, in particular, sensitive to whether the loss (gain) occurs in the homogeneous background medium or in the random scatterers, and thus do not occur in $d=1$ layered structures. (3) We predict that due to loss or gain the intensity correlation length can be substantially smaller than the scattering mean-free path ℓ, constituting a finite coherence volume. Possible implications on coherent feedback in random lasers are discussed.

II. MODEL AND TRANSPORT THEORY

We consider a system of randomly positioned spherical scatterers in a background medium with dielectric constants ϵ_s and ϵ_b, respectively. On the semiclassical level, linear absorption and the stimulated part of emission are represented by a positive or negative imaginary part of the index of refraction, $n_s = \sqrt{\epsilon_s}$, $n_b = \sqrt{\epsilon_b}$. Since we will focus on the fundamental problems posed above rather than a quantitative description, we will neglect the vector nature (polarization) of light and consider scalar, classical waves only. These obey the wave equation

$$\frac{\omega^2}{c^2}\epsilon(\vec{r})\Psi_\omega(\vec{r}) + \nabla^2\Psi_\omega(\vec{r}) = -i\omega\frac{4\pi}{c^2}j_\omega(\vec{r}), \quad (1)$$

where c denotes the vacuum speed of light and $j_\omega(\vec{r})$ an external source. The dielectric constant $\epsilon(\vec{r}) = \epsilon_b + \Delta\epsilon V(\vec{r})$, $\Delta\epsilon = \epsilon_s - \epsilon_b$, describes the arrangement of scatterers through the function $V(\vec{r}) = \Sigma_R S_R^-(\vec{r}-\vec{R})$, with $S_R^-(\vec{r})$ a localized shape function at random locations $\vec{R}$. The Fourier transform of the retarded, disorder averaged Green's function of Eq. (1) reads

$$G_{\vec{k}}^\omega = \frac{1}{\epsilon_b(\omega/c)^2 - |\vec{k}|^2 - \Sigma_{\vec{k}}^\omega}, \quad (2)$$

where the retarded self-energy $\Sigma_{\vec{k}}^\omega$ arises from scattering off the random "potential" $-(\omega/c)^2(\epsilon_s - \epsilon_b)V(\vec{r})$. The mode density is given by $N(\omega) = -(\omega/\pi)\,\mathrm{Im}\,G_0^\omega$, $G_0^\omega \equiv \int d^3k/(2\pi)^3 G_{\vec{k}}^\omega$.

To develop a transport theory we now turn to the four-point intensity correlation function, defined in terms of the nonaveraged Green's functions $\hat{G}$, $\hat{G}^*$ and the disorder average $\langle\cdots\rangle$ as $\Phi_{\vec{q}\vec{q}'}^\omega(\vec{Q},\Omega) = \langle\hat{G}_{\vec{q}_+\vec{q}'_+}^{\omega_+}\hat{G}_{\vec{q}_-\vec{q}'_-}^{\omega_-*}\rangle$. It is determined by the kinetic equation, see, e.g., Ref. 20,

$$\left[\omega\Omega\frac{\mathrm{Re}\,\epsilon_b}{c^2} - Q(\vec{q}\times\hat{Q}) + \frac{i}{c^2\tau^2}\right]\Phi_{\vec{q}\vec{q}'}^\omega$$
$$= -i\,\mathrm{Im}\,G_{\vec{q}}^\omega\left[1 + \int\frac{d^3q''}{(2\pi)^3}\gamma_{\vec{q}\vec{q}''}^\omega\Phi_{\vec{q}''\vec{q}'}^\omega\right]. \quad (3)$$

Here, we have introduced the usual[20] center-of-mass and relative frequencies and momenta: The variables Ω, $\vec{Q}$ are associated with the time and position dependence of the averaged energy density, with $\hat{Q} = \vec{Q}/|\vec{Q}|$, while $\omega_\pm = \omega \pm \Omega/2$ and $\vec{q}_\pm = \vec{q} \pm \vec{Q}/2$, etc. are the frequencies and momenta of in- and outgoing waves, respectively. In order to analyze the correlation function's long-time ($\Omega \to 0$) and long-distance ($|\vec{Q}| \to 0$) behavior, terms of $O(\Omega^2, Q^3, \Omega Q)$ have been neglected here and throughout this paper. Equation (3) contains both the *total* quadratic momentum relaxation rate $1/\tau^2 = c^2\,\mathrm{Im}\,(\epsilon_b\omega^2/c^2 - \Sigma^\omega)$ (due to absorption or gain in the background medium as well as impurity scattering) and the irreducible two-particle vertex function $\gamma_{\vec{q}\vec{q}'}^\omega(\vec{Q},\Omega)$. The energy conservation is implemented in a field theoretical sense by a Ward identity (WI) which has been derived for the photonic case in Ref. 20, and which for scalar waves takes the exact form

$$\Sigma_{\vec{q}_+}^{\omega_+} - \Sigma_{\vec{q}_-}^{\omega_-*} - \int\frac{d^3q'}{(2\pi)^3}[G_{\vec{q}'_+}^{\omega_+} - G_{\vec{q}'_-}^{\omega_-*}]\gamma_{\vec{q}'\vec{q}}^\omega(\vec{Q},\Omega)$$
$$= f_\omega(\Omega)\left[\mathrm{Re}\,\Sigma_{\vec{q}}^\omega + \int\frac{d^3q'}{(2\pi)^3}\,\mathrm{Re}\,G_{\vec{q}'}^\omega\gamma_{\vec{q}'\vec{q}}^\omega(\vec{Q},\Omega)\right]. \quad (4)$$

The right-hand side of Eq. (4) represents reactive effects (real parts), originating from the explicit ω^2 dependence of the photonic random "potential." In conserving media ($\mathrm{Im}\,\epsilon_b = \mathrm{Im}\,\epsilon_s = 0$) these terms renormalize the energy transport velocity v_E relative to the average phase velocity

c_p without destroying the diffusive long-time behavior.[3,20] In the presence of loss or gain, however, these effects are enhanced via the prefactor $f_\omega(\Omega) = (\omega\Omega\,\mathrm{Re}\,\Delta\epsilon + i\omega^2\,\mathrm{Im}\,\Delta\epsilon)/(\omega^2\,\mathrm{Re}\,\Delta\epsilon + i\omega\Omega\,\mathrm{Im}\,\Delta\epsilon)$, which now does not vanish in the limit $\Omega \to 0$. As a consequence, more severe renormalizations of the diffusion coefficient D and a finite intensity correlation length ξ_a are induced, as seen in Sec. III.

III. SELF-CONSISTENT SOLUTION

As long as wave interference effects are not taken into account, the self-energy and the irreducible vertex may be evaluated within a local approximation, like the independent scatterer approximation employed in Ref. 21 or modifications of the coherent potential approximation (CPA).[22] In this case, these quantities become q independent, $\Sigma_{\vec{q}}^\omega \to \Sigma_0^\omega$, $\gamma_{\vec{q}'\vec{q}}^\omega(\vec{Q},\Omega) \to \gamma_0^\omega$, and Eq. (3) may be solved in a straightforward way[20] with the help of Eq. (4). However, to account for Anderson localization effects, the q dependence of $\gamma_{\vec{q}'\vec{q}}^\omega(\vec{Q},\Omega)$ is known to be essential.[14,15] To solve the kinetic equation(3) for this case, $\Phi_{\vec{q}\vec{q}'}^\omega$ is expanded[23] into its moments with respect to the longitudinal current vertex $(c_p\vec{p}\times\hat{Q})$, i.e., into the energy density correlator $P_E^\omega(\vec{Q},\Omega)$ and the energy current density correlator $J_E^\omega(\vec{Q},\Omega)$,[3,20]

$$P_E^\omega(\vec{Q},\Omega) = \left(\frac{\omega}{c_p}\right)^2 \int\frac{d^3q\,d^3q'}{(2\pi)^6}\Phi_{\vec{q}\vec{q}'}^\omega(\vec{Q},\Omega) \quad (5)$$

$$J_E^\omega(\vec{Q},\Omega) = \frac{\omega v_E}{c_p}\int\frac{dq\,dq'}{(2\pi)^6}(\vec{p}\times\hat{Q})\Phi_{\vec{q}\vec{q}'}^\omega(\vec{Q},\Omega). \quad (6)$$

In these definitions the phase velocity c_p of the disordered medium is defined via the dispersion obtained from the averaged single-particle Green's function, Eq. (2), as

$$c_p = \mathrm{Re}\,\frac{c}{\sqrt{\epsilon_b - \Sigma_0^\omega c^2/\omega^2}}, \quad (7)$$

and the transport velocity v_E will be given below. Inserting the expansion by moments into the kinetic equation (3) and employing the WI, one obtains the exact continuity equation and the current relaxation equation, respectively,

$$\Omega P_E^\omega + QJ_E^\omega = \frac{4\pi i\omega N(\omega)}{g_\omega^{(1)}[1+\Delta(\omega)]c_p^2} + \frac{i[g_\omega^{(0)} + \Lambda(\omega)]}{g_\omega^{(1)}[1+\Delta(\omega)]}P_E^\omega \quad (8)$$

$$\left[\omega\Omega\frac{\mathrm{Re}\,\epsilon_b}{c^2} + \frac{i}{c^2\tau^2} + iM(\Omega)\right]J_E^\omega + \tilde{A}QP_E^\omega = 0. \quad (9)$$

The expression for the transport velocity is now determined by the condition that the density and the current correlators must appear on the left-hand side of the continuity equation (8) without additional prefactors. One obtains[3]

$$v_E = \frac{(c/\mathrm{Re}\,\epsilon_b)(c/c_p)}{1+\Delta(\omega)}. \quad (10)$$

The expressions for the coefficients $\tilde{A}$, Δ, Λ, $g_\omega^{(0)}$, $g_\omega^{(1)}$ follow in analogy to Ref. 20,

$$\tilde{A} = \frac{c_p v_E}{\omega}\left[1 - \frac{\omega^2\,\mathrm{Im}\,\epsilon_b}{u_\epsilon\,\mathrm{Im}\,G_0} + \frac{r_\epsilon A_\epsilon}{u_\epsilon\,\mathrm{Im}\,G_0}\right],$$

$$\Delta(\omega) = B_\epsilon A_\epsilon + i r_\epsilon \partial_\Omega A_\epsilon(\Omega),$$

$$\Lambda(\omega) = i\omega^2\,\mathrm{Im}\,\epsilon_b - i r_\epsilon A_\epsilon,$$

$$g_\omega^{(0)} = \frac{2\omega}{c^2}\,\mathrm{Im}\,\epsilon_b,$$

$$g_\omega^{(1)} = \frac{4\omega}{c^2}\,\mathrm{Re}\,\epsilon_b.$$

They enter into the transport quantities given explicitly below. Equation (8) represents the local energy conservation with corrections due to absorption or gain as well as reactive corrections. Equation (9) describes the time evolution of the current density (J_E^ω) induced by a density distribution (P_E^ω) due to momentum relaxation and the memory kernel $M(\Omega)$. $M(\Omega)$ is a functional of the irreducible two-wave vertex $\gamma_{\vec{q}'\vec{q}}^\omega(Q,\Omega)$. Its characteristic Ω dependence describes memory effects due to the diffusion and interference as well as amplification or absorption of waves in the medium; see below. Equations (8) and (9) are easily solved to obtain the diffusion pole form for the density correlator,

$$P_E^\omega(Q,\Omega) = \frac{4\pi i N(\omega)/(g_\omega^{(1)}[1 + \Delta(\omega)]c_p^2)}{\Omega + iQ^2 D + i\xi_a^{-2}D}, \quad (11)$$

with the Ω-dependent diffusion coefficient $D(\Omega)$,

$$D(\Omega)[1 - i\Omega\omega\tau^2\,\mathrm{Re}\,\epsilon_b] = D_0^{tot} - c^2\tau^2 D(\Omega)M(\omega), \quad (12)$$

and the absorption or gain induced correlation length ξ_a of the diffusive modes,

$$\xi_a^{-2} = \frac{r_\epsilon A_\epsilon - 2\omega^2\,\mathrm{Im}\,\epsilon_b}{2\,\mathrm{Re}\,\epsilon_b - A_\epsilon B_\epsilon/\omega}\frac{1}{\omega D(\Omega)}. \quad (13)$$

The diffusion constant without memory effects, $D_0^{tot} = D_0 + D_b + D_s$, consists of the bare diffusion constant,[3]

$$D_0 = \frac{2v_E c_p}{\pi N(\omega)}\int\frac{\mathrm{d}^3k}{(2\pi)^3}[\vec{k}\times\hat{Q}]^2(\mathrm{Im}\,G_{\vec{k}}^\omega)^2, \quad (14)$$

and renormalizations from absorption or gain in the background medium (D_b) and in the scatterers (D_s),

$$D_b = \frac{1}{4}(\omega\tau)^2\mathrm{Im}\,\epsilon_b\tilde{D}_0 \quad (15)$$

$$D_s = \frac{1}{8}r_\epsilon A_\epsilon\tau^2\tilde{D}_0, \quad (16)$$

where $\tilde{D}_0$ is the same as in Eq. (14), with $(\mathrm{Im}\,G_{\vec{k}}^\omega)^2$ replaced by $(\mathrm{Re}\,G_{\vec{k}}^{\omega2})$. In Eqs. (13)–(16) the following short-hand notations have been introduced:

$$u_\epsilon = \frac{\mathrm{Im}(\Delta\epsilon\Sigma^\omega)}{\mathrm{Im}(\Delta\epsilon G_0^\omega)}, \quad r_\epsilon = \mathrm{Im}\,\Delta\epsilon/\mathrm{Re}\,\Delta\epsilon,$$

$$A_\epsilon = 2[u_\epsilon\,\mathrm{Re}\,G_o + \mathrm{Re}\,\Sigma_o],$$

$$B_\epsilon = \frac{(\mathrm{Re}\,\Delta\epsilon)^2 + (\mathrm{Im}\,\Delta\epsilon)^2}{2\omega^2(\mathrm{Re}\,\Delta\epsilon)^2}.$$

We have carefully analyzed $\gamma_{\vec{q}'\vec{q}}^\omega$ for the self-consistent calculation of $M(\Omega)$,[15,24] exploiting time-reversal symmetry of propagation in the active medium. In the long-time limit ($\Omega\to0$) the dominant contributions to $\gamma_{\vec{q}'\vec{q}}^\omega$ are the same maximally crossed diagrams (Cooperons) as for conserving media, however now acquiring the absorption (gain)-induced decay (growth) rate $\xi_a^{-2}D$. $M(\Omega)$ reads

$$M(\Omega) = -\frac{(2v_\mathrm{E}c_p)^2 u_\epsilon[2\pi\omega u_\epsilon N(\omega) + r_\epsilon A_\epsilon - 2\omega^2\,\mathrm{Im}\,\epsilon_b]}{\pi\omega N(\omega)D_0 D(\Omega)}$$

$$\times\int\frac{\mathrm{d}^3q}{(2\pi)^3}\int\frac{\mathrm{d}^3q'}{(2\pi)^3}$$

$$\times\frac{[\vec{q}\times\hat{Q}]|\mathrm{Im}\,G_q|(\mathrm{Im}\,G_{q'})^2[\vec{q}'\times\hat{Q}]}{\dfrac{-i\Omega}{D(\Omega)} + (\vec{q}+\vec{q}')^2 + \xi_a^{-2}}. \quad (17)$$

Equations (12)–(17) constitute the self-consistency equations for the diffusion coefficient $D(\Omega)$ and the correlation length ξ_a in the presence of absorption or gain. We have evaluated them below, employing the independent scatterer approximation of Ref. 21 for the single-particle quantities. More sophisticated approximation schemes, suitable for high scatterer concentrations, like the coherent potential approximation (CPA),[22] will be considered elsewhere.

IV. DISCUSSION

Let us first discuss the kernel $M(\Omega)$. It describes the enhanced backscattering. For a conserving medium ($\mathrm{Im}\,\epsilon_b = \mathrm{Im}\,\epsilon_s = 0$), where $\xi_a^{-2} = 0$, it drives the AL transition due to its negative, infrared divergent contribution.[3,15] For illustration we show in Fig. 1 the AL phase diagram and $D(\Omega=0)$ for $\epsilon_b = 1$ and various real values of ϵ_s, displaying strong suppression near the Mie resonances, albeit for rather high dielectric contrast $\Delta\epsilon$. Introducing now absorption or gain, the quadratic inverse correlation length ξ_a^{-2} becomes nonzero and can assume both positive and negative values. True AL due to repeated enhanced backscattering is no longer possible in this case, because, through self-consistency, a vanishing $D(0)$ would drive $\xi_a^{-2}\to\infty$ and $D(0)M(0)\to0$. Inspection of $M(\Omega)$ shows analytically that, to leading order in ξ_a^{-2}, the real part of the integral in Eq. (17) is symmetrical with respect to $\xi_a^{-2}\leftrightarrow-\xi_a^{-2}$, i.e., AL is suppressed by absorption or gain in a symmetrical way. This is in agreement with the surprising absorption-gain duality of Ref. 16, which is valid for small gain and short times. However, we do find important deviations from this result for systems where there is no symmetry between background
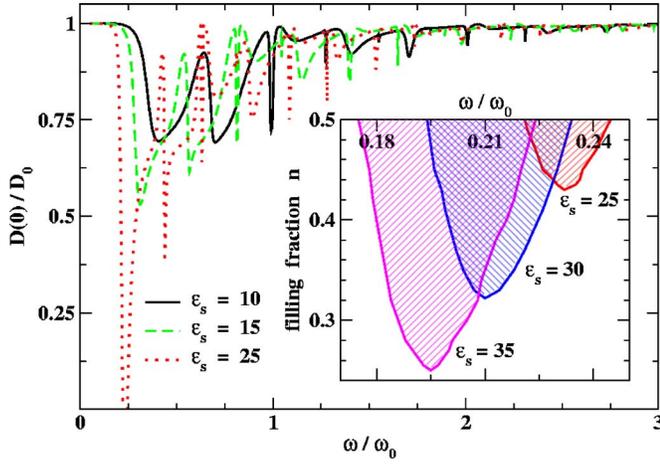
FIG. 1. (Color online) Normalized diffusion constant $D(0)$ for $\epsilon_b=1$ and scatterer volume fraction $\nu=50\%$. The frequency unit in all figures is $\omega_0=2\pi c/r_0$, with r_0 the scatterer radius. The localization transition is reached for $\epsilon_s \gtrsim 24$. Inset: phase diagram in the (ν,ω) plane within the first Mie resonance for $\epsilon_b=1$ and various ϵ_s. Shaded areas: localized phases.

and scattering medium (like in our $d=3$ case): (1) As seen from Eq. (13), ξ_a^{-2} itself is not symmetric in the sign of Im ϵ_b or Im ϵ_s and can be positive even for purely emissive media, Im $\epsilon_b<0$, Im $\epsilon_s<0$. (2) The full diffusion coefficient has additional contributions D_b, D_s from loss or gain in the background and in the scatterers, Eqs. (15), which to our knowledge have not been reported before. While D_b is always positive for absorption and negative for gain as expected, D_s has a complicated dependence on the signs of Im ϵ_b, Im ϵ_s and depends sensitively on whether absorption and/or gain occurs in the background medium or in the scatterers. This is because, in contrast to D_b, the impurity scattering contribution D_s results from an intricate interplay between elastic momentum relaxation and absorption or gain processes. We emphasize that the existence and the form of D_b, D_s, and ξ_a^{-2} are a direct consequence of the nonconserving terms in the WI, Eq. (4), and, thus, are exact.

The complete scenario of localization effects is now as follows. Even though in the presence of absorption or gain there are no true Anderson localized modes because of the finite ξ_a^{-2}, the contributions D_b and D_s can strongly suppress the total diffusion coefficient $D(0)$. This is shown in Fig. 2 for a system of absorbing scatterers embedded in air. Even moderate absorption drastically decreases the diffusion constant $D(0)$ close to the low-order Mie resonances. At the same frequencies the correlation length ξ_a is suppressed even more dramatically. For the case of a purely absorbing system, ξ_a may be identified with the effective absorption length for diffusive modes. For the case of emission, e.g., in the background, two scenarios are possible. (i) $\xi_a^{-2}>0$; this may occur due to the subtle interplay with momentum relaxation processes and absorption in the scatterers. Both $D(0)$ and the intensity correlation length ξ_a are real and finite, but suppressed near the Mie resonances, as shown in Fig. 3. It is also seen that, e.g., absorption in the scatterers can be partially compensated by emission in the background. (ii) Re $\xi_a^{-2}<0$; this is realized for sufficiently strong gain. It

implies a pole on the real axis in the integration range of Eq. (17) and, hence, complex $D(0)$ and ξ_a. Fourier transforming Eq. (11), this means an exponential intensity growth for long times with rate $1/\tau_a=\mathrm{Re}[(\xi_a^{-2}+Q^2)D(0)]$, in qualitative agreement with Ref. 17, thus reconciling these long-time results with the weak gain or short-time results of Ref. 16. This growth is modulated by temporal and spatial oscillations with characteristic frequency $\Omega_D=-Q^2\,\mathrm{Im}\,D(0)$ and wave number $k_D=\mathrm{Im}\,1/\xi_a$, respectively. We interpret this oscillatory behavior as a memory effect, originating from the competition between the enhanced backscattering of waves and their amplified propagation in the surrounding medium. Self-induced oscillations have been found before in other driven systems with competing dynamics.

V. CONCLUSION

In conclusion, we have presented a semianalytical theory for the interplay of strong localization effects and absorption

FIG. 2. (Color online) (a) Diffusion constant $D(0)$ for $\nu=50\%$. The scatterers exhibit different amounts of absorption as indicated. (b) Absorption length ξ_a in units of the scatterer radius for the systems in (a).
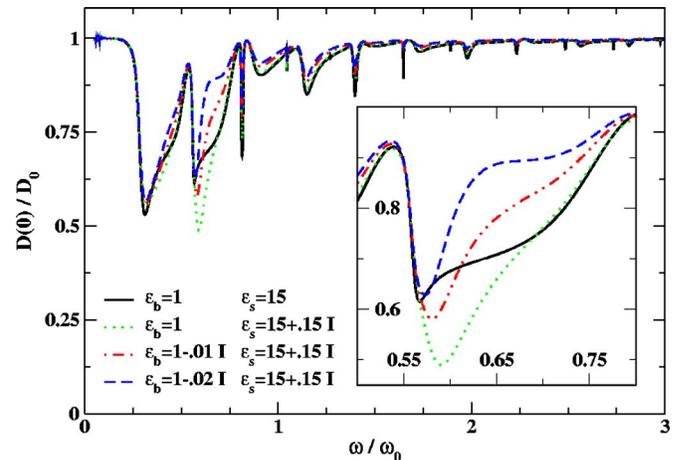
FIG. 3. (Color online) Diffusion constant $D(0)$ for absorbing scatterers ($\nu=50\%$) in a host medium with optical gain, as indicated. Parameters are chosen such that $\xi_a^2>0$.

or gain. True AL is not possible in the presence of either loss or gain. However, strong renormalizations of the diffusion constant D arise from the violation of the conservation laws. These renormalizations depend sensitively on whether absorption or gain occurs in the scatterers or the background. Intimately connected with the suppression of D is the appearance and reduction of a finite intensity correlation length ξ_a, even though there are no truly localized modes. ξ_a includes effects of both impurity scattering and loss or gain in the medium, and thus can be shorter than the scattering mean-free path ℓ. For example, in a pure medium with loss or gain ξ_a would characterize the absorption or gain length and would be finite, while ℓ would obviously be infinite. It should be emphasized that the present theory incorporates both the physics of multiple impurity scattering valid at length scales larger than ℓ (diffusion and Cooperon contributions) and the physics of coherent amplification present at all length scales. Therefore, it is expected to give an accurate estimate for ξ_a. We conjecture that in random lasers this finite length scale ξ_a might define the coherence volume necessary for resonant feedback that is observed experimentally[12] and that appears to be smaller than ℓ in those experiments. In order to substantiate this conjecture, the present localization theory should be coupled self-consistently to the laser rate equations, which will yield results for the position-dependent dielectric function above the lasing threshold (Im $\epsilon_{b,s} \neq 0$) and, hence, for the lasing mode volume. This will be the subject of further research. Oscillatory behavior in space and time is also predicted for sufficiently strong gain, although it is presumably difficult to observe, as it occurs only during the exponential intensity growth between the laser threshold and saturation.

[1] P. W. Anderson, Phys. Rev. **109**, 1492 (1958).

[2] S. John and M. J. Stephen, Phys. Rev. B **28**, 6358 (1983).

[3] J. Kroha, C. M. Soukoulis, and P. Wölfle, Phys. Rev. B **47**, 11093 (1993).

[4] Y. Kuga and J. Ishimaru, J. Opt. Soc. Am. B **1**, 831 (1984); M. P. Van Albada and A. Lagendijk, Phys. Rev. Lett. **55**, 2692 (1985); P. E. Wolf and G. Maret, Phys. Rev. Lett. **55**, 2696 (1985).

[5] M. P. van Albada, B. A. van Tiggelen, A. Lagendijk, and A. Tip, Phys. Rev. Lett. **66**, 3132 (1991).

[6] B. A. van Tiggelen, A. Lagendijk, M. P. van Albada, and A. Tip, Phys. Rev. B **45**, 12233 (1992).

[7] D. S. Wiersma, P. Bartolini, A. Lagendijk, and R. Righini, Nature (London) **390**, 671 (1997).

[8] F. Scheffold, R. Lenke, R. Tweer, and G. Maret, Nature (London) **398**, 206 (1999).

[9] D. S. Wiersma, J. Gomez Rivas, P. Bartolini, A. Lagendijk, and R. Righini, Nature (London) **398**, 207 (1999).

[10] H. Cao, Waves Random Media **13**, R1 (2003).

[11] H. Cao, J. Y. Xu, D. Z. Zhang, S. H. Chang, S. T. Ho, E. W. Seelig, X. Liu, and R. P. H. Chang, Phys. Rev. Lett. **84**, 5584 (2000).

[12] H. Cao, Y. Ling, J. Y. Xu, C. Q. Cao, and P. Kumar, Phys. Rev. Lett. **86**, 4524 (2001).

[13] V. M. Apalkov, M. E. Raikh, and B. Shapiro, Phys. Rev. Lett. **89**, 016802 (2002).

[14] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).

[15] D. Vollhardt and P. Wölfle, Phys. Rev. Lett. **45**, 842 (1980); Phys. Rev. B **22**, 4666 (1980).

[16] J. C. J. Paasschens, T. Sh. Misirpashaev, and C. W. J. Beenakker, Phys. Rev. B **54**, 11887 (1996).

[17] X. J. Jiang, Q. Li, and C. M. Soukoulis, Phys. Rev. B **59**, R9007 (1999).

[18] X. Jiang and C. M. Soukoulis, Phys. Rev. E **65**, 025601(R) (2002); A. Yamilov and H. Cao, Phys. Rev. E **70**, 037603 (2004).

[19] A. Yamilov, S. Chang, A. Burin, A. Taflove, and H. Cao, Phys. Rev. B **71**, 092201 (2005); A. Yamilov, X. Wu, H. Cao, and A. L. Burin, Opt. Lett. **30**, 2430 (2005).

[20] A. Lubatsch, J. Kroha, and K. Busch, Phys. Rev. B **71**, 184201 (2005).

[21] B. A. van Tiggelen, A. Lagendijk, and A. Tip, Phys. Rev. Lett. **71**, 1284 (1993).

[22] K. Busch and C. M. Soukoulis, Phys. Rev. Lett. **75**, 3442 (1995); Phys. Rev. B **54**, 893 (1996).

[23] T. Kopp, J. Phys. C **17**, 1897 (1984); **17**, 1918 (1984).

[24] J. Kroha, Physica A **167**, 231 (1990).