Magnetotransport along a barrier: Multiple quantum interference of edge states

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Transport in a two-dimensional (2D) electron gas subject to an external magnetic field is analyzed in the presence of a *longitudinal barrier*. We show that *quantum interference of the edge states* bound by the longitudinal barrier results in a drastic change of the electron motion: the degenerate discrete Landau levels are transformed into an alternating sequence of energy bands and energy gaps. These features of the electron spectrum should result in a high sensitivity of thermodynamic and transport properties of the 2D electron gas to external fields. In particular, we predict giant oscillations of the ballistic conductance and discuss nonlinear current-voltage characteristics, coherent Bloch oscillations, and effects of impurities.

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I. INTRODUCTION

Great attention has been attracted during the last decades to study of transport properties of various mesoscopic systems, e.g., tunnel junctions, quantum dots, etc.[1,2] Such systems display fascinating quantum-mechanical behavior on a macroscopic scale, which results in quantization of the conductance,[1,3] Coulomb blockade,[2] weak localization,[1,2] mesoscopic conductance fluctuations,[4] macroscopic quantum tunneling,[5] etc. All quantum-mechanical effects are enhanced in low-dimensional systems, such as a two-dimensional electron gas (2DEG), quasi-one-dimensional quantum wires, systems of coupled small tunnel junctions. Moreover, since the application of an external magnetic field allows one to transform the continuous spectrum of electrons to discrete Landau levels (in a two-dimensional electron gas), various quantum-mechanical effects, such as Shubnikov–de Haas oscillations,[1] integer and fractional quantum Hall effects,[1,2] etc. have been observed in magnetotransport measurements in these systems.

It is clear that if a potential barrier is placed across the direction of the electron motion, the current would flow only due to tunneling through the barrier. Such tunneling under Hall effect conditions was studied theoretically[6] and experimentally in papers[7,8] where the energy gaps were discovered in the spectrum of the edge states. However, what can happen if the barrier is created along the direction of the current? Of course, the problem is not very interesting in the absence of a magnetic field but the situation drastically changes if a magnetic field is applied perpendicularly to the plane of the 2DEG.

In this paper, we show that the quantum interference of the edge states qualitatively changes the dynamics of electrons moving along the barrier resulting in extreme sensitivity of the electron "longitudinal" kinetics to weak external fields. We predict giant oscillations of the conductance in the direction parallel to the barrier with a change of the Fermi level under a weak gate voltage, strong nonlinear current-voltage characteristics, coherent Bloch oscillations under weak electric fields, and specific quantum traps for electrons in the presence of a weak smooth scattering potential.

To be specific, we consider 2DEG subject to an external magnetic field H. We assume that a potential barrier of a narrow width separates the systems into two parts (left and right). What is important? The barrier should be penetrable such that electrons can tunnel from one part of the system to the other. The tunneling through the barrier can generally be characterized by a reflection amplitude r that can vary from zero to one. A schematic setup is shown in Fig. 1(a). Here, we would like to emphasize that such a setup is realistic and similar systems have been produced by using a split-gate technique or cleaved edge fabrication method.[9]

We start our discussion with a qualitative analysis of the energy spectrum of the electrons. Effects of the external magnetic field are considered in the Landau gauge, i.e., the vector-potential $\mathbf{A}=(-Hy,0,0)$, where the axis y is perpendicular to the barrier. In this gauge, the component p_x of the momentum conserves even in the presence of the longitudinal barrier along the x axis.

In the limit of a completely transparent barrier, $r=0$, all states are just Landau levels (the size of the system in the y direction is assumed to be large), and the electron spectrum is $\epsilon_n(p_x)=\hbar\omega_c(n+1/2)$, where $\omega_c=eH/mc$ is the cyclotron frequency and m is the electron mass. Such a spectrum is shown by dashed lines in Fig. 1(b). In this limit, the energy spectrum is independent of p_x.

In the opposite limit of the zero barrier transparency, $r=1$, the electron motion near the barrier considerably changes and can be described in terms of independent *edge states* in the left and right parts of the system [see Fig. 1(a)]. In this case, the degeneracy of the Landau levels is lifted and the spectrum of the edge states near the barrier depends on p_x. The corresponding spectrum is represented in Fig. 1(b) by solid lines. A peculiar property of such a spectrum is the presence of "crossing points," the number of which grows with an increase of the quasiclassical parameter $\alpha=\epsilon_F/(\hbar\omega_c)$, where ϵ_F is the Fermi energy of electrons in the absence of magnetic field. Indeed, "the distance" between the neighboring crossing points is $\delta p_x\simeq\hbar/R_c$ with $R_c=cp_F/(eH)$ being the cyclotron radius of electron orbits. As the momentum p_x is restricted by the Fermi momentum p_F, the number of the crossing points is determined by $p_F/\delta p_x=\alpha\gg 1$.

The case of a finite barrier transparency $(0<r<1)$ is of the most interest. In this case, the *quantum interference* be-

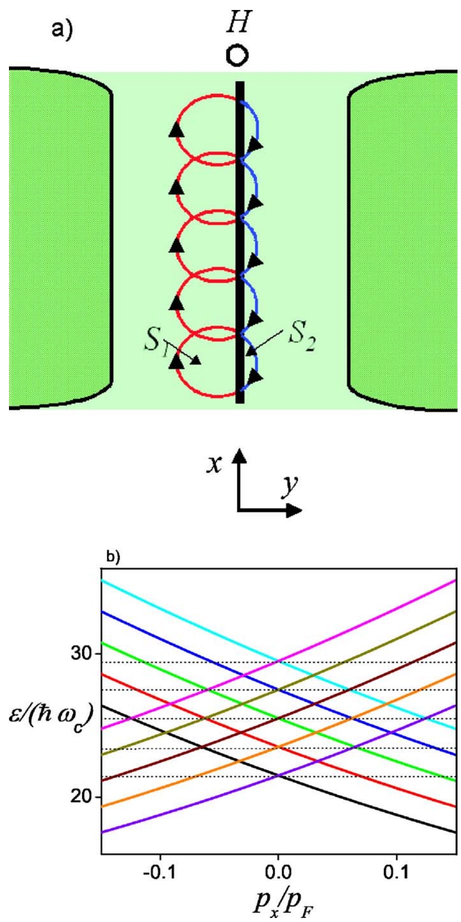

FIG. 1. (Color online) (a) Schematic of a two-dimensional electron gas in an external magnetic field H applied perpendicular to the electron gas confinement plane. The longitudinal barrier together with two edge states are shown. S_1 and S_2 are the areas of left and right edge states. (b) A part of the spectrum in the case of a zero barrier transmission, $r=1$. Dashed lines are degenerate Landau levels, i.e., the spectrum of the system at $r=0$. We use the quasiclassical parameter $\alpha=25$.

tween the edge states lifts the degeneracy in the crossing points, and narrow "energy bands" and "energy gaps" appear in the electron spectrum $\epsilon_n(p_x)$. For $r\sim 1$, the widths of the energy bands $\Delta\epsilon$ and the energy gaps Δ_g are of the order $\hbar\omega_c \ll \epsilon_F$. Note that the electron states in the bands are delocalized, and thus, electrons move along the barrier with the velocity $v=d\epsilon_n(p_x)/dp_x \sim \Delta\epsilon/\delta p_x \sim v_F$ (n is the band number).

II. DYNAMICS OF ELECTRONS BOUND TO A LONGITUDINAL JUNCTION: QUASI-CLASSICAL DESCRIPTION

In this section, we analyze quantitatively wave functions and the spectrum of the system. For this purpose, we solve the Schrödinger equation for a two-dimensional electron gas [in the plane (x,y)] in the presence of a magnetic field H and of the barrier described by a potential $V(y)$

$$\left\{\frac{1}{2m}\left(p_x+\frac{eHy}{c}\right)^2-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2}+V(y)-\epsilon\right\}\Psi(x,y)=0. \quad (1)$$

While writing Eq. (1), we used the Landau gauge [$\mathbf{A}=(-Hy,0,0)$] in which the component p_x of the electron momentum conserves in the presence of both the magnetic field and the potential $V(y)$, and the electron wave function is given by

$$\Psi(x,y)=e^{ip_x x}\psi(y). \quad (2)$$

For simplicity sake, we assume that the characteristic width of the barrier l_b is much smaller than the cyclotron radius, $l_b \ll R_c$.

In the absence of the magnetic field, the scattering of electrons by the potential barrier is described by a 2×2 scattering matrix

$$\hat{\rho}=ie^{i\varphi}\begin{pmatrix}|r|e^{-i\chi} & t\\ -t^* & |r|e^{i\chi}\end{pmatrix}; \quad |r|^2+|t|^2=1, \quad (3)$$

where r and t are the probability amplitudes for an incident electron to be reflected back and to be transmitted through the barrier; φ and χ are the scattering phases. Solving Eq. (1) in the quasi-classical approximation at distances $|y|\gg l_b$, where the potential barrier is negligibly small, and matching the wave functions of the electron on the left ($y<0$) and the right ($y>0$) sides of the barrier with the help of the scattering matrix, Eq. (3), we come to the following dispersion equation (see all details of this analysis in Appendix A).[10]

$$D\equiv\cos[\Phi_+(\varepsilon)+\varphi]-|r|\cos[\Phi_-(\varepsilon,p_x)+\chi]=0, \quad (4)$$

where the phases $\Phi_\pm$ are expressed as $\Phi_\pm=\frac{\pi HS_\pm}{2\phi_0}$, $S_\pm=S_1\pm S_2$, and $S_{1,2}$ are two areas bounded by the electron orbits [see Fig. 1(a)]; $\phi_0=hc/2e$ is the flux quantum. Although Eq. (4) is valid for an arbitrary dispersion relation of electrons, it becomes an extremely transparent for the parabolic spectrum of electrons: the complete orbit is a circle with the radius $R_c=c\sqrt{2m\varepsilon}/(eH)$ and the centrum shifted on the distance cp_x/eH along the y axis [see Fig. 1(a)].

The spectrum $\varepsilon_n(p_x)$ obtained from Eq. (4) depends on both the electron momentum p_x and a discrete quantum number n (the band number). It displays gaps and bands with an *almost periodic dependence* in a wide region of p_x [see Fig. 2(a)]. In this sense, it resembles the energy spectrum of electrons in semiconducting superlattices. However, in our case the spectrum can be tuned by an external magnetic field and/or the reflection coefficient r controlled by the gate voltage. Moreover, the energy levels ε_n for a fixed value of p_x are distributed in a pseudorandom way. The typical distribution of energy levels is presented in Fig. 2(b).

Using Eq. (4), we calculate the electron density of states (DOS) $\nu(\varepsilon)$ and the ballistic conductance $G(\varepsilon_F)$. We consider only the contribution of the states bound to the longitudinal barrier. If the energy ε is located between the Landau levels in the bulk, another well-known contribution comes from the edge states on the boundaries of the sample. However, this contribution is a smooth function of the energy and is not interesting for us.

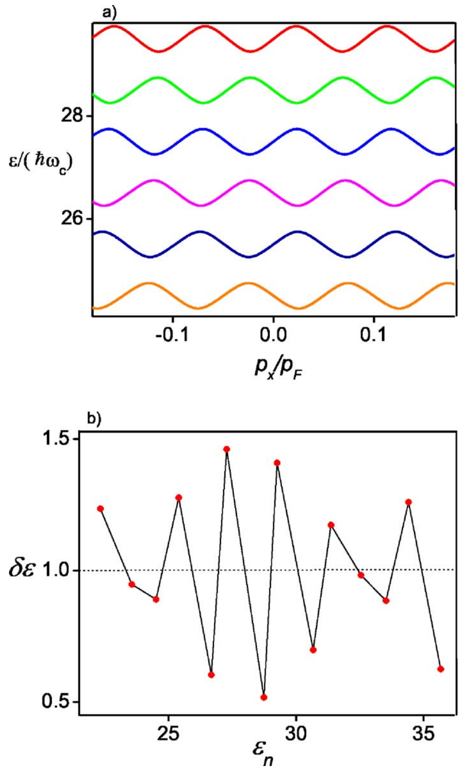

FIG. 2. (Color online) (a) A part of the spectrum in the case of an intermediate barrier transmission, $r=0.7$. The phases φ and χ are obtained for the model of the δ-function barrier, $V(y)=V\delta(y)$. (b) The dependence of the energy level difference $\delta\varepsilon=\varepsilon_{n+1}-\varepsilon_n$ on the energy ε_n (energy-level distribution). We use the quasi-classical parameter $\alpha=25$, and a particular value of $p_x/p_F=0.3$. The points are connected by a thin line just for clarity.

In order to explicitly calculate the density of states, we use the approach developed by Slutskin for analogous spectra of electrons under the magnetic break-down.[11]

Using the identity

$$\sum_n \delta(\epsilon-\epsilon_n)=|\partial D/\partial\epsilon|\delta(D), \tag{5}$$

the electron DOS

$$\nu(\varepsilon)=\frac{1}{2R_c}\int\frac{dp_x}{2\pi\hbar}\sum_n \delta[\varepsilon-\varepsilon_n(p_x)]$$

can be rewritten in the form

$$\nu(\varepsilon)=\frac{1}{2R_c}\int\frac{dp_x}{2\pi\hbar}\left|\frac{\partial D}{\partial\varepsilon}\right|\delta(D). \tag{6}$$

Substituting (4) into (6) and expanding the integrand of it into the Fourier series in Φ_-, one finds DOS to be

$$\nu(\varepsilon)=\frac{1}{2R_c}\sum_{l=-\infty}^{+\infty}\int\frac{dp_x}{2\pi\hbar}C_l(\varepsilon,p_x)e^{il\Phi_-(\varepsilon,p_x)}, \tag{7}$$

where the Fourier coefficients are

$$C_l=\int_{-\pi}^{\pi}\frac{d\omega}{2\pi}|\Phi_+'\sin(\Phi_++\varphi)-|r|\Phi_-'\sin\omega| \times\delta(\cos(\Phi_++\varphi)-|r|\cos\omega)e^{-il\omega}. \tag{8}$$

Here,

$$\Phi_+'\equiv\frac{d}{d\varepsilon}\Phi_+(\varepsilon);\quad \Phi_-'\equiv\frac{\partial}{\partial\varepsilon}\Phi_-(\varepsilon,p_x).$$

Since at $\epsilon\sim\epsilon_F$, one has $\Phi_-(\varepsilon,p_x)\sim\alpha\gg 1$, the exponents in (7) are fast oscillating functions of p_x (on the scale of $\hbar/R_c\ll p_F$). At the same time, the Fourier coefficients $C_l(\varepsilon,p_x)$ are smooth functions of p_x [see Eq. (8)], and hence, the term with $l=0$ in (7) gives the main contribution to the density of states, i.e.,

$$\nu(\varepsilon)=\frac{1}{2R_c}\int\frac{dp_x}{2\pi\hbar}C_0(\varepsilon,p_x). \tag{9}$$

Carrying out integration in (8) at $l=0$ and using (9), we obtain

$$\nu(\varepsilon)=\frac{1}{8\pi^2\hbar R}\frac{\theta(|r|^2-\cos^2\tilde{\Phi}_+)}{\sqrt{|r|^2-\cos^2\tilde{\Phi}_+}}\int\frac{dp_x}{2\pi\hbar} \times\sum_{n=1}^{2}|T_+\sin\tilde{\Phi}_++(-1)^nT_-\sqrt{|r|^2-\cos^2\tilde{\Phi}_+}|, \tag{10}$$

where $\tilde{\Phi}_+=\Phi_++\varphi$ and $T_\pm=2\pi\hbar\partial\Phi_\pm/\partial\epsilon$; $\theta(x)$ is the step function. Because of the inequality $\cos^2\tilde{\Phi}_+\leqslant|r|^2$, the second term in (10) drops out from the sum, and finally, the density of states for electrons bound to the longitudinal junction is expressed in the form (here and below, we drop the tilde sign):

$$\nu(\varepsilon)=\frac{m}{(\pi\hbar)^2}\frac{|\sin\Phi_+(\varepsilon)|}{\sqrt{|r|^2-\cos^2\Phi_+(\varepsilon)}}\theta[|r|^2-\cos^2\Phi_+(\varepsilon),], \tag{11}$$

where $\Phi_+(\varepsilon)=2\pi\varepsilon/(\hbar\omega_c)+\varphi$. Therefore, there are "gaps" in the DOS which can be found from the condition $\cos^2(2\pi\varepsilon/\hbar\omega_c+\varphi)>|r|^2$.

Such a dramatic transformation of the electron spectrum has to lead to various effects in transport properties of 2DEG. As an example, we analyze next the ballistic transport along the x direction.

We use the standard Landauer-Büttiker approach, based on the relationship between the conductance and the transmission probability in propagating channels,[2] which permits one to write the linear conductance as follows:

$$G=2e^2\frac{1}{kT}\sum_n\int\frac{dp_x}{2\pi\hbar}|v(\varepsilon_n,p_x)|\cosh^{-2}\frac{\varepsilon_n(p_x)-\varepsilon_F}{2kT}. \tag{12}$$

We rewrite it in the form

$$G = 2e^2 \frac{1}{kT} \int d\varepsilon \cosh^{-2} \frac{\varepsilon - \varepsilon_F}{2kT} \times \int \frac{dp_x}{2\pi\hbar} |v(\varepsilon,p_x)| \sum_n \delta[\varepsilon - \varepsilon_n(p_x)]. \qquad (13)$$

Here, it is convenient to write the velocity of electrons along the junction as

$$v(\varepsilon_n,p_x) = \frac{\partial \varepsilon_n}{\partial p_x} = -\left.\frac{\partial D/\partial p_x}{\partial D/\partial \varepsilon}\right|_{\varepsilon=\varepsilon_n}. \qquad (14)$$

Substituting it into (13) and using (4), one obtains

$$G = 2e^2 \frac{|r|}{kT} \int d\varepsilon \cosh^{-2} \frac{\varepsilon - \varepsilon_F}{2kT} \int \frac{dp_x}{2\pi\hbar} \left| \frac{\partial \Phi_-}{\partial p_x} \right| \times |\sin \Phi_-| \delta[\cos(\Phi_+) - |r| \cos(\Phi_-)] \qquad (15)$$

Similarly to the calculation of the density of states, expanding the integrand in Eq. (15) into the Fourier series in Φ_- and neglecting the fast oscillating terms in the integral with respect to p_x, we obtain

$$G = 2e^2 \frac{r}{kT} \int d\varepsilon \cosh^{-2} \frac{\varepsilon - \varepsilon_F}{2kT} \int \frac{dp_x}{2\pi\hbar} \left| \frac{\partial \Phi_-}{\partial p_x} \right| F_0(\varepsilon,p_x), \qquad (16)$$

where

$$F_0 = \int_{-\pi}^{\pi} \frac{d\omega}{2\pi} |\sin \omega| \delta(\cos \Phi_+ - |r| \cos \omega) \qquad (17)$$

Calculating the integrals in (16) and (17), the dependence of the linear conductance on the Fermi energy ε_F is expressed in the form

$$G = \frac{16e^2}{h} \frac{\varepsilon_F}{\hbar\omega_c} \sum_n \left(\tanh \frac{\varepsilon_n^{(t)} - \varepsilon_F}{2kT} - \tanh \frac{\varepsilon_n^{(b)} - \varepsilon_F}{2kT} \right), \quad (18)$$

where $\varepsilon_n^{(t)} = (n\pi + \pi - \arccos r)\hbar\omega_c/2$ and $\varepsilon_n^{(b)} = (n\pi + \arccos r)\hbar\omega_c/2$ are the top and the bottom of the nth energy band, respectively.

The conductance G [see, Eq. (18)] becomes very sensitive to the Fermi energy ε_F. Experimentally, this energy can be tuned by applying a gate voltage. The typical dependence of G on the gate voltage displaying *giant oscillations of the conductance* is shown in Fig. 3. These oscillations of the conductance reflect the presence of the bands and gaps in the spectrum of the edge states, thus proving the quantum interference of the edge states. The oscillations are smeared by temperature (compare the curves in Fig. 3).

The oscillations found here resemble those predicted[12] and observed[13] in the conductance of a junction between a superconductor and a two-dimensional electron gas. However, in that case the quantum interference between hole and electron edge states occurred due to Andreev reflection on the boundary.

As the main features of the electron motion under consideration are due to the quantum interference, thermodynamics and transport properties of the system are extremely sensitive to the influence of weak external fields.[11] Thus, in the ballistic regime, an increase of the transport voltage should lead to "giant steps" in the current-voltage characteristics (CVC). The width of the voltage steps is determined by the width of the gaps in the electron spectrum, i.e., $\Delta V \simeq \Delta_g / e$.

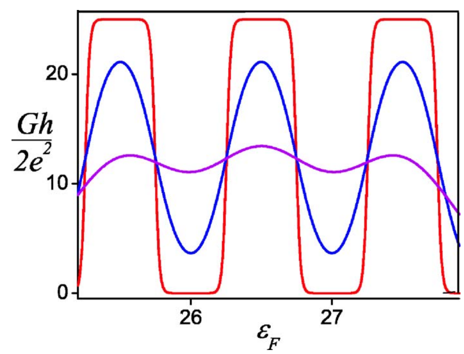

FIG. 3. (Color online) The giant oscillations of the conductance as a function of the Fermi energy (the gate voltage). The influence of temperature is shown: $k_B T_1 = 0.03(\hbar\omega_c)$, $k_B T_2 = 0.2(\hbar\omega_c)$, and $k_B T_3 = 0.5(\hbar\omega_c)$. We use the quasi-classical parameter $\alpha = 25$ and the reflection amplitude $r = 0.7$.

III. DIFFUSIVE TRANSPORT ALONG LONGITUDINAL JUNCTION

In order to analyze the transport properties of electrons in the presence of impurities, we start with the equation for the density matrix $\hat{\rho}$ in the τ approximation

$$\frac{i}{\hbar}[\hat{\rho},\hat{H}] - \frac{i}{\hbar}[\hat{\rho},e\mathcal{E}x] + \frac{\hat{\rho} - f_0(\hat{H})}{\tau} = 0. \qquad (19)$$

Here, $\hat{H}$ is the Hamiltonian corresponding to the Schrödinger equation (1), f_0 is the Fermi distribution function, $\mathcal{E}$ is the electric field along the junction, and τ is the electron scattering time. Writing the density matrix in the form $\hat{\rho} = f_0(\hat{H}) + \hat{\rho}^{(1)}$, we obtain

$$\frac{i}{\hbar}[\hat{\rho}^{(1)},\hat{H}] - \frac{i}{\hbar}[\hat{\rho}^{(1)},e\mathcal{E}x] + \frac{\hat{\rho}^{(1)}}{\tau} = -e\mathcal{E}\hat{v}\frac{df_0}{d\varepsilon}, \qquad (20)$$

where $\hat{v}$ is the quantum-mechanical operator of the electron velocity. In the quasi-classical limit and assuming that the characteristic width of the energy bands is much larger than the energy broadening caused by the impurities, i.e., $r \gg 1/(\omega_c \tau)$, the density matrix $\hat{\rho}^{(1)}$ reduces to the distribution function $\rho_n(p_x)$ satisfying the kinetic equation

$$\frac{d\rho_n}{dp_x} + \frac{\rho_n}{e\mathcal{E}\tau} = -v_n \left.\frac{\partial f_0}{\partial \epsilon}\right|_{\epsilon=\epsilon_n}. \qquad (21)$$

Here $v_n = \partial \epsilon_n / \partial p_x$ is the quantum-mechanical averaged electron velocity. The total current along the junction (which is carried by the electrons bound to the junction) is written as follows:

$$I = 2e\sum_n \int \frac{dp_x}{2\pi\hbar}\rho_n v_n. \tag{22}$$

Using Eqs. (21) and (22), we show that due to the quantum interference of the edge states, the nonlinear current-voltage characteristics for electron bound to a longitudinal junction under a magnetic field arises in relatively weak electric fields $\mathcal{E}\sim\hbar\omega_c/e\ell$, where ℓ is the electron mean free path.

A. Nonlinear response: Bloch oscillation regime

A crucial property of the obtained electron spectrum is oscillations of the electron energy $\varepsilon_n(p_x)$ as a function of the momentum along the junction p_x. Using an analogy with the electron transport in metals under magnetic breakdown,[14] a twinned plate,[15] and semiconducting superlattices,[16] one can expect *coherent Bloch oscillations*, and hence, an N-shaped nonlinear CVC under relatively weak electric fields.

We consider the Bloch oscillations under the condition that the electron mean free path ℓ satisfies an inequality $R_c<\ell<L$ (L is the length of the sample along the barrier). In the presence of an electric field $\mathcal{E}$ parallel to the barrier, the electron momentum changes in time $p_x(t)=p_x^{(0)}+e\mathcal{E}t$ ($p_x^{(0)}$ is the initial momentum), and hence, nearly periodic dependence of $\epsilon_n(p_x)$ results in *localization* of electrons along the x direction. The localization length can be estimated as $L_{\mathrm{loc}}=v_F\tau_{\mathrm{loc}}\sim v_F\delta p_x/(e\mathcal{E})\sim\hbar v_F/(e\mathcal{E}R_c)$. The localization takes place if $L_{\mathrm{loc}}<\ell$, which occurs in electric fields $\mathcal{E}>\hbar\omega_c/(e\ell)$.

The conductivity G is obtained as $\sigma=n_e e^2 u$, where n_e is the density of the charge carriers and the mobility u is determined by the Einstein relation $u=D/\varepsilon_F$ (D is the diffusion constant). In the case under consideration, the localized electrons diffusively move along the barrier over the distance L_{loc} during the electron scattering time τ due to collisions with impurities, and hence $D\sim L_{\mathrm{loc}}^2/\tau$. Therefore, the current carried by the localized electrons is

$$I = BR_c\sigma_0\left(\frac{\hbar}{\tau e\mathcal{E}R_c}\right)^2 \quad \mathcal{E} = BR\frac{\sigma_0}{\mathcal{E}}\left(\frac{\hbar}{e\tau R_c}\right)^2 \tag{23}$$

where B is a constant of order unity and σ_0 is the Drude conductivity at $H=0$. Since a nearly periodic dependence of $\epsilon_n(p_x)$ takes place for $p_x<p_F$, the CVC with the negative differential resistance [Eq. (23)] realizes in a wide region of electric field $\hbar\omega_c/\ell<\mathcal{E}<\epsilon_F/e\ell$.

Next we quantitatively analyze the nonlinear CVC of a junction in the whole range of electric fields following the procedure elaborated in.[17,18] This analysis includes both the linear and the Bloch oscillations regimes. First, the electron velocity [see Eq. (14)] is expressed as

$$v_n = |r|\frac{\partial\Phi_-}{\partial p_x}\left.\frac{\sin\Phi_-}{\Phi_+'\sin\Phi_+-|r|\Phi_-'\sin\Phi_-}\right|_{\epsilon=\epsilon_n} \tag{24}$$

under the condition $\cos\Phi_+-|r|\cos\Phi_-=0$. Here, $\Phi_{+(-)}'$ are slowly varying functions of p_x and ε.

Substituting (24) into (22) and using (5), one gets

$$I = 2e|r|\int\frac{dp_x}{2\pi\hbar}\int d\epsilon\frac{df_0}{d\epsilon}\frac{\partial\Phi_-}{\partial p_x}\sin\Phi_-(\epsilon,p_x) \times\rho(\vec{\Phi}(\epsilon,p_x),p_x)\frac{|\partial D/\partial\epsilon|}{\partial D/\partial\epsilon}\delta(D[\vec{\Phi}(\epsilon,p_x)]), \tag{25}$$

where $\vec{\Phi}=(\Phi_+,\Phi_-)$. Next, we express the distribution function as

$$\rho_n(p_x) = \rho[\vec{\Phi}(\epsilon,p_x),p_x]\frac{df_0}{d\varepsilon},$$

where ρ is a 2π-periodic function of $\vec{\Phi}$ and a slow varying function of the last argument with the characteristic interval $\delta p_x\sim p_F$. Such a dependence of the distribution function on p_x is justified by the fact that it follows the p_x dependence of the electron velocity v_n in Eq. (24), which forms the right-hand side of the kinetic equation Eq. (21). For systems displaying oscillating dependence $\varepsilon(p_x)$, it is convenient to rewrite the expression for the current in the following form (for the details of such a procedure, see Appendix B):

$$I = -\frac{e|r|^2}{2\pi}\int\frac{dp_x}{2\pi\hbar}\int d\epsilon\frac{\partial f_0}{\partial\varepsilon}\frac{\partial\Phi_-}{\partial p_x} \times\left\langle\frac{\sin\Phi_-}{\sqrt{1-r^2\cos^2\Phi_-}}\tilde{\rho}(\Phi_-)\right\rangle, \tag{26}$$

where $\tilde{\rho}(\Phi_-)=\rho(\Phi_+^{(2)},\Phi_-)-\rho(\Phi_+^{(1)},\Phi_-)$ and the brackets mean the averaging

$$\langle\dots\rangle = \frac{1}{2\pi}\int_0^{2\pi}\dots d\Phi_-.$$

Here, indexes 1(2) correspond to electrons moving with positive (negative) velocities along the junction [see Eq. (B4)].

Next, we rewrite the kinetic equation (21) as

$$\frac{\partial\Phi_-}{\partial p_x}\frac{d\rho^{(\alpha)}}{d\Phi_-}+\frac{\rho^{(\alpha)}}{e\mathcal{E}\tau} = -v^{(\alpha)}, \tag{27}$$

where superscript $\alpha=1,2$ marks two solutions $\Phi_+^{(1,2)}(\Phi_-)$ of the dispersion relationship $\cos\Phi_+-r\cos\Phi_-=0$. Here, we have neglected the partial derivative $\partial\rho/\partial p_x$ that is small in the quasi-classical limit.

Using Eq. (24), we write the equation for the distribution function $\tilde{\rho}(\Phi_-)$ in the following form:

$$\frac{d\tilde{\rho}}{d\Phi_-}+\frac{\tilde{\rho}}{e\mathcal{E}\gamma} = -w, \tag{28}$$

where $\gamma(p_x)=\tau\partial\Phi_-/\partial p_x\sim\tau R_c/\hbar$ and

$$w = \frac{v^{(2)}-v^{(1)}}{\partial\Phi_-/\partial p_x}.$$

The expression for w is written as

$$w = 2|r|\sin\Phi_-\left(\frac{S_+'\sqrt{1-|r|^2\cos 2\Phi_-}}{|t|^2S_+'^2+2|r|^2S_1'S_2'\sin 2\Phi_-}\right). \tag{29}$$

Expanding the distribution function in the Fourier series in Φ_- and using Eq. (28) one finds it to be

$$\rho(\Phi_-) = -\sum_{l=-\infty}^{+\infty} \frac{e\mathcal{E}\gamma}{1+(e\mathcal{E}\gamma l)^2}\langle w(\Phi)e^{-il\Phi}\rangle e^{il\Phi_-} \tag{30}$$

Inserting Eq. (30) into Eq. (26), we obtain the final expression for the current

$$I = \frac{2e|r|^2}{\pi}\int \frac{dp_x}{2\pi\hbar}\frac{\partial S_-}{\partial p_x}S'_+\sum_{l=1}^{\infty}\frac{e\mathcal{E}\gamma(p_x)}{1+[le\mathcal{E}\gamma(p_x)]^2} \times \left\langle \frac{\sin\Phi_-\sqrt{1-|r|^2\cos\Phi_-}e^{-il\Phi_-}}{|t|^2S_+'^2+2|r|^2S_1'S_2'\sin^2\Phi_-}\right\rangle\left\langle\frac{\sin\Phi_- e^{il\Phi_-}}{\sqrt{1-|r|^2\cos\Phi_-}}\right\rangle, \tag{31}$$

where all the quantities are taken at the Fermi energy.

An estimation of Eq. (31) shows that the total current I along the junction is of the order of

$$I \sim R_c\sigma_0\frac{\mathcal{E}}{1+(e\mathcal{E}\tau/\hbar\omega_c)^2}, \tag{32}$$

where $\sigma_0 = ne^2\tau/m$ is the conventional conductivity of the 2DEG in the absence of the magnetic field.

IV. DISCUSSION AND CONCLUSIONS

The most interesting consequence of the Eq. (31) is the nonlinear current-voltage characteristics (CVC) for the diffusive electron transport along the barrier. Indeed, the CVC displays a linear dependence at weak electric fields $e\mathcal{E}\ell \ll \hbar\omega_c$, and this behavior changes to $1/\mathcal{E}$ in electric fields $e\mathcal{E}\ell \gg \hbar\omega_c$. This limit corresponds to the Bloch oscillation regime. The current maximum reaches at electric fields $e\mathcal{E}\ell \sim \hbar\omega_c$. Note here that the crucial assumption for the appearance of the Bloch oscillation regime, and therefore the CVC with the negative differential resistance, is a small probability of Landau-Zener transitions between bands. Actually, we can neglect Landau-Zener transitions in rather wide range of electric fields, i.e., $e\mathcal{E} < \Delta/R_c$, where Δ is the characteristic value of the gap between bands in the spectrum of electrons.

Finally, we note that in the conventional situation (in the absence of the longitudinal barrier), a smooth (on the scale of the Fermi wave length $\lambda_F \sim \hbar/p_F$) random scattering potential $U(x,y)$ does not change quasi-classical transport properties of 2DEG and is not usually seen in experiments. In contrast, in the presence of the longitudinal barrier, the same smooth potential can qualitatively change the electron motion along the barrier. Arguments analogous to those presented for the Bloch oscillations lead to the conclusion that the electrons are *trapped* by this potential (or by an inhomogeneity of the magnetic field[18]), the size of the trap (that is the localization length) being $L_{\text{trap}} \sim (\Delta\varepsilon/U_0)L_{\text{rand}}$ (U_0 and L_{rand} are the characteristic value and the correlation radius of the random potential U, accordingly) as soon as $\Delta\varepsilon \ll U_0$. Estimations show that the giant oscillations (Fig. 3) can be observed if $U_0 < \Delta\varepsilon \sim \hbar\omega_c$. Actually, this is the condition for the observation of Schubnikov–de Haas oscillations and the integer quantum Hall effect.

In conclusion, we demonstrated that dynamics and kinetics of electrons moving along a longitudinal barrier in the presence of a magnetic field are extremely sensitive to external fields that results in a number unusual effects in a 2D electron gas. A crucial property of such a system is the quantum interference of electron edge states propagating along the barrier that gives rise to narrow energy bands and gaps in the electron spectrum. The spectrum is characterized by a nearly periodic dependence of $\varepsilon_n(p_x)$ (with the period $\sim\hbar/R_c \ll p_F$) in a wide region of p_x. The widths of the bands and the gaps can be tuned by the magnetic field and the gate voltage. Many interesting effects in the electron transport, such as giant oscillations of the ballistic conductance of 2DEG as a function of the gate voltage, nonlinear CVC, etc., are possible in such a system.

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APPENDIX A: DISPERSION EQUATION

As the Fermi energy ϵ_F is larger than $\hbar\omega_c$, we can use the quasi-classical approximation in order to calculate the wave functions and the spectrum of electrons. The quasi-classical wave functions of an electron at distances $|y| \gg l_b$ are written as (the indexes 1, 2 correspond to the left and right parts of the system, accordingly):

$$\psi_1(y) = \frac{C_1}{\sqrt{p(y)}}\left[\exp\left(\frac{i}{\hbar}\int_{y_-}^{y}p(y')dy' - \frac{i\pi}{4}\right) + \exp\left(-\frac{i}{\hbar}\int_{y_-}^{y}p(y')dy' + \frac{i\pi}{4}\right)\right] \tag{A1}$$

for $y<0$, and

$$\psi_2(y) = \frac{C_2}{\sqrt{p(y)}}\left[\exp\left(\frac{i}{\hbar}\int_{y}^{y_+}p(y')dy' - \frac{i\pi}{4}\right) + \exp\left(-\frac{i}{\hbar}\int_{y}^{y_+}p(y')dy' + \frac{i\pi}{4}\right)\right] \tag{A2}$$

for $y>0$. Here, we introduce the turning points $y_\pm = \frac{c}{eH}(-p_x \pm \sqrt{2m\epsilon})$, and the classical momentum $p(y)$ is

$$p(y) = \sqrt{2m\epsilon - \left(p_x + \frac{eH}{c}y\right)^2}. \tag{A3}$$

The constants $C_{1,2}$ are determined by the matching of the wave functions at the barrier and the normalization condition. Expanding the integrals in (A1) and (A2), one finds that in the barrier vicinity these wave functions are of the form of plane waves

$$\psi_1(y) = \frac{1}{\sqrt{p(0)}}(A_1e^{ip(0)y} + B_1e^{-ip(0)y}) \tag{A4}$$

$$\psi_2(y) = \frac{1}{\sqrt{p(0)}}(A_2 e^{-ip(0)y} + B_2 e^{-ip(0)y}), \tag{A5}$$

where the amplitudes of the incident $A_{1(2)}$ and reflected $B_{1(2)}$ plane waves

$$A_{1(2)} = C_{1(2)} e^{i(\Phi_{1(2)}-\pi/4)},$$

$$B_{1(2)} = C_{1(2)} e^{-i(\Phi_{1(2)}-\pi/4)}. \tag{A6}$$

Here, $p(0)=\sqrt{2m\epsilon-p_x^2}$ and

$$\Phi_{1(2)} = \pm\frac{1}{\hbar}\int_0^{y_\pm} p(y')dy' = \pm\frac{c}{eH\hbar}\int_{p_x}^{\pm p_\epsilon}\sqrt{2m\epsilon-p_x'^2}\,dp_x' \tag{A7}$$

are the quasi-classical phases, which the electron obtains moving between the turning points and the barrier in the magnetic field. The amplitudes $A_{1(2)}$ and $B_{1(2)}$ are found by using the scattering matrix (3) as

$$\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \hat{\rho}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix}. \tag{A8}$$

Using (3) and (A8), one finds the following set of matching equations:

$$B_1 - i|r|e^{i(\varphi-\chi)}A_1 - ite^{i\varphi}A_2 = 0$$

$$B_2 + it^* e^{i\varphi}A_1 - i|r|e^{i(\varphi+\chi)}A_2 = 0. \tag{A9}$$

Equations (A7) and (A9) are a set of linear homogeneous algebraic equations. Equating the determinant $D(\varepsilon,p_x)$ of it to zero we obtain the dispersion equation that determines the spectrum of the electrons bound to the longitudinal junction in the presence of magnetic field, i.e., Eq. (4).

APPENDIX B: NONLINEAR CVC

Since the integrand in Eq. (25) is a 2π-periodic function of Φ_+ and Φ_-, we expand it in a double Fourier series

$$\sin\Phi_-\rho(\vec{\Phi})\frac{|\partial D/\partial\epsilon|}{\partial D/\partial\epsilon}\delta(D(\vec{\Phi})) = \sum_{m=-\infty}^{\infty} A_{l,m}\exp\{il\Phi_+(\epsilon) + im\Phi_-(\epsilon,p_x)\}. \tag{B1}$$

Here, the Fourier coefficients $A_{l,m}(\epsilon,p_x)$ are smooth functions of ϵ and p_x (with the characteristic intervals of variations $\Delta\epsilon\sim\epsilon_F$ and $\Delta p\sim p_F$, respectively) while the exponents $\exp[il\Phi_+(\epsilon)]$ and $\exp[im\Phi_-(\epsilon,p_x)]$ are fast oscillating functions of these variables with the characteristic intervals $\delta\epsilon\sim\hbar\omega_c$ and $\delta p\sim\hbar/R_c$. Therefore, the zero harmonics give the main contribution to the integrals with respect to ϵ and p_x in Eq. (25), and in the limit of $kT\gg\hbar\omega_c$, we obtain the expression for the current as

$$j = 2e|r|\int\frac{dp_x}{2\pi\hbar}\int d\epsilon\frac{df_0}{d\epsilon}\frac{\partial\Phi_-}{\partial p_x}A_{0,0}(\epsilon,p_x), \tag{B2}$$

where

$$A_{0,0} = \int_0^{2\pi}\int_0^{2\pi}\frac{d\Phi_+ d\Phi_-}{(2\pi)^2}\sin\Phi_-\rho(\vec{\Phi})\frac{|\partial D/\partial\epsilon|}{\partial D/\partial\epsilon}\delta[D(\vec{\Phi})] \tag{B3}$$

Performing integration in (B3) with respect to Φ_+ and taking into account that the equation $D(\vec{\Phi})=\cos\Phi_+ - |r|\cos\Phi_-=0$ has two solutions,

$$\Phi_+^{(1,2)}(\Phi_-) = \pm\arccos(|r|\cos\Phi_-) + 2\pi l, \tag{B4}$$

one may write Eq. (B3) as follows:

$$A_{0,0} = \int_0^{2\pi}\frac{d\Phi_-}{(2\pi)^2}\sin\Phi_-[\rho(\Phi_+^{(1)},\Phi_-) - \rho(\Phi_+^{(2)},\Phi_-)]. \tag{B5}$$

Substituting (B5) into (B2), we obtain Eq. (26).

[1] Th. Heinzel, *Mesoscopic Electronics in Solid State Nanostructures* (Wiley, New York, 2003).

[2] Th. Dittrich, G.-L. Ingold, G. Schön, P. Hänggi, B. Kramer, and W. Zwerger, *Quantum Transport and Dissipation* (Wiley, New York, 1998).

[3] C. W. J. Beenakker and H. van Houten, in *Solid State Physics*, edited by H. Ehrenreich and D. Turnbull (Academic, New York, 1991), Vol. 44.

[4] J. A. Folk, S. R. Patel, S. F. Godijn, A. G. Huibers, S. M. Cronenwett, C. M. Marcus, K. Campman, and A. C. Gossard, Phys. Rev. Lett. **76**, 1699 (1996); A. M. Chang, H. U. Baranger, L. N. Pfeiffer, K. W. West, and T. Y. Chang, *ibid.* **76**, 1695 (1996).

[5] M. H. Devoret, J. M. Martinis, and J. Clarke, Phys. Rev. Lett. **55**, 1908 (1985); J. M. Martinis, M. H. Devoret, and J. Clarke, Phys. Rev. B **35**, 4682 (1987).

[6] Tin-Lun Ho, Phys. Rev. B **50**, 4524 (1994).

[7] N. F. Deutscher, J. M. Ryan, and D. K. Ferry, Semicond. Sci. Technol. **9**, 1354 (1994).

[8] W. Kang, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Nature (London) **403**, 59 (2000).

[9] C. C. Eugster, J. A. del Alamo, M. J. Rooks, and M. R. Melloch, Appl. Phys. Lett. **64**, 3157 (1994); O. M. Auslaender, A. Yacoby, R. de Picciotto, K. W. Baldwin, L. N. Pfeiffer, and K. W. West, Science **295**, 825 (2002).

[10] In the general case, if we do not use the quasi-classical condition, matching two independent solutions of the Schrödinger equation at the barrier leads to the dispersion equation in the following form: $V\sqrt{\frac{\hbar R_c}{p_F}}D_\varepsilon\left(p_x\sqrt{\frac{R_c}{\hbar p_F}}\right)D_\varepsilon\left(-p_x\sqrt{\frac{R_c}{\hbar p_F}}\right)=1$, where $D_\varepsilon(x)$ is the parabolic cylinder function and V is determined by the properties of the barrier.

[11] An analogous situation takes place in bulk metals under magnetic breakdown; see review paper, M. I. Kaganov and A. A. Slutskin, Phys. Rep. **98**, 189 (1983).

[12] H. Hoppe, U. Zülicke, and G. Schön, Phys. Rev. Lett. **84**, 1804 (2000).
[13] D. Uhlisch, S. G. Lachenmann, Th. Schäpers, A. I. Braginski, H. Lüth, J. Appenzeller, A. A. Golubov, and A. V. Ustinov, Phys. Rev. B **61**, 12463 (2000).
[14] A. A. Slutskin and A. M. Kadigrobov, JETP Lett. **28**, 201 (1978).
[15] A. M. Kadigrobov and I. V. Koshkin, Sov. J. Low Temp. Phys. **12**, 249 (1986).
[16] C. Waschke, H. G. Roskos, R. Schwedler, K. Leo, H. Kurz, and K. Köhler, Phys. Rev. Lett. **70**, 3319 (1993).
[17] A. A. Slutskin and A. M. Kadigrobov, JETP Lett. **32**, 338 (1980).
[18] A. A. Slutskin and A. M. Kadigrobov, JETP Lett. **8**, 17 (1968).