Role of disorder in the Mott-Hubbard transition

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We investigate the role of disorder in the Mott-Hubbard transition based on the slave-rotor representation of the Hubbard model, where an electron is decomposed into a fermionic spinon for a spin degree of freedom and a bosonic rotor (chargon) for a charge degree of freedom. In the absence of disorder, the Mott-Hubbard insulator is assumed to be the spin liquid Mott insulator in terms of gapless spinons near the Fermi surface and gapped chargons interacting via U(1) gauge fields. We found that the Mott-Hubbard critical point becomes unstable as soon as disorder is turned on. As a result, a disorder critical point appears to be identified with the spin liquid glass insulator to the Fermi liquid metal transition, where the spin liquid glass consists of the U(1) spin liquid and the chargon glass. We expect that glassy behaviors of charge fluctuations can be measured by the optical spectra in the insulating phase of an organic material $\kappa-(BEDT\text{-}TTF)_2Cu_2(CN)_3$. Furthermore, since the Mott-Anderson critical point depends on the spinon conductivity, universality in the critical exponents may not be found.

I. INTRODUCTION

The metal-insulator transition (MIT) is one of the most studied subjects in condensed matter physics. However, even the existence of the MIT is not convincingly proven at zero temperature in two spatial dimensions [(2+1)D], especially when both interaction and disorder coexist.[1] The Mott-Hubbard MIT has been claimed to occur at the critical interaction strength in the Hubbard model without disorder.[2] On the other hand, it is believed that the Anderson MIT does not arise in two spatial dimensions for the case of noninteracting electrons.[3] The common belief seems to be that the MIT in the presence of both interaction and disorder would not appear in (2+1)D because both interaction and disorder increase an insulating tendency. Recent experiments challenge this belief.[4] In a Si metal-oxide-semiconductor field-effect transistor, and other high mobility semiconductor devices, unexpected MITs have been reported although these transitions are questioned to be truly quantum phase transitions owing to the temperature ranges in these experiments.[5]

In the present paper, we investigate the role of disorder in the Mott-Hubbard MIT based on the Hubbard model. The main questions in this paper are (i) the nature of the insulating phase and (ii) the nature of the MIT, where the nonmagnetic insulating phase is assumed to be a spin liquid state with a Fermi surface in the absence of disorder. The core in these questions is the fate of gapped charge fluctuations in the Mott-Hubbard insulator when disorder is introduced.

Using the slave-rotor representation of the Hubbard model,[6] we obtain an effective field theory for the MIT in the presence of disorder, given by a U(1) gauge theory in terms of fermionic spinons and bosonic collective excitations interacting via U(1) gauge fields.[7,8] Disorder couples to charge fluctuations, and affects their dynamics severely. Using a renormalization-group (RG) analysis, we argue that the Mott-Hubbard MIT turns into the Mott-Anderson MIT since the pure MIT critical point becomes unstable as soon as disorder is turned on, resulting in a nontrivial stable fixed point with a finite disorder. Accordingly, the resulting insulating

phase with disorder is expected to be a Bose glass state, where gapped charge excitations in the Mott-Hubbard insulator are gapless in the presence of disorder. As a result, the U(1) spin liquid state would coexist with the Bose glass phase of charge fluctuations, thus called the spin liquid charge glass.

The present study is expected to apply to geometrically frustrated lattices such as an organic material κ-(BEDT-TTF)$_2$Cu$_2$(CN)$_3$ since the U(1) spin liquid Mott insulator is believed to appear in this material.[7] Our study implies that although spin dynamics is little affected by weak disorder, charge dynamics is severely modified from the Bose-Mott insulator to the Bose glass. This will be measured by charge spectra in optical conductivity experiments.

II. SLAVE-ROTOR THEORY WITH DISORDER

A. Formulation

We consider the Hubbard model with disorder

$$H = -t\sum_{ij\sigma} c_{i\sigma}^\dagger c_{j\sigma} + u\sum_i \left(\sum_\sigma c_{i\sigma}^\dagger c_{i\sigma}\right)^2 - \sum_i v_i \left(\sum_\sigma c_{i\sigma}^\dagger c_{i\sigma}\right),$$
(1)

where t is a hopping integral, u the strength of on-site Coulomb interaction, and v_i a random potential introduced by disorder.

A usual methodology treating the Hubbard u term is a Hubbard-Stratonovich (HS) transformation. Using the coherent state representation, and performing the HS transformation, we obtain the partition function

$$Z = \int D[c_{i\sigma}, \varphi_i] \exp\left[-\int d\tau \left\{\sum_{i\sigma} c_{i\sigma}^*(\partial_\tau - \mu)c_{i\sigma} - t\sum_{ij\sigma} c_{i\sigma}^* c_{j\sigma} + \sum_i \left(\frac{1}{4u}\varphi_i^2 - (i\varphi_i + v_i)\sum_\sigma c_{i\sigma}^* c_{i\sigma}\right)\right\}\right],$$
(2)

where φ_i is an order parameter associated with a charge den-

sity wave (CDW), and μ is the chemical potential of electrons. Physically, the φ_i field corresponds to an effective electric potential. In the usual mean-field manner, the CDW order parameter is given by $-i\bar{\varphi}_i = 2u\langle\Sigma_\sigma c_{i\sigma}^* c_{i\sigma}\rangle$.

We consider the gauge transformation for an electron field

$$c_{i\sigma} = e^{-i\theta_i}f_{i\sigma}. \tag{3}$$

Here $e^{-i\theta_i}$ is assigned to be an annihilation operator of an electron charge, and $f_{i\sigma}$ is an annihilation operator of an electron spin. In this paper, we call $e^{-i\theta_i}$ and $f_{i\sigma}$ chargon and spinon, respectively. Inserting this decomposition into Eq. (2), we obtain

$$Z = \int D[f_{i\sigma}, \theta_i, \varphi_i]$$

$$\times \exp\left[-\int d\tau\left\{\sum_{i\sigma} f_{i\sigma}^*(\partial_\tau - \mu - i\partial_\tau\theta_i)f_{i\sigma}\right.\right.$$

$$- t\sum_{ij\sigma} f_{i\sigma}^* e^{i(\theta_i - \theta_j)}f_{j\sigma}$$

$$\left.\left. + \sum_i\left(\frac{1}{4u}\varphi_i^2 - (i\varphi_i + v_i)\sum_\sigma f_{i\sigma}^* f_{i\sigma}\right)\right\}\right]. \tag{4}$$

Performing the HS transformation $(1/4u)\varphi_i^2 \to uL_i^2 + iL_i\varphi_i$, and shifting φ_i into $\varphi_i \to \varphi_i + iv_i - \partial_\tau\theta_i$, we obtain the following expression for the partition function:

$$Z = \int D[f_{i\sigma}, \theta_i, \varphi_i, L_i]e^{-\int d\tau L},$$

$$L = \sum_{i\sigma} f_{i\sigma}^*(\partial_\tau - \mu)f_{i\sigma} - t\sum_{ij\sigma} f_{i\sigma}^* e^{i(\theta_i - \theta_j)}f_{j\sigma}$$

$$+ \sum_i\left[uL_i^2 - iL_i(\partial_\tau\theta_i - iv_i) + i\varphi_i\left(L_i - \sum_\sigma f_{i\sigma}^* f_{i\sigma}\right)\right]. \tag{5}$$

In this expression, the CDW order parameter φ_i plays the role of a Lagrange multiplier field to impose the rotor constraint $L_i = \Sigma_\sigma f_{i\sigma}^\dagger f_{i\sigma}$. Interestingly, the θ_i field is canonically conjugate to the charge density $L_i = \Sigma_\sigma f_{i\sigma}^\dagger f_{i\sigma}$ as shown by the coupling term $-iL_i\partial_\tau\theta_i$. This allows us to interpret the $e^{-i\theta_i}$ operator as an annihilation operator of an electron charge. It should be noted that Eq. (5) is just another representation of Eq. (1) via the transformation Eq. (3). Integrating over the potential field φ_i and the density field L_i in Eq. (5), and performing the gauge transformation Eq. (3), one can recover the Hubbard model Eq. (1).

Integrating out the density variable L_i, Eq. (5) reads

$$Z = \int D[f_{i\sigma}, \theta_i, \varphi_i]e^{-\int d\tau L},$$

$$L = \sum_{i\sigma} f_{i\sigma}^*(\partial_\tau - \mu - i\varphi_i)f_{i\sigma} - t\sum_{ij\sigma} f_{i\sigma}^* e^{i(\theta_i - \theta_j)}f_{j\sigma}$$

$$+ \frac{1}{4u}\sum_i (\partial_\tau\theta_i - \varphi_i - iv_i)^2. \tag{6}$$

Note that the random potential v_i couples to the charge density represented by $\partial_\tau\theta_i$. Decomposing the hopping term by using the HS transformation, one can obtain the effective Lagrangian

$$L = L_0 + L_f + L_\theta,$$

$$L_0 = t\sum_{\langle ij\rangle} (\alpha_{ij}\beta_{ij}^* + \beta_{ij}\alpha_{ij}^*),$$

$$L_f = \sum_{i\sigma} f_{i\sigma}^*(\partial_\tau - \mu - i\varphi_i)f_{i\sigma} - t\sum_{\langle ij\rangle\sigma} (f_{i\sigma}^*\beta_{ij}^* f_{j\sigma} + f_{j\sigma}^*\beta_{ij}f_{i\sigma}),$$

$$L_\theta = \frac{1}{4u}\sum_i (\partial_\tau\theta_i - \varphi_i - iv_i)^2 - t\sum_{\langle ij\rangle} (e^{i\theta_i}\alpha_{ij}e^{-i\theta_j} + e^{i\theta_j}\alpha_{ij}^* e^{-i\theta_i}), \tag{7}$$

where α_{ij} and β_{ij} are spinon and chargon hopping order parameters, respectively. A remark on notation: each nearest-neighbor bond is counted only once in $\langle ij\rangle$, whereas ij implies a sum over all i and all the nearest-neighbor sites j with respect to i.

A saddle-point analysis results in the self-consistent equations

$$\left\langle\sum_\sigma f_{i\sigma}^* f_{i\sigma}\right\rangle = 1,$$

$$-i\varphi = -i\langle\partial_\tau\theta_i\rangle + 2u\left\langle\sum_\sigma f_{i\sigma}^* f_{i\sigma}\right\rangle = 2u,$$

$$\alpha_{ij} = \left\langle\sum_\sigma f_{i\sigma}^* f_{j\sigma}\right\rangle, \quad \beta_{ij} = \langle e^{i\theta_j}e^{-i\theta_i}\rangle. \tag{8}$$

Considering low-energy fluctuations around this saddle point,[9] one can set

$$\alpha_{ij} = \alpha e^{ia_{ij}}, \quad \beta_{ij} = \beta e^{ia_{ij}}, \quad \varphi_i = \varphi + a_{i\tau}, \tag{9}$$

where $\alpha = |\langle\Sigma_\sigma f_{i\sigma}^* f_{j\sigma}\rangle|$ and $\beta = |\langle e^{i\theta_j}e^{-i\theta_i}\rangle|$ are amplitudes of the hopping order parameters, and a_{ij} and $a_{i\tau}$ are spatial and time components of U(1) gauge fields.

Inserting Eq. (9) into Eq. (7), we find an effective U(1) gauge theory for the Mott-Anderson transition,

$$L_f = \sum_{i\sigma} f_{i\sigma}^*(\partial_\tau - ia_{i\tau})f_{i\sigma} - t\beta\sum_{\langle ij\rangle\sigma} (f_{i\sigma}^* e^{-ia_{ij}}f_{j\sigma} + \text{H.c.}),$$

$$L_\theta = \frac{1}{4u}\sum_i (\partial_\tau\theta_i - a_{i\tau} - iv_i)^2 - 2t\alpha\sum_{\langle ij\rangle}\cos(\theta_j - \theta_i - a_{ij}), \tag{10}$$

where the mean-field potential φ cancels the chemical potential at half-filling in the fermion Lagrangian, and the Berry

phase term $S_B = -i\Sigma_i \int_0^\beta d\tau \partial_\tau \theta_i$ resulting from the mean-field potential φ has no physical effects at half-filling in the boson Lagrangian, thus safely ignored.[10]

B. Discussion in the mean-field level

It is interesting to note that spinon dynamics is decoupled to chargon dynamics in the mean-field scheme ignoring gauge fluctuations, governed by

$$L_f = \sum_{i\sigma} f_{i\sigma}^* \partial_\tau f_{i\sigma} - t\beta \sum_{\langle ij\rangle\sigma} (f_{i\sigma}^* f_{j\sigma} + \text{H.c.}),$$

$$L_\theta = \frac{1}{4u} \sum_i (\partial_\tau \theta_i - iv_i)^2 - 2t\alpha \sum_{\langle ij\rangle} \cos(\theta_j - \theta_i). \quad (11)$$

Since the chargon Lagrangian corresponds to the quantum *XY* model in this level of approximation, the coherent-incoherent transition of the phase fields occurs in the absence of disorder, belonging to the *XY* universality class at zero temperature. The incoherent phase with charge gap but no spin gap is identified with the Mott-Hubbard insulator of a spin liquid with a Fermi surface. In the spin liquid, there is no coherent quasiparticle peak at zero energy, and only an incoherent hump is observed near the correlation energy $\pm u/2$.[6] On the other hand, the coherent phase is understood as a Fermi liquid with a coherent quasiparticle peak at zero energy.[6]

Realizing that the quantum *XY* model L_θ in Eq. (11) is equivalent to the boson Hubbard model for critical phenomena, one can find its effective field theoretic action given by[11]

$$S = \int d\tau d^2 r \left[|\partial_\mu \psi|^2 + m_\psi^2 |\psi|^2 + \frac{u_\psi}{2} |\psi|^4 + v\psi^\dagger \partial_\tau \psi + w|\psi|^2 \right]. \quad (12)$$

Here $\psi \sim \langle e^{i\theta}\rangle$ is the effective chargon field with its mass $m_\psi^2 \sim u/t - (u/t)_c$, where $(u/t)_c$ is the critical strength of local interactions, associated with the Mott transition in the mean field level.[6] u_ψ is a phenomenologically introduced parameter for local chargon interactions. v is a Gaussian random variable resulting from the Berry phase contribution, and w is a Gaussian random mass originating from the random chemical potential in the boson Hubbard model, where they satisfy $\langle v(r)\rangle = 0, \langle v(r)v(r')\rangle = V\delta(r-r')$ and $\langle w(r)\rangle = 0, \langle w(r)w(r')\rangle = W\delta(r-r')$, respectively.

To take into account the random variables, one can utilize the standard replica method, then obtain

$$Z = \int D\psi_l e^{-S},$$

$$S = \sum_l \int d\tau d^2 r \left[|\partial_\mu \psi_l|^2 + m_\psi^2 |\psi_l|^2 + \frac{u_\psi}{2} |\psi_l|^4 \right]$$

$$- \sum_{l,l'} \int d\tau d\tau' \int d^2 r \frac{V}{2} (\psi_{l\tau}^\dagger \partial_\tau \psi_{l\tau})(\psi_{l'\tau'}^\dagger \partial_{\tau'} \psi_{l'\tau'})$$

$$- \sum_{l,l'} \int d\tau d\tau' \int d^2 r \frac{W}{2} |\psi_{l\tau}|^2 |\psi_{l'\tau'}|^2, \quad (13)$$

where $l, l' = 1, \ldots, N$ are replica indices, and the limit of $N \to 0$ is performed in the final stage of calculations.

The pure Mott critical point $(m_\psi^{2*} = 0, u_\psi^* \neq 0, V^* = 0,$ and $W^* = 0)$ can be easily checked to be unstable against the presence of disorder $(W \neq 0)$. In Eq. (13), the bare scaling dimension of ψ is given by $[\psi] = L^{1/2}$ owing to the quadratic derivative in the kinetic energy term, where L is a length scale. Thus, we obtain $[W] = L^2$, indicating that disorder is relevant at the pure Mott critical point. $[V] = L^0$ is obtained, thus marginal. In the $1/N_\psi$ approximation where N_ψ is the flavor number of boson fields, it was shown that V is irrelevant and W is relevant so that the RG flow goes to a strong disorder regime.[12] But, recent RG calculations exhibit a weak disorder fixed point in the case of $V = 0$, identified with the Bose glass to superfluid transition instead of the Bose-Mott insulator to superfluid transition.[13] The weak disorder fixed point can also be shown to exist in the dual vortex formulation, where the quantum *XY* model is mapped into the scalar quantum electrodynamics in $(2+1)$D (QED_3) in terms of vortices interacting via vortex gauge fields. A random mass term for the vortices is also induced by disorder, making unstable the pure Mott critical point associated with the Bose-Mott insulator to superfluid transition. A stable disorder fixed point appears to be identified with the Bose glass to superfluid transition in the dual formulation.[14] In this respect, the spin liquid Mott insulator turns into the spin liquid Bose glass in the mean-field level.

Beyond the mean-field level, dynamics of spinons and chargons is coupled via U(1) gauge fluctuations. In this case, several important issues arise even in the absence of disorder. One can doubt the stability of the spin liquid phase against U(1) gauge fluctuations, especially instanton excitations allowed by the compactness of the U(1) gauge field. Recently, the present author revisited the confinement-deconfinement problem in the presence of the Fermi surface, and proposed the stability of the spin liquid phase when the spinon conductivity is sufficiently large.[15] In addition, the *XY* transition nature in the mean-field level without disorder should be modified by spinon excitations. Especially, gapless spinon excitations result in dissipative dynamics of the U(1) gauge field. These damped gauge fluctuations are expected to turn the *XY* transition into the other. Furthermore, the presence of disorder makes the MIT much more complex.

III. RENORMALIZATION-GROUP ANALYSIS

A. Boson-only effective action

To investigate the role of spinon excitations in the coherent-incoherent transition of chargon fields, we obtain the effective chargon-gauge action in the continuum limit by integrating out spinons in Eq. (10),

$$S_{\text{eff}} = \int d\tau d^2 r \left[\frac{1}{4u} (\partial_\tau \theta - a_\tau - iv)^2 - 2t\alpha \cos(\nabla\theta - \mathbf{a}) \right]$$

$$+ \frac{1}{\beta} \sum_{\omega_n} \int dq_r \frac{1}{2} a_\mu(q_r, i\omega_n) D_{\mu\nu}^{-1}(q_r, i\omega_n) a_\nu(-q_r, -i\omega_n), \quad (14)$$

where $D_{\mu\nu}(q_r, i\omega_n)$ is the renormalized gauge propagator, given by

TABLE I. Effective action depending on the spinon conductivity.

	$\sigma_0 \to \infty$ Spinon perfect metal	$0 < \sigma_0 < \infty$ Spinon metal	$\sigma_0 \to 0$ Spinon insulator
Effective chargon action	XY	QED$_3$+spinon-gauge correction	QED$_3$
Dual vortex action	QED$_3$	QED$_3$+spinon-gauge correction	XY

$$D_{\mu\nu}(q_r, i\omega_n) = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) D(q_r, i\omega_n),$$

$$D^{-1}(q_r, i\omega_n) = D_0^{-1}(q_r, i\omega_n) + \Pi(q_r, i\omega_n). \quad (15)$$

Here $D_0^{-1}(q_r, i\omega_n) = (q_r^2 + \omega_n^2)/g^2$ is the bare gauge propagator given by the Maxwell gauge action, resulting from integration of high-energy fluctuations of spinons and chargons. g is an internal gauge charge of the spinon and chargon. $\Pi(q_r, i\omega_n)$ is the self-energy of the gauge field, given by the correlation function of spinon charge (number) currents. Since the current-current correlation function is calculated in the noninteracting fermion ensemble, its structure is well known,[16,17]

$$\Pi(q_r, i\omega_n) = \sigma(q_r)|\omega_n| + \chi q_r^2. \quad (16)$$

Here the spinon conductivity $\sigma(q_r)$ is given by $\sigma(q_r) \approx k_0/q_r$ in the clean limit while it is $\sigma(q_r) \approx \sigma_0 = k_0 l$ in the dirty limit, where k_0 is of order k_F (Fermi momentum), and l the spinon mean free path determined by disorder scattering. The diamagnetic susceptibility χ is given by $\chi \sim m_f^{-1}$, where $m_f \sim (t\beta)^{-1}$ is the band mass of spinons. The frequency part of the kernel $\Pi(q, i\omega_n)$ shows the dissipative propagation of the gauge field owing to particle-hole excitations of spinons near the Fermi surface.

Recently, the present author reinvestigated the Mott-Hubbard MIT based on the effective chargon-gauge action Eq. (14) without disorder.[8] In this study, we found that dissipative gauge fluctuations result in a dissipative critical point, depending on the spinon conductivity σ_0 that determines the strength of dissipation. In the limit of $\sigma_0 \to \infty$ identified with a perfect metal of spinons, gauge fluctuations are completely screened by spinon excitations, thus safely ignored. The resulting chargon action is nothing but the XY Lagrangian, yielding the XY transition. On the other hand, in the limit of $\sigma_0 \to 0$ considered as an insulator of spinons, only the Maxwell gauge action is expected to appear from high-energy contributions of spinons and chargons. The resulting chargon-gauge action coincides with the scalar QED$_3$, yielding the inverted XY (IXY) transition[18] owing to gauge excitations. Varying the spinon conductivity, these two limits would be connected.

Emergence of the dissipative charged fixed point can be easily understood from the effective gauge-only action at the critical point. Integrating over critical chargon fluctuations in Eq. (14), the critical gauge action can be obtained in a highly schematic form at the critical point

$$S_g = \frac{1}{\beta} \sum_{\omega_n} \int d^2 q_r \frac{1}{2} \mathbf{a}^T(q_r, i\omega_n) \Pi(q_r, i\omega_n) \mathbf{a}^T(-q_r, -i\omega_n),$$

where $\mathbf{a}^T(q_r, i\omega_n)$ represent the transverse components of the gauge fields. The gauge kernel $\Pi(q_r, i\omega_n)$ is given by

$$\Pi(q_r, i\omega_n) = \frac{N_\theta}{8} \sqrt{q_r^2 + \omega_n^2} + \sigma_0 |\omega_n|,$$

where N_θ is the flavor number of the chargon field, here $N_\theta = 1$. The first term results from critical chargon fluctuations while the second originates from gapless spinon excitations near the Fermi surface. This gauge action can be easily checked to be scale-invariant at the tree level, giving the IXY fixed point in the $\sigma_0 \to 0$ limit and the XY one in the $\sigma_0 \to \infty$ limit. It is important to notice that the critical gauge action is well defined for any values of the spinon conductivity, and regularly linked as a function of σ_0. Thus, the spinon contribution characterized by the spinon conductivity σ_0 connects these two fixed points smoothly. A finite conductivity causes a dissipative critical point between the XY and IXY fixed points. In the present paper, we examine the role of disorder in the dissipative fixed point.

Before closing this section, we summarize the effective boson-only action depending on the spinon conductivity σ_0 in Table I.

B. Dual vortex action with disorder

Disorder effects produce random Berry phase to chargon fields. Because the Berry phase term leads to a complex phase factor to the partition function of Eq. (14), it is not easy to handle the partition function in the chargon representation. Duality transformation is generally performed to treat the Berry phase term.[19] The dual vortex action of Eq. (14) is obtained to be

$$S_v = \int d\tau d^2 r \Bigg[|(\partial_\mu - ic_\mu)\Phi|^2 + m_v^2 |\Phi|^2 + \frac{u_v}{2}|\Phi|^4 + u(\partial \times c)_\tau^2$$
$$+ \frac{1}{4t\alpha}(\partial \times c)_r^2 - v(\partial \times c)_\tau - ia_\mu(\partial \times c)_\mu \Bigg]$$
$$+ \frac{1}{\beta} \sum_{\omega_n} \int dq_r \frac{1}{2} a_\mu(q_r, i\omega_n) D_{\mu\nu}^{-1}(q_r, i\omega_n) a_\nu(-q_r, -i\omega_n).$$

$$(17)$$

Here Φ is a vortex field and c_μ is a vortex gauge field. m_v is a vortex mass, given by $m_v^2 \sim (u/t)_c - u/t$ with the mean-field

MIT critical point $(u/t)_c$, and u_v a phenomenologically introduced parameter for local interactions between vortices. The random potential v plays the role of random magnetic fields in vortices.

Since Eq. (17) is quadratic in gauge fluctuations a_μ, one finds the effective vortex-gauge action by performing the Gaussian integration for the gauge fields a_μ,

$$Z_v = \int D[\Phi, c_\mu] e^{-S_v},$$

$$S_v = \int d\tau d^2 r \left[|(\partial_\mu - ic_\mu)\Phi|^2 + m_v^2 |\Phi|^2 + \frac{u_v}{2}|\Phi|^4 \right.$$

$$+ u(\partial \times c)_\tau^2 + \frac{1}{4t\alpha}(\partial \times c)_r^2 - v(\partial \times c)_\tau \right]$$

$$+ \int d\tau d\tau_1 d^2 r d^2 r_1$$

$$\times \frac{1}{2} c_\mu(r,\tau) K_{\mu\nu}(r-r_1, \tau - \tau_1) c_\nu(r_1, \tau_1), \qquad (18)$$

where the renormalized gauge propagator $K_{\mu\nu}(r-r_1, \tau - \tau_1)$ is given by an energy-momentum space

$$K_{\mu\nu}(q_r, i\omega_n) = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) K(q_r, i\omega_n),$$

$$K(q_r, i\omega_n) = \frac{q_r^2 + \omega_n^2}{(q_r^2 + \omega_n^2)/g^2 + \sigma(q_r)|\omega_n| + \chi q_r^2}$$

$$\approx \frac{q_r^2 + \omega_n^2}{(q_r^2 + \omega_n^2)/\bar{g}^2 + \sigma(q_r)|\omega_n|}. \qquad (19)$$

Here $\bar{g}$ is a redefined variable including the susceptibility. In the following, we consider dirty cases characterized by $\sigma(q_r) = \sigma_0$.

In order to take into account the random potential by disorder, we use the replica trick to average over disorder. The random magnetic field v in the vortex action Eq. (18) would cause

$$-\sum_{l,l'} \int d\tau d\tau_1 \int d^2 r \frac{\Im}{2}(\partial \times c_l)_\tau (\partial \times c_{l'})_{\tau_1}$$

for the Gaussian random potential satisfying $\langle v(r) \rangle = 0$ and $\langle v(r)v(r_1) \rangle = \Im \delta(r-r_1)$ with the strength $\Im$ of the random potential. However, inclusion of only this correlation term is argued to be not enough for disorder effects. Because the gauge-field propagator has off-diagonal components in replica indices, the vortex-gauge interaction of the order $\Im^2 e_v^4$ generates a quartic term including the couplings of different replicas of vortices even if this term is absent initially.[14] Here e_v is a vortex charge. The resulting disordered vortex action is obtained to be

$$Z_v = \int D[\Phi_l, c_{\mu l}] e^{-S_v},$$

$$S_v = \sum_l \int d\tau d^2 r \left[|(\partial_\mu - ic_{\mu l})\Phi_l|^2 + m_v^2 |\Phi_l|^2 + \frac{u_v}{2}|\Phi_l|^4 \right.$$

$$+ u(\partial \times c_l)_\tau^2 + \frac{1}{4t\alpha}(\partial \times c_l)_r^2 \right]$$

$$+ \sum_l \sum_{\omega_n} \int dq_r$$

$$\times \frac{1}{2} c_{\mu l}(q_r, i\omega_n) K_{\mu\nu}(q_r, i\omega_n) c_{\nu l}(-q_r, -i\omega_n)$$

$$- \sum_{l,l'} \int d\tau d\tau_1 \int d^2 r \frac{W}{2} |\Phi_{l\tau}|^2 |\Phi_{l'\tau_1}|^2$$

$$- \sum_{l,l'} \int d\tau d\tau_1 \int d^2 r \frac{\Im}{2}(\partial \times c_l)_\tau (\partial \times c_{l'})_{\tau_1} \qquad (20)$$

with $W > 0$. The correlation term induced by disorder

$$- \sum_{l,l'} \int d\tau d\tau_1 \int d^2 r \frac{W}{2} |\Phi_{l\tau}|^2 |\Phi_{l'\tau_1}|^2$$

has the same form as the term resulting from a random mass term. Equation (20) is our starting action for studying the role of disorder in the Mott-Hubbard MIT.

C. Renormalization-group analysis

We perform an RG analysis for Eq. (20). Anisotropy in the Maxwell gauge action for the vortex gauge field is assumed to be irrelevant, and only the isotropic Maxwell gauge action is considered by replacing $u, 1/4t\alpha$ with $1/(2e_v^2)$, where e_v is a vortex charge. In the limit of small anisotropy, the anisotropy was shown to be irrelevant at the one-loop level.[14] Furthermore, the correlation term between random magnetic fluxes is also ignored. In the small $\Im$ limit, this term was shown to be exactly marginal at the one-loop level.[14] To address the quantum critical behavior at the Mott transition, we introduce the scaling $r = e^l r'$ and $\tau = e^l \tau'$, and consider the renormalized theory at the transition point $m_v^2 = 0$,

$$S_v = \sum_l \int d\tau' d^2 r' \left[Z_\Phi |(\partial_\mu' - ie_v c_{\mu l})\Phi_l|^2 + Z_u \frac{u_v}{2}|\Phi_l|^4 \right.$$

$$+ \frac{Z_c}{2}(\partial' \times c_l)^2 \right]$$

$$- \sum_{l,l'} \int d\tau' d\tau_1' \int d^2 r' Z_W \frac{W}{2} |\Phi_{l\tau}|^2 |\Phi_{l'\tau_1}|^2, \qquad (21)$$

where Z_Φ, Z_u, Z_c, and Z_W are the renormalization factors defined by

$$\Phi = e^{-(1/2)l} Z_\Phi^{1/2} \Phi_r, \quad c_\mu = e^{-(1/2)l} Z_c^{1/2} c_{\mu r},$$

$$e_v^2 = e^{-l} Z_c^{-1} e_{vr}^2, \quad u_v = e^{-l} Z_u Z_\Phi^{-2} u_{vr},$$

$$W = e^{-2l} Z_W Z_\Phi^{-2} W_r. \tag{22}$$

In the renormalized action Eq. (21), the subscript r implying "renormalized" is omitted for a simple notation.

Evaluating the renormalization factors at the one-loop level, the RG equations are expected to be[13,14,18,20,21]

$$\frac{de_v^2}{dl} = e_v^2 - \left(\lambda + \frac{\zeta}{\sigma_0}\right)e_v^4,$$

$$\frac{du_v}{dl} = u_v + [h(\sigma_0, e_v^2)e_v^2 + \gamma W]u_v - \rho u_v^2 - g(\sigma_0, e_v^2)e_v^4,$$

$$\frac{dW}{dl} = 2W + [h(\sigma_0, e_v^2)e_v^2 - \kappa u_v]W + \eta W^2. \tag{23}$$

Here $\lambda, \zeta, \gamma, \rho, \kappa, \eta$ are positive numerical constants, and $h(\sigma_0, e_v^2), g(\sigma_0, e_v^2)$ are analytic and monotonically increasing functions of σ_0, as will be explained below.

The first RG equation for the vortex charge can be understood in the following way. Integrating out critical vortex fluctuations, we obtain the singular contribution for the effective gauge action,

$$S_c = \frac{1}{\beta}\sum_{\omega_n}\int d^2q_r \frac{1}{2}c_\mu(q_r, i\omega_n)\Xi_{\mu\nu}(q_r, i\omega_n)c_\nu(-q, -i\omega_n),$$

$$\Xi_{\mu\nu}(q_r, i\omega_n) = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Xi(q_r, i\omega_n),$$

$$\Xi(q_r, i\omega_n) = \frac{N_v}{8}\sqrt{q_r^2 + \omega_n^2} + K(q_r, i\omega_n)$$

$$\approx \frac{N_v}{8}\sqrt{q_r^2 + \omega_n^2} + \frac{q_r^2 + \omega_n^2}{\sigma_0|\omega_n|},$$

where N_v is the flavor number of the vortex field, here $N_v = 1$. The first term in the kernel $\Xi(q_r, i\omega_n)$ results from the screening effect of the vortex charge via vortex polarization, causing the $-\lambda e_v^4$ term in the RG equation, while the second originates from that via spinon excitations, yielding the $-(\zeta/\sigma_0)e_v^4$ term. The first e_v^2 term in the RG equation denotes the bare scaling dimension of the vortex charge in $(2+1)D$. We note that the above critical gauge action results in the relativistic dispersion $\omega \sim q_r$.

For the second and third RG equations, unfortunately, we do not know the exact functional forms of $h(\sigma_0, e_v^2)$ and $g(\sigma_0, e_v^2)$ owing to the complexity of the gauge kernel. Owing to the spinon contribution $K(q_r, i\omega_n)$ [Eq. (19)], the kernel of the gauge propagator (c_μ),

$$D_c(q_r, i\omega_n) = \frac{1}{q_r^2 + \omega_n^2 + e_v^2 K(q_r, i\omega_n)} \approx \frac{\sigma_0|\omega_n|}{(q_r^2 + \omega_n^2)(e_v^2 + \sigma_0|\omega_n|)},$$

should be utilized instead of the Maxwell propagator in calculating one-loop diagrams. Note the dependence of the vortex charge e_v^2 in the effective gauge propagator. This gives the dependence of the vortex charge to the analytic functions $h(\sigma_0, e_v^2)$ and $g(\sigma_0, e_v^2)$. Although the exact functional forms

are not known, the limiting values of these functions can be found.

1. $\sigma_0 \to \infty$

In the limit of $\sigma_0 \to \infty$, the gauge kernel is reduced to the Maxwell propagator

$$D_c(q_r, i\omega_n) = \frac{1}{(q_r^2 + \omega_n^2)}$$

because gauge fluctuations a_μ are completely screened via spinon excitations in the perfect spinon metal ignored, and the resulting chargon action is nothing but the quantum XY model, causing the scalar QED_3 as an effective vortex-gauge action. See Table I. Thus, $h(\sigma_0 \to \infty, e_v^2) \to c_1$ and $g(\sigma_0 \to \infty, e_v^2) \to c_2$ are obtained, where c_1 and c_2 are positive numerical constants. Then, Eqs. (23) become the RG equations of the scalar QED_3 with a random mass term

$$\frac{de_v^2}{dl} = e_v^2 - \lambda e_v^4,$$

$$\frac{du_v}{dl} = (1 + c_1 e_v^2 + \gamma W)u_v - \rho u_v^2 - c_2 e_v^4,$$

$$\frac{dW}{dl} = (2 + c_1 e_v^2 - \kappa u_v)W + \eta W^2. \tag{24}$$

These are formally the same as the RG equations studied in Ref. 14, where the existence of the weak disorder fixed point was nicely discussed, guaranteeing the presence of the Mott-Anderson MIT.

In the absence of disorder ($W^* = 0$), a stable charged critical point ($e_v^{*2} \neq 0$) is expected to appear, associated with the Mott insulator to superfluid transition, although there is a delicate issue about the existence of the charged fixed point when the flavor number of complex matter fields is one, corresponding to the superconducting transition. This issue is well discussed in Ref. 18. In this paper, we assume the existence of the charged Mott critical point. This fixed point becomes unstable as soon as disorder is turned on, as shown in the third RG equation for W. A nontrivial stable fixed point is found with a finite disorder ($W^* \neq 0$), identified with the Bose glass to superfluid critical point.[14]

2. $\sigma_0 \to 0$

In the spinon insulator of $\sigma_0 \to 0$, the chargon-gauge action is given by the scalar QED_3 with disorder, as discussed before. The resulting vortex action becomes the Φ^4 model with a random mass term since vortex gauge fluctuations c_μ are gapped owing to the presence of long-range interactions mediated by the U(1) gauge fields a_μ, thus ignored in the low energy limit. This coincides with the fact that the gauge kernel $D_c(q_r, i\omega)$ vanishes. As a result, $h(\sigma_0 \to 0, e_v^2) \to 0$ and $g(\sigma_0 \to 0, e_v^2) \to 0$ are obtained. Accordingly, Eqs. (23) are reduced to the RG equations of the Φ^4 theory with a random mass term

$$\frac{du_v}{dl} = (1 + \gamma W)u_v - \rho u_v^2,$$

$$\frac{dW}{dl} = (2 - \kappa u_v)W + \eta W^2. \tag{25}$$

The existence of the weak disorder fixed point can be shown in Eq. (25),[13,22] although there are some papers claiming that there is no weak disorder fixed point in this model.[11,12] If only the two parameters corresponding to u_v and W are considered in the RG equations of Ref. 14, one finds that there indeed exists the weak disorder fixed point, describing the Mott-Anderson transition. One can perform the RG analysis based on the chargon-gauge QED$_3$ action instead of the vortex Φ^4 action. The chargon QED$_3$ with a random mass term is formally equivalent to the vortex QED$_3$ with a random mass term if we correspond chargons, chargon gauge fields, and chargon random mass to vortices, vortex gauge fields, and vortex random mass. According to the previous discussion in the vortex QED$_3$, a disorder critical point would be found in the chargon QED$_3$, implying that the Bose glass to superfluid transition also appears in this case.

3. $0 < \sigma_0 < \infty$

In the small σ_0 limit $(\sigma_0|\omega_n| \ll e_v^2)$, the gauge kernel is given by

$$D_c(q_r, i\omega_n) \approx \frac{\sigma_0}{e_v^2}\frac{|\omega_n|}{q_r^2 + \omega_n^2},$$

thus resulting in $h(\sigma_0, e_v^2) = c_h\sigma_0/e_v^2$ and $g(\sigma_0, e_v^2) = c_g\sigma_0^2/e_v^4$, where c_h and c_g are positive numerical constants. The corresponding RG equations are obtained to be

$$\frac{de_v^2}{dl} = e_v^2 - \left(\lambda + \frac{\zeta}{\sigma_0}\right)e_v^4,$$

$$\frac{du_v}{dl} = (1 + c_h\sigma_0 + \gamma W)u_v - \rho u_v^2 - c_g\sigma_0^2,$$

$$\frac{dW}{dl} = (2 + c_h\sigma_0 - \kappa u_v)W + \eta W^2, \tag{26}$$

where the RG flows of e_v^2 and u_v, W are decoupled in this limit. If σ_0 is replaced with e_v^2 in the $\sigma_0 \to \infty$ limit, Eq. (26) coincides with Eq. (24). The replacement of σ_0 with e_v^2 is justified by the fact that the above gauge kernel should be reduced to that in the $\sigma_0 \to \infty$ limit. In this respect, Eq. (26) can be considered to be a bridge between Eqs. (24) and (25).

Ignoring the σ_0^2 term in the second RG equation, one finds the weak disorder fixed point depending on the spinon conductivity.[23] This fixed point coincides with that of Eq. (25) in the $\sigma_0 \to 0$ limit. With increasing σ_0, we expect that the fixed point of Eq. (26) gets close to that of Eq. (24) because Eq. (26) should correspond to Eq. (24), as discussed above. In other words, the Mott-Anderson critical point is expected to move from the disorder fixed point of the $\sigma_0 \to 0$ limit to that of the $\sigma_0 \to \infty$ limit, depending on the
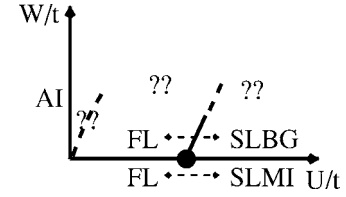

FIG. 1. A schematic phase diagram in the slave-rotor representation of the Hubbard model with disorder: Here SLBG is the spin liquid Bose glass, SLMI the spin liquid Mott insulator, FL the Fermi liquid metal, and AI the Anderson insulator. U is the strength of local interactions, and W that of disorder. t is a hopping integral of electrons. The regions indicated by question marks and dashed lines are beyond the scope of this theory. See the text.

spinon conductivity. This can be understood in the following way. The pure Mott critical points between the $\sigma_0 \to \infty$ and $\sigma_0 \to 0$ limits are smoothly connected by controlling the spinon conductivity, as discussed before. The presence of disorder makes the pure Mott critical points unstable, resulting in disorder fixed points. Thus, it is natural that these disorder fixed points are also connected smoothly through varying the spinon conductivity, as clearly shown in the small[23] and large σ_0 limits.[24] Since the critical points depend on the spinon conductivity, the concept of universality is not applied to the Mott-Anderson transition from the spin liquid charge glass to the Fermi liquid metal.

D. Phase diagram and discussion

We summarize our results in the schematic phase diagram Fig. 1, where SLBG is the spin liquid Bose glass, SLMI the spin liquid Mott insulator, FL the Fermi liquid metal, and AI the Anderson insulator. It should be noted that our approach cannot cover the whole range of the phase diagram. The regions indicated by question marks in Fig. 1 are beyond the scope of this theory. Strictly speaking, although the slave-rotor theory can produce meaningful physics in the Fermi liquid regime $[u/t < (u/t)_c]$,[6] the RG equations in this paper would not be applied because chargon condensation $\langle e^{i\theta_i}\rangle \neq 0$ allows only electron excitations owing to confinement between condensed chargons and spinons. When the strength of disorder becomes large, the RG equations would not work because the present analysis is based on the perturbation theory for weak disorder. Furthermore, strong disorder decreases the spinon conductivity, making the spin liquid phase unstable against instanton excitations, as discussed before.[15] Remember that the spin liquid state can be stable in the sufficiently good spinon metal. Thus, our RG analysis can be applied to a limited range of the phase diagram near the MIT in the presence of weak disorder, marked by dotted arrow lines. In the clean limit $(W \to 0)$, the Mott-Hubbard MIT is obtained between SLMI and FL.[6] On the other hand, in the small disorder limit the Mott-Hubbard MIT is shown to turn into the Mott-Anderson MIT between SLBG and FL. The chargon Mott insulator is expected to evolve into the chargon Bose glass as soon as disorder is turned on, as discussed earlier. Since the chargon superfluidity appears in the presence of disorder, the resulting electronic phase may be iden-

tified with the Fermi liquid metal. We note that in the weak interaction limit $u/t \ll (u/t)_c$ the Anderson transition from FL to AI is expected to occur by increasing disorder, although this is beyond the scope of the slave-rotor theory.

A recent dynamical mean-field theory (DMFT) study shows that the nonmagnetic phase with weak disorder is still a Mott insulator.[25] The Mott insulating phase in the DMFT study seems to be in contrast to our claim that the paramagnetic phase is a gapless insulator of the Anderson type instead of the Mott one. We argue that this difference is not a contradiction because the physics of the DMFT approach differs from that of our approach. The paramagnetic Mott insulator in the DMFT study is different from the spin liquid Mott insulator in the slave-rotor theory in that (i) elementary spin excitations carry the spin quantum number 1 instead of the fractionalized spin 1/2, and (ii) there is no spin-charge separation physics. As mentioned in the Introduction, the spin liquid Mott insulator is possible to appear in the triangular lattice such as an organic material κ-(BEDT-TTF)$_2$Cu$_2$(CN)$_3$.[7] The present slave-rotor theory is expected to apply to the triangular lattice while the DMFT study would explain the square lattice. In this respect, these two approaches see different systems, thus the resulting disordered insulators can be different.

We should point out an important issue that the nature of the insulating phase in the boson Hubbard model with weak disorder is not completely understood. The Bose-Mott insulator was claimed at commensurate filling instead of the Bose glass insulator.[26] One different thing from our vortex formulation is that the present vortex action includes anomalous gauge interactions resulting from the spinon contribution.

IV. FERMION-ONLY EFFECTIVE THEORY WITH DISORDER

There is an alternative way to treat disorder in the slave-rotor representation of the Hubbard model. In the effective gauge Lagrangian Eq. (10), the gauge shift $a_{i\tau} \rightarrow a_{i\tau} - iv_i$ results in

$$L_f = \sum_{i\sigma} f_{i\sigma}^*(\partial_\tau - ia_{i\tau})f_{i\sigma} - \sum_{i\sigma} v_i f_{i\sigma}^\dagger f_{i\sigma}$$
$$- t\beta \sum_{\langle ij \rangle \sigma} (f_{i\sigma}^* e^{-ia_{ij}} f_{j\sigma} + \text{H.c.}),$$

$$L_\theta = \frac{1}{4u} \sum_i (\partial_\tau \theta_i - a_{i\tau})^2 - 2t\alpha \sum_{\langle ij \rangle} \cos(\theta_j - \theta_i - a_{ij}). \quad (27)$$

Interestingly, the effect of disorder appears as the random chemical potential of spinons instead of the random Berry phase of chargons. Since the chargon dynamics does not couple to disorder directly, the glass phase is not likely to appear in this approach. Only the Mott insulator to superfluid transition is expected to occur in the chargon Lagrangian.

Integrating out the gapped chargon excitations in the Mott insulating phase, we obtain an effective spinon-gauge action in the continuum limit,

$$S_f = \int d\tau d^2r \left[\sum_\sigma \left(f_\sigma^\dagger(\partial_\tau - ia_\tau)f_\sigma \right. \right.$$
$$\left. \left. + \frac{1}{2m_f}|(\partial_r - ia_r)f_\sigma|^2 - v f_\sigma^\dagger f_\sigma \right) + \frac{1}{2g^2}|\partial \times a|^2 \right], \quad (28)$$

where $m_f \sim (t\beta)^{-1}$ is a spinon band mass. The main question in this effective spinon action is about the role of the random chemical potential in the spinon dynamics.

The role of nonmagnetic disorder in the QED$_3$ without the Fermi surface was investigated.[27,28] In contrast to the (2+1)D free Dirac theory, long-range gauge interactions are shown to reduce the strength of disorder, and induce a delocalized state in the QED$_3$. The presence of disorder destabilizes the free Dirac fixed point. The RG flow goes away from the fixed point, indicating localization.[29,30] On the other hand, the charged fixed point in the QED$_3$ remains stable at least against weak randomness. An unstable fixed point separating delocalized and localized phases is found.[27,28] The RG flow shows that the effects of random potentials vanish if we start from sufficiently weak disorder. The stability of the charged critical point against weak disorder in the QED$_3$, physically, results from the fact that the fermionic spinons feel the effective dimensionality higher than 2 because of the long-range gauge interactions at the charged critical point, thus killing the effects of weak disorder.[28]

In the spinon-gauge critical theory Eq. (28),[15] a similar result is expected. Deconfined spinons near the Fermi surface would remain delocalized at least against weak randomness owing to long-range gauge interactions while noninteracting spinons without gauge interactions are localized by random potentials according to the scaling theory.[3] However, it should be considered that the presence of nonmagnetic disorder reduces the spinon conductivity σ_0. Thus, even if the charged fixed point can be stable against weak disorder in the case of noncompact U(1) gauge fields, the fixed point can be unstable against instanton excitations owing to the reduction of the conductivity.[15] As mentioned earlier, the spin liquid phase can be stable when the spinon conductivity is sufficiently large. In the spinon bad metal, the spinons would be confined owing to the presence of disorder. This is the main reason why the application of the present slave-rotor formulation should be limited within weak disorder.

The above discussion suggests that the spin liquid Mott insulator remains stable against weak randomness in contrast to the emergence of the spin liquid Bose glass in the prior treatment.[31] To interpret this inconsistency between the two approaches in a consistent manner, we claim that the U(1) spin liquid of spinons is stable against weak randomness, but the Bose-Mott insulator of chargons is not. The resulting insulator is identified with the spin liquid charge glass.

V. SUMMARY

In the present paper, we examined the role of disorder in the Mott-Hubbard metal-insulator transition based on the slave-rotor formulation of the Hubbard model. In this representation, the Mott-Hubbard insulator is understood as the spin liquid Mott insulator in terms of gapless spinons and

gapped chargons interacting via U(1) gauge fields. We found that the Mott-Hubbard critical point becomes unstable as soon as disorder is turned on, resulting in a disorder critical point interpreted as the spin liquid glass insulator to the Fermi liquid metal transition. The glassy behaviors of charge fluctuations[11] can be measured by the optical spectra in the insulating phase of an organic material κ-(BEDT-TTF)$_2$Cu$_2$(CN)$_3$. Furthermore, since the Mott-Anderson critical points depend on the spinon conductivity, universal-

ity in the critical exponents may not be found.

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[9] Here amplitude fluctuations of order parameters, corresponding to high-energy modes, are safely ignored, and phase (transverse) fluctuations are mainly considered since they are gapless excitations.

[10] The Berry phase term arises from the phase-fluctuation term in the chargon Lagrangian L_θ,

$$S_B = -\sum_i \int_0^\beta d\tau \frac{1}{2u}\varphi\partial_\tau\theta_i$$

$$= -\sum_i \int_0^\beta d\tau\left(\frac{1}{2u}\langle\partial_\tau\theta_i\rangle\partial_\tau\theta_i + i\left\langle\sum_\sigma f_{i\sigma}^\dagger f_{i\sigma}\right\rangle\partial_\tau\theta_i\right).$$

At half-filling, the Berry phase does not play any roles because time-reversal symmetry considered in this paper leads to $\langle\partial_\tau\theta_i\rangle=0$, and the average occupation number of spinons is given by $\langle\sum_\sigma f_{i\sigma}^\dagger f_{i\sigma}\rangle=1$. Inserting this into the expression of the Berry phase, one obtains $S_B=-\sum_i\int_0^\beta d\tau i\partial_\tau\theta_i=-2\pi i\sum_i q_i$, where q_i is an integer representing an instanton number, here a vortex charge. Thus, the contribution of the Berry phase to the partition function is nothing because of $e^{-S_B}=1$. Away from half-filling, the Berry phase action is obtained to be $S_B=2\pi i\delta\sum_i q_i$ with modular 2π, where δ is hole concentration. This results in a complex phase factor to the partition function, given by $Z=\sum_Q e^{2\pi i\delta Q}Z_Q$, where $Q=\sum_i q_i$ is a total instanton number, and Z_Q, the partition function for a fixed Q. The role of the Berry phase away from half-filling was investigated in Ref. 8.

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[21] One can rewrite Eq. (22) as

$$e_{vr}^2 = e^l e^{lnZ_c}e_v^2, \quad u_{vr} = e^l e^{lnZ_u^{-1}}e^{lnZ_\Phi^2}u_v, \quad W_r = e^{2l}e^{lnZ_W^{-1}}e^{lnZ_\Phi^2}W.$$

Differentiating the above scaling equations with respect to the scaling parameter l, one obtains the exact scaling equations

$$\frac{de_{vr}^2}{dl} = e_{vr}^2 + \frac{dlnZ_c}{dl}e_{vr}^2, \quad \frac{du_{vr}}{dl} = u_{vr} + \frac{dlnZ_u^{-1}}{dl}u_{vr} + \frac{dlnZ_\Phi^2}{dl}u_{vr},$$

$$\frac{dW_r}{dl} = W_r + \frac{dlnZ_W^{-1}}{dl}W_r + \frac{dlnZ_\Phi^2}{dl}W_r.$$

The renormalization factors result from singular contributions of dynamical processes, formally given in the one-loop approximation,[13,14,18,20]

$$Z_c \approx \exp\left[-\left(\lambda + \frac{\zeta}{\sigma_0}\right)e_{vr}^2 l\right],$$

$$Z_u \approx \exp\left[-\left([h(\sigma_0, e_{vr}^2) - 2c_\Phi]e_{vr}^2\right.\right.$$

$$\left.\left. + \gamma W_r - \rho u_{vr} - g(\sigma_0, e_{vr}^2)\frac{e_{vr}^4}{u_{vr}}\right)l\right],$$

$$Z_W \approx \exp(-\{[h(\sigma_0, e_{vr}^2) - 2c_\Phi]e_{vr}^2 - \kappa u_{vr} + \eta W_r\}l),$$

$$Z_\Phi \approx \exp[c_\Phi e_{vr}^2 l],$$

where all numerical values are positive.

[22] The weak disorder fixed point is given by $u_v^* = (\eta - 2\gamma)/(\eta\rho - \gamma\kappa)$ and $W^* = (\kappa - 2\gamma)/(\eta\rho - \gamma\kappa)$. For this fixed point to exist, the positive numerical constants should satisfy $\eta\rho < \gamma\kappa < 2\gamma\rho$ or $\eta\rho > \gamma\kappa > 2\gamma\rho$. Using the numerical values in Refs. 13 and 14, one can check that $\eta\rho < \gamma\kappa < 2\gamma\rho$ is satisfied.

[23] The weak disorder fixed point is given by $e_v^{2*} = \sigma_0/(\lambda\sigma_0 + \zeta)$, $u_v^* = (\eta - 2\gamma + (\eta - \gamma)c_h\sigma_0)/(\eta\rho - \gamma\kappa)$, and $W^* = (\kappa - 2\gamma + (\kappa - \gamma)c_h\sigma_0)/(\eta\rho - \gamma\kappa)$, where $\eta\rho < \gamma\kappa < \gamma\rho$ is satisfied.

[24] We admit that our calculations are only truly valid for small and large σ_0 limits, thus it is not completely clear for intermediate values of σ_0. In this respect, other solutions may be possible.

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[31] This difference between the fermion-gauge and boson-gauge systems for the role of disorder originates from the fact that the scaling dimension of the boson fields differs from that of fermion fields. As mentioned before, the quadratic derivative in the boson action yields the bare scaling dimension of $L^{1/2}$ to the boson field while the linear derivative in the fermion action, resulting from the expansion near the Fermi surface, gives L to the fermion field. This difference causes the bare scaling dimension of disorder to be L^2 in the boson system while it is L^0 in the fermion system. Thus, the charged critical point in the boson-gauge action becomes unstable as soon as disorder is turned on, as discussed in the text. On the other hand, the charged fixed point in the fermion system remains stable against weak disorder because gauge interactions make the scaling of disorder weakly negative.[28] This is the statement that fermions feel the effective dimensionality higher than 2, thus killing the effects of weak disorder. This concept is analogous to the Harris criterion determining the stability of the critical point against weak randomness.