Long-range order of vortex lattices pinned by point defects in layered superconductors

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How the vortex lattice orders at long range in a layered superconductor with weak point pinning centers is studied through a duality analysis of the corresponding frustrated *XY* model. Vortex-glass order emerges out of the vortex liquid across a macroscopic number of weakly coupled layers in a perpendicular magnetic field as the system cools down. Further, the naive magnetic-field scale determined by the Josephson coupling between adjacent layers is found to serve as an upper bound for the stability of any possible conventional vortex lattice phase at low temperature in the extreme type-II limit.

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I. INTRODUCTION

It is well known that an external magnetic field can penetrate a type-II superconductor in the form of lines of flux quanta.[1] The repulsive forces that such flux lines experience favor the creation of a triangular vortex lattice, while the quenched point disorder present to some degree in all superconductors frustrates that tendency. Three thermodynamic ground states are then likely. Either the triangular vortex lattice is robust to weak point pinning and assumes a Bragg glass state with no lines of dislocations that thread it,[2] or it will transit into a defective state with quenched-in lines of dislocations that thread it. The latter, in turn, has two possible outcomes: a vortex glass state that retains macroscopic phase coherence of the superconducting order parameter,[3] or a pinned liquid state that does not.[2]

High-temperature superconductors, in particular, are extremely type-II and layered.[1] Below, we shall study how a vortex lattice pinned by material point defects orders at long range in such materials. The vortex lattice in layered superconductors with weak random point pins shall be described theoretically in terms of the phase of the superconducting order parameter via the corresponding frustrated *XY* model.[4] This model notably neglects the effects of magnetic coupling between layers, while it treats the Josephson coupling between them exactly. The growth of long-range order across layers is then computed from the *XY* model through a duality analysis,[5] where the ratio of the energy of the Josephson coupling between adjacent layers to the temperature emerges as a small parameter. We find first that the correlation length for vortex-glass order[3] across weakly coupled layers diverges as temperature cools down from the vortex liquid toward the two-dimensional (2D) ordering transition. The divergence signals a transition to a vortex glass phase.[6–9] Second, we find no evidence for the divergence of conventional superconducting phase correlations across layers from inside the latter vortex glass to lowest order in the interlayer Josephson coupling. This indicates ultimately that the naive decoupling field for the pristine vortex lattice[10] serves as an upper bound for a stable Bragg glass phase[2] in the extreme type-II limit. Comparisons with previous numerical,[4] theoretical,[11] and experimental[12] determinations of the stability line for the Bragg glass in layered superconductors are made at the end of the paper.

II. TWO DIMENSIONS

The *XY* model with uniform frustration is the minimum theoretical description of vortex matter in extremely type-II superconductors. Both fluctuations of the magnetic induction and of the magnitude of the superconducting order parameter are neglected within this approximation. The model hence is valid deep inside the interior of the mixed phase. The thermodynamics of an isolated layer with uniform frustration is determined by its superfluid kinetic energy

$$E_{XY}^{(2)} = -\sum_{\vec{r}} \sum_{\mu=x,y} J_\mu \cos\left[\Delta_\mu \phi - A_\mu\right]|_{\vec{r}}, \tag{1}$$

which is a functional of the phase of the superconducting order parameter, $e^{i\phi}$, over the square lattice, $\vec{r}$. Here, J_x and J_y are the local phase rigidities that are equal and constant, except over links in the vicinity of a pinning center. The vector potential $\vec{A} = (0, 2\pi fx/a)$ represents the magnetic induction oriented perpendicular to the layers, $B_\perp = \Phi_0 f/a^2$. Here a denotes the square lattice constant, which is of order the coherence length of the Cooper pairs, Φ_0 denotes the flux quantum, and f denotes the concentration of vortices per site.

Analytical and numerical work indicates that the 2D vortex lattice is invaded by quenched-in dislocations in the presence of any degree of random point pinning.[13] The author has argued[14] that the dislocations quenched into each 2D vortex lattice described by the frustrated *XY* model (1) notably do *not* line up to form low-angle grain boundaries, however (cf. Ref. 15). That argument is based on the incompressible nature of 2D vortex matter in the extreme type-II limit. The absence of grain boundaries is consistent with Monte Carlo simulations[16] of the equivalent 2D Coulomb gas ensemble with random point pins,[8] as well as with Monte Carlo simulations of the frustrated *XY* model in three dimensions with randomly located columnar pins.[17] Secondly, a net superfluid density is predicted at zero temperature for perpendicular magnetic fields above the collective-pinning threshold, $B_{cp}^{(2D)}$, in which case the number of pinned vortices is greater than the number of isolated dislocations quenched into the 2D vortex lattice.[18] Here, the scale of the Larkin domains[1] is set by the separation between neighboring dislocations quenched into the vortex lattice. A variational calculation by Mullock and Evetts yields the estimate $B_{cp}^{(2D)} \sim (4f_p/\varepsilon_0 d)^2 \Phi_0$ for the threshold field,[19] where f_p de-

notes the maximum pinning force, where $\varepsilon_0 = (\Phi_0/4\pi\lambda_L)^2$ is the maximum tension of a fluxline in the superconductor, and where d denotes the separation between adjacent layers. Here λ_L represents the London penetration depth. The pinning of the vortex lattice in isolated layers shall be assumed to be collective henceforth: $B_\perp > B_{\rm cp}^{\rm (2D)}$.

The previous paragraph indicates that a hexatic vortex glass characterized by a homogeneous distribution of quenched-in dislocations and by a net superfluid density exists in isolated layers of the frustrated XY model (1) with weak random point pins at zero temperature.[18] The transition temperature $T_g^{\rm (2D)}$ that separates the low-temperature hexatic vortex glass from the high-temperature vortex liquid must therefore be equal to zero or greater. Recent current-voltage measurements of 2D arrays of Josephson junctions in weak external magnetic field indicate that the 2D superconducting/normal transition at $T = T_g^{\rm (2D)}$ is second order,[20] with $T_g^{\rm (2D)}$ much larger than the 2D melting temperature of the pristine vortex lattice, $T_m^{\rm (2D)} \cong J/20$. Since the previous is a faithful realization of the frustrated XY model (1) in 2D with random point pinning centers, we shall assume henceforth that the hexatic vortex glass melts into a vortex liquid at temperature $T_g^{\rm (2D)} > 0$ via a second-order phase transition.

III. THREE DIMENSIONS

We shall now demonstrate how long-range vortex-glass order emerges across layers from the vortex liquid phase of layered superconductors with weak random point pins. Let us first couple the layers through the Josephson effect by adding a term $-J_z \cos(\Delta_z\phi - A_z)$ to the internal energy of the frustrated XY model for each nearest-neighbor link across adjacent layers. The component of the magnetic induction parallel to the layers is taken to be null throughout. At weak coupling, $J_z \ll k_B T$, phase correlations across N layers can then be determined from the quotient

$$\left\langle \exp\left[i\sum_r p(r)\phi(r)\right]\right\rangle = Z_{\rm CG}[p]/Z_{\rm CG}[0] \qquad (2)$$

of partition functions for a layered Coulomb gas (CG) ensemble,[5]

$$Z_{\rm CG}[p] = \sum_{\{n_z(r)\}} y_0^{N[n_z]} \Pi_l C_l[q_l] e^{-i\Sigma_r n_z A_z}. \qquad (3)$$

Above, $n_z(\vec{r}, l)$ is a dual charge/integer field that lives on links between adjacent layers l and $l+1$, located at 2D points $\vec{r}$, and $p(r) = \delta_{\vec{r},0} \cdot (\delta_{l,1} - \delta_{l,N})$ is the external integer probe field. The ensemble is weighted by a product of phase autocorrelation functions for isolated layers l,

$$C_l[q] = \left\langle \exp\left[i\sum_{\vec{r}} q(\vec{r})\phi(\vec{r}, l)\right]\right\rangle_{J_z=0}, \qquad (4)$$

probed at the dual charge that accumulates onto that layer,

$$q_l(\vec{r}) = p(\vec{r}, l) + n_z(\vec{r}, l-1) - n_z(\vec{r}, l). \qquad (5)$$

It is also weighted by a bare fugacity y_0 that is raised to the power $N[n_z]$ equal to the total number of dual charges,

$n_z = \pm 1$. The fugacity of the dual CG ensemble (3) is given by $y_0 = J_z/2k_B T$ in the selective high-temperature regime, $J_z \ll k_B T$, reached at large model anisotropy. It is small compared to unity in such a case, which implies a dilute concentration of dual n_z charges.[5] The dual CG ensemble (3) is valid in that regime.

The above duality analysis is particularly natural and effective in the vortex-liquid phase, where autocorrelations of the superconducting order parameter in isolated layers (4) are short range. They shall be assumed to take to the form that is characteristic of a hexatic vortex liquid between points $\vec{r}_1$ and $\vec{r}_2$ in an isolated layer l,[18,21]

$$C_l(1,2) = g_0 e^{-r_{1,2}/\xi_{\rm 2D}} e^{-i\phi_0(1)} e^{i\phi_0(2)}. \qquad (6)$$

Here $e^{i\phi_0}$ is the superconducting order parameter of layer l in isolation at zero temperature, $\xi_{\rm 2D}$ denotes the phase correlation length of the 2D hexatic vortex liquid, and g_0 is a prefactor of order unity. Also, $\vec{r}_{1,2} = \vec{r}_1 - \vec{r}_2$ is the displacement between the probes within layer l. To lowest order in the (dual) fugacity, y_0, Eqs. (2) and (3) then yield the expression

$$\overline{\langle e^{i\phi_{l,l+n}}\rangle} \cong y_0^n \sum_1 \cdots \sum_n \overline{C_l(0,1)C_{l+1}(1,2)\cdots C_{l+n}(n,0)} \quad (7)$$

for the bulk average (overbar) of the gauge-invariant autocorrelation function of the conventional superconducting order parameter $e^{i\phi}$ across n layers, at zero parallel field. Above and hereafter, we take the gauge $A_z = 0$. The uncorrelated nature of point pinning centers across layers implies the form

$$\overline{\Pi_{m=0}^n e^{-i\phi_0(\vec{r}_m, l+m)} e^{i\phi_0(\vec{r}_{m+1}, l+m)}} = \Pi_{m=0}^n e^{-r_{m,m+1}/2l_\phi} \qquad (8)$$

for the bulk average of the relevant product of zero-temperature order parameters, with matching end points $\vec{r}_0 = \vec{r}_{n+1}$. Here, l_ϕ is a quenched disorder scale that is set by the density of lines of dislocations quenched into the vortex lattice at $J_z = 0$ that begin or end at a given layer. We remind the reader that l_ϕ is believed to be finite (in the absence of interlayer coupling) for any nonzero strength of quenched point disorder.[13] Substitution of Eq. (8) into expression (7) then yields the principal dependence[22]

$$\overline{\langle e^{i\phi_{l,l+n}}\rangle} \propto \{g_0(J/k_B T)[(l_\phi^{-1} + \xi_\phi^{-1})^{-1}/\Lambda_0]^2\}^n \qquad (9)$$

for the correlation of the conventional superconducting order parameter across n layers. Here, J is the macroscopic phase rigidity of an isolated layers at zero temperature, $\Lambda_0 = (J/J_z)^{1/2}a$ is the Josephson penetration depth, and $\xi_\phi = \xi_{\rm 2D}/2$. Notice that the existence of the disorder scale l_ϕ implies that the perturbative result (9) above does *not* diverge with the 2D phase correlation length $\xi_{\rm 2D}$ in the vicinity of the 2D ordered phase. We conclude that conventional superconducting phase coherence across many layers ($n \to \infty$) does not emerge out of the vortex liquid at weak Josephson coupling between adjacent layers.

The growth of macroscopic vortex-glass order across layers[3] from inside of the vortex liquid is still possible, however. We shall test for it by computing the corresponding autocorrelation function,[3] which is given by

$$\overline{|\langle e^{i\phi_{l,l+n}}\rangle|^2} \cong y_0^{2n} \sum_{1,\bar{1}} \cdots \sum_{n,\bar{n}} \overline{C_l(0,1)C_l^*(0,\bar{1})C_{l+1}(1,2)C_{l+1}^*(\bar{1},\bar{2})\cdots C_{l+n}(n,0)C_{l+n}^*(\bar{n},0)} \tag{10}$$

to lowest order in the (dual) fugacity, y_0. It is natural to look for vortex-glass order to emerge from within the 2D critical regime: $\xi_{2D} \gg 2l_\phi$ at $T > T_g^{(2D)}$, where $T_g^{(2D)}$ denotes the transition temperature of the 2D hexatic vortex glass. The bulk average of the product of zero-temperature order parameters that appears in the integrand above can then be approximated by the corresponding product of the bulk averages limited to adjacent layers, $l' = l+m-1$ and $l'+1$, only,

$$\overline{\exp[i\phi_{l',l'+1}^{(0)}(m)]\exp[-i\phi_{l',l'+1}^{(0)}(\bar{m})]} = e^{-r_{m,\bar{m}}/l_\phi}. \tag{11}$$

Here $\phi_{l',l'+1}^{(0)}(\vec{r}) = \phi_0(\vec{r},l'+1) - \phi_0(\vec{r},l') - A_z(\vec{r})$ is the quenched interlayer phase difference. Converting to center-of-mass variables among the interlayer coordinates, $\vec{r}_m$ and $\vec{r}_{\bar{m}}$, then yields the principal dependence[22]

$$\overline{|\langle e^{i\phi_{l,l+n}}\rangle|^2} \propto [g_0(J/k_BT)(l_\phi\xi_\phi/\Lambda_0^2)]^{2n} \tag{12}$$

for the vortex-glass correlations across layers in the 2D critical regime, $\xi_{2D} \gg 2l_\phi$, at zero parallel field. The corresponding correlation length ($\xi_\perp$) is equal to the layer spacing (d) when the argument in brackets above is set to $1/e$. This occurs at a crossover field

$$B_\times \sim g_0(J/k_BT)(l_\phi\xi_\phi/a_{vx}^2)(\Phi_0/\Lambda_0^2) \tag{13}$$

that separates two-dimensional from three-dimensional (3D) vortex-liquid behavior (see Table I). Above, a_{vx} denotes the square root of the area per vortex inside of a given layer. Also, the argument between brackets on the right-hand side of Eq. (12) notably diverges with ξ_{2D} in the vicinity of the 2D ordering transition. This indicates that a transition to a vortex glass that orders across a macroscopic number of layers,[4,6] $\xi_\perp \to \infty$, occurs at a critical temperature T_g that lies inside of the window $[T_g^{(2D)}, T_\times]$. Indeed, setting the argument of the exponent on the right-hand side of Eq. (12) to unity yields a critical field $B_g = B_\times/e$, below which a vortex glass exists (see Table I).

Lastly, recall that the superfluid density across layers, $\rho_s^\perp = -\mathcal{N}^{-1}k_BT\partial^2 \ln Z_{CG}/\partial A_z^2|_0$, is given by the expression[5]

$$\rho_s^\perp = \mathcal{N}^{-1}\left\langle \left[\sum_{\vec{r},l} n_z(\vec{r},l)\right]^2 \right\rangle k_BT, \tag{14}$$

where $\mathcal{N}$ counts the number of nearest-neighbor links between layers, and where periodic boundary conditions are assumed across layers. Study of Eqs. (2)–(5) yields that the tension for a line across layers of dual n_z quanta is equal to $\xi_\perp^{-1}$, where $\xi_\perp$ denotes the correlation length for vortex-glass order across layers. The corresponding superfluid density (14) is then null in the limit of a macroscopic number of layers inside of the vortex liquid, where $\xi_\perp < \infty$ (see Table I).

The previous result (12) clearly demonstrates that a selective high-temperature expansion in powers of the fugacity y_0 necessarily breaks down in the 2D ordered phase, $T \le T_g^{(2D)}$, where ξ_{2D} is infinite. A direct analysis of the frustrated XY model for an isolated layer finds, in particular, that long-range correlations of the superconducting order parameter (4) decay algebraicly instead at such low temperatures,[8,21]

$$C_l[q] = g_0^{n_+} \exp\left[\eta_{2D}\sum_{(1,2)} q(\vec{r}_1)\ln(r_{1,2}/r_0)q(\vec{r}_2)\right]$$
$$\cdot \exp\left[i\sum_1 q(\vec{r}_1)\phi_0(\vec{r}_1,l)\right]. \tag{15}$$

The exponent η_{2D} that characterizes the algebraic decay of 2D phase coherence is related to the 2D superfluid density by $\rho_s^{(2D)} = k_BT/2\pi\eta_{2D}$. Above, $g_0 = \rho_s^{(2D)}/J$ is the ratio of the 2D phase stiffness with its value at zero temperature, J, while n_+ counts half the number of probes in $q(\vec{r})$. Also, r_0 denotes the natural ultraviolet scale. It is important to observe at this stage that the loop excitations in the (completely) dual representation of the 3D XY model[5] lose their integrity in the ordered phase. This translates into the absence of charge conservation in the (partially) dual CG ensemble (3). In other words, the dual n_z charges form a plasma in the ordered phase. A Hubbard-Stratonovich transformation of the CG partition function (3) followed by the unrestricted summation of configurations of charges with values $n_z = 0, \pm 1$ then yields the equivalent partition function[23] $Z_{LD}[p]$

TABLE I. Listed are the conventional phase correlation length (L_ϕ) and the vortex-glass phase correlation length ($\xi_\perp$) across equally spaced (d) layers, as well as the corresponding "cosine" and phase rigidity (see Ref. 8), for the various regimes found inside the mixed phase of an extremely type-II superconductor at weak Josephson coupling between layers, with weak point pinning.

Disorder index	Regime/phase	$\overline{\langle\cos\phi_{l,l+1}\rangle}$	$\rho_s^\perp/J_z$	L_ϕ/d	$\xi_\perp/d$
1	Bragg glass	unity	unity	∞	∞
2	Defective vortex lattice	unity	unity	unity, or greater	∞
3	Vortex glass	fraction	fraction	fraction	∞
4	Critical vortex liquid	fraction	0	fraction	unity, or greater
5	Decoupled vortex liquid	fraction	0	fraction	fraction

$= \int \mathcal{D}\theta e^{-E_{LD}/k_B T + i\Sigma_p \theta}$ for a renormalized Lawrence-Doniach (LD) model that shows no explicit dependence on the perpendicular magnetic field. Its energy functional is specifically given by[5,8]

$$E_{LD} = \rho_s^{(2D)} \int d^2 r \sum_l \left[\frac{1}{2}(\vec{\nabla}\theta_l)^2 - \Lambda_0^{-2} \cos\theta_{l,l+1} \right], \quad (16)$$

where $\theta_{l,l+1} = \phi_{l,l+1}^{(0)} + \theta_{l+1} - \theta_l$. The above continuum description is understood to have an ultraviolet cut off r_0 of order the intervortex spacing a_{vx}.

We can now determine the growth of correlations across layers of the conventional superconducting order parameter deep inside of the vortex glass phase, $T < T_g^{(2D)}$, at weak Josephson coupling between layers, $\Lambda_0 \to \infty$. The physics described by the original layered XY model coincides directly with that of the renormalized LD model described above at large scales in distance compared to the ultraviolet cutoff, r_0. Asymptotic correlations of the conventional superconducting order parameter across layers, for example, are identical to those of the LD model: $\lim_{n\to\infty} \overline{\langle e^{i\phi_{l,l+n}} \rangle} = \overline{\langle e^{i\theta_{l,l+n}} \rangle}$. The configuration that optimizes E_{LD} must be determined first in order to compute the later near-zero temperature. The LD energy functional (16) implies that it satisfies the field equation

$$-\nabla^2[\theta_{l'+1}^{(0)} - \theta_{l'}^{(0)}] = \Lambda_0^{-2}\sin\theta_{l'+1,l'+2}^{(0)} - 2\Lambda_1^{-2}\sin\theta_{l',l'+1}^{(0)}$$
$$+ \Lambda_0^{-2}\sin\theta_{l'-1,l'}^{(0)}, \quad (17)$$

where $\Lambda_1 = \Lambda_0$ (cf. Refs. 24 and 25). The phase angles $\theta_{l'}^{(0)}$ are then constant inside of a given layer l' in the weak-coupling limit,[8] $\Lambda_0, \Lambda_1 \to \infty$. Next, if $\delta\theta_{l'}^{(0)}$ denotes the fluctuation in the phase angles, the autocorrelation function for conventional superconducting order across many layers is then approximated by the expression

$$\overline{e^{i\theta_{l,l+n}}} \cong \Pi_{l'=l}^{l+n-1} e^{i\theta_{l',l'+1}^{(0)}} i[\delta\theta_{l'+1}^{(0)} - \delta\theta_{l'}^{(0)}] \quad (18)$$

near zero temperature, to lowest order in the fluctuation.[26] After inverting the field equation (17) for the fluctuation of the phase difference between adjacent layers, substitution into the expression above yields the result

$$\overline{e^{i\theta_{l,l+n}}} \cong a_n \Lambda_1^{-2n} \left[\Pi_{m=1}^n \int d^2 r_m G^{(2)}(0,m) \right]$$
$$\times \overline{\Pi_{m=1}^n e^{i\phi_{l+m-1,l+m}^{(0)}(0)} e^{-i\phi_{l+m-1,l+m}^{(0)}(m)}} \quad (19)$$

for the autocorrelation of the superconducting order parameter across layers. The prefactor on the right-hand side satisfies the recursion relation $a_{n+1} = a_n + (\Lambda_1^2/2\Lambda_0^2)^2 a_{n-1}$, with $a_0 = 1$ and $a_{-1} = 0$. Also,

$$G^{(2)} = [-\nabla^2 + 2\Lambda_1^{-2}\cos\theta_{l',l'+1}^{(0)}]^{-1} \quad (20)$$

is the 2D Green's function. The eigenstates of the latter operator within brackets are localized, with a localization length[27] $R_0 \sim \Lambda_1^2/l_\phi$. We therefore have $G^{(2)}(1,2) = (2\pi)^{-1}\ln(R_0/r_{1,2})$ at separations $r_{1,2} \ll R_0$ in the weak-coupling limit, $\Lambda_0, \Lambda_1 \to \infty$ (cf. Ref. 7). A scale transforma-

tion $\vec{r}_m = l_\phi \cdot \vec{x}_m$ of the $2n$-dimensional integral above (19) yields the final result

$$\overline{e^{i\theta_{l,l+n}}} \sim [(l_\phi/\Lambda_1)^2 \ln(\Lambda_1/l_\phi)^2]^n \quad (21)$$

for the asymptotic correlations of the superconducting order parameter across layers near zero temperature. The weakly coupled vortex-glass crosses over to a 3D vortex lattice threaded by lines of dislocations when the phase correlation length across layers, L_ϕ, exceeds the spacing between adjacent layers, d. This crossover occurs at a magnetic field

$$B_D(0) \sim (l_\phi/a_{vx})^2(\Phi_0/\Lambda_1^2) \quad (22)$$

near zero temperature, at which point the argument between brackets on the right-hand side of Eq. (21) is set to $1/e$. The defective vortex lattice is decoupled across layers at perpendicular magnetic fields above B_D (see Table I), where $l_\phi \ll \Lambda_1$.

Consider again very weak Josephson coupling between adjacent layers, such that $l_\phi \ll \Lambda_1$. Notice that this limit necessarily requires high perpendicular magnetic fields compared to the naive decoupling scale, Φ_0/Λ_1^2, by the inequality $a_{vx} < l_\phi$. Equation (21) then predicts short-range correlations of the superconducting order parameter across layers, with a correlation length L_ϕ that is less than the layer spacing d. Imagine next that the quenched disorder is reduced, such that $l_\phi \gg \Lambda_1$. The argument in brackets on the right-hand side of Eq. (21) then notably does *not* diverge toward positive infinity with the ratio l_ϕ/Λ_1 because of the logarithmic factor that originates from the 2D Green's function. Instead, it attains a maximum value of order unity at $l_\phi \sim \Lambda_1$. Like in the cooldown from the vortex liquid, Eq. (9), these observations indicate that the correlation length L_ϕ for conventional superconducting order across layers does not diverge at perpendicular magnetic fields above the naive decoupling scale, $B_\perp > \Phi_0/\Lambda_1^2$. Unlike the case of vanishing thermal disorder ($\xi_\phi \to \infty$) in Eq. (9), however, the argument in brackets on the right-hand side of Eq. (21) diverges toward negative infinity with vanishing quenched disorder ($l_\phi \to \infty$) because of the logarithmic factor. That divergence is spurious. The 2D Green's function (20) is given by $G^{(2)}(1,2) = (2\pi)^{-1}K_0(r_{1,2}/R_0)$ in the limit $l_\phi \to \infty$, where $\cos\theta_{l',l'+1}^{(0)} = 1$. Here, $K_0(x)$ is a modified Bessel function, and $R_0 = \Lambda_1/2^{1/2}$. Inspection of the original expression (19) for the autocorrelator across layers of the quenched superconducting order parameter then yields the asymptotic result $\lim_{n\to\infty} a_n(R_0/\Lambda_1)^{2n} = (\{1 + [1 + (\Lambda_1/\Lambda_0)^4]^{1/2}\}/4)^n$ for that quantity as l_ϕ diverges. Notice that the latter argument raised to the power n instead saturates to a value that lies inside of the range $[0.5, 0.6]$, which is notably less than unity. No evidence for conventional superconducting order of the vortex lattice across a macroscopic number of layers therefore emerges from the above perturbative analysis to lowest nontrivial order in the Josephson coupling between layers, at $B_\perp > \Phi_0/\Lambda_1^2$.

IV. DISCUSSION AND CONCLUSIONS

In conclusion, a duality analysis of the frustrated XY model for the mixed phase of layered superconductors with

weak point defects finds that long-range vortex-glass order across layers emerges out of the vortex liquid at weak Josephson coupling between layers. This is consistent with recent Monte Carlo simulations of the same *XY* model that find evidence for a thermodynamic vortex glass phase.[4,6] It also potentially accounts for the recent observation of a thermodynamic vortex glass state in the mixed phase of high-temperature superconductors that show extreme layer anisotropy.[9] The analysis also indicates that the naive decoupling scale,[25] Φ_0/Λ_1^2, serves as an upper bound for the stability of the Bragg glass phase as a function of perpendicular magnetic field in the extreme type-II limit. Previous theoretical work on layered superconductors predicts that the Bragg glass is stable to weak point pinning in general at the extreme type-II limit.[11] The discrepancy with the present work is likely due to the use there of a criterion for the destruction of the Bragg glass phase that is too stringent. In particular, the length L_ϕ along the field over which the vortex lattice tilts by a lattice constant is not divergent in Ref. 11. Also, the general robustness of the Bragg glass predicted by Ref. 11 at weak pinning conflicts with the belief that the Bragg glass is generally *unstable* to invasion by dislocations in the limit of decoupled layers,[13] $\Lambda_1 \rightarrow \infty$. A Bragg glass is also reported at fields beyond the naive decoupling scale in Ref. 4, where the same *XY* model is studied numerically by Monte Carlo simulation. The discrepancy with the stability bound established here is likely due to a combination of finite-size effects and of intrinsic pinning by the grid in each 2D *XY* model (1). The last effect has been neglected here throughout. Finally, Bragg peaks in neutron scattering that signal conventional vortex-lattice order at long range have been observed in the mixed phase of extremely layered high-temperature superconductors,[12] at fields below 500 G. That threshold is consistent with the stability bound established here, Φ_0/Λ_1^2, if the Josephson penetration depth is bounded by $\Lambda_0 < 200$ nm. Note that high layer anisotropy implies that the correction due to magnetic screening (λ_c) suggested by Ref. 25 can be ignored: $\Lambda_1 \cong \Lambda_0$.

The two theoretical results just reviewed depend critically on the existence of a vortex-glass state for isolated layers in the vicinity of zero temperature. Although recent experimental determinations of the current-voltage characteristic in 2D arrays of Josephson junctions in weak magnetic field obtain evidence for melting of the 2D vortex lattice at transition temperatures $T_g^{(2D)}$ that are in fact much greater than the 2D melting temperature of the pristine vortex lattice,[20] theoretical arguments suggest that a perfectly conducting vortex glass can exist only at zero temperature in two dimensions.[10] Let us therefore consider the worst-case scenario, $T_g^{(2D)} \rightarrow 0$. The emergence of long-range vortex-glass order across layers from inside the weakly coupled vortex liquid (12) survives this limit, since the 2D phase correlation length ξ_{2D} remains divergent. Secondly, it is important to notice that the field equation (17) used to obtain conventional phase correlations across layers (21) inside of the vortex glass is independent of the superfluid density $\rho_s^{(2D)}$. This indicates that the stability bound in perpendicular magnetic field for the conventional vortex lattice, Φ_0/Λ_1^2, survives the limit $T_g^{(2D)} \rightarrow 0$ as well.

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