Phase states in dc SQUIDs with inhomogeneous junction parameters

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By means of perturbation analysis, assuming small inductance values, the dynamical equations for the gauge-invariant superconducting phase differences in a dc superconducting quantum interference device (SQUID), containing junctions with different resistive and coupling parameters, are reduced to a single non-linear differential equation. The resulting effective reduced potential shows that degenerate phase states exist for half-integer values of the applied flux number Ψ_{ex} and for zero bias current. It can be also shown that, by small variations of the externally applied flux, this degeneracy can be removed and, for sufficiently low values of the SQUID parameter β, different magnetic states, characterized by opposite magnetic moments, can be reversibly realized.

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I. INTRODUCTION

The static and dynamic properties of dc superconducting quantum interference devices (SQUIDs) have been widely investigated in the literature.[1–3] Recently, the two-junction interferometer has been proposed as a phase qubit,[4] a basic unit for quantum computing. Devices containing three[5] or more[6] Josephson junctions (JJs) have also been proposed to obtain better defined minima in their potential energy, when compared to the two-junction interferometer states. The use of π junctions has also been proposed[7–9] in order to have "quiet" devices, operated around null magnetic field values, which are more robust with respect to flux fluctuations. The analyses of these systems are, in general, carried out by assuming negligible values of the inductance L of a single branch of the device, so that $\beta = LI_J/\Phi_0 \approx 0$, where Φ_0 is the elementary flux quantum, and $I_J = (I_{J1} + I_{J2})/2$ is the mean value of the maximum Josephson currents of the JJs. In this limit, indeed, considering a two-junction interferometer, one can write a single effective dynamical equation for the superconducting phase differences ϕ_1 and ϕ_2 across the two JJs, considerably simplifying the analysis, which should be carried out, in different limits, by considering two coupled nonlinear differential equations for ϕ_1 and ϕ_2. By reducing the dimensionality of the mathematical problem to one, by simply setting $\beta = 0$, however, important features of the system as a whole are lost. For instance, appearance of half-integer Shapiro steps, already observed by Vanneste *et al.*,[10] cannot be predicted. Recently, Romeo and De Luca,[11] starting from the full dynamical system, assuming a series solution for the magnetic flux variable in terms of the perturbation parameter β, have noticed that, in the overdamped limit, the two-junction interferometer model still reduces to a single nonlinear ordinary differential equation for the superconducting phase variable $\varphi = (\phi_1 + \phi_2)/2$. This perturbation approach allows derivation of the SQUID dynamical properties in analogy with the well-known results obtained for a single overdamped Josephson junction, provided that, in carrying out the analogy, an effective unconventional current-phase relation is attributed to the one-junction model for the device. By this perturbation approach the authors calculate, in closed analytic form, the amplitude of the half-integer Shapiro steps appearing in these devices. By a similar approach, applied to a multijunction interference device containing N JJs on each branch, the same authors predict appearance of half-integer Shapiro steps only for odd values of N.[12] Furthermore, by the same analytical method, the dynamics of one-dimensional overdamped Josephson junction arrays with an arbitrary number of JJs is described by a single nonlinear differential equation.[13]

The same perturbation approach is adopted, in the present work, to define the phase states of the system by an effective potential analysis, which correctly gives the reduced dynamical equations in the small β limit, as obtained by applying the resistively shunted junction (RSJ) model to both JJs. By retaining finite values of the parameters characterizing the inhomogeneity in the junction coupling energy and resistance, we find that the metastable energy states of the system for applied fluxes close to $\Phi_0/2$ present a double-well degeneracy, which is not detectable in the $\beta = 0$ limit.

The paper is thus organized as follows. In the next section the effective potential approach, in conjunction with the derivation of dynamical equations of the complete model of a dc SQUID, is presented. In the third section, by means of a first-order perturbation analysis, the two-dimensional effective potential is reduced to a one-dimensional potential, where the parameters characterizing the resistive and coupling inhomogeneity of the junctions are seen to play an important role in the system dynamics. Starting from the reduced effective potential of the system, the phase states are defined for null values of the bias current and for normalized fluxes close to half-integer values in the fourth section. Conclusions are drawn in the last section and possible applications of the present study are briefly mentioned.

II. EFFECTIVE POTENTIAL FOR A dc SQUID

In the present section we shall show that the dynamical equations of a symmetric dc SQUID, containing Josephson junctions with nonhomogeneous junction parameters, can be written from the notion of an effective potential U_{eff} defined in terms of the superconducting phase variable φ and of the normalized flux variable (or flux number) $\Psi=[(\phi_2-\phi_1)/2\pi]+n$, with n being an integer.

Let us assume that the inductance L associated to one branch of the dc SQUID is such that $\beta=LI_J/\Phi_0\ll 1$. We start by noticing that the variables φ and Ψ play an important role in defining the electrodynamic properties of the system. For example, the instantaneous voltage V across the device can be expressed as $V=(\Phi_0/2\pi)(d\varphi/dt)$, and the dc magnetic susceptibility $\chi_{d.c.}$ as $(\Psi-\Psi_{ex})/\Psi_{ex}$, where $\Psi_{ex}=\mu_0 HS_0/\Phi_0$, with μ_0 being the permeability of vacuum, H is the applied magnetic field, and S_0 is the geometric area of the SQUID.

The effective potential of the SQUID must be related to the Josephson coupling energy and to the electrodynamic energy due to the presence of the normalized applied magnetic flux Ψ_{ex} and to the bias current $i_B=I_B/I_J$. Indeed, being ϕ_1 and ϕ_2 the gauge-invariant phase differences across the two JJs, we may define the Josephson coupling energy E_J as follows:

$$E_J=\frac{\Phi_0 I_J}{2\pi}\sum_{k=1}^{2}(1+\varepsilon_k)(1-\cos\phi_k), \tag{1}$$

where we have written the maximum Josephson current of the kth JJ as $I_{Jk}=I_J(1+\varepsilon_k)$, with $\varepsilon_1=\varepsilon$ and $\varepsilon_2=-\varepsilon$. Furthermore, the electrodynamic potential energy can be written as follows:

$$E_B=-\frac{\Phi_0 I_J}{2\pi}\sum_{k=1}^{2}\int i_k d\phi_k, \tag{2}$$

where $i_k=I_k/I_J$ is the normalized current flowing in the kth $(k=1,2)$ SQUID branch. By fluxoid quantization, the same definition of the flux variable Ψ is recovered,

$$\phi_1-\phi_2+2\pi\Psi=2\pi n, \tag{3}$$

where n is an integer. Furthermore, the total flux Ψ is the result of the superposition of the applied flux Ψ_{ex} and of the induced flux, so that

$$\Psi=\Psi_{ex}+\beta(i_1-i_2). \tag{4}$$

By now defining Ψ in terms of the phase difference $(\phi_2-\phi_1)$ from Eq. (3) and considering Eq. (4), by a change of variables in Eqs. (1) and (2), we obtain

$$U_{eff}(\varphi,\Psi)=E_J+E_B=E_0\left[2-(-1)^n 2(\cos\pi\Psi\cos\varphi+\varepsilon\sin\pi\Psi\sin\varphi)-i_B\varphi+\frac{\pi}{2\beta}(\Psi-\Psi_{ex})^2\right], \tag{5}$$

where $E_0=\Phi_0 I_J/2\pi$. The time evolution of the variables ϕ_1 and ϕ_2 in the overdamped regime can be obtained by setting

$$\frac{1}{1+\delta}\frac{d\phi_1}{d\tau}=-\frac{\partial u_{eff}}{\partial\phi_1}, \tag{6a}$$

$$\frac{1}{1-\delta}\frac{d\phi_2}{d\tau}=-\frac{\partial u_{eff}}{\partial\phi_2}, \tag{6b}$$

where $u_{eff}=U_{eff}/E_0$, $\tau=(2\pi RI_J/\Phi_0)t$ is a normalized time, and the parameter δ takes account of the inhomogeneity of the resistive junction parameters R_1 and R_2, so that $R_1=(1+\delta)R$, $R_2=(1-\delta)R$, where $R=(R_1+R_2)/2$. The set of equations [Eqs. (6a) and (6b)] can be intuitively understood in terms of the dynamics of a massless point particle moving in a fictitious two-dimensional space (ϕ_1,ϕ_2) under the action of velocity dependent damping and of a potential u_{eff}. In order to prove the validity of Eqs. (6a) and (6b), it suffices to show that they give rise to the correct phase dynamics for the two JJs. Indeed, first notice that $u_{eff}(\phi_1,\phi_2)=(E_J+E_B)/E_0$, where E_J and E_B are given by Eq. (1) and Eq. (2), respectively. By differentiating $u_{eff}(\phi_1,\phi_2)$ first with respect to ϕ_1 and then with respect to ϕ_2, and by substituting the results in Eq. (6a) and in Eq. (6b), respectively, we find

$$\frac{1}{1+\delta}\frac{d\phi_1}{d\tau}+(1+\varepsilon)\sin\phi_1=i_1, \tag{7a}$$

$$\frac{1}{1-\delta}\frac{d\phi_2}{d\tau}+(1-\varepsilon)\sin\phi_2=i_2. \tag{7b}$$

Equations (7a) and (7b) represent the dynamical evolution of the phase variables ϕ_1 and ϕ_2 as given by the RSJ model, as it was to be proven.

By the following change of variables:

$$\varphi=\frac{\phi_1+\phi_2}{2}, \tag{8a}$$

$$\Psi=\frac{\phi_2-\phi_1}{2\pi}+n, \tag{8b}$$

we can rewrite Eqs. (6a) and (6b) as follows:

$$\frac{d\varphi}{d\tau}=-\frac{1}{2}\left(\frac{\partial u_{eff}}{\partial\varphi}-\frac{\delta}{\pi}\frac{\partial u_{eff}}{\partial\Psi}\right), \tag{9a}$$

$$\pi\frac{d\Psi}{d\tau}=-\frac{1}{2}\left(\frac{1}{\pi}\frac{\partial u_{eff}}{\partial\Psi}-\delta\frac{\partial u_{eff}}{\partial\varphi}\right). \tag{9b}$$

The above equations, together with Eq. (5), give the following dynamical equations for the variables φ and Ψ:

$$\frac{d\varphi}{d\tau}+(-1)^n[(1+\delta\varepsilon)\cos\pi\Psi\sin\varphi-(\varepsilon+\delta)\sin\pi\Psi\cos\varphi]=\frac{i_B}{2}+\frac{\delta}{2\beta}(\Psi-\Psi_{ex}), \tag{10a}$$

$$\pi\frac{d\Psi}{d\tau}+(-1)^n[(1+\delta\varepsilon)\sin\pi\Psi\cos\varphi-(\varepsilon+\delta)\cos\pi\Psi\sin\varphi]=-\frac{1}{2\beta}(\Psi-\Psi_{ex})-\delta\frac{i_B}{2}. \tag{10b}$$

The above set of equations represents the complete dynamical model, expressed in terms of the variables φ and Ψ, for a symmetric dc SQUID with inhomogeneous junction parameters in the overdamped limit.

III. REDUCED MODELS

Considering the dynamical equations of the system as written in Eqs. (10a) and (10b), we notice that, for $\beta=0$, the two equations decouple: Equation (10b) becomes the trivial identity $\Psi=\Psi_{ex}$, while Eq. (10a) defines the time evolution of the average superconducting phase and can be rewritten as follows:

$$\frac{d\varphi}{d\tau}+(-1)^n[(1+\delta\varepsilon)\cos\pi\Psi_{ex}\sin\varphi-(\varepsilon+\delta)\sin\pi\Psi_{ex}\cos\varphi] = \frac{i_B}{2}. \tag{11}$$

According to what stated in the previous section, the normalized effective potential $u_{eff}=U_{eff}/E_0$ becomes a function of φ, and the problem becomes effectively one-dimensional, since

$$u_{eff}=u_{eff}(\varphi,\Psi_{ex})=2-(-1)^n 2(\cos\pi\Psi_{ex}\cos\varphi + \varepsilon\sin\pi\Psi_{ex}\sin\varphi)-i_B\varphi, \tag{12}$$

where Ψ_{ex} plays the role of a parameter. We notice that in the $\beta=0$ case, the only relevant parameter is ε, since the parameter δ, which is linked to the fluxon dynamics through the dynamical equation for Ψ [Eq. (10b)], does not enter the expression for the potential u_{eff}, given that Ψ is assumed not to depend on time. In Figs. 1(a) and 1(b), we represent the effective potential for $i_B=0$ and for various values of the applied flux Ψ_{ex}. In particular, in Fig. 1(a) we report the u_{eff} versus φ curves for $\varepsilon=0.1$, while in Fig. 1(b) the parameter ε is doubled. The system is seen to behave much like a symmetric SQUID with perfectly identical junctions, so that no new features are detected in its physical properties. In particular, we notice that for small field values [take, for example, the $\Psi_{ex}=0.1$ curve in Fig. 1(a)], the system, when prepared under zero-field-cooling conditions, rests in a phase state $\varphi=\tilde{\varphi}\approx 0$. At $\Psi_{ex}=0.5$, however, the phase state $\tilde{\varphi}$ disappears, and the new equilibrium value $\hat{\varphi}=\pi/2$ appears. As the normalized applied flux value goes over $\Psi_{ex}=0.5$, phase states closer to $\varphi=\pi$ arise.

We must thus abandon the trivial case with $\beta=0$ in order to obtain well-defined nontrivial flux states in the device, yet keeping β small. We thus develop a first-order perturbation solution in β of the dynamical equation for the flux variable Ψ, so that we write

$$\Psi(\tau)=\Psi_{ex}+\beta\Psi_1(\tau)+O(\beta^2). \tag{13}$$

By substituting Eq. (13) into Eq. (10b) we find,

$$\Psi_1(\tau)=-2(-1)^n[(1+\delta\varepsilon)\sin(\pi\Psi_{ex})\cos\varphi - (\varepsilon+\delta)\cos(\pi\Psi_{ex})\sin\varphi]-\delta i_B. \tag{14}$$

In this way, Eq. (10a) becomes

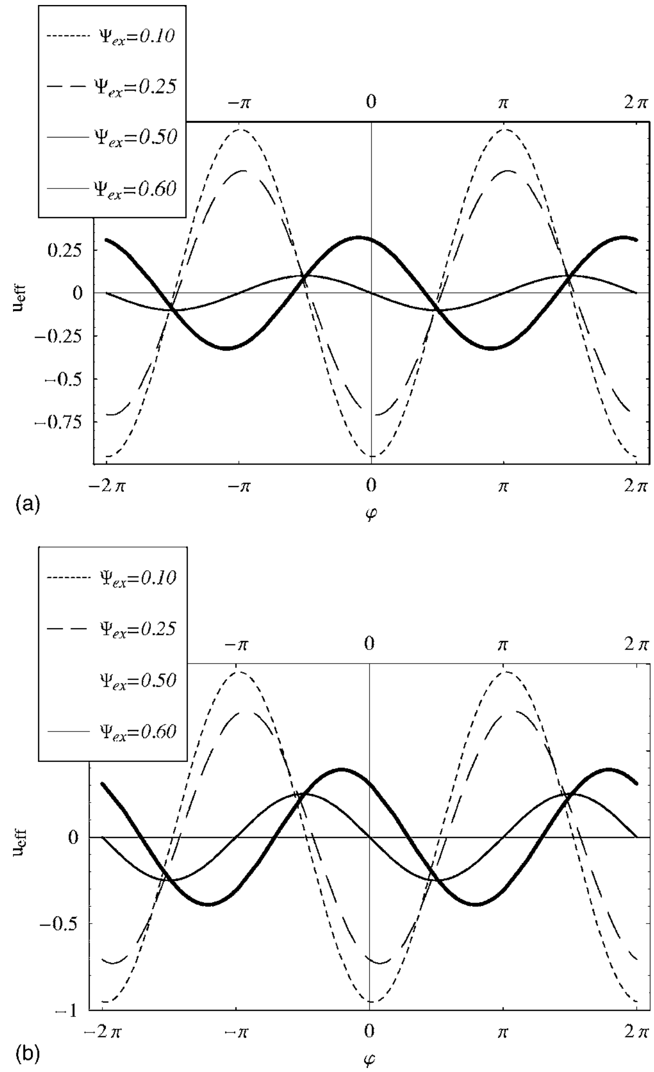

FIG. 1. Effective potential u_{eff} for $i_B=0$ and for various values of the applied flux Ψ_{ex} (in the legend) as a function of the average superconducting phase φ. (a) u_{eff} vs φ curves for $\varepsilon=0.1$, (b) u_{eff} vs φ curves for $\varepsilon=0.2$.

$$\frac{d\varphi}{d\tau}+b_1\sin\varphi+b_2\sin 2\varphi+a_1\cos\varphi+a_2\cos 2\varphi=\frac{i_B}{2}(1-\delta^2), \tag{15}$$

where

$$\begin{aligned} b_1&=(-1)^n[(1-\delta^2)\cos(\pi\Psi_{ex})+\delta(1+\varepsilon\delta)\pi\beta i_B\sin(\pi\Psi_{ex})],\\ b_2&=\pi\beta[(1+\varepsilon\delta)^2\sin^2(\pi\Psi_{ex})-(\varepsilon+\delta)^2\cos^2(\pi\Psi_{ex})],\\ a_1&=(-1)^n[\delta(\varepsilon+\delta)\pi\beta i_B\cos(\pi\Psi_{ex})-\varepsilon(1-\delta^2)\sin(\pi\Psi_{ex})],\\ a_2&=\pi\beta(\varepsilon+\delta)(1+\varepsilon\delta)\sin(2\pi\Psi_{ex}). \end{aligned} \tag{16}$$

In Eq. (15), which is the reduced version of the symmetric SQUID model presenting inhomogeneity in the junction parameters, we notice the appearance of second-harmonic terms. Solving the stationary part of Eq. (15) for i_B and maximizing this quantity with respect to φ, we find the normal-

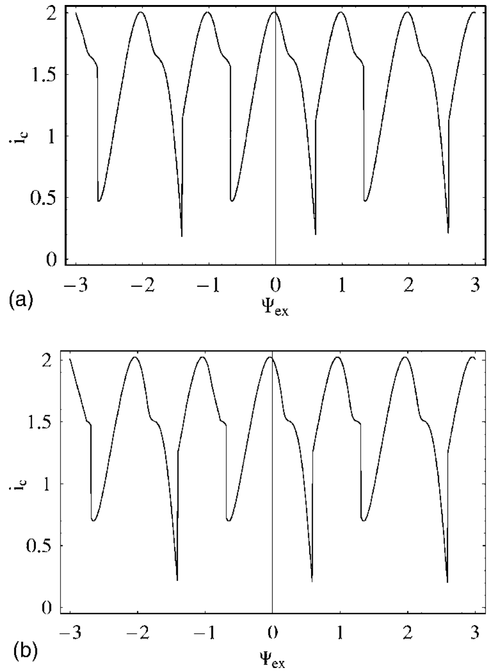

FIG. 2. Normalized critical current for $n=0$ and $\beta=0.1$. In (a) the inhomogeneity parameters are $\delta=0.4$ and $\varepsilon=0.1$, while in (b) the inhomogeneity parameters are $\delta=0.4$ and $\varepsilon=0.2$.

ized critical current of the device, which is represented in Figs. 2(a) and 2(b) for $n=0$ and $\beta=0.1$. In Fig. 2(a) the parameters values are $\delta=0.4$ and $\varepsilon=0.1$, while in Fig. 2(b) the parameters are $\delta=0.4$ and $\varepsilon=0.2$. We notice a first important feature of the system: The periodicity of the critical current of the device with respect to Ψ_{ex} is twice the periodicity observed in dc SQUIDs with very low inductance. This is just a first appearance of nonordinary phenomena in these rather interesting systems. The change in the dimensionality of the problem (we are actually also considering the dynamics of flux states) is introducing new interesting properties in the system.

IV. REDUCED POTENTIAL AND PHASE STATES

The reduced effective potential in the overdamped case can be obtained by rewriting Eq. (15) in the following form:

$$\frac{d\varphi}{d\tau} = -\frac{\partial u_{red}}{\partial \varphi}, \tag{17}$$

so that

$$u_{red} = -b_1 \cos\varphi - \frac{b_2}{2}\cos 2\varphi + a_1 \sin\varphi + \frac{a_2}{2}\sin 2\varphi - \frac{i_B}{2}(1-\delta^2)\varphi. \tag{18}$$

A representation of the reduced potential is given in Figs. 3(a) and 3(b) for $n=0$, $i_B=0$, and $\beta=0.1$. In Fig. 3(a) the parameters characterizing the inhomogeneity of the JJs are $\delta=0.6$ and $\varepsilon=0.3$, while in Fig. 3(b) we have $\delta=0.4$ and $\varepsilon=0.1$. Notice that, at the flux value $\Psi_{ex}=1/2$, the potential

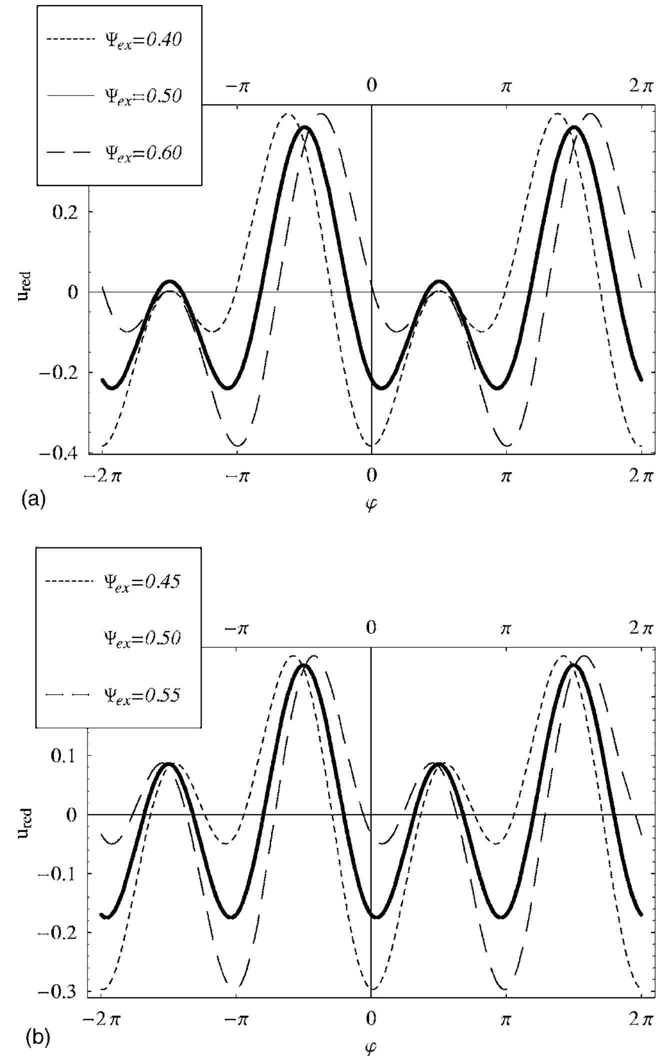

FIG. 3. Effective potential $u_{\rm eff}$ for $i_B=0$, $n=0$, and $\beta=0.1$ and for various values of the applied flux Ψ_{ex} (in the legend) as a function of the average superconducting phase φ. (a) u_{eff} vs φ curves for $\delta=0.6$ and $\varepsilon=0.3$, (b) u_{eff} vs φ curves for $\delta=0.4$ and $\varepsilon=0.1$.

presents a double well with a barrier that is comparable to the Josephson energy $E_0=\Phi_0 I_J/2\pi$ (which in normalized units is 1). Starting from one of these degenerate states, one at $\varphi_0=\sin^{-1}(1/\lambda)$, one $\varphi_1=\pi-\sin^{-1}(1/\lambda)$, where $\lambda=b_2/2|a_1|$, we can smoothly go from one to the next by small variations of the externally applied flux, as also suggested by Figs. 3(a) and 3(b). In Fig. 3(a), indeed, the potential obtained for $\Psi_{ex}=1/2$ is represented along with other two curves, one for $\Psi_{ex}=0.4$ and one for $\Psi_{ex}=0.6$. For $\Psi_{ex}=1/2$ the energy barrier height can be easily calculated to be $\Delta u_{red}=b_2-|a_1|$. Therefore, in order to have $\Delta u_{red}>0$ the following inequality must be satisfied: $\beta>-\varepsilon(1-\delta^2)/\pi(1+\varepsilon\delta)^2$, where we have assumed $\varepsilon,\delta<1$. We notice that, while for $\Psi_{ex}=0.4$, the metastable state with phase φ_1 is unfavored with respect to the stable state $\varphi=\varphi_0$, the contrary is true for $\Psi_{ex}=0.6$. The same can also be noticed in Fig. 3(b), where the field variation necessary to produce the crossover can be argued to be smaller.

If we compare these features with the ordinary SQUID behavior, reported in Figs. 1(a) and 1(b), we notice that states

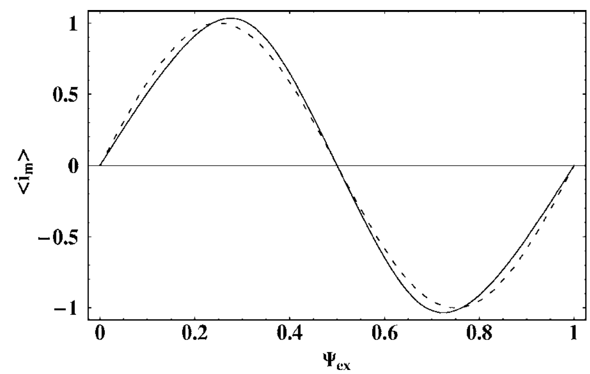

FIG. 4. Average time value of the circulating current $i_m = \Psi_1(\tau)$ as a function of the applied flux for $n=0$, $i_B=0$, $\beta=0.1$ and for the inhomogeneity parameters $\delta=0.4$ and $\varepsilon=0.1$ (full line). The dashed line represents the curve $\langle i_m \rangle = \sin(2\pi\Psi_{ex})$ for comparison.

close to the zero value of the average superconducting phase φ are neither stable nor metastable for $\Psi_{ex}=1/2$. Furthermore, while the reduced SQUID model for $\beta=0$ does not allow current states (zero magnetic moment can be associated to the SQUID loop for every value of the externally applied flux), we can see that different current states apply to the phase states φ_0 and φ_1, as it can be inferred from Fig. 4, where the average time value of the circulating current $i_m = (i_1 - i_2) = (\Psi - \Psi_{ex})/\beta = \Psi_1(\tau)$ is graphed as a function of the applied flux for $n=0$, $i_B=0$, $\beta=0.1$, and for $\delta=0.4$ and $\varepsilon=0.1$. All states are realized by starting from zero trapped field inside the SQUID loop and by gradually increasing the applied flux from zero. After each small increase in the Ψ_{ex}, we let the system reach its equilibrium phase state, from which we start to further increase Ψ_{ex} and to recalculate the phase state realized for such flux value. In this curve it is important to notice how the circulating current, which is zero at $\Psi_{ex}=1/2$, goes from positive to negative values for increasing fields around half-integer values of Ψ_{ex}. This feature is important for detection of the phase state in the dc SQUID by an external readout unit. Indeed, by starting from a zero circulating current state and increasing the external applied flux, the metastable φ_0 state becomes energetically unfavorable, until the system switches to the φ_1 state in a time that can be roughly estimated by the Arrhenius formula. Another important aspect to point out is that the system shows reversible behavior for $\beta=0.1$ around $\Psi_{ex}=1/2$, as it can be numerically detected.

V. CONCLUSIONS

We have solved the analytic problem related to finding the phase states of a symmetric dc SQUID containing junctions with inhomogeneous parameters under the influence of a nonzero applied magnetic flux and zero bias current for low β values. The analytic investigation of the problem is made possible by a perturbation solution of the dynamics of the flux number Ψ, which is used to find the approximated time evolution of the average superconducting phase difference φ.

The solution of the present problem allows us to characterize phase states by means of a one-dimensional effective reduced potential. From the potential curves at different values of the applied magnetic flux, it can be argued that degenerate phase states exist for half-integer values of the applied flux number Ψ_{ex}. Nondegenerate states with positive, or negative, magnetic moments can be realized by increasing, or decreasing, the applied magnetic flux starting from the zero circulating current state at $\Psi_{ex}=1/2$. These features might suggest that the device has the characteristics of a phase qubit, in which the degenerate phase states are obtained at half-integer values of the normalized applied magnetic flux. However, in order to adapt the above approach to a genuine quantum system, one needs to remove the assumption of overdamped phase dynamics. Therefore, in the case of finite values of the capacitance of the Josephson junctions, reduction of the problem is not possible, so that the potential energy of the system needs to be defined in the two-dimensional space (ϕ_1, ϕ_2). Future work will be devoted to investigate the system beyond the overdamped limit. Nevertheless, as for an immediate application of the present analysis, we notice that the proposed physical system may provide an experimental tool to investigate the stochastic dynamics of asymmetrical bistable systems.

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