Vortex-antivortex configurations in a superconducting film due to a ferromagnetic strip: Edge barrier versus annihilation barrier

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Magnetic response of a hybrid structure containing a thin ferromagnetic (FM) strip, placed above a superconducting (SC) film of finite width, is studied. It is shown that depending on the system parameters the maximum value of the Meissner current may be reached either at the SC film edges or at its symmetry axis. Total energy of the symmetrical vortex-antivortex (VA) configuration (which is only possible due to the symmetry of a system) is calculated depending on the FM magnetization. It is found that the emergence of the VA pairs is controlled by two potential barriers: A Bean-Livingston-like edge barrier and the "annihilation" barrier. The former one controls the VA pair entry from the SC film edges, the latter determines the creation of the VA pair at the center of the SC film. The threshold magnetization is obtained at which the transition of the SC film from the Meissner state to the VA-pair-formed mixed state, takes place. The conditions are found at which the VA pair is not allowed to exist inside the SC film.

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I. INTRODUCTION

The interaction between superconductivity and magnetism in the heterogeneous magnetic-superconducting system (HMSS) has been the subject of numerous studies (see, e.g. Refs. 1–18). Such systems contain usually ferromagnetic (FM) and superconducting (SC) parts separated by a thin layer of insulating oxide to avoid the proximity effect. Both in-plane and out-of-plane magnetization effects such as the Abrikosov vortices creation and pinning were experimentally studied in such systems. An inhomogeneous magnetization of the FM produces the magnetic field which penetrates into the superconductor, induces supercurrent and subsequently modifies the resultant magnetic field. The magnetic flux from magnetic textures may create the Abrikosov vortices inside the superconductor and even pin them, thus changing drastically the magnetic response of the system. To study the flux-pinning ability of type-II superconducting films, several experimental groups have fabricated periodic arrays of magnetic dots and antidots placed above or under the superconducting film.[3–6] The results indicate that pinning properties and vortex dynamics in thin superconducting films can be manipulated by tuning the configuration of the magnetic dot array. It was shown recently by Lange *et al.* that a nanoengineered lattice of magnetic dots (dipoles) on top of the superconducting film can be used to selectively enhance the critical field of the superconducting film.[7] Magnetic-field-induced superconductivity was observed in such hybrid superconductor/ferromagnet systems due to the compensation of the applied field by the stray field of the dipoles. As it was noted in Ref. 7 the dipole array field compensator can also be used to design logical devices in which superconductivity is controlled by switching between the two polarities of the magnetized dot array. Recent progress in the development of magnetic thin-film technology has triggered a lot of interest in the field. Indeed, systems in which both the FM and SC parts are made of thin film are extremely important for technological applications, such as devices which can be tuned by a weak magnetic field. Miloševič *et al.*[8] proposed a FM-SC device consisting of a submicron superconducting sample combined with an in-plane ferromagnet where both the critical field and critical current may be substantially enhanced.

In the majority of previous theoretical investigations on the FM-SC heterostructures there practically was not considered the possibility of the existence of an edge barrier controlling the vortex penetration conditions into (or exiting from) the SC film. Recently, Erdin[9] had studied the example of a HMSS, consisting of a semi-infinite FM film on a top of a semi-infinite SC film. It was shown that the system manifests the Bean-Livingston type surface barrier for the vortex entry/exit, which is controlled by the FM film magnetization and by the Ginsburg parameter of the SC film.[9] Meanwhile, the magnetic response of hybrid systems consisting of type-II finite-width superconducting film and of a magnetic strip placed nearby, have not been considered exhaustively. Since the strip geometry is quite common in superconducting applications, this problem is also of current technological relevance.

In this paper we study the vortex states due to a ferromagnetic strip, possessing an in-plane magnetization, placed above a SC film of finite thickness d and width w. Similar geometry was studied experimentally and theoretically in Ref. 10. Considerable enhancement was found of the critical current in the SC strip due to a magnetic strip on top on it. We consider the narrow film at which the Pearl length $\lambda_\perp = 2\lambda^2/d$ is much greater than w: $\lambda_\perp \gg w$ (λ is the London penetration depth). Such narrow superconducting films deserve appreciable attention from the experimental and theoretical viewpoints. Recently, it was shown[19] that the superconducting devices designed using narrow wires quite efficiently suppress the noise generation due to the vortex motion. In addition, narrow current-carrying thin-film bridges offer a unique opportunity to study both thermal activation and quantum tunneling through the well-designed potential barrier.[20]

In Sec. II we consider the Meissner state of the superconducting film generated by a ferromagnet. Our calculations

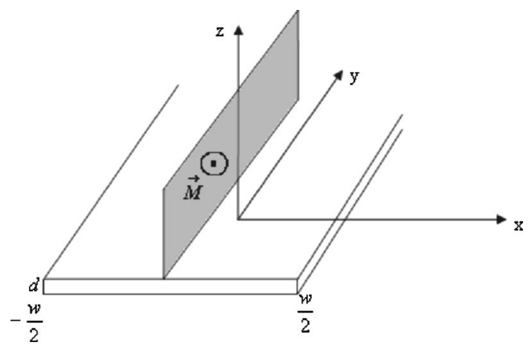

FIG. 1. Thin magnetic strip with height H and magnetization M located at distance z_0 above the superconducting film with thickness d and width w.

show that the value of the screening current is maximal either at the center of the SC film or at its edges. In Sec. III we study the interaction energy between a vortex and the magnetic structure and discuss the possibility of the magnetic pinning of a vortex. In Sec. IV the appearance of the symmetrical vortex-antivortex configuration in the considered HMSS is studied. We investigate how such a vortex state in the SC may occur depending on the parameter of the FM. Our results are summarized in Sec. V.

II. MEISSNER STATE OF A SUPERCONDUCTING FILM

Consider a narrow pin-free superconducting film of width $w(-w/2 \leqslant x \leqslant w/2)$ and thickness $d < \xi \ll \lambda$ ($w \ll \lambda_\perp = \lambda^2/d$, ξ is the coherence length) which are infinite in the y direction (Fig. 1). A thin magnetic strip of height H with the magnetization $\vec{M}$ directed along the plane of the superconducting film (for simplicity, we use the term the "in-plane magnetization")

$$\vec{M} = M_0 \delta(x) \theta(z_1 - z) \theta(z - z_0) \vec{e}_x \tag{1}$$

is located at a distance z_0 above the SC($z_1 = z_0 + H$), M_0 denotes the magnetic moment per unit surface. For the film of thickness less than the coherence length one may neglect the influence of the parallel field components H_x, H_y in the SC film and consider the two-dimensional problem.

To find the supercurrent induced by the FM we use the Maxwell-London equation[21–23]

$$\frac{2\pi}{c}\lambda_\perp \frac{di_m}{dx} + \frac{2}{c}\int_{-w/2}^{w/2} \frac{i_m(t)dt}{t - x} = H_z(x), \tag{2}$$

where $i_m(x) = j_y d$ is the sheet current density, satisfying the condition

$$\int_{-w/2}^{w/2} i_m(x)dx = 0, \tag{3}$$

$H_z(x)$ is the magnetic field produced by ferromagnet [Eq. (1)] at $z=0$

$$H_z(x) = \frac{2M_0 x(z_0^2 - z_1^2)}{(x^2 + z_0^2)(x^2 + z_1^2)}. \tag{4}$$

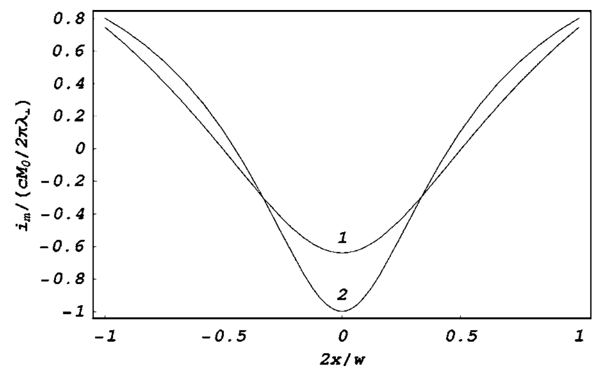

FIG. 2. Meissner current density in SC film at different values of parameters $\alpha_0 = 2z_0/w$ and $h = 2H/w$ for (1) $\alpha_0 = 0.5$, $h = 1.5$ [$|i_m(0)| < i_m(1)$], (2) $\alpha_0 = 0.3$, $h = 0.7$ [$|i_m(0)| > i_m(1)$].

For the case of a narrow film $\lambda_\perp/w \gg 1$ the self-field of the current [the second term in Eq. (2)] may be neglected. After that we obtain from Eqs. (2)–(4)

$$i_m(t) = -\frac{cM_0}{2\pi\lambda_\perp}\left(\ln\frac{t^2 + \alpha_1^2}{t^2 + \alpha_0^2} - g(1)\right), \tag{5}$$

where

$$g(u) = u \ln\frac{u^2 + \alpha_1^2}{u^2 + \alpha_0^2} - 2\alpha_0 \tan^{-1}\frac{u}{\alpha_0} + 2\,\alpha_1 \tan^{-1}\frac{u}{\alpha_1}. \tag{6}$$

In Eqs. (5) and (6) and below the distance is measured in units of a half-width of the superconducting film: $t = 2x/w$, $\alpha_0 = 2z_0/w$, $\alpha_1 = \alpha_0 + h$, where $h = 2H/w$ is a dimensionless height of the magnetic strip. The dependence $i_m(t)$ for different values of α_0 and h is shown in Fig. 2. Depending on the parameters of α_0 and h the value of current density at $t=0$ may be less or greater than its value at the edges $t = \mp 1$ (Fig. 3).

Let us discuss the situation when the superconducting film is initially in the Meissner state with the sheet current density $i_m(t)$, which is determined by Eq. (5). The vortex state in such a film may appear in two ways. Consider first the case

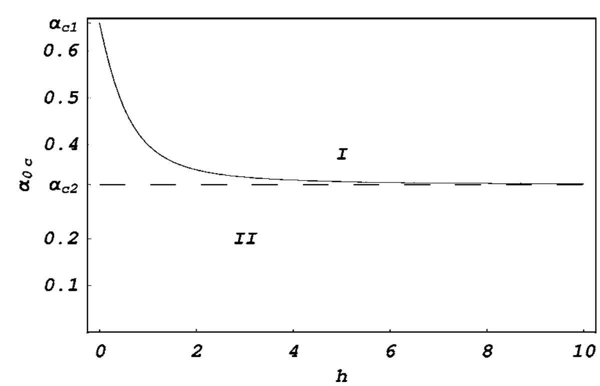

FIG. 3. Height dependence of the critical distance $\alpha_{0c}(h)$. The region I [$\alpha_{0c} > \alpha_{0c}(h)$] corresponds to the inequality $|i_m(0)| < i_m(1)$, the region II [$\alpha_{0c} < \alpha_{0c}(h)$]–$|i_m(0)| > i_m(1)$; at $\alpha = \alpha_{0c}(h)$ $|i_m(0)| = i_m(1)$. Here $\alpha_{0c} = 2z_{0c}/w$, $h = 2H/w$.

when the function $i_m(t)$ satisfies the condition $|i_m(0)| < i_m(1)$ and the magnetization of the FM is strong enough. In this situation the value of the current density near the film edge may first reach the order of the pair-breaking current density $i_s = \frac{c\Phi_0}{6\sqrt{3}\pi^2\xi\lambda_\perp}$ (Φ_0 is the flux quantum). According to the conventional scenario, the vortices (or antivortices) may nucleate at the left (right) edge and penetrate into the film. In the opposite case $|i_m(0)| > i_m(1)$, provided the magnetization M_0 is sufficiently high, the creation of a vortex-antivortex pair may occur at the center of the superconducting film. Therefore the critical curve $\alpha_0 = \alpha_{0c}(h)$ shown in Fig. 3 separates the two regions in the parameters plane (α_0, h), where the vortex and antivortex can appear close to the edges of the SC (region I in Fig. 3) or at the center of the superconducting film (region II in Fig. 3). Note that the creation of a VA pair in a superconducting film with a ferromagnet disk on top of it was studied in Ref. 24 on the basis of the nonlinear Ginzburg-Landau theory. It was shown microscopically that the VA pair creation occurs in the region where maximal current density reaches the value i_s. In that which follows we obtain the condition for the VA pair creation inside SC film by analyzing the behavior of the energy barriers.

III. MAGNETIC STRIP—VORTEX INTERACTION

The interaction energy E_{v-f} between a vortex in an infinite type-II superconducting film and the magnetic structure is given by (Refs. 11, 15, and 16)

$$E_{v-f} = \frac{1}{2c}\int dV(\vec{j}_m \vec{\Phi}_v) - \frac{1}{2}\int dV(\vec{h}_v \vec{M}). \tag{7}$$

Here $\vec{\Phi}_v = (\Phi_\rho, \Phi_\varphi, 0) = (0, \Phi_0/2\pi\rho, 0)$ is the part of the total vector potential related to the vortex,[25] $\vec{h}_v$ denotes the vortex-related magnetic field, and $\vec{j}_m$ is the density of Meissner current generated in the SC by the FM. The integration in the first term of Eq. (7) is performed over the volume of the superconductor, where $\vec{\Phi}_v \neq 0$, while the second integration is performed over the volume of the ferromagnet, where $\vec{M} \neq 0$. As it was noted in Ref. 15 the contributions of the first and second (including minus sign) terms in Eq. (7) are equal. Thus, we have

$$E_{v-f} = \frac{1}{c}\int dV(\vec{j}_m \vec{\Phi}_v). \tag{8}$$

For the superconducting film of width w containing a single vortex at the position $\vec{r}_0 = (x_0, 0)$ the expression for the $E_{v-f}(t)$ $(t = 2x_0/w)$ takes the form (see Appendix A)

$$E_{v-f}(t) = \frac{\Phi_0 w}{2c}\int_t^1 i_m(u)du. \tag{9}$$

Using Eq. (5) for the sheet current density $i_m(u)$ one finds from Eq. (9)

$$E_{v-f}(t) = \frac{m\varepsilon_0}{2}[g(t) - tg(1)], \tag{10}$$

where we denote

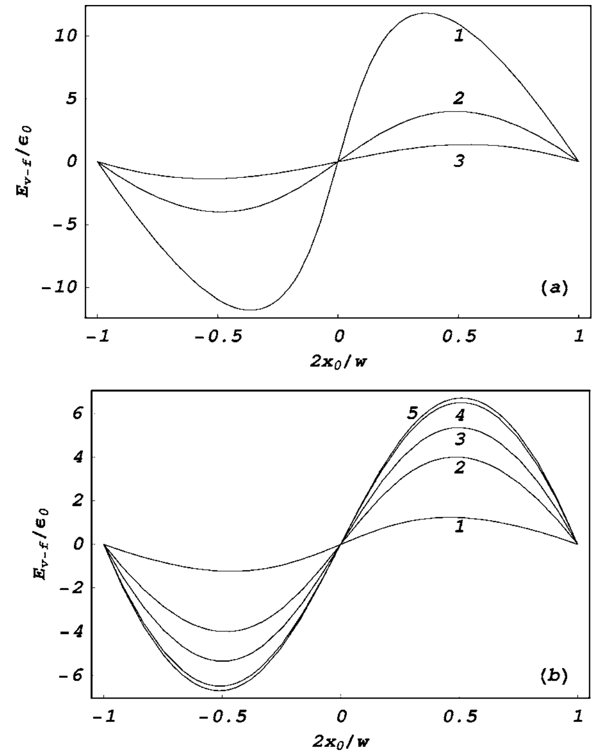

FIG. 4. Shown is the dependence of the normalized magnetic strip-vortex interaction energy E_{v-f} in units of $\varepsilon_0 = \Phi_0^2/(4\pi^2\lambda_\perp)$ as a function of the position of the vortex (x_0): (a) for the FM with the magnetization $m=60$ and the height $h=0.5$ and for distance $\alpha_0 = 0.1$, 0.5, 1 (curves 1, 2, and 3, correspondingly); (b) for the FM with the same magnetization $m=60$ placed at $\alpha_0=0.5$ above the SC and for different height $h=0.1$, 0.5, 1, 3, 6 (the curves 1, 2, 3, 4, and 5). Here $m = 2\pi w M_0/\Phi_0$, $\alpha_0 = 2z_0/w$, $h = 2H/w$.

$$\varepsilon_0 = \frac{\Phi_0^2}{4\pi^2\lambda_\perp}, \quad m = \frac{2\pi w M_0}{\Phi_0}, \tag{11}$$

and the function $g(t)$ is given by Eq. (6).

It is well known that the in-plane magnetized dipole pins the vortex at its negative pole, where the magnetic field of the vortex is parallel to that one of the dipole[17] (an antivortex present in the system would be pinned near the opposite pole). A similar qualitative behavior for the in-plane magnetizated strip [Eq. (1)] placed outside an infinite SC film was revealed in Ref. 16. The magnetic strip-vortex interaction energy $E_{v-f}(t)$, across the narrow SC film is shown in Fig. 4. The interaction energy depends on the height of the FM strip $h = 2H/w$ as well as on its separation from the SC film $\alpha_0 = 2z_0/w$. As one can see from Fig. 4(a) the amplitude of the interaction energy reduces if the magnetic strip is displaced further above the SC. The increase of the height of the FM strip leads to the increase of the total magnetic moment of the FM structure. As it follows from Eq. (10), the magnet strip-vortex interaction energy increases with h in the height range $0 \leq h \leq 10\alpha_0$. However, for a large enough strip height $h > 10\alpha_0$ the interaction energy becomes rather independent of h [see Fig. 4(b)]. The reason is that in this limit the z—component of the magnetic field $H_z(x)$ produced by the

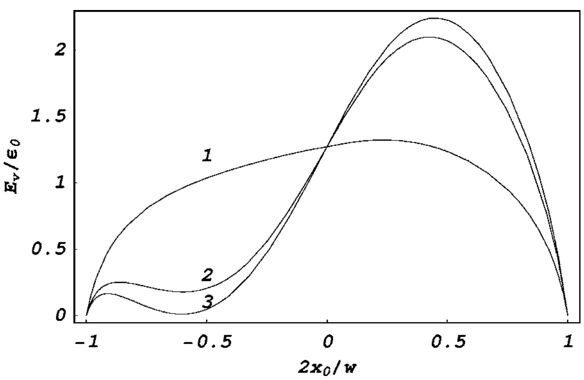

FIG. 5. Normalized total energy $[\varepsilon_0=\Phi_0^2/(4\pi^2\lambda_\perp)]$ of a vortex as a function of its position (x_0) for the magnetic strip (of height $h=0.5$ and separation $\alpha_0=0.5$) with different magnetization m for (1) $m=1.5$, (2) $m=14$, (3) $m=m_{c1}=16.7$. Here $m=2\pi w M_0/\Phi_0$, $\alpha_0=2z_0/w$, $h=2H/w$.

FM near the SC-plane ($z=0$) [see Eq. (4)] remains practically independent of size h and is equal to

$$H_z(x) = -\frac{2M_0 x}{x^2+z_0^2}, \quad |x| \leq w/2. \tag{12}$$

The asymptotic expression for the interaction energy in the limit $h\to\infty$ can be derived from Eqs. (6) and (10)

$$E_{v-f}(t) = \frac{m\varepsilon_0}{2}\left[t \ln\frac{1+\alpha_0^2}{t^2+\alpha_0^2} + 2\alpha_0\left(t \tan^{-1}\frac{1}{t} - \tan^{-1}\frac{t}{\alpha_0}\right)\right]. \tag{13}$$

In our case the equilibrium position of the vortex (or antivortex) is determined by both the interaction energy Eq. (10) and the self-energy E_{sv} (Ref. 26)

$$E_{sv}(x_0) = \frac{\Phi_0^2}{8\pi^2\lambda_\perp}\ln\left[\frac{w}{\pi\xi}\cos\left(\frac{\pi x_0}{w}\right)\right], \quad |x_0| \leq w/2-\xi. \tag{14}$$

The latter energy E_{sv} results from the attraction between the vortex and its images.[27]

Thus, if the superconducting film contains some vortices,[28] they are localized at the equilibrium line $x^* = \frac{w}{2}t^*(x^*<0)$, the position of which is defined by the equation

$$\left.\frac{\partial E_v(m,t)}{\partial t}\right|_{t=t^*} = 0, \tag{15}$$

where the total vortex energy of $E_v=E_{v-f}+E_{sv}$ is given by

$$E_v(m,t) = \frac{\varepsilon_0}{2}\left\{\ln\left[\frac{2}{\pi\xi}\cos\left(\frac{\pi t}{w}\right)\right] + m[g(t)-tg(1)]\right\}. \tag{16}$$

The solution of Eq. (15) defines the position $t^*(m)$ of the local minimum of the vortex energy $E_v(t^*)$ which is separated by the potential barriers from the edge and the center of the film (Fig. 5). If the vortex energy is positive: $E_v(t^*)>0$, then the vortex state is metastable. The critical magnetization $m_{c1}(\alpha_0,h)$, at which $E_v(t^*)=0$, so that the existence of the vortex (or antivortex) inside the SC becomes energetically favorable, may be found from the conditions

$$\left.\frac{\partial E_v(t)}{\partial t}\right|_{t=t^*} = 0, \quad E_v(t^*) = 0. \tag{17}$$

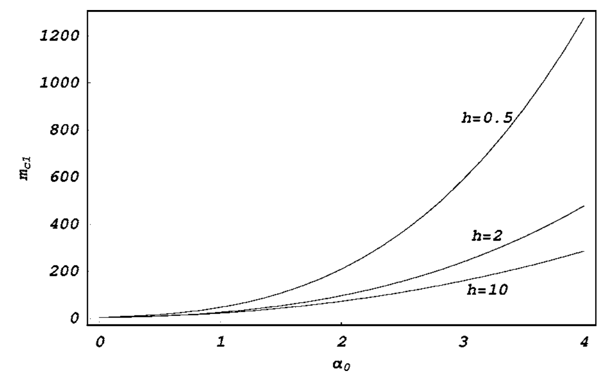

FIG. 6. Shown is the dependence of the normalized critical magnetization for a vortex $m_{c1}=\frac{2\pi w M_{c1}}{\Phi_0}$ as a function of the distance between magnetic strip and the superconducting film $\alpha_0=2z_0/w$. $h=2H/w$ is a dimensionless height of the magnetic strip.

In Fig. 6 we plot the magnetization m_{c1} obtained from Eq. (17) versus the distance of the FM over the SC plane for the magnetic strips of different height. It is clear that if the magnetic strip is displaced higher above the superconducting film, the interaction reduces and it is necessary to increase the value m_{c1} to satisfy the condition $E_v(t^*)=0$. The solution of Eq. (15) corresponding to the minimum of vortex energy exists only for a large enough value of magnetization $m > m_1^*$. In the magnetization range $0<m<m_1^*$ the barrier for the vortex exit from the film disappears and the vortex leaves the film. The quantity $m_1^*=m_1^*(\alpha_0,h)$ denotes the magnetization of the FM at which the dependence of the function $E_v(t)$ in the region $-1+\xi\leq t\leq 0$ becomes monotonic, i.e., satisfies the equations (Ref. 26)

$$\left.\frac{\partial E_v(t)}{\partial t}\right|_{t=t^*} = 0, \qquad \left.\frac{\partial^2 E_v(t)}{\partial t^2}\right|_{t=t^*} = 0, \tag{18}$$

where $E_v(t)$ is given by Eq. (16).

Thus, at $m\leq m_1^*$ vortices (or antivortices) are absolutely unstable in the superconducting film. Plots of the functional dependence $m_1^*(\alpha_0,h)$ on the distance α_0 are shown in Fig. 7 for different height h. As it follows from Fig. 7, for a given magnetic strip, there exists a maximal separation from the SC beyond which the vortex (or antivortex) cannot remain in the film.

IV. THE APPEARANCE OF THE VORTEX-ANTIVORTEX CONFIGURATION IN THE SC

Let us assume that the initially superconducting film is in the Meissner state. To formulate the creation conditions of a vortex-antivortex (VA) pair inside SC film and to analyze an edge effect in the considering FM-SC heterostructures one should calculate the total energy of such a pair. Due to the

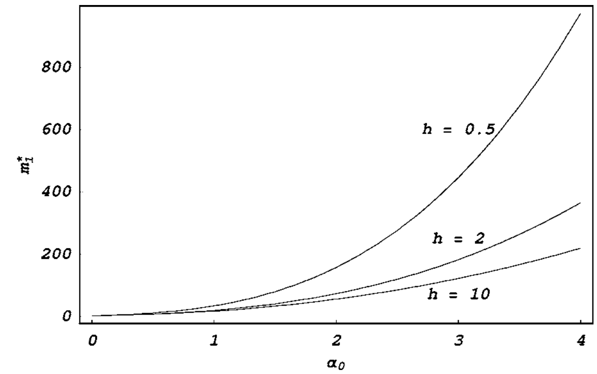

FIG. 7. Dependence of the normalized magnetization $m_1^* = \frac{2\pi w M_1^*}{\Phi_0}$ for a vortex as a function of the distance between the in-plane magnetized strip and the superconducting film $\alpha_0 = 2z_0/w$. Below the curve $m_1^*(\alpha_0,h)$ the vortices are unstable. $h=2H/w$ is a dimensionless height of the magnetic strip.

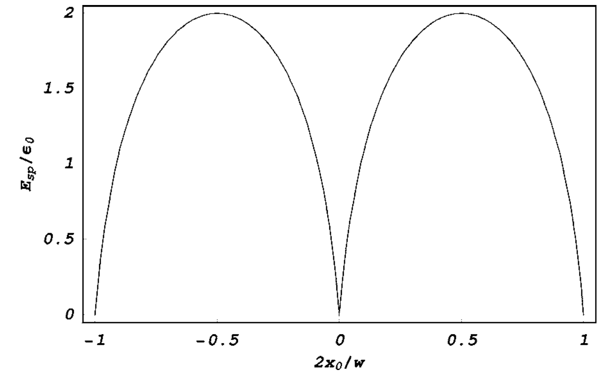

FIG. 8. The self-energy of the vortex-antivortex pair located in the narrow superconducting film in units of $\varepsilon_0 = \Phi_0^2/(4\pi^2\lambda_\perp)$ as a function of the vortex position (x_0). (We take the dimensionless coherent length $\zeta = 2\xi/w = 0.05$).

symmetry of the problem we may assume that the coordinates of the vortex and antivortex are: $\vec{r}_v = (x_0, 0)$ and $\vec{r}_{av} = (-x_0, 0)$. Then the total energy of the symmetrical VA configuration can be represented by a sum:

$$E(x_0) = E_{sv}(x_0) + E_{s-av}(-x_0) + E_{v-f}(x_0) + E_{av-f}(-x_0) + E_{v-av}(x_0;-x_0). \tag{19}$$

Here $E_{sv}(x_0)$, $E_{s-av}(-x_0)$ denote the self-energy of the vortex [Eq. (14)] and antivortex, respectively; it is quite obvious that $E_{s-av}(-x_0) = E_{sv}(x_0)$. The third and fourth terms in the expression (19) are the interaction energy of the FM with vortex and antivortex, respectively; it is obvious that $E_{av-f}(-x_0) = E_{v-f}(x_0)$. The last term in Eq. (19) $E_{v-av}(x_0;-x_0)$ describes the interaction energy between the vortex and antivortex [see Appendix B, Eq. (B18)]. After substituting corresponding expressions into Eq. (19) we obtain the total energy of VA pair $E(t)$ as a function of the dimensionless coordinate $t = 2x_0/w$,

$$E(t) = E_{sp}(t) + E_{p-f}(t) = \varepsilon_0\left[\ln\left(\frac{\sin \pi|t|}{\pi\xi}\right] + m[g(t) - tg(1)]\right), \quad \xi \leqslant t \leqslant 1-\zeta, \tag{20}$$

where ε_0 and m are given by Eq. (11) and $\zeta = 2\xi/w$. The first term in Eq. (20) is the self-energy of a vortex-antivortex pair in the narrow superconducting film $E_{sp}(t)$. For convenience it is useful to redefine the expression for $E_{sp}(t)$ so that it becomes valid within the London approach for an arbitrary value of $t(-1 \leqslant t \leqslant 1)$

$$E_{sp}(t) = \varepsilon_0 \ln\left(\frac{2}{\pi\xi} \sin\frac{\pi(|t|+\xi)}{2} \cos\frac{\pi(|t|-\xi)}{2}\right). \tag{21}$$

The coordinate dependence of $E_{sp}(t)$ is shown in Fig. 8. Due to the symmetry of the problem it is sufficient to consider only one half of the superconducting film. In what follows, for concreteness, we analyze the left one ($-\frac{w}{2} \leqslant x_0 \leqslant 0$ or $-1 \leqslant t \leqslant 0$), where the vortices reside.

The interaction of the pair with the magnetic strip [the term E_{p-f} in Eq. (20)] reduces the total energy of the system. Therefore, if the inhomogeneous magnetic field of the FM is strong enough (at $m > m^*$) the total energy of the pair possesses a local minimum $E(t^*)$ inside the SC which defines the equilibrium position t^* of the vortex: $\frac{\partial E}{\partial t}|_{t=t^*} = 0$.

The magnetization $m^* = m^*(\alpha_0, h)$ determines the stability boundary of the vortex-antivortex configuration. Similar to the case of a single vortex the value $m^*(\alpha_0,h)$ may be found from Eqs. (18) in which the vortex energy $E_v(t)$ is replaced by the energy of the VA pair [Eq. (21)]. The energy E_{va} as a function of the transverse vortex coordinate is shown in Fig. 9 for two different cases (see below). At a small enough magnetization $m > m^*$ the vortex-antivortex state is metastable provided its energy $E(t^*) > 0$. Indeed in this case the energy of the VA pair at the equilibrium position t^* exceeds its value $E(-1) = E(0) = 0$. However, the motion of the vortex to the edge $t=-1$ or to the center of the film $t=0$ (we consider the left side of the film) is prevented by either the edge or annihilation energy barrier, respectively. As it follows from Eq. (20) the relative height of each potential barrier ($\Delta u_{edge}, \Delta u_{an}$) depends on the relation between the values of the Meissner current density $i_m(t)$ at the edge and that in the center of the film $i_m(0)$. When $|i_m(0)| < i_m(-1)$ (see region I in Fig. 3) the edge potential barrier (E-barrier) is lower than the annihilation one: $\Delta u_{edge} < \Delta u_{an}$. In the opposite case $|i_m(0)| > i_m(-1)$ (region II in Fig. 3) the annihilation barrier (A-barrier) is lower than the E-barrier: $\Delta u_{edge} > \Delta u_{an}$. It is clear that if the magnetization becomes equal to m^* the lowest potential barrier disappears. In the first of the considered cases the vortex and antivortex tend to leave the film through its edges. In the second case they tend to meet each other (and subsequently annihilate) at the center of the film. Milošević *et al.*[17] have discussed the stability of the vortex-antivortex pair located in the infinite superconducting film and interacting with a pointlike in-plane magnetized dipole. It was shown that for this configuration the second opportunity only is possible, i.e., the annihilation of vortex and antivortex takes place at $m = m^*$.

Until now we discussed the metastable vortex-antivortex configuration with $E(t^*) > 0$. The emergence of the vortex-

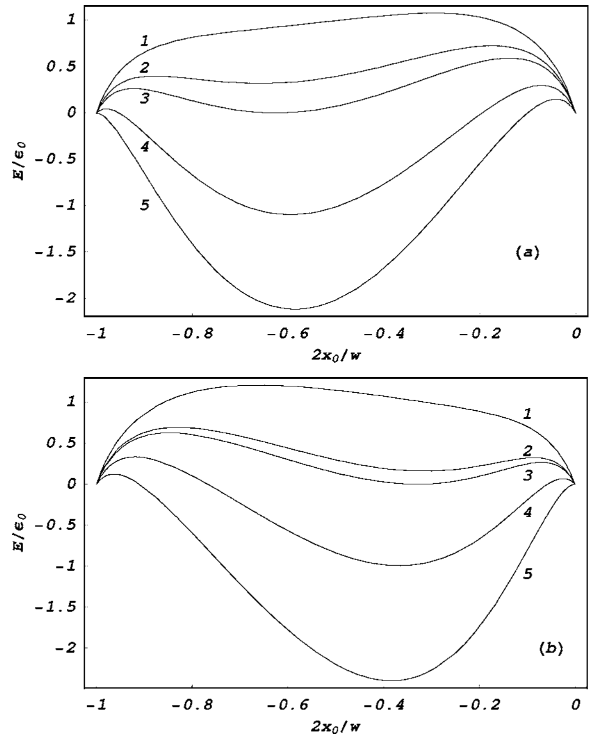

FIG. 9. The energy of the vortex-antivortex configuration in units of $\varepsilon_0=\Phi_0^2/(4\pi^2\lambda_\perp)$ as a function of the vortex position (x_0) for different values of the FM magnetization $m=2\pi w M_0/\Phi_0$, for (a) the magnetic strip of height $h=2H/w=50$ separated by the distance $\alpha_0=2z_0/w=1.7$ from the SC [in this case $i_m(-1)>|i_m(0)|$]. Curve (1) corresponds to $m=25<m^*$, $m^*=33.8$; (2) - $m=40>m^*$; (3) - $m=m_c=48$; (4) - $m=75<m_{s1}$; (5) - $m=m_{s1}=100$, (b) the magnetic strip of height $h=0.5$ for $\alpha_0=0.2$ [so that $i_m(-1)<|i_m(0)|$]. (1) - $m=3<m^*$, $m^*=4.63$; (2) - $m=6>m^*$; (3) - $m=m_c=6.6$; (4) - $m=10<m_{s2}$; (5) - $m=m_{s2}=14.8$.

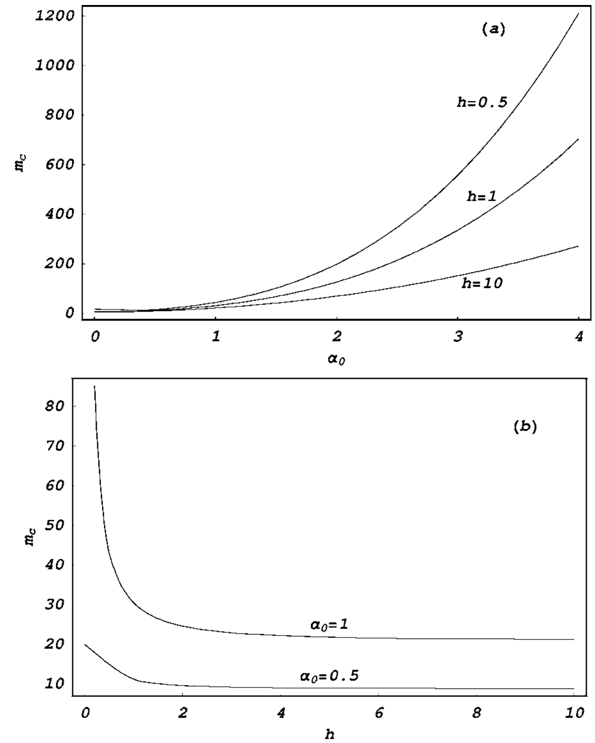

FIG. 10. Shown is the dependence of the normalized critical magnetization for a vortex-antivortex pair $m_c=2\pi w M_c/\Phi_0$ as a function (a) of the distance between the magnetic strip and the superconducting film ($\alpha_0=2z_0/w$), (b) of the height of the magnetic strip ($h=2H/w$). The creation of the vortex-antivortex pair is energetically favorable in the region above the curve $m=m_c(\alpha_0,h)$ and unfavorable below it.

antivortex pair becomes energetically favorable in the magnetization range $m\geqslant m_c>m^*$, for which the total energy of the VA pair becomes negative (Fig. 9). Hence, in order to obtain critical magnetization m_c at which the VA pair may occur inside the SC, one should solve a set of equations

$$E(t^*)=0, \qquad \left.\frac{\partial E(t)}{\partial t}\right|_{t=t^*}=0, \tag{22}$$

where $E(t)$ is given by Eqs. (20) and (6). The results of the numerical solution of Eqs. (22) are presented in Fig. 10 for the configuration shown in Fig. 1.

It is physically obvious that when the magnet is positioned higher above the SC (i.e., the parameter α_0 increases), the magnet-vortex interaction weakens and therefore critical value of magnetization m_c, at which the creation of the vortex-antivortex pair becomes energetically favorable, increases [Fig. 10(a)]. If the FM strip is made wider (the parameter h increases), the magnet-vortex interaction energy, as was mentioned above, increases strongly with h, and then reaches saturation. Therefore, the value of $m_c(h)$ initially strongly decreases with h, and then remains constant for all h exceeding some value $h_0>1$ [Fig. 10(b)].

If the magnetization $m(m>m_c)$ is not too large, the energy minimum is separated both from the edge and from the center of the superconducting strip by the energy barriers which prevent the penetration of the vortices and antivortices inside the film [the curves 4 in Figs. 9(a) and 9(b)]. The edge barrier may be overcome under some conditions by the thermal activation, or by quantum tunneling. It results in a penetration of the vortices and antivortices into the sample, where they reside in the vicinity of the energy minimum at $x=x_0^*$ and $x=-x_0^*$ correspondingly ($x_0^*<0$). If we do not consider the thermal activation process, then the VA pair penetration into SC film may occur at a significantly higher magnetization $m\geqslant m_s>m_c$ when the edge barrier (E barrier) for the vortex (or antivortex) entry is suppressed. Two possible cases may be distinguished further.

(i) Let the sheet current density $i_m(x)$ at the edges exceed its value at the center of the superconducting film [region I in the parameter plane (α_0,h) in Fig. 3]. In this case the value of the magnetization $m_s=m_{s1}$ corresponds to the condition at which the E barrier is suppressed [the curve 5 in Fig. 9(a)]

$$\left.\frac{\partial E}{\partial t}\right|_{t=-1}=0. \tag{23}$$

Using Eq. (20) which determines the energy $E(t)$ we find from Eq. (23) the vortex-entry magnetization $m_s=m_{s1}$

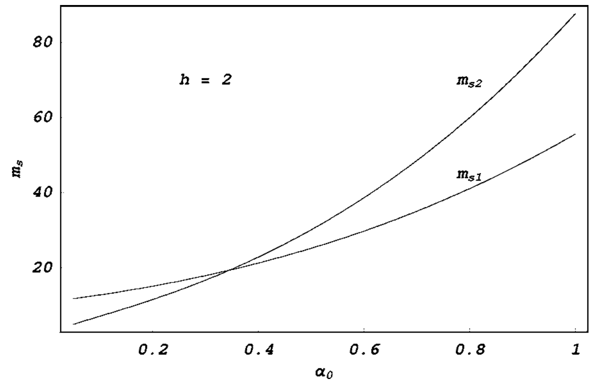

FIG. 11. The normalized barrier suppression magnetization $m_s = 2\pi w M_s/\Phi_0$ as a function of the position of a magnetic strip $\alpha_0 = 2z_0/w$: $m_s = \min\{m_{s1}(\alpha_0, h=2), m_{s2}(\alpha_0, h=2)\}$. Here $h = 2H/w$ and $m_{s1}(\alpha_0, h), m_{s2}(\alpha_0, h)$ are given by Eqs. (24) and (26).

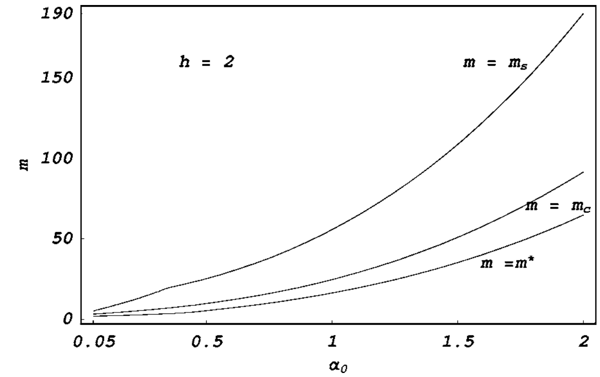

FIG. 12. Phase diagram for the vortex-antivortex configuration induced by the FM. At $m < m^*(\alpha_0, h)$ the SC film is in Meissner state; at $m^*(\alpha_0, h) < m < m_c(\alpha_0, h)$ the film with one VA pair is in metastable state; at $m_c(\alpha_0, h) < m < m_s(\alpha_0, h)$ the vortex and antivortex are in the equilibrium state; at $m > m_s(\alpha_0, h)$ the SC film is in the mixed state. Here $m = 2\pi w M_0/\Phi_0$, $\alpha_0 = 2z_0/w$, $h = 2H/w$.

$$m_{s1}(\alpha_0, h) = \frac{1}{2\zeta\left[(\alpha_0 + h)\tan^{-1}\dfrac{1}{\alpha_0 + h} - \alpha_0 \tan^{-1}\dfrac{1}{\alpha_0}\right]}. \tag{24}$$

(ii) If the size h of the FM strip and its distance from the SC α_0 belong to region II in Fig. 3, the Meissner current density has a maximum in the center of the film. When the value $i_m(0)$ at $m_s = m_{s2}$ reaches the value of the depairing current density i_s, the spontaneous creation of a vortex-antivortex pair at $x = 0$ may occur. Physically, this corresponds to the suppression of the annihilation barrier for the vortex (antivortex) creation at the center of the SC [see the curve 5 in Fig. 9(b)]

$$\left.\frac{\partial E}{\partial t}\right|_{t=0} = 0. \tag{25}$$

Combination of Eqs. (6), (20), and (25) brings the magnetization $m_s = m_{s2}$, which corresponds to the A-barrier suppression

$$m_{s2}(\alpha_{0,h}) = \frac{1/\xi}{\ln\left(\dfrac{(\alpha_0 + h)^2(1 + \alpha_0^2)}{\alpha_0^2[1 + (\alpha_0 + h)^2]}\right) - 2(\alpha_0 + h)\tan^{-1}\dfrac{1}{\alpha_0 + h} + 2\alpha_0 \tan^{-1}\dfrac{1}{\alpha_0}} \tag{26}$$

Thus, for the FM of fixed height h the curve $m_s = m_s(\alpha_0, h)$ consists of two parts:

$$m_s(\alpha_0, h) = \begin{cases} m_{s2}(\alpha_0, h), & \text{for } \alpha_0 \leqslant \alpha_{0c}(h) \\ m_{s1}(\alpha_0, h), & \text{for } \alpha_0 \geqslant \alpha_{0c}(h), \end{cases} \tag{27}$$

where $\alpha_{0c}(h)$ satisfies the condition $|i_m(0)| = i_m(1)$ (see Fig. 3). Therefore, the curve $m_s(\alpha_0, h)$ suffers a crossover at $\alpha_0 = \alpha_{0c}(h)$ (Fig. 11). The height h dependence of the functions $m_{s1}(\alpha_0, h), m_{s2}(\alpha_0, h)$ (at fixed α_0) are analogous to the corresponding dependence of the critical magnetization $m_c(\alpha_0, h)$.

Figure 12 shows the state diagram of the vortex-antivortex configuration. By changing the FM distance above the SC for the given magnetic strip with fixed m and height $h = 2$ it is possible to observe several transitions to different VA states. For example, when α_0 decreases from $\alpha_{0\max} \approx 2$ (keeping $m \leqslant 50$, see Fig. 12) the Meissner state stability threshold which is described by the curve $m = m^*(\alpha_0)$ [$m^*(\alpha_0) = m^*(\alpha_0, h=2)$], will be crossed first. With a further decrease of α_0, the line $m = m_c(\alpha_0)$ will be crossed next, which separates the metastable VA configuration and the equilibrium VA configuration. Finally, the line $m = m_s(\alpha_0)$ describing the mixed state threshold will be crossed. We should point out that in case $m < \bar{m}_s[\bar{m}_s = m_s(\alpha_{0c}(h), h)]$ VA pairs are created at the symmetry axis $x = 0$. In the opposite case $m > \bar{m}_s$ the resultant VA structure is formed when vortices and antivortices enter the SC from the film edges.

V. SUMMARY

Using the London approach we investigated the HMSS system consisting of a thin superconducting film of finite width $w \ll \lambda_\perp$ and an in-plane magnetized thin ferromagnetic strip placed above it. Depending on the dimensionless parameters: the height $h=2H/w$ of the FM and its distance $\alpha_0=2z_0/w$ from the SC we obtained exact analytical expressions for the screening Meissner current and for the FM-vortex interaction energy. We ascertained that similar to the case of the in-plane magnetized point dipole or in-plane magnetized strip over the infinite superconducting film,[16,17] in our case the vortex is attracted by a negative pole of the magnet $(-w/2\leqslant x\leqslant 0)$ and repelled at $0\leqslant x\leqslant w/2$ (for antivortex—vice versa). Employing the image method[26,27] we calculated the total energy of the symmetrical vortex and antivortex configuration and found that the VA pair may exist inside the SC provided the normalized magnetization of the FM $m=2\pi wM_0/\Phi_0$ exceeds some value $m^*=m^*(\alpha_0,h)$: $m>m^*$ (Fig. 9). Therefore by moving the FM [for which $m>m^*(\alpha_0,h)$] far away from superconducting film we increase the exit magnetization m^* (see Fig. 12). At some distance $\alpha_0'=2z_0'/w$ between the FM and the SC the magnetization of the FM m becomes equal $m^*(\alpha_0',h)$ so the vortices and antivortices are expelled completely from the SC. It was shown that two possible scenarios of the flux exit/penetration may take place depending on the maximum value of screening current density $i_m(t)$ (i.e., on the parameters m, α_0 and h):

(i) $i_{\max}=i_m(\mp 1)$ [region I in the plane (α_0,h) in Fig. 3]. Then at the magnetization $m<m^*(\alpha_0,h)$ the edge potential barrier for the vortex exit disappears and vortex (or antivortex) leaves the film from its left (right) edge, respectively. At $m>m_{s1}(\alpha_0,h)$ the edge barrier for the VA pair entry is suppressed. Thus, vortices and antivortices created at opposite edges penetrate into the film;

(ii) $i_{\max}=|i_m(0)|$ [region II in the plane (α_0,h) in Fig. 3]. In this case the processes of the VA annihilation [at $m<m^*(\alpha_0,h)$] and VA pair creation [at $m>m_{s2}(\alpha_0,h)$] occur at the symmetry axis of the film. Note, that the lower magnetization $(m_{s2}<m_{s1})$ is needed in order to create this pair at the center.

In this paper we have considered a most simple geometry of the HMSS, which possesses the symmetry plane $x=0$. A rich variety of possible scenarios of the flux penetration/exit may take place if the system is placed in the perpendicular magnetic field or if the magnetic strip is shifted in the transverse direction along the x axis with respect to superconducting film.

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APPENDIX A

For a SC film of width w the interaction energy of vortex centered at $\vec{r}_0=(x_0,0)$ with FM [Eq. (8)] takes the form:

$$E_{v-f}(x_0)=\frac{1}{c}\int_{-w/2}^{w/2}dx\, i_m(x)\int_{-\infty}^{\infty}dy\Phi_{vy}(x,y), \tag{A1}$$

where $\vec{\Phi}_v(x,y)$ satisfies the equations

$$\mathrm{rot}\,\vec{\Phi}_v(\mathrm{x},\mathrm{y})=\Phi_0\delta(\mathrm{y})\delta(\mathrm{x}-\mathrm{x}_0)\vec{\mathrm{e}}_{\mathrm{z}}, \tag{A2}$$

$$\mathrm{div}\,\vec{\Phi}_{\mathrm{v}}(\mathrm{x},\mathrm{y})=0, \tag{A3}$$

and the boundary conditions

$$\Phi_{\mathrm{vx}}\left(\mathrm{x}=\mp\frac{\mathrm{w}}{2},\mathrm{y}\right)=0. \tag{A4}$$

Using of the method of images one obtains from Eqs. (A2)–(A4) (Ref. 27):

$$\vec{\Phi}_{\mathrm{v}}(\vec{\mathrm{r}},\vec{\mathrm{r}}_0)=\frac{\Phi_0}{2\pi}\sum_{\mathrm{k}}\frac{(-1)^{\mathrm{k}}[\vec{\mathrm{e}}_{\mathrm{z}}\cdot(\vec{\mathrm{r}}-\vec{\mathrm{r}}_k)]}{|\vec{\mathrm{r}}-\vec{\mathrm{r}}_{\mathrm{k}}|^2}, \tag{A5}$$

where $\vec{r}_k=kw\vec{e}_x+(-1)^k\vec{r}_0$ denotes the position of kth vortex (antivortex).

As it follows from Eqs. (A2)–(A4):

$$\int_{-\infty}^{\infty}\Phi_{vy}(x,y)dy=\Phi_0\left(\theta(x-x_0)+\frac{x_0-w/2}{w}\right). \tag{A6}$$

Indeed, by integrating Eq. (A2) over x one finds

$$\int_{-\infty}^{\infty}\Phi_{v,y}(x,y)dy=\Phi_0(\theta(x-x_0)+A) \tag{A7}$$

where

$$\theta(x)=\begin{cases}1, & x>0\\ 0, & x<0.\end{cases}$$

After the integration of Eq. (A3) with due account for Eq. (A4) it is easy to show that

$$\int_{-w/2}^{w/2}\Phi_{vy}(x,y)dx=\mathrm{const}=0 \tag{A8}$$

[note that according to Eq. (A7) $\int\int\Phi_y(x,y)dxdy<\infty$]. Taking into account Eq. (A7) one finds the constant $A=(x_0-w/2)/w$, and, finally, derives Eq. (A6).

Substituting Eq. (A6) into Eq. (A1) and using Eq. (A2) we find

$$E_{v-f}(x_0)=\frac{\Phi_0}{c}\int_{x_0}^{w/2}i_m(x)dx. \tag{A9}$$

Note that the formula coincides with the corresponding equation for the energy of a single vortex in the SC film of width w interacting with the Meissner current induced by the external homogeneous perpendicular magnetic field.[27,29]

APPENDIX B

In this appendix the energy of interaction between the vortex and antivortex located at $x=x_0$ and $x=-x_0$, respectively, in the narrow SC film will be derived. The expression for this energy is

$$E_{v-av}(x_0,-x_0)=\frac{1}{c}\int_{-w/2}^{w/2}\vec{i}_v(\vec{r},\vec{r}_0)\vec{\Phi}_{av}(\vec{r},-\vec{r}_0)d^2r. \tag{B1}$$

For narrow film of width $w\ll\lambda_\perp$ the current density of the vortex $\vec{i}_v$ can be written as (Ref. 27) follows:

$$\vec{i}_v(\vec{r},\vec{r}_0) = \frac{c\vec{\Phi}_v(\vec{r},\vec{r}_0)}{2\pi\lambda_\perp}, \tag{B2}$$

where $\vec{\Phi}_v(\vec{r},\vec{r}_0)$ is determined by Eq. (A5). Corresponding expression for the antivortex is

$$\vec{\Phi}_{av}(\vec{r},-\vec{r}_0) = -\vec{\Phi}_v(\vec{r},-\vec{r}_0). \tag{B3}$$

Substituting Eqs. (B2), (B3) into Eq. (B1) we obtain

$$E_{v-av}(x_0,-x_0) = -\frac{1}{2\pi\lambda_\perp}\int_{-w/2}^{w/2} dx \int_{-\infty}^{\infty} dy \vec{\Phi}_v(x,x_0,y) \times\vec{\Phi}_v(x,-x_0,y). \tag{B4}$$

Using a one-dimensional Fourier transformation,

$$\vec{\Phi}_v(x,x_0,q) = \int_{\infty}^{\infty} e^{iqy}\vec{\Phi}_v(x,x_0,y)dy, \tag{B5}$$

we derive from Eq. (B4)

$$E_{v-av}(x_0,-x_0) = -\frac{1}{4\pi^2\lambda_\perp}\int_{-\infty}^{\infty} dq \int_{-w/2}^{w/2} dx \vec{\Phi}_v(x,x_0,q) \times\vec{\Phi}_v(x,-x_0,-q). \tag{B6}$$

The components of the Fourier transform $\vec{\Phi}_v(x,x_0,q)$ from Eqs. (A5) and (B5) as follows:

$$\Phi_{vx}(x,x_0,q) = \frac{-i\Phi_0}{2}\,\text{sign}\, q[e^{-|q||x_0-x|} + \textstyle\sum_x (x,x_0,|q|)], \tag{B7}$$

$$\Phi_{vy}(x,x_0,q) = \frac{\Phi_0}{2}\left[\frac{x-x_0}{|x-x_0|}e^{-|q||x_0-x|} + \textstyle\sum_y (x,x_0,|q|)\right], \tag{B8}$$

where

$$\textstyle\sum_x (x,x_0,|q|) = 2S[ch|q|(x-x_0) - e^{-|q|w}ch|q|(x+x_0)], \tag{B9}$$

$$\textstyle\sum_y (x,x_0,|q|) = 2S[-sh|q|(x-x_0) + e^{|q|w}sh|q|(x+x_0)], \tag{B10}$$

$$S = \sum_{n=1}^{k_0/2} \exp(-2nw|q|), \tag{B11}$$

where $k_0=\lambda_\perp/w \gg 1$.

The direct substitution of the vector-potential $\vec{\Phi}_v$ into Eq. (B6) brings the following result for integration over x:

$$\int_{-w/2}^{w/2} dx\vec{\Phi}_v(x,x_0,q)\vec{\Phi}_v(x,-x_0,-q) = \frac{\Phi_0^2}{2|q|}[e^{-2|q||x_0|} - e^{-|q|w} + 2S(ch2|q|x_0 - ch|q|w)]. \tag{B12}$$

Then integrating Eq. (B12) over q we arrive (after some algebra) at the following equation for the interaction energy E_{v-av}:

$$E_{v-av}(x_0,-x_0) = -\frac{\Phi_0^2}{4\pi^2\lambda_\perp}\left[\ln\frac{w}{2|x_0|} + \sum_{n=1}^{k_0/2}\ln\frac{n^2-1/4}{n^2-x_0^2/w^2}\right], \quad \xi \leqslant |x_0| \leqslant w/2. \tag{B13}$$

Denote the sum appearing in Eq. (B13) $S_m(t)$ ($t=x_0/w$),

$$S_m(t) = \sum_{n=1}^{k_0/2}\ln\frac{n^2-1/4}{n^2-t^2}. \tag{B14}$$

Differentiating Eq. (B14) with respect to t we find[30]

$$\frac{dS_m}{dt} = \sum_{n=0}^{\frac{k_0}{2}-1}\left(\frac{1}{n+1-t} - \frac{1}{n+1+t}\right) = \psi\left(\frac{k_0}{2}+1-t\right) - \psi(1-t) - \psi\left(\frac{k_0}{2}+1+t\right) + \psi(1-t), \tag{B15}$$

where $\psi(z)=\frac{d\ln\Gamma(z)}{dz}$ and $\Gamma(z)$ denotes the gamma function. Using here the asymptote of $\psi(z)$ at $z \gg 1$ $\psi(z) \approx \ln(z)$, as well as functional relations from Ref. 30, we obtain

$$\frac{dS_m}{dt} = \frac{1}{t} - \pi ctg\pi t. \tag{B16}$$

Integrating this equation over t and using the condition $S_m(1/2)=0$ one finds

$$S_m\left(t=\frac{x_0}{w}\right) = \ln\frac{2x_0}{w\sin\frac{\pi x_0}{w}}. \tag{B17}$$

After substitution Eq. (B17) into Eq. (B13) we obtain finally

$$E_{v-av}(x_0,-x_0) = \frac{\Phi_0^2}{4\pi^2\lambda_\perp}\ln\left(\sin\frac{\pi|x_0|}{w}\right), \quad \xi \leqslant |x_0| \leqslant w/2. \tag{B18}$$

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