

Josephson effect in junctions of ferromagnetic superconductors with equal spin pairing symmetry

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We theoretically investigate the dc Josephson current in the junctions of ferromagnetic superconductors (FSs) with the equal spin pairing symmetry, which has been suggested for the $ZrZn_2$. The low-temperature anomaly of the Josephson current is either presented or absent, depending on the different forms of the pair potential. It is also shown that the Josephson junctions can be used as the switches by tuning the magnetization configuration of the two FS layers.

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I. INTRODUCTION

The Josephson effects in the junctions made of the unconventional superconductors has been a most interesting subject for many years.^{1,2} The pairing symmetry of the superconductor has a great influence on the transport properties of the Josephson junctions. In the anisotropic superconductors, the sign change of the pair potential enables the formation of the zero-energy state (ZES) at the normal-metal–superconductor interface.^{3–6} Those ZESs are found to be responsible for the low-temperature anomaly of the Josephson current in the junctions of the d -wave superconductors.^{7,8} The Josephson junctions consisting of spin-triplet superconductors have also attracted much attention.^{9–13} It is shown that the triplet-superconductor–ferromagnet–triplet-superconductor junctions can be used to build Josephson current switches.¹³

Recently, a new category of superconductor, namely, the ferromagnetic superconductor (FS), has been revealed in bulk materials, such as UGe_2 ,¹⁴ $ZrZn_2$,¹⁵ and $URhGe$.¹⁶ In addition, the coexistence between the ferromagnetism and the superconductivity has also been suggested in the ferromagnet-superconductor hybrid structures due to the proximity effects.^{17–20} The Josephson current in the FS tunnel junctions has been discussed in a number of studies.^{17,21} It is found that the critical current in the junction can be enhanced by increasing the exchange energy in the FS on condition of strong barrier strength, low temperatures, and the antiparallel configuration of the two magnetizations in the FS layers. In their studies,^{17,21} the FS is composed of the thin ferromagnet–superconductor bilayer and the pairing symmetry in the FS is assumed as s -wave. The possibility to control spin and charge supercurrent by tuning magnetic configuration in the Josephson junction between two nonuniform Fulde-Ferrel-Larkin-Ovchinnikov superconductors with helimagnetic order has also been proposed and analyzed.²² On the other hand, the pairing symmetry in the uniform bulk FS, such as UGe_2 , $ZrZn_2$, and $URhGe$, is difficult to classify. Although there were some arguments that the pairing symmetry in those materials is s -wave,^{23–25} it seems more reasonable that only the spin-triplet superconductivity could coexist with the itinerant ferromagnetism in the uniform samples to avoid the large depairing influence of the exchange field.^{26–31} In fact, the study on the various hybrid structures concerning the triplet superconductor has already

attracted much interest.^{9–13,32} Therefore, the study of the dc Josephson effect in the FS junctions with the triplet pairing symmetry is highly desirable.

In this paper, the dc Josephson effect in a clean FS-insulator-FS junction is investigated theoretically. To be specific, we assume the equal spin pairing symmetry in the FS, which has been suggested for the $ZrZn_2$.^{31,33} The Josephson current is calculated from the Andreev reflection coefficients by a Furusaki-Tsukada-like formula,^{1,21,34–36} and the Andreev reflection coefficient is obtained from the Bogoliubov–de Gennes equation.^{37–39} In this formalism, the influence of the ZES on the Josephson current has been taken into account automatically because that the Josephson current is expressed by the Andreev reflection coefficients.³⁵ For the FS with the equal spin pairing symmetry, such as $ZrZn_2$, it is found that the Josephson current is finite if the two magnetizations in the FS layers are in parallel configuration and zero if the magnetizations are in antiparallel configuration. The meaning of this result is twofold. First, this switch effect can be used to classify whether the Cooper pair in $ZrZn_2$ is in the equal spin pairing state^{31,33} or in the s -wave pairing state.^{23–25} Second, the Josephson junction made of the equal spin pairing FS can be used as the two-level system tuned by the magnetization configuration, which is of great interest in the quantum information technology.^{13,40} For the junctions under the parallel configuration, the temperature dependence of the critical Josephson current is obtained. It is shown that the low-temperature anomaly of the Josephson current resulting from the formation of the ZESs is either presented or absent, depending on the different selections of the pair potentials.

The organization of this paper is as follows. In Sec. II, the formula of the Josephson current is given. The numerical results of the Josephson current are discussed in Sec. III. Finally, we briefly summarize our results in Sec. IV.

II. JOSEPHSON CURRENT FORMULA

The system under our consideration is an FS-insulator-FS junction. The layers are assumed to be the x – y plane and to be stacked along the z direction. The thin insulator layer at $z=0$ is modeled by a δ -type barrier potential with the strength V_0 . The FS is described by the effective single-particle Hamiltonian $H_0 = -(\hbar^2/2m)\nabla^2 + V_0\delta(z) - E_F$ with E_F

the Fermi energy and m the effective mass. In addition, there are also the exchange energy h_0 and the pair potential Δ in the FS. The two FS layers are the same except for a phase difference. The magnetizations of the two FS layers can be either parallel or antiparallel to each other. In general, the pair potential with triplet pairing symmetry is very complicated and in a matrix form $\Delta = \begin{pmatrix} \Delta_{\uparrow\uparrow} & \Delta_{\uparrow\downarrow} \\ \Delta_{\downarrow\uparrow} & \Delta_{\downarrow\downarrow} \end{pmatrix}$. Here, we only consider the simple case of the equal spin pairing symmetry. The pair potential has only one component $\Delta_{\uparrow\uparrow}$. This case is proposed for the ZrZn_2 where only one of the exchange-split bands is superconducting.^{31,33} We assume that the length of the junction is less than the spin-flipping length. As a result, both of the spin-up bands in the two FS layers are superconducting and neither of the spin-down bands is superconducting if the magnetizations of the two FS layers are in the parallel configuration. On the other hand, each of the spin-split band is superconducting in the one FS layer and retains normal in the other FS layer if the magnetizations are in the antiparallel configuration. Therefore, in the antiparallel configuration, either the spin-up or spin-down channel can be treated as a superconductor–normal-metal junction and there is only a finite tunnel current carried by the quasiparticles, but zero Josephson current carried by the Cooper pairs. The finite Josephson current can only be observed in the spin-up channel with the two FS layers in the parallel configuration.

In order to calculate the Josephson current in the parallel configuration, the specific form of the pair potential is needed. Now, the pair potential is anisotropic in the momentum space. The possible choice of the $\Delta_{\uparrow\uparrow}$ can be obtained by the symmetry consideration. In our calculation, the magnetization of the ZrZn_2 is assumed along the $[001]$ direction and two of the possible pair potentials can be written as^{31,41}

$$\begin{aligned} \Delta_{\uparrow\uparrow, A} &= i\Delta_0 \cos \frac{k_z a}{2} \left(\sin \frac{k_y a}{2} + i \sin \frac{k_x a}{2} \right), \\ \Delta_{\uparrow\uparrow, 2E} &= i\Delta_0 \sin \frac{k_z a}{2} \left(\cos \frac{k_x a}{2} + \cos \frac{k_y a}{2} \right), \end{aligned} \quad (1)$$

where Δ_0 is the strength of the pair potential, \mathbf{k} is the wave vector and a is the lattice constant. These two pair potentials correspond to the p -wave equal spin pairing superconducting states proposed for ZrZn_2 .^{31,41} The subscripts A and ²E denote the corresponding irreducible representations of the symmetry group, respectively.

The quasiparticles in the spin-up channel in the tunnel junction satisfy the Bogoliubov–de Gennes equation³⁸

$$\begin{bmatrix} H_0 - h_0 & \Delta \\ \Delta^* & -H_0 + h_0 \end{bmatrix} \begin{pmatrix} u_{\uparrow} \\ v_{\uparrow} \end{pmatrix} = E \begin{pmatrix} u_{\uparrow} \\ v_{\uparrow} \end{pmatrix}, \quad (2)$$

where Δ is one of the pair potentials in Eq. (1) and $\begin{pmatrix} u_{\uparrow} \\ v_{\uparrow} \end{pmatrix}$ is the wave function of the quasiparticles in the spin-up channel of energy E measured from the Fermi level. Considering an electronlike quasiparticle (ELQ) incoming from the left FS layer, the solution of Eq. (2) reads

$$\begin{aligned} \Psi_L &= \begin{pmatrix} u \\ v e^{-i\phi_L - i\gamma_+} \end{pmatrix} \exp(ik_{Lz}^e z) + a \begin{pmatrix} v \\ u e^{-i\phi_L - i\gamma_+} \end{pmatrix} \exp(ik_{Lz}^h z) \\ &\quad + b \begin{pmatrix} u \\ v e^{-i\phi_L - i\gamma_-} \end{pmatrix} \exp(-ik_{Lz}^e z), \\ \Psi_R &= c \begin{pmatrix} u \\ v e^{-i\phi_R - i\gamma_+} \end{pmatrix} \exp(ik_{Rz}^e z) + d \begin{pmatrix} v \\ u e^{-i\phi_R - i\gamma_+} \end{pmatrix} \exp(-ik_{Rz}^h z), \end{aligned} \quad (3)$$

where

$$\begin{aligned} u &= \frac{1}{\sqrt{2}} \left(1 + \frac{\sqrt{E^2 - |\Delta|^2}}{E} \right)^{1/2}, \\ v &= \frac{1}{\sqrt{2}} \left(1 - \frac{\sqrt{E^2 - |\Delta|^2}}{E} \right)^{1/2}. \end{aligned} \quad (4)$$

The wave function in the left FS, Ψ_L , includes three terms: the incident ELQ, the Andreev reflected holelike quasiparticle (HLQ)³⁷ with the amplitude a , and the normal reflected ELQ with the amplitude b . The wave function in the right FS, Ψ_R , includes two terms: the transmitted ELQ with the amplitude c and the transmitted HLQ with the amplitude d . The k_{Lz}^e , k_{Lz}^h , k_{Rz}^e , and k_{Rz}^h are the z components of the wave vectors of the ELQ and HLQ in the left and right FS layers, respectively. Since the pair potential and exchange energy are much smaller than the Fermi energy, all of those wave vectors can be approximated as $\mathbf{k}_F \cos \theta$ with the Fermi wave vector \mathbf{k}_F and the incident angle θ . The ϕ_L and ϕ_R are the external phase of the left FS and that of the right FS, respectively. The internal phases of the quasiparticles in each process, γ_{\pm} , are defined by $\exp(i\gamma_{\pm}) = \Delta(\pm k_z) / |\Delta(\pm k_z)|$,¹ respectively. The left FS and the right FS are the same here. From Eq. (1), one finds that $|\Delta(\mathbf{k}_z)|^2 = |\Delta(-\mathbf{k}_z)|^2$ in our calculation. Consequently, in this simple case, the amplitudes (u, v) for the ELQ and that for the HLQ in the left and right FS layers are all the same^{1,35} in spite that the pair potential is anisotropic in the momentum space.

The Andreev reflection coefficient a can be determined by matching the boundary conditions

$$\Psi_R|_{z=0^+} = \Psi_L|_{z=0^-},$$

$$\left. \frac{\partial \Psi_R}{\partial z} \right|_{z=0^+} - \left. \frac{\partial \Psi_L}{\partial z} \right|_{z=0^-} = \frac{2mV_0}{\hbar^2} \Psi_L(z=0). \quad (5)$$

From Eqs. (3)–(5), one obtains

$$\begin{aligned} a_A(E) &= \frac{E(\cos \phi - 1) - i\sqrt{E^2 - |\Delta|^2} \sin \phi}{2E^2 \left[1 + \left(\frac{Z}{\cos \theta} \right)^2 \right] - |\Delta|^2 \left[1 + \cos \phi + 2 \left(\frac{Z}{\cos \theta} \right)^2 \right]} |\Delta|, \end{aligned} \quad (6)$$

for the pair potential $\Delta_{\uparrow\uparrow, A}$ and

$$a_{2E}^2(E) = \frac{E \left[\cos \phi - 1 - 2 \left(\frac{Z}{\cos \theta} \right)^2 \right] - i \sqrt{E^2 - |\Delta|^2} \sin \phi}{2E^2 \left[1 + \left(\frac{Z}{\cos \theta} \right)^2 \right] - |\Delta|^2 (1 + \cos \phi)} |\Delta|, \quad (7)$$

for the pair potential $\Delta_{\uparrow\uparrow,2E}$, where $\phi = \phi_R - \phi_L$ denotes the phase difference and $Z = mV_0/(\hbar^2 k_F)$ is the dimensionless barrier strength. The expressions of the Andreev reflection coefficients for the two pair potentials are different. For the $\Delta_{\uparrow\uparrow,A}$, we have $\Delta(k_z) = \Delta(-k_z)$. Therefore, all of the internal phases in Eq. (3) are the same and can be omitted. The Andreev reflection coefficient defined in Eq. (6) is similar to that for the s -wave Josephson junction.³⁴ For the $\Delta_{\uparrow\uparrow,2E}$, we have $\Delta(k_z) = -\Delta(-k_z)$ and $\exp(i\gamma_+)/\exp(i\gamma_-) = -1$. This situation is similar to that of the d -wave Josephson junctions when the angle between the normal of the interface and the a axis of the superconductor is $\pi/4$.¹ The resultant Andreev reflection coefficient defined in Eq. (7) becomes very large

when the phase difference approaches π for the zero energy quasiparticles.

With help of the Andreev reflection coefficients, the dc Josephson current can be obtained by a Furusaki-Tsukada-like formula^{1,34-36}

$$I = \frac{e}{\hbar} \sum_{k_x, k_y} \sum_{\omega_n} \frac{k_B T |\Delta|}{\Omega_n} [a(i\omega_n) - \tilde{a}(i\omega_n)], \quad (8)$$

where $\Omega_n = \sqrt{\omega_n^2 + |\Delta|^2}$, with the temperature T and the Matsubara frequency $\omega_n = \pi k_B T (2n+1)$. The Andreev reflection coefficient $a(i\omega_n)$ describes the process where an incoming ELQ from the left FS is reflected as an HLQ and can be obtained from Eq. (6) or (7) by the analytic continuation $E \rightarrow i\omega_n$. The Andreev reflection coefficient $\tilde{a}(i\omega_n)$ describes the process where an incoming HLQ from the left FS is reflected as an ELQ. Because of the symmetry present in our system, the $\tilde{a}(i\omega_n)$ can be found by replacing the phase difference ϕ in $a(i\omega_n)$ with $(-\phi)$ and therefore, one finally obtains

$$R_N I_A = \frac{\pi \bar{R}_N k_B T}{e} \sum_{k_x, k_y} \sum_{\omega_n} \frac{|\Delta|^2 \sin \phi}{2\omega_n^2 \left[1 + \left(\frac{Z}{\cos \theta} \right)^2 \right] + |\Delta|^2 \left[1 + \cos \phi + 2 \left(\frac{Z}{\cos \theta} \right)^2 \right]}, \quad (9)$$

for the pair potential $\Delta_{\uparrow\uparrow,A}$ and

$$R_N I_{2E} = \frac{\pi \bar{R}_N k_B T}{e} \sum_{k_x, k_y} \sum_{\omega_n} \frac{|\Delta|^2 \sin \phi}{2\omega_n^2 \left[1 + \left(\frac{Z}{\cos \theta} \right)^2 \right] + |\Delta|^2 (1 + \cos \phi)}, \quad (10)$$

for the pair potential $\Delta_{\uparrow\uparrow,2E}$, where R_N denotes the normal-state resistance,⁴² $\bar{R}_N^{-1} = \int_0^{\pi/2} \cos^3 \theta \sin \theta / (Z^2 + \cos^2 \theta) d\theta$. The Josephson currents in Eqs. (9) and (10) have already been averaged over the incident angle which enters the equations from the summation over the momentums k_x and k_y .

III. RESULTS AND DISCUSSION

The numerical results of the Josephson currents in Eqs. (9) and (10) are presented by taking the pair potentials in Eq. (1) and the Fermi surface parameter $k_F a \approx 1.6$.⁴³ The Josephson current I is plotted as a function of the phase difference ϕ at $T = 0.01 T_c$ in Fig. 1, with T_c the superconducting critical temperature. For the pair potential $\Delta_{\uparrow\uparrow,A}$, the Josephson current is proportion to $\sin \phi$, which is similar to what occurs in an s -wave Josephson junction. For the pair potential $\Delta_{\uparrow\uparrow,2E}$, the Josephson current deviates significantly from the $\sin \phi$ relation as shown in Fig. 1. The Josephson current becomes very large, at first, and then suddenly drops to zero when the phase difference is close to π . From Eqs. (7) and (10), one can find that this result stems from the large Andreev reflection

coefficients for the zero energy quasiparticles, which is due to the sign changes in the internal phases of the pair potential.

The temperature dependence of the critical Josephson current I_c is shown in Fig. 2, where we assume a BCS temperature dependence of the pair potential. It is shown that the I_c increases as decreasing temperature and is saturated at zero temperature for the case of the pair potential $\Delta_{\uparrow\uparrow,A}$, which is similar to what occurs in an s -wave Josephson junction. As for the pair potential $\Delta_{\uparrow\uparrow,2E}$, one finds that the critical Josephson current increases drastically at low temperatures. This situation, namely, the low-temperature anomaly of the Josephson current, is similar to what exists in the d -wave Josephson junctions when the angle between the normal of the interface and the a axis of the superconductor is $\pi/4$.^{1,36} The low-temperature anomaly of the Josephson current is ascribed to the formation of the ZESs at the normal-metal-superconductor interface, which is a result of the sign changes in the internal phases of the pair potential in the anisotropic superconductor.^{1,36} In our simple model, the current is along the z direction. For the $\Delta_{\uparrow\uparrow,A}$, we have $\Delta(k_z) = \Delta(-k_z)$ and therefore all of the internal phases in Eq. (3) are

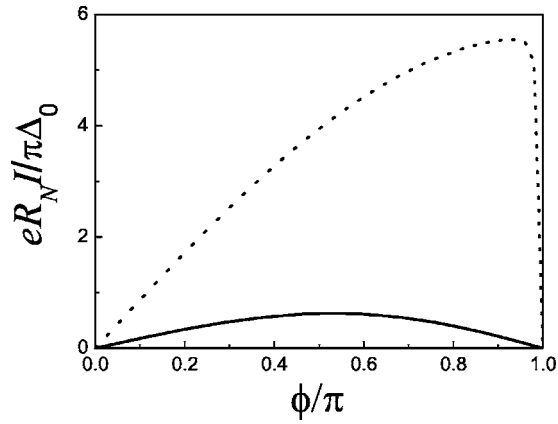


FIG. 1. The Josephson current I is plotted as a function of the phase difference ϕ for $Z=1$. The solid line represents the result for the pair potential $\Delta_{\uparrow\uparrow,A}$ and the dashed line for $\Delta_{\uparrow\uparrow,E}$.

the same and can be omitted. As for the $\Delta_{\uparrow\uparrow,E}$, we have $\Delta(k_z) = -\Delta(-k_z)$ and $\exp(i\gamma_+)/\exp(i\gamma_-) = -1$, indicating the sign changes of the internal phases. Consequently, the low-temperature anomaly of the Josephson current is observed for the case of the pair potential $\Delta_{\uparrow\uparrow,E}$ but absent for the case of the $\Delta_{\uparrow\uparrow,A}$.

We note that the Josephson currents shown in Figs. 1 and 2 are the results for the equal spin pairing FS tunnel junction in the parallel configuration. If the two FS layers are in the antiparallel configuration, either the spin-up or spin-down quasiparticles feel a normal-metal-superconductor junction, which results in a finite normal tunnel current but zero supercurrent. If we fix the orientation of the magnetization of the left FS and tune that of the right FS by applying a local magnetic field, a Josephson current switch comes into being. The Josephson current induced by the phase difference, but independent on the bias voltage, can be observed when the two FS layers are in the parallel configuration. If the two FS layers are in the antiparallel configuration, then there is no Josephson current, but the normal current, which is induced by the bias voltage and independent on the external phases of the superconductors. Although the model we considered is very simple, the switch effect proposed here is highly non-trivial. First, such Josephson current switches can be used as the two-level systems in the quantum information technology. Second, the switch effect can be used to detect whether the pairing symmetry in some FSs, such as ZrZn_2 , is equal spin pairing or not.

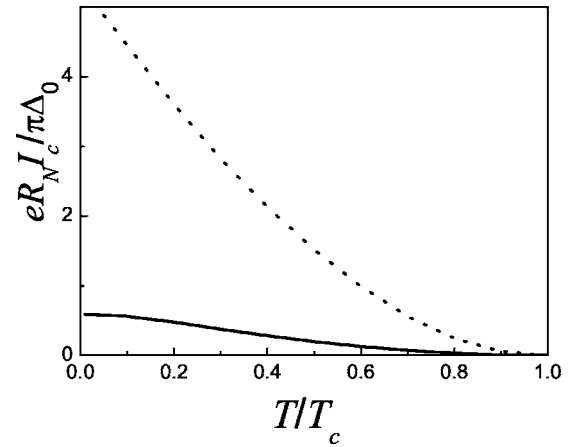


FIG. 2. The critical Josephson current I_c is plotted as a function of the temperature for $Z=1$. The solid line represents the result for the pair potential $\Delta_{\uparrow\uparrow,A}$ and the dashed line for $\Delta_{\uparrow\uparrow,E}$.

IV. CONCLUSION

In summary, the dc Josephson effect in a clean FS-insulator-FS junction with the equal spin pairing symmetry is investigated theoretically. The Josephson current is calculated from the Andreev reflection coefficients by a Furusaki-Tsukada-like formula. It is shown that the low-temperature anomaly of the Josephson current is either presented or absent, depending on the different selections of the pair potentials. It is also shown that there is a switch effect tuned by the magnetic configurations of the two FS layers which can be used to classify the pairing symmetry in the FS.

The switch effect proposed in this paper is a general property of the Josephson junction made of the equal spin pairing FSs. A possible candidate to achieve this effect experimentally is the junction made of ZrZn_2 below the superconducting critical temperature of 0.29 K.¹⁵ It is usually assumed in the literature that only one of the exchange-split bands is superconducting in ZrZn_2 .^{31,33} However, the real pairing symmetry in the superconducting crystal ZrZn_2 is probably more complicated than considered simplified equal spin pairing. In order to fully confirm the validity of this Josephson switch, further theoretical and experimental investigation is necessary.

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