Mesoscopic anisotropic magnetoconductance fluctuations in ferromagnets

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The conductance of a ferromagnetic particle depends on the relative orientation of the magnetization with respect to the direction of current flow. This phenomenon is known as "anisotropic magnetoresistance." Quantum interference leads to an additional random dependence of the conductance on the magnetization direction. These "mesoscopic anisotropic magnetoresistance fluctuations" are caused by the interplay of random impurity scattering and spin-orbit scattering, which couples the electron motion to the exchange field in the ferromagnet. We report a calculation of the dependence of the conductance autocorrelation function on the rotation angle of the magnetization direction.

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One hallmark of phase-coherent transport is the phenomenon of "universal conductance fluctuations," random, but reproducible variations in a sample's conductance as a function of the applied magnetic field or the Fermi energy.[1–5] The magnitude of the conductance fluctuations is of order unity, in units of the conductance quantum e^2/h, and does not depend on specific sample properties, such as the impurity concentration, the material, shape, or method of preparation.

Recently there has been both theoretical and experimental interest in mesoscopic transport in itinerant ferromagnets. The experimental interest stems from the ability to fabricate ferromagnetic conductors small enough that transport through the magnet is predominantly coherent.[6,7] The theoretical interest is motivated by the rich variety of ways through which random impurity scattering can affect the properties of an itinerant ferromagnet. Theoretical predictions exist for the effect of domain walls on weak localization and conductance fluctuations[8,9] as well as for the combined effect of spin-orbit interaction and impurity scattering on weak localization[10] and magnetic anisotropy.[11] Although disordered ferromagnetic conductors display different phenomena than their normal-metal counterparts, the theoretical framework to describe them is rather similar. Indeed, the methods of diagrammatic perturbation theory developed for electron transport in disordered metals can be applied to ferromagnets by modifying the single particle Hamiltonian taking into account the exchange field and/or spin-orbit interactions.

In this Report, we address the mesoscopic contribution to a ferromagnet's anisotropic magnetoresistance in diffusive samples. Anisotropic magnetoresistance is the phenomenon that a magnet's resistance depends on the orientation of the magnetization resulting from a combination of spin-orbit coupling and orbital magnetic effects.[12] For a single domain magnet, the resistance is a smooth function of the magnetization direction. The mesoscopic effect described here consists of an additional and faster random dependence on the magnetization direction that is different for each sample, but reproducible for a given sample. Its origin is the coherent multiple scattering off impurities in the ferromagnet. As a function of magnetization direction, the mesoscopic correction will show a quick succession of minima and maxima, superimposed on the smooth material-dependent anisotropic magnetoresistance of the bulk material. Such features were seen in the recent experiment of Ref. 7. This situation is not very different from the case of standard universal conductance fluctuations in a normal metal, where the random magnetic-field dependent fluctuations are superimposed on a systematic magnetoconductance.

The mesoscopic anisotropic magnetoresistance we consider here is related to but different from the anisotropic magnetoresistance of a ferromagnetic point contact, which has received attention recently. Ferromagnetic junctions of only a few atoms wide were measured to have anomalously large anisotropic magnetoresistance.[13–16] Theoretically, nanoscale ferromagnetic junctions were predicted to show the ferromagnetic analog of conductance quantization.[17] Although the magnetoresistance in nanoscale junctions differs from its bulk counterpart, the resistance is predicted to remain a monotonic and material dependent (but not sample dependent) function of the magnetization direction if the junction is regular.[17] On the other hand, the effect we consider is a nonmonotonic sample-dependent feature superimposed on the bulk anisotropic magnetoresistance. Although the same phenomena may occur in nanoscale junctions because of backscattering from the electrodes, our theory is aimed at disordered ferromagnets with a conductance much larger than the conductance quantum.

There are two possible mechanisms through which the magnetization direction can affect the interference correction to the conductance. First, a change of the magnetization direction causes a change of the internal magnetic field, which directly affects the orbital motion of the electrons via a change of Aharonov-Bohm phases. Second, a change of the magnetization direction causes a change of the exchange field, which affects the motion of the electrons via spin-orbit scattering. The two effects scale differently with sample size L or the phase coherence length L_ϕ, with the orbital effect becoming the weaker one at smaller length scales; see Ref. 10 and the concluding paragraph below. The orbital effect starts to dominate if the magnetic flux through an area $l_{so} \times \min(L, L_\phi)$ is of the order of the flux quantum, where l_{so} is the spin-orbit length. For many magnetic materials, L_ϕ and l_{so} can be small and the orbital effect can be neglected.[7,18] In what follows, we assume that this condition holds and that the second effect dominates the mesoscopic anisotropic magnetoresistance. For the same reason, we ignore any effect of

an applied magnetic field used to change the magnetization direction.

We consider an ensemble of ferromagnetic particles, each with a different configuration of impurities and calculate the conductance autocorrelation function

$$\mathcal{C}(\theta) = \langle G(\hat{\mathbf{m}})G(\hat{\mathbf{m}}')\rangle - \langle G(\hat{\mathbf{m}})\rangle^2, \tag{1}$$

where θ is the angle between the magnetization directions $\hat{\mathbf{m}}$ and $\hat{\mathbf{m}}'$ and the brackets $\langle\cdots\rangle$ denote the ensemble average. The vectors $\hat{\mathbf{m}}$ and $\hat{\mathbf{m}}'$ are defined to have unit length. The Hamiltonian for a ferromagnet with spin-orbit scattering is

$$\mathcal{H}_{\alpha\beta} = \left(\frac{p^2}{2m} - \mu\right)\delta_{\alpha\beta} - E_Z\sigma^z_{\alpha\beta} + \mathcal{V}_{\alpha\beta}, \tag{2}$$

where α and β are spin indices, σ_z is the Pauli matrix, the magnetization direction $\hat{\mathbf{m}}$ is taken as the spin quantization axis, and $E_Z = \mu_B B_{\rm ex}$ is the Zeeman energy corresponding to the exchange field $B_{\rm ex}$. We perform the ensemble average at a fixed chemical potential μ and exchange field $B_{\rm ex}$, rather than at a self-consistently determined μ and $B_{\rm ex}$. Although the omission of the self-consistency conditions is known to affect averaged quantities, it is believed not to affect fluctuations.[19–21]

The random potential $\mathcal{V}$ in Eq. (2) describes the effect of elastic impurity scattering and spin-orbit scattering, respectively. Its Fourier transform is

$$\mathcal{V}_{\alpha\mathbf{k},\beta\mathbf{k}'} = V_{\mathbf{k}-\mathbf{k}'} - iV^{\rm so}_{\mathbf{k}-\mathbf{k}'}(\mathbf{k}'\times\mathbf{k})\cdot(\hat{\mathbf{m}}\sigma^z + \hat{\mathbf{e}}_1\sigma^x + \hat{\mathbf{e}}_2\sigma^y)_{\alpha\beta}, \tag{3}$$

where $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$ are unit vectors perpendicular to each other and to $\hat{\mathbf{m}}$ such that $\hat{\mathbf{e}}_1\times\hat{\mathbf{e}}_2 = \hat{\mathbf{m}}$. The random potentials V and $V^{\rm so}$ are assumed to be uncorrelated and Gaussian white noise, with rms strength v and $v^{\rm so}$, respectively,

$$\langle V_{\mathbf{q}}V_{\mathbf{q}'}\rangle = v^2\delta(\mathbf{q}-\mathbf{q}'), \quad \langle V^{\rm so}_{\mathbf{q}}V^{\rm so}_{\mathbf{q}'}\rangle = v^2_{\rm so}\delta(\mathbf{q}-\mathbf{q}'). \tag{4}$$

In the leading order Born approximation, the scattering time τ_α for spin-independent impurity scattering of electrons with spin α is given by

$$\frac{1}{2\pi\nu_\uparrow\tau_\uparrow} = v^2, \quad \frac{1}{2\pi\nu_\downarrow\tau_\downarrow} = v^2, \tag{5}$$

where ν_α is the density of states of electrons with spin α. Similarly, for spin-conserving and spin-flip scattering off $V^{\rm so}$, one has the mean free times

$$\frac{1}{2\pi\nu_\uparrow\tau_{\uparrow\parallel}} = \frac{2}{9}v^2_{\rm so}k^4_{\rm F\uparrow}, \quad \frac{1}{2\pi\nu_\downarrow\tau_{\downarrow\parallel}} = \frac{2}{9}v^2_{\rm so}k^4_{\rm F\downarrow},$$

$$\frac{1}{2\pi\nu_\downarrow\tau_{\uparrow\perp}} = \frac{1}{2\pi\nu_\uparrow\tau_{\downarrow\perp}} = \frac{2}{9}v^2_{\rm so}k^2_{\rm F\uparrow}k^2_{\rm F\downarrow}, \tag{6}$$

respectively, where $k_{\rm F\alpha}$ is the Fermi wave vector for spin α electrons. In a realistic ferromagnet, the kinetic energy and the random potential will not have the simple form assumed in our calculation, which implies that the relationships between the scattering times implied by Eqs. (5) and (6) need not hold. Although we use the simple model described above to set up our calculation and to define the scattering times, these are then considered independent of the rest of the calculation [except for the equality in the second line of Eq. (6), which follows from detailed balance].

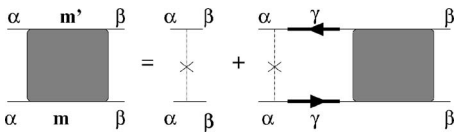

FIG. 1. Dyson equation for the Diffuson ladder. The dotted line indicates a scattering event.

Throughout the calculation, we assume that $\tau \ll \tau_\parallel, \tau_\perp$. This implies that all Green's functions appearing in intermediate phases of the calculation can be averaged over all directions of the momentum. We also assume that phase coherence is preserved over the entire sample. In a sample with size L larger than the phase coherence length L_ϕ, our answer would be modified as $\mathcal{C}(\theta, L) \sim \mathcal{C}(\theta, L_\phi)(L_\phi/L)$. In this case, the angle over which the conductance typically fluctuates is then determined by L_ϕ instead of L.

We now describe the details of our calculation. For the retarded Green's function $\mathcal{G}^R$, averaged over the random potential and over all directions of the momentum, we find

$$\langle \mathcal{G}^R_\alpha(\omega, k, \hat{\mathbf{m}})\rangle^{-1} = \omega - \varepsilon_\alpha(k) + \frac{i}{2\tau_\alpha} + \frac{i}{2\tau_{\alpha\parallel}} + \frac{i}{\tau_{\alpha\perp}}, \tag{7}$$

where $\varepsilon_\alpha(k) = \hbar^2k^2/2m - \mu - E_Z\sigma^z_{\alpha\alpha}$ is the energy of an electron with spin α and momentum $\hbar\mathbf{k}$. In order to calculate the conductance autocorrelation function (1), we need to consider the Diffuson and Cooperon propagators of diagrammatic perturbation theory. Again, in view of the inequality $\tau \ll \tau_\parallel, \tau_\perp$, we only need Diffuson and Cooperon propagators averaged over all momentum directions. Since the Cooperon and Diffuson propagators are related by time reversal,

$$C(\omega, \mathbf{q}, \theta) = D(\omega, \mathbf{q}, \pi - \theta), \tag{8}$$

it will be sufficient to calculate the Diffuson only.

The Diffuson propagator is defined by the ladder diagrams shown in Fig. 1. The solid arrows in Fig. 1 denote the impurity-averaged Green functions (7). The two legs of the ladder refer to the two magnetization directions $\hat{\mathbf{m}}$ and $\hat{\mathbf{m}}'$. For both magnetization directions we use the convention that the magnetization direction is the spin quantization axis. This is the natural choice for ferromagnets: Since $E_Z\tau \gg 1$ in a typical ferromagnet, with this convention only ladder diagrams for which the spin indices of retarded and advanced Green functions are pairwise equal at all times need to be considered; contributions with different spin index for retarded and advanced Green's functions dephase within a mean free time and do not contribute to the Diffuson propagator. One should note, however, that this convention implies that the directions of "spin up" and "spin down" in the upper and lower legs of the ladder correspond to different physical directions if $\hat{\mathbf{m}} \neq \hat{\mathbf{m}}'$.

Summing the ladder diagrams of Fig. 1, we then find that the Diffuson obeys the 2×2 matrix equation

$$\sum_{\gamma=\uparrow,\downarrow} K_{\alpha\gamma} D(\omega,\mathbf{q},\theta)_{\gamma\beta} = \delta_{\alpha\beta} \frac{1}{2\pi\nu_\alpha \tau_\alpha}. \tag{9}$$

Here K is a 2×2 matrix, with diagonal elements given by

$$\hat{K}_{\alpha\alpha} = \tau_\alpha \left[D_\alpha q^2 + i\omega + \frac{2}{\tau_{\alpha\perp}} + \frac{1-\cos\theta}{\tau_{\alpha\parallel}} \right], \tag{10}$$

where $D_\alpha = v_{F\alpha}^2 \tau_\alpha / 3$ is the diffusion constant. The off-diagonal matrix elements contain a phase factor that depends on the precise choice of coordinate axes perpendicular to $\hat{\mathbf{m}}$ and $\hat{\mathbf{m}}'$, cf. Eq. (3). In all final expressions, the off-diagonal elements of K only enter through their product, which is independent of this choice,

$$K_{\uparrow\downarrow} K_{\downarrow\uparrow} = \frac{\tau_\uparrow \tau_\downarrow}{\tau_{\uparrow\perp}\tau_{\downarrow\perp}} (1+\cos\theta)^2. \tag{11}$$

Once the Diffuson is known, the Cooperon is calculated via Eq. (8). For the special case $\theta=0$, the result for C was previously obtained by Dugaev *et al.*[10]

We can now proceed to calculate the conductance correlation function $\mathcal{C}(\theta)$. We are interested in the conductance correlations at zero temperature, which allows us to set $\omega=0$ in our expressions for the Diffuson and Cooperon propagators. We consider a coherent rectangular sample with sides L_x, L_y, and L_z, with a current in the z direction. Following the formalism of Refs. 2 and 5, we then find the conductance autocorrelation function

$$\mathcal{C}(\theta) = \frac{6e^4}{\pi^4 h^2} \sum_{\mathbf{q}} \sum_{\pm} \left[\frac{1}{((L_z q/\pi)^2 + a_\pm(\theta))^2} + \frac{1}{((L_z q/\pi)^2 + a_\pm(\pi-\theta))^2} \right], \tag{12}$$

where

$$a_\pm(\theta) = \frac{1}{\tau_{\uparrow\perp}E_\uparrow} + \frac{1}{\tau_{\downarrow\perp}E_\downarrow} + \frac{\tau_{\uparrow\parallel}E_\uparrow + \tau_{\downarrow\parallel}E_\downarrow}{2\tau_{\uparrow\parallel}\tau_{\downarrow\parallel}E_\uparrow E_\downarrow}(1-\cos\theta) \pm \sqrt{\frac{(1+\cos\theta)^2}{\tau_{\uparrow\perp}\tau_{\downarrow\perp}E_\uparrow E_\downarrow} + \left[\frac{1}{\tau_{\uparrow\perp}E_\uparrow} - \frac{1}{\tau_{\downarrow\perp}E_\downarrow} - \frac{\tau_{\uparrow\parallel}E_\uparrow - \tau_{\downarrow\parallel}E_\downarrow}{2\tau_{\uparrow\parallel}\tau_{\downarrow\parallel}E_\uparrow E_\downarrow}(1-\cos\theta) \right]^2} \tag{13}$$

and $E_\alpha = D_\alpha(\pi/L_z)^2$ is the Thouless energy for spin α. Note that the parameter that governs the importance of spin-orbit scattering is the product $\tau_{\alpha\perp}E_\alpha$ or $\tau_{\alpha\parallel}E_\alpha$, which is the ratio of the spin-orbit time and the Thouless time, which is the time to diffuse through the sample.

The expression for $a_\pm(\theta)$ simplifies in two limiting cases. If $\theta=0$, one has $a_+ = 2/\tau_{\uparrow\perp}E_\uparrow + 2/\tau_{\downarrow\perp}E_\downarrow$ and $a_- = 0$, showing the presence of universal conductance fluctuations in a ferromagnet. The corresponding eigenvalues for the Cooperon contribution are found by setting $\theta=\pi$, $a_+(\pi) = 2/\tau_{\uparrow\perp}E_\uparrow + 2/\tau_{\uparrow\parallel}E_\uparrow$ and $a_-(\pi) = 2/\tau_{\downarrow\perp}E_\downarrow + 2/\tau_{\downarrow\parallel}E_\downarrow$. Another simple limit is that of a half-metal, a ferromagnet with vanishing density of states for the minority spins. For a half metal, the only relevant time and energy scales are the scattering time $\tau_{\uparrow\parallel}$ for spin-preserving spin-orbit scattering of majority electrons and the majority electron Thouless energy $E_\uparrow$. One then finds that only one root $a_\pm$ is relevant, $a(\theta) = (1-\cos\theta)/\tau_{\uparrow\parallel}E_\uparrow$. The sum over wave vectors in Eq. (12) can be performed analytically for a quasi-one-dimensional sample. Setting $n_x = n_y = 0$ in the summation, one finds

$$\mathcal{C}(\theta) = \sum_{\pm} [F(\pi\sqrt{a_\pm(\theta)}) + F(\pi\sqrt{a_\pm(\pi-\theta)})], \tag{14}$$

where $F(x) = 3e^4(-2 + x\coth x + x^2 \sinh^{-2} x)/2x^4h^2$. Note that for $\theta=0$, Eq. (14) reproduces the known results $\mathrm{var}\, G = (e^2/h)^2(1/15)$ for strong spin-orbit scattering and $\mathrm{var}\, G = (e^2/h)^2(4/15)$ for weak spin-orbit scattering. For quasi-2D and 3D samples $\mathcal{C}(\theta)$ can be computed numerically. The dependence on the spin-orbit scattering is qualitatively similar for all these cases. Shown in Fig. 2 is $\mathcal{C}(\theta)$ for a half metal with $L_x = L_y = L_z$. The top dashed line in Fig. 2 is the variance of the conductance in the absence of spin-orbit scattering. Without spin-orbit scattering, there is no angle-dependent mesoscopic correction to the conductance, so $\mathcal{C}(\theta)$ is independent of θ. For $\tau_{\uparrow\parallel} \ll 1/E_\uparrow$, conductance fluctuations saturate at half their value without spin-orbit scattering. Changing the magnetization by a small angle θ_c changes the mesoscopic conductance correction enough to lose all conductance correlations. Our calculation shows

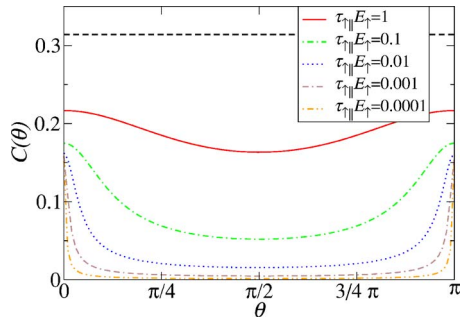

FIG. 2. (Color online) The correlation function of the conductance at different directions of the magnetization, for various strengths of the spin-orbit scattering. Results shown here are for a half metal with cubic geometry.

$$\theta_c \sim (\tau_{\uparrow\parallel} E_\uparrow)^{1/2} \sim l_{so}/L, \tag{15}$$

where l is the mean free path and $l_{so} \sim l(\tau_{\uparrow\parallel}/\tau_\uparrow)^{1/2}$ is the spin-orbit length. In a realistic ferromagnet, the quantitative form of $\mathcal{C}(\theta)$ is different, although the qualitative picture, including the estimate for the correlation angle θ_c is the same as for the half metal [see Eq. (13)].

Let us estimate the correlation angle θ_c for the spin-orbit induced mesoscopic conductance fluctuations. For the highly disordered ferromagnetic wires used in the experiments of Refs. 6 and 7, the mean free path l is of the order of a few nm. Taking the spin-orbit times $\tau_\parallel$ and $\tau_\perp$ within an order of magnitude of the elastic scattering time τ [as is appropriate for Co (see Ref. 18)], we find $\theta_c \sim (1 \times 10^{-8}\ \mathrm{m})/L$. (Recall that L has to be replaced by the phase coherence length L_ϕ if $L_\phi < L$.) This would be sufficiently small to explain the few conductance oscillations seen in the experiment of Ref. 7, for which $L_\phi \sim 30$ nm and the conductance was measured as a function of an external magnetic field that changed the magnetization direction.

It is instructive to compare the correlation angle θ_c for spin-orbit induced conductance fluctuations considered here to the correlation angle arising from the coupling of the electron's charge to the internal magnetic field. The latter is $\sim \Phi_0/\Phi$, where Φ is the magnetic flux through the sample and Φ_0 is the flux quantum. Taking the internal magnetic field to be ~ 2 T, as is appropriate for Co, one finds a correlation angle $\sim (2 \times 10^{-15}\ \mathrm{m}^2)/L^2$. Hence, with the parameters taken above, the orbital effect will dominate for samples with size $L \gtrsim 2 \times 10^{-7}$ m. This is in agreement with Ref. 7, where it was shown that the orbital effect alone cannot account for the observed conductance fluctuations.[7]

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[1] P. A. Lee and A. D. Stone, Phys. Rev. Lett. **55**, 1622 (1985).
[2] P. A. Lee, A. D. Stone, and H. Fukuyama, Phys. Rev. B **35**, 1039 (1987).
[3] B. L. Altshuler, JETP Lett. **41**, 648 (1985).
[4] B. L. Altshuler and D. E. Khmelnitskii, JETP Lett. **42**, 359 (1986).
[5] B. L. Altshuler and B. I. Shklovskii, Sov. Phys. JETP **64**, 127 (1986).
[6] S. Lee, A. Trionfi, and D. Natelson, Phys. Rev. B **70**, 212407 (2004).
[7] Y. G. Wei, X. Y. Liu, L. Y. Zhang, and D. Davidović, Phys. Rev. Lett. **96**, 146803 (2006).
[8] G. Tatara and H. Fukuyama, Phys. Rev. Lett. **78**, 3773 (1997).
[9] Y. Lyanda-Geller, I. L. Aleiner, and P. M. Goldbart, Phys. Rev. Lett. **81**, 3215 (1998).
[10] V. K. Dugaev, P. Bruno, and J. Barnás, Phys. Rev. B **64**, 144423 (2001).
[11] P. W. Brouwer and D. A. Gorokhov, Phys. Rev. Lett. **95**, 017202 (2005).
[12] R. C. O'Handley, *Modern Magnetic Materials* (Wiley, New York, 2000).
[13] M. Viret, S. Berger, M. Gabureac, F. Ott, D. Olligs, I. Petej, J. F. Gregg, C. Fermon, G. Francinet, and G. LeGoff, Phys. Rev. B **66**, 220401(R) (2002).
[14] C.-S. Yang, C. Zhang, J. Redepenning, and B. Doudin, Appl. Phys. Lett. **84**, 2865 (2004).
[15] Z. K. Keane, L. H. Yu, and D. Natelson, Appl. Phys. Lett. **88**, 062514 (2006).
[16] K. I. Bolotin, F. Kuemmeth, and D. C. Ralph, cond-mat/0602251 (2006).
[17] J. Velev, R. F. Sabirianov, S. S. Jaswal, and E. Y. Tsymbal, Phys. Rev. Lett. **94**, 127203 (2005).
[18] L. Piraux, S. Dubois, C. Marchal, J. M. Beuken, J. F. D. L. Filipozzi, K. Ounadjel, and A. Fert, J. Magn. Magn. Mater. **156**, 317 (1996).
[19] H. Bouchiat and G. Montambaux, J. Phys. (Paris) **50**, 2695 (1989).
[20] H.-F. Cheung, E. K. Riedel, and Y. Gefen, Phys. Rev. Lett. **62**, 587 (1989).
[21] B. L. Altshuler, Y. Gefen, and Y. Imry, Phys. Rev. Lett. **66**, 88 (1991).