PHYSICAL REVIEW B **73**, 201303(R) (2006)

Electronic transport in an array of quasiparticles in the $\nu=5/2$ non-Abelian quantum Hall state

Eytan Grosfeld and Ady Stern
Department of Condensed Matter, Weizmann Institute of Science, Rehovot 76100, Israel
(Received 11 April 2006; published 4 May 2006)

The Moore-Read Pfaffian $\nu=5/2$ quantum Hall state is a p-wave superconductor of composite fermions. Small deviations from $\nu=5/2$ result in the formation of an array of vortices within this superconductor, each supporting a Majorana zero mode near its core. We consider how tunneling between these cores is reflected in the electronic response to an electric field of a nonzero wave-vector $\mathbf{q}$ and frequency ω. We find a mechanism for dissipative transport at frequencies below the $\nu=5/2$ gap, and calculate the $\mathbf{q},\omega$ dependence of the dissipative conductivity. The contributions we find depend exponentially on $|\nu-5/2|^{-1/2}$.

DOI: 10.1103/PhysRevB.73.201303 PACS number(s): 73.43.Cd, 71.10.Pm

The $\nu=5/2$ fractional quantum Hall state is expected to be characterized by quasiparticles obeying non-Abelian statistics. There are strong indications that this state is well described by the Moore-Read Pfaffian wave function,[1] which may be formulated within the composite-fermion theory (each electron is bound to two flux quanta) as a p-wave superconductor of composite fermions (CFs) at a zero magnetic field. Excitations in this superconductor are vortices carrying half a flux quantum and an electric charge of $e/4$, and fermions created in twos by breaking pairs with an appropriate energy gap.[2,3] The Bogoliubov–de Gennes (BdG) equation describing the fermionic excitations of a two-dimensional (2D) p-wave superconductor admits zero-energy solutions in the presence of well-separated vortices, one solution near each vortex' core; these solutions are Majorana fermions γ, satisfying $\gamma^\dagger=\gamma$. As a consequence, the ground state is degenerate; for $2N$ well-separated vortices, the ground state degeneracy is 2^N. The adiabatic interchange of the two vortices induces a unitary transformation within the subspace of the degenerate ground states. Two such transformations do not necessarily commute; hence vortex excitations obey non-Abelian statistics. A related spin model showing similar non-Abelian excitations was recently studied by Kitaev.[4]

Experimental support for the Moore-Read theory is still needed. Relating the theory, and in particular the non-Abelian nature of the quasiparticles, to measurable observables, is a major theoretical challenge. Interference experiments may be a venue toward that goal.[5–7]

In this work, we pursue a different method to probe the ground-state degeneracy as well as some of the properties of the Majorana excitations by considering the response of a quantum Hall system near a filling factor of $\nu=5/2$ to an external electric field of wave-vector $\mathbf{q}$ and frequency ω. In a fractional quantum Hall system at a filling factor of $\nu=5/2\pm\varepsilon$ ($\varepsilon\ll 1$), the density deviation from $\nu=5/2$ is accommodated by means of quasiparticles (vortices) whose density is $8\varepsilon n$, where n is the density of electrons. For a perfectly clean system, these quasiparticles form a lattice, and when their density is large enough, tunneling between their cores should be taken into account. The degeneracy of the ground state is partially removed by this tunneling, and a band is formed with a width of the order of the tunneling strength. The tunneling also breaks the particle-hole symmetry of the localized γ_i's.

We study the electronic transport through that band for square and triangular lattices. We find that due to the existence of the band, there is a dissipative part to the conductivity below the $\nu=5/2$ energy gap, with a unique $\mathbf{q},\omega$ dependence. This contribution to the conductivity, which does not involve a motion of the vortices, depends exponentially on $|\varepsilon|^{-1/2}$, due to its origin in tunneling. There is a qualitative difference between the two lattice types. The square lattice is described by an effective massless Dirac Hamiltonian, while the triangular lattice shows a gap of a fraction of the bandwidth. We calculate the dissipative part of the conductivity of the CFs using Kubo's formula,[14] and then map it to the electronic conductivity by a Chern-Simon transformation[8] $(\sigma^e)^{-1}=(\sigma^{cf})^{-1}+\frac{2h}{e^2}\hat{\epsilon}$ (with $\hat{\epsilon}$ being the antisymmetric tensor). For the square lattice, we find that the longitudinal and transverse CF conductivities are, respectively,

$$\mathrm{Re}(\sigma^{cf}_{\square,\parallel},\sigma^{cf}_{\square,\perp})=\frac{e^2}{\hbar}\frac{\vartheta^2(aq)^2}{16}\left(\frac{|\omega|}{\eta_\square^{1/2}},\frac{3\eta_\square^{1/2}}{|\omega|}\right)\theta(\eta_\square), \quad (1)$$

where $\eta_\square=\omega^2-v_0^2q^2$, a is the lattice constant, v_0 is the velocity characterizing the Dirac spectrum, and ϑ, to be defined below, is related to the tunneling strength. For the triangular lattice we find

$$\mathrm{Re}\ \sigma^{cf}_{\triangle,\parallel}=\mathrm{Re}\ \sigma^{cf}_{\triangle,\perp}=\frac{e^2}{\hbar}\frac{\vartheta^2(3aq)^2\eta_\triangle\theta(\eta_\triangle)}{8(|\hbar\omega|/\sqrt{3}t)}, \quad (2)$$

where $\eta_\triangle=\frac{|\hbar\omega|}{\sqrt{3}t}-2-\frac{a^2q^2}{4}$. As we explain below, the electronic conductivities are suppressed by a factor of ω^2 relative to the CF conductivities.

There are four steps in the calculation leading to these response functions. First, we specify the Hamiltonian describing the array. This Hamiltonian turns out to be closely related to the Azbel-Hofstadter (A-H) Hamiltonian,[9,10] describing electrons on a tight-binding lattice in a magnetic field. Second, we calculate the spectrum of the Hamiltonian. Third, we find how the system couples to gauge fields by expressing the density and current operators in terms of the Majorana operators γ_i (with i the vortex index); we also present a physical picture of this coupling. Finally, we calculate the response functions.

Based solely on the requirement of hermiticity and on the relation $\gamma_i=\gamma_i^\dagger$, a lattice of well-separated vortices is generally described by a tight-binding Hamiltonian

$$H = it\sum_{ij} s_{ij}\gamma_i\gamma_j, \tag{3}$$

where γ_i are the Majorana operators satisfying $\{\gamma_i,\gamma_j\}=\delta_{ij}$, and where i,j are nearest-neighbor lattice site indices. The tunneling strength t is real and positive. The matrix $s_{ij}=\pm$ is antisymmetric and indicates the sign of the tunneling along the bond (i,j). While the freedom to redefine $\gamma_i\to-\gamma_i$ makes the elements s_{ij} gauge dependent, the product of s_{ij} over bonds creating a closed path is gauge independent. We now show that this product is determined by a nontrivial phase, a Majorana fermion accumulates when encircling a plaquette, and give a simple formula for the effective flux per plaquette. This formula fixes the matrix s_{ij} up to a choice of gauge.

In the absence of tunneling between vortex cores, the localized solution to a 2D p-wave BdG equation near a vortex embedded in a lattice of vortices is given by

$$\chi_i(\mathbf{r}) = \begin{bmatrix} e^{-i\pi/4+(i/2)\int_{\mathbf{P}_i}^{\mathbf{r}}\nabla\Phi_i(\mathbf{l})\cdot d\mathbf{l}}g(\mathbf{r}-\mathbf{R}_i) \\ e^{i\pi/4-(i/2)\int_{\mathbf{P}_i}^{\mathbf{r}}\nabla\Phi_i(\mathbf{l})\cdot d\mathbf{l}}g(\mathbf{r}-\mathbf{R}_i)\end{bmatrix}. \tag{4}$$

This is an approximate zero-energy eigenstate of the first quantized 2D p-wave Hamiltonian H_{BdG} (see Refs. 2 and 11) of an order parameter $\Delta_0(\mathbf{r})\exp i\Omega(\mathbf{r};\{\mathbf{R}_i\})$, where $\mathbf{r}$ is the 2D-space coordinate and $\{\mathbf{R}_i\}$ are the vortices' positions and the phase $\Omega(\mathbf{r};\{\mathbf{R}_i\})$ has the property of increasing by 2π around any closed path surrounding one vortex (clockwise). The phase appearing in the solution (4) is given by $\Phi_i(\mathbf{r};\{\mathbf{R}_i\})=\Omega(\mathbf{r};\{\mathbf{R}_i\})+\arg(\mathbf{r}-\mathbf{R}_i)$, where the first term originates from the order parameter and the second term originates from the p_x+ip_y pairing, which induces a relative particle-hole angular momentum. The point $\mathbf{P}_i$ is arbitrarily chosen close to the vortex core. The real wave function $g(r)$ is localized at the vortex core. The tunneling matrix elements for nearest neighbors are purely imaginary, and are given by $\pm it$ where $t=|\text{Im}\int_{\mathbf{r}}\chi(\mathbf{r}-a\hat{x})[H_{\text{BdG}}(\mathbf{r})-H^{(0)}_{\text{BdG}}(\mathbf{r})]\chi(\mathbf{r})|$ is the tunneling strength, and where $H^{(0)}_{\text{BdG}}$ is the Hamiltonian in the absence of tunneling, of which Eq. (4) is an exact zero-energy eigenvector. For well-separated vortices, t decreases exponentially with $a\sim\varepsilon^{-1/2}$.

To determine the matrix elements s_{ij}, we consider a Majorana operator hopping between n vortices along a closed path, which forms a polygon whose edges connect the vortices. We show that there exists a nontrivial phase related to this path, given by *half the sum of the interior angles of the polygon*. The origin of this phase is in an interplay between the phase of the order parameter and the p-wave pairing. First, we calculate the tunneling matrix elements $\langle\chi_i|H_{\text{BdG}}|\chi_j\rangle=t_{ij}\exp i\psi_{ij}$, where we use the tight-binding assumption to neglect the spatial dependence of the phase and explicitly set $\mathbf{r}=(\mathbf{R}_i+\mathbf{R}_j)/2\equiv\mathbf{C}_{ij}$. For all bonds $t_{ij}=t$, while

$$\psi_{ij} = \frac{1}{2}\int_{\mathbf{P}_i}^{\mathbf{P}_j}\nabla\Omega(\mathbf{l})\cdot d\mathbf{l} + \frac{1}{2}\int_{\mathbf{P}_i}^{\mathbf{C}_{ij}}\nabla\arg(\mathbf{l}-\mathbf{R}_i)\cdot d\mathbf{l} - \frac{1}{2}\int_{\mathbf{P}_j}^{\mathbf{C}_{ij}}\nabla\arg(\mathbf{l}-\mathbf{R}_j)\cdot d\mathbf{l}. \tag{5}$$

The first term depends only on the order parameter; it measures the change of the phase of the spinor due to vortices enclosed in the path. The second and third terms are the contributions due to the relative particle-hole angular momentum induced by the p_x+ip_y pairing; they measure changes in the direction of the path. Considering n tunneling events $t^n\exp i[\psi_{i_1i_2}+\psi_{i_2i_3}+\cdots+\psi_{i_ni_1}]$, the total phase is given by $\frac{1}{2}\oint\nabla\Omega(\mathbf{l})\cdot d\mathbf{l}+\frac{1}{2}\Sigma_{i=1}^{n}A_i$, where A_i is the angle subtended by the path with respect to the ith vortex, positive for anticlockwise traversal. The first term gives a π winding for each vortex enclosed in the path. For each of these enclosed vortices, A_i is given by minus the exterior angle, which can be written as $-(2\pi-I_i)$, where I_i is the interior angle. For all other vortices $A_i=I_i$. Therefore, we get $\frac{1}{2}\oint\nabla\Omega\cdot d\mathbf{l}+\frac{1}{2}\Sigma_{i=1}^{n}A_i=\frac{1}{2}\Sigma_i I_i$, i.e., half the sum of interior angles of the polygon. This result is independent of whether the core of a vortex on the path is inside or outside of the polygon: if a path is deformed as to cross a vortex' core, both the term related to the order parameter and the relevant angle $A_i/2$ acquire an extra π, and these two contributions cancel each other. For a general polygon of n vortices, we get a phase of $\pi n/2-\pi$; consequently, for a lattice whose plaquette is a polygon of n vortices we get $n/4-1/2$ flux quanta per plaquette.

We note that for the A-H problem of tight-binding electrons on the same lattice with the same flux per plaquette, the Hamiltonian is

$$H^h = it\sum_{ij} s_{ij}c_i^\dagger c_j. \tag{6}$$

The Hamiltonians (3) and (6) share the same Harper's equation, their spectra are identical, but they differ considerably in the way they couple to gauge fields. Yet, there exist relations between their response functions.

Our determination of the effective flux in (3) then singles out a chain of A-H-type problems, one for each value of n, where the flux per plaquette is determined by the geometry of the lattice. There is a qualitative difference between the triangular lattice, with an odd n, and the square lattice, with an even n; the former breaks time-reversal symmetry in the effective A-H problem, while the latter does not. The honeycomb lattice, for which $n=6$, was considered in Ref. 4.

In the next step, we calculate the spectrum and eigenvectors of the Hamiltonian (3). After identifying the flux per plaquette, we choose a gauge which complies with it, commonly breaking translational symmetry. Translational symmetry is restored by choosing a unit cell, which contains an integer multiple of the flux quantum. The sites of the unit cell are numbered $z=1,\ldots,s$; in this way, we divide our lattice into s sublattices. We aim at finding an operator $\Gamma^\dagger$ satisfying the equation $[H,\Gamma^\dagger]=E\Gamma^\dagger$. We expand it in local site operators as $\Gamma^\dagger=\Sigma_i\lambda_i\gamma_i$ ending with the following equation

$$it\sum_j s_{ij}\lambda_j = E\lambda_i. \quad (7)$$

Using translational symmetry, the solution for each sublattice z can be written as $\lambda_i = e^{i\mathbf{k}\cdot\mathbf{R}_i}\lambda_{z(i)}(\mathbf{k})$, where $z(i)$ is the sublattice to which the site $\mathbf{R}_i$ belongs. The equation for λ_z is given by

$$\tilde{H}_{zz'}\lambda_{z'} \equiv it\sum_{z'}\sum_{j\in z} s_{ij}e^{i\mathbf{k}\cdot(\mathbf{R}_j-\mathbf{R}_i)}\lambda_{z'} = E\lambda_z, \quad (8)$$

where the site i is an arbitrarily chosen lattice site that belongs to the z' sublattice. We denote the eigenvectors of $\tilde{H}$ by $\lambda^{(\alpha)}(\mathbf{k})$. This results in the following operators

$$\Gamma_{\mathbf{k}}^{(\alpha)\dagger} = \sum_i \lambda_i^{(\alpha)}(\mathbf{k})\gamma_i = \sum_{z=1}^{s}\lambda_z^{(\alpha)}(\mathbf{k})\sum_{i\in z} e^{i\mathbf{k}\cdot\mathbf{R}_i}\gamma_i, \quad (9)$$

which obey the usual fermionic anticommutation relations $\{\Gamma_{\mathbf{k}}^{(\alpha)\dagger},\Gamma_{\mathbf{k}'}^{(\beta)}\}=\delta_{\alpha\beta}\delta(\mathbf{k}-\mathbf{k}')$ and $\{\Gamma_{\mathbf{k}}^{(\alpha)\dagger},\Gamma_{\mathbf{k}'}^{(\beta)\dagger}\}=0$ for positive energy modes. In terms of these operators, the Hamiltonian is diagonal, $H=\Sigma_{\mathbf{k}\alpha}E_{\mathbf{k}\alpha}\Gamma_{\mathbf{k}}^{(\alpha)\dagger}\Gamma_{\mathbf{k}}^{(\alpha)}$.

For the square lattice, the A-H Hamiltonian has half a quantum of flux per plaquette. We choose a gauge for which $s_{ij}=+$ along columns and has alternating signs between adjacent rows. Having translational invariance in doubled lattice vectors, we may split the lattice sites into four sublattices, numbered $z=1,\ldots,4$. The Hamiltonian H may be written in a 4×4 matrix notation as

$$\tilde{H}_{\square} = 2t\sigma_x\otimes\tau_z\sin(ak_x) + 2t\sigma_x\otimes\tau_x\sin(ak_y). \quad (10)$$

In the limit $|k|a\to 0$, the Hamiltonian (10) has a doubly degenerate gapless isotropic Dirac spectrum $\epsilon_{k\alpha}=\mathrm{sgn}(\alpha)v_0|k|$, with $\alpha=\pm 1,\pm 2$ and the characteristic velocity $v_0=2at$.[13] The eigenvectors of Eq. (10) are

$$\lambda_{-\mathbf{k}}^{(1)*} = \lambda_{\mathbf{k}}^{(-2)} = \lambda_{-\mathbf{k}}^{(2)} = \lambda_{\mathbf{k}}^{(-1)*} = \frac{(ie^{i\theta_k},e^{i\theta_k},-i,1)}{2}, \quad (11)$$

where $e^{i\theta_k}=(k_x+ik_y)/|k|$.

For the triangular lattice, the A-H Hamiltonian has a quarter of a flux quantum per plaquette. The Hamiltonian in the sublattice representation is given by

$$\tilde{H}_{\triangle} = 2t\sum_{i=1}^{2}\xi_i\sin(\mathbf{a}_i\cdot\mathbf{k}) + \xi_3\cos(\mathbf{a}_3\cdot\mathbf{k}), \quad (12)$$

where $\xi_1=I\otimes\tau_x$, $\xi_2=\sigma_y\otimes\tau_y$, $\xi_3=\sigma_y\otimes\tau_z$, and $\mathbf{a}_1=(a\hat{x}-\sqrt{3}a\hat{y})/2$, $\mathbf{a}_2=(a\hat{x}+\sqrt{3}a\hat{y})/2$, $\mathbf{a}_3=a\hat{x}$ are the three lattice directions. There is a doubly degenerate spectrum indexed again by α, $\epsilon_{\mathbf{k}\alpha}=\mathrm{sgn}(\alpha)\epsilon_{\triangle,\mathbf{k}}$, where

$$\epsilon_{\triangle,\mathbf{k}} = \sqrt{2}t\sqrt{3+\cos(2ak_x)-2\cos(ak_x)\cos(\sqrt{3}ak_y)}. \quad (13)$$

The spectrum is gapped,[12] and there are two minima at $\mathbf{k}_0=(\pm\pi/3a,0)$, around which it is quadratic $\epsilon_{\mathbf{k}_0+\boldsymbol{\kappa},\alpha}\simeq\mathrm{sgn}(\alpha)\sqrt{3}t\left(1+\frac{1}{2}a^2\kappa^2\right)$. The eigenvectors of Eq. (12) are

$$\lambda_{\mathbf{k}}^{(1)} = \lambda_{-\mathbf{k}}^{(-2)*} = \frac{1}{N_{\mathbf{k}}}(iB_{-\mathbf{k}},-iB_{-\mathbf{k}},1,1), \quad (14)$$

$$\lambda_{\mathbf{k}}^{(2)} = \lambda_{-\mathbf{k}}^{(-1)*} = \frac{1}{N_{\mathbf{k}}}(-iB_{\mathbf{k}},-iB_{\mathbf{k}},-1,1), \quad (15)$$

where $N_{\mathbf{k}}$ is a normalization factor and

$$B_{\mathbf{k}} = \frac{\epsilon_{\triangle}(k)/2t + \sin(\mathbf{a}_1\cdot\mathbf{k})}{\cos(\mathbf{a}_3\cdot\mathbf{k}) + i\sin(\mathbf{a}_2\cdot\mathbf{k})}. \quad (16)$$

The coupling of the Majorana states of the Hamiltonian (3) to an electric field is very different from that of the electrons in the A-H Hamiltonian (6), due to the particle-hole symmetry of the operators γ_i. While each Majorana state (4) is electrically neutral, when tunneling between vortex cores is switched on, a nonzero density of charge appears *between* the vortices. Projected to the subspace of the Majorana states, the density operator may be written as $\rho(\mathbf{r})=\Sigma_{ij}\rho_{ij}(\mathbf{r})$ where

$$\rho_{ij}(\mathbf{r}) = is_{ij}g(\mathbf{r}-\mathbf{R}_i)g(\mathbf{r}-\mathbf{R}_j)\gamma_i\gamma_j. \quad (17)$$

The operator $i\gamma_i\gamma_j$ has two eigenvalues ± 1, which describe the sign of the charge mostly sitting at the center of the bond; however, the operators ρ_{ij} do not commute if they share a common Majorana operator, and consequently, one cannot specify the charge at all bonds simultaneously. The excitations acquire a charge by the following mechanism. When the Majorana fermions are allowed to tunnel between cores, the states move away from zero energy and the amplitudes of the particlelike and the holelike parts of the excitation are no longer equal.[2,11] This asymmetry is maximal at the midpoint between two vortices. Indeed, two nearest-neighbor spinors χ_i and χ_j are exactly orthogonal $\langle\chi_i|\chi_j\rangle=0$ due to a π phase difference between the overlap of the particles and the overlap of the holes; however, they do support nonzero matrix elements of the charge operator $\langle\chi_i|\sigma_z|\chi_j\rangle=is_{ij}\vartheta$, where $\vartheta=\int_{\mathbf{r}}g(\mathbf{r}-a\hat{x})g(\mathbf{r})$; the charge is proportional to the tunneling strength, and therefore, to the energy. Consequently, the excitation $\Gamma_k^{(\alpha)\dagger}$carries a charge of $\vartheta\epsilon_{k\alpha}/t$.

Next, we identify the current operator. At $q=0$, the current is found using the identity

$$\mathbf{j}_{\mathbf{q}=0} = i[H,\mathbf{d}] = -i\vartheta t\sum_{ijl}s_{ij}s_{jl}\left(\frac{\mathbf{R}_l-\mathbf{R}_i}{2}\right)\gamma_i\gamma_l, \quad (18)$$

where $\mathbf{d}=\int_{\mathbf{r}}\mathbf{r}\rho(\mathbf{r})$ is the total dipole operator. The sum $\Sigma_j s_{ij}s_{jl}\neq 0$ only for sites i and l separated by a doubled lattice vector. The current may be transformed to a k space by inverting (9) and substituting it into (18). The $\mathbf{q}=0$ current is a conserved quantity. To see that, we examine the commutator of $\mathbf{j}_{\mathbf{q}=0}$ with the Hamiltonian

$$[H,\mathbf{j}_{\mathbf{q}=0}] \propto \sum_{ijlm}s_{ij}s_{jl}s_{lm}\left(\frac{\mathbf{R}_i+\mathbf{R}_m}{2}-\frac{\mathbf{R}_j+\mathbf{R}_l}{2}\right)\gamma_i\gamma_m,$$

which is described by paths composed of three consecutive bonds connecting the vortices given by i, j, l, m. The sum over all paths is zero, as each path interferes destructively with a second path formed by starting from one of its ends and reversing the order of steps to the other end.

At finite q we find the current by calculating $\rho(\mathbf{q}) = \int_{\mathbf{r}} e^{i\mathbf{q}\cdot\mathbf{r}}\rho(\mathbf{r})$ and using charge conservation. The current operator is, in momentum space,

$$\mathbf{j}(\mathbf{q}) = \sum_{\mathbf{k}\alpha\beta} \tilde{e}_k \mathbf{v}_{\mathbf{k}} n^{\alpha\beta}_{\mathbf{k},\mathbf{q}} \Gamma^{(\alpha)\dagger}_{\mathbf{k}+\mathbf{q}/2}\Gamma^{(\beta)}_{\mathbf{k}-\mathbf{q}/2}, \tag{19}$$

where $n^{\alpha\beta}_{\mathbf{k},\mathbf{q}} = \lambda^{(\alpha)*}_{\mathbf{k}+\mathbf{q}/2}\cdot\lambda^{(\beta)}_{\mathbf{k}-\mathbf{q}/2}$ are the density matrix elements of the associated A-H problem, and $\tilde{e}_k = e\vartheta\epsilon_k/t$, $\mathbf{v}_{\mathbf{k}} = \partial_{\mathbf{k}}\epsilon_k$ are the charge and velocity of the quasiparticle, respectively. For comparison, the longitudinal component of the current in the A-H Hamiltonian (6) is

$$j^h_{\parallel}(\mathbf{q}) = \sum_{\mathbf{k}\alpha\beta} J^h_{\alpha\beta}(\mathbf{k},\mathbf{q}) c^{(\alpha)\dagger}_{\mathbf{k}+\mathbf{q}/2} c^{(\beta)}_{\mathbf{k}-\mathbf{q}/2}, \tag{20}$$

where for the relevant transitions $J^h_{1,-1} = J^{h*}_{2,-2} = \frac{ev_0}{2}(e^{i\theta_{\mathbf{k}+\mathbf{q}/2}} - e^{-i\theta_{\mathbf{k}-\mathbf{q}/2}})$ for the square lattice, and $J^h_{1,-2} = -J^h_{2,-1} = \frac{eat}{2\sqrt{2}}(3+3\sqrt{3}i)$ for the triangular lattice near the bottom of the band. Over all, the matrix elements of the current operator in the Majorana problem (19) are smaller by a factor of qa relative to (20).

Having calculated the spectrum and identified the relevant operators, the response functions of the array of Majorana states is readily calculated employing the Kubo formula, with the results given by Eqs. (1) and (2). This result affirms the existence of dissipative conductivity, hence the flow of in-phase current, even at frequencies below the $\nu=5/2$ energy gap.

Two steps need to be taken to transform the composite fermion conductivities (1) and (2) into the measurable electronic conductivity. First, the imaginary part of the CF conductivity, $i\rho_s e^2/\omega$ (with ρ_s being the superfluid density of the CFs), originating from the superconductivity of the CF condensate, should be added to the calculated real part. Second, the Chern-Simon transformation should be used to transform the CF conductivity into the electronic one. In the limit $\omega \to 0$, these steps result in

$$\mathrm{Re}(\sigma^e_{\parallel},\sigma^e_{\perp}) = \left(\frac{\omega}{2h\rho_s}\right)^2 \mathrm{Re}(\sigma^{cf}_{\perp},\sigma^{cf}_{\parallel}). \tag{21}$$

At finite temperature, assuming $v_0 q \ll \omega$, the conductivity satisfies $\sigma[T] = \sigma[T=0]\mathrm{sgn}(\omega)\tanh\frac{\hbar\omega}{4k_BT}$.

Before closing, we note that the same methods may be used to find the response functions of the associated A-H problems. For the square lattice, the conductivity is $\mathrm{Re}(\sigma^h_{\square,\parallel},\sigma^h_{\square,\perp}) = \frac{1}{8}\frac{e^2}{\hbar}\left(\frac{|\omega|}{\eta^{1/2}_{\square}},\frac{\eta^{1/2}_{\square}}{|\omega|}\right)\theta(\eta_{\square})$. The value of the conductivity at the $q\to 0$ limit is a universal $e^2/8\hbar$ (Ref. 13); the origin of the universality lies in an exact cancellation of the dependence on v_0 due to the linear density of states $\propto \omega/v_0^2$. The dependence of the conductivity on temperature is $\sigma^h_{\square} = \frac{1}{8}\frac{e^2}{\hbar}\,\mathrm{sgn}(\omega)\tanh\frac{\hbar\omega}{4k_BT}$. For the triangular lattice, the conductivity at the bottom of the band is again universal, $\frac{3}{4}\frac{e^2}{\hbar}$.

In summary, we calculated the electronic response of an array of immobile quasiparticles of the $\nu=5/2$ state to an electric field of nonzero $\mathbf{q},\omega$, due to the tunneling of Majorana fermions between their cores. We found a contribution to the dissipative conductivity, Eq. (21), that is of a unique $\mathbf{q},\omega$ dependence, and a strong exponential dependence on the deviation of ν from $5/2$. Our analysis neglected disorder, which will be discussed elsewhere.

We acknowledge financial support from the U.S.-Israel BSF (Contract No. 2002-238) and the Minerva Foundation. We thank A. Kitaev and Z. Tesanovic for instructive discussions.

[1] G. Moore and N. Read, Nucl. Phys. B **360**, 362 (1991).
[2] N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000).
[3] K. C. Foster, N. E. Bonesteel, and S. H. Simon, Phys. Rev. Lett. **91**, 046804 (2003.)
[4] A. Kitaev, cond-mat/0506438 (unpublished).
[5] S. Das Sarma, M. Freedman, and C. Nayak, Phys. Rev. Lett. **94**, 166802 (2005).
[6] A. Stern and B. I. Halperin, Phys. Rev. Lett. **96**, 016802 (2006).
[7] P. Bonderson, A. Kitaev, and K. Shtengel, Phys. Rev. Lett. **96**, 016803 (2006).
[8] B. I. Halperin, P. A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993).
[9] M. Ya Azbel, Zh. Eksp. Teor. Fiz. **46**, 929 (1964) [Sov. Phys. JETP **19**, 634 (1964)].
[10] D. Hofstadter, Phys. Rev. B **14**, 2239 (1976).
[11] A. Stern, F. von Oppen, and E. Mariani, Phys. Rev. B **70**, 205338 (2004).
[12] A. Ioselevich, D. A. Ivanov, and M. V. Feigelman, Phys. Rev. B **66**, 174405 (2002).
[13] M. Fisher and E. Fradkin, Nucl. Phys. B **251**, 457 (1985).
[14] R. Kubo, M. Toda, and N. Hashitsume, *Statistical Physics II*, 2nd ed. (Springer, Berlin, 1991).